CBSE Test Paper 04

CH-09 Sequences and Series

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1. The sum of 40 A.M.'s between two number is 120. The sum of 50 A.M.'s between them is equal to
   a. 130
   b. 150
   c. 160
   d. 140
2. The number of terms common to the Arithmetic progressions 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is
   a. 14
   b. 51
   c. 21
   d. 28
3. pth term of an A.P. is q and qth term is p, its (p+ q)th term is
   a. p - q
   b. - (p + q)
   c. 0
   d. p + q
4. The values of 'a' for which the roots of the equation sin θ = a in A.P. are
   a. 0 and 1
   b. 0, 1 and – 1
   c. - 1 and 1
   d. none of theses
5. If A and G denote respectively, the A.M. and G.M. between two positive numbers a and b, then A - G is equal to
   a. $\frac{1}{2}\left(\sqrt{a}-\sqrt{b}\right)^2$
   b. a + b
   c. a – b
   d. $\frac{2\ a\ b}{a+b}$
6. Fill in the blanks:

Common ratio of a geometric sequence cannot be equal to ________.

7. Fill in the blanks:

   The sum of the series: 2 + 4 + 6 + 8 + ...+ 2n is ________.

8. Find the sum to infinity of GP: 6, 1.2, 0.24,.... ∞.

9. Which term of the sequence 2, $2\sqrt{2}$, 4,........ is 128?

10. The third term of GP is 4. Find the product of its first 5 terms.

11. If arithmetic mean and geometric mean between two numbers is 5 and 4 respectively, then find the two numbers.

12. How many terms in the AP - 9, - 6, - 3, ... must be added together so that the sum may be 66?

13. The Fibonacci sequence is defined by 1 = a_1 = a_2 and a_n = a_{n-1} + a_{n-2}, n > 2. Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5.

14. The 2nd, 31st and last term of an AP are $7\frac{3}{4}$, $\frac{1}{2}$ and - $6\frac{1}{2}$, respectively. Find the first term and the number of terms.

15. Find four numbers in GP, whose sum is 85 and product is 4096.

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Solution

1. (b) 150

   Explanation: sum of n number of A.M's between two numbers a and b is given by

   $Sum = n(\frac{a+b}{2})$

   so, $\Rightarrow 120 = 40(\frac{a+b}{2})$
   $\Rightarrow (\frac{a+b}{2}) = 3$

   therefore sum of 50 A.M's between a and b is,

   $Sum = n(\frac{a+b}{2}) = 50 \times 3 = 150$

2. (a) 14

   Consider the arithmetic progressions 3, 7, 11,15,19,23,27,31,35,39,43,47,51, ..., 407 and 2, 9, 16, 23,30,37,44,51...., 709

   By inspection we have the common terms in the two sequences are,
   $23, 51, 79$........which is again an A.P with $a = 23, d = 28$

   But the last term of this A.Pshould not exceed 407, therefore
   $a_n \leq 407$
   $23 + (n - 1)28 \leq 407$
   $28n - 5 \leq 407$
   $28n \leq 412$
   $n \leq 412/28$
   $n \leq 14\frac{20}{28}$
   n=14

3. (c) 0

   Explanation: Using the general term formula for A.P
   $T_p = a + (p - 1)d = q$ ....(i)
   $T_q = a + (q - 1)d = p$.....(ii)

subtract equation (i) from (ii), we get

$(q-p)d=p-q$

$\Rightarrow d=-1$ ....(iii)

Now from (i) and (iii), we get

a-(p-1)=q

$\Rightarrow a=q+p-1$....(iv)

Now we have

T_{p+q}= a+(p+q-1) d=(q+p-1)+(p+q-1)(-1)$\Rightarrow T_{p+q}=0$

4. (b) 0, 1 and – 1

   Explanation:

   We have $\sin\theta$ is a periodic function whose value oscillates between -1 and 1

   Also $\sin\theta=0$ when $\theta=0^{\circ}$, $\sin\theta=1$ when $\theta=90^{\circ}$, and $\sin\theta=-1$

   But we have -1,0,1 is and A.P

   Hence the values of x are -1,0,1

5. Given the numbers are a and b,then we have

   $A.M=A=\frac{a+b}{2}, G.M=G=\sqrt{ab}$

   Then $A-G=\frac{a+b}{2}-\sqrt{ab}=\frac{(\sqrt{a}-\sqrt{b})^{2}}{2}$

6. 1

7. n(n + 1)

8. The given GP is 6, 1.2, 0.24,... ∞

   Here, a = 6 and ratio(r) = $\frac{T_2}{T_1}=\frac{1.2}{6}$ = 0.2

   Since, $|0.2|$ = 0.2 < 1

   $\therefore$ S_{∞} = $\frac{a}{1-r}=\frac{6}{1-0.2}=\frac{6}{0.8}$ = 7.5

9. Here a = 2, r = $\frac{2\sqrt{2}}{2}=\sqrt{2}$ and a_n = 128

   $\therefore a_n = ar^{n-1}$

   $\Rightarrow$128 = $2\times(\sqrt{2})^{n-1}$

   $\Rightarrow$ 64 = $(\sqrt{2})^{n-1}$

   $\Rightarrow(\sqrt{2})^{12}=(\sqrt{2})^{n-1}$

   $\Rightarrow$n - 1 = 12

$\Rightarrow$n = 13

Therefore, 13th term of the given G.P. is 128.

10. Let a be the first term and r be the common ratio.

    Given, third term = 4 i.e., T_3 = 4 $\Rightarrow$ ar^2 = 4 ...(i)

    Now, product of first 5 terms

    = $a_1 \times a_2 \times a_3 \times a_4 \times a_5$

    = a (ar) (ar^2) (ar^3) (ar^4) = $a^5 r^{10}$

    = $(ar^2)^5$ = $(4)^5$ [from Eq. (i)]

11. Given, arithmetic mean, A = 5

    and geometric mean, G = 4

    Let the two numbers be a and b.

    We know that,

    a = A + $\sqrt{A^2 - G^2}$ and b = A - $\sqrt{A^2 - G^2}$

    $\therefore$ a = 5 + $\sqrt{5^2 - 4^2}$ and b = 5 - $\sqrt{5^2 - 4^2}$

    $\Rightarrow$ a = 5 + $\sqrt{25 - 16}$ and b = 5 - $\sqrt{25 - 16}$

    $\Rightarrow$ a = 5 + $\sqrt{9}$ and b = 5 - $\sqrt{9}$

    $\Rightarrow$ a = 5 + 3 and b = 5 - 3

    $\Rightarrow$ a = 8 and b = 2

    Hence, the required numbers are 2 and 8.

12. Let 66 be the sum of n terms

    We have, a = - 9

    and d = - 6 - (- 9) = -6 + 9 = 3

    $\because S_n = \frac{n}{2}$ [2 (- 9) + (n - 1)3]

    $\therefore$ 66 = $\frac{n}{2}$ [- 18 + 3n - 3]

    $\Rightarrow$ 132 = n [3n - 21]

    $\Rightarrow$ 44 = n [n - 7] [divide both sides by 3]

    $\Rightarrow$ n^2 - 7n - 44 = 0

    $\Rightarrow$ (n - 11) (n + 4) = 0

    $\therefore$ n = 11, - 4

    Rejecting n = - 4 because number of terms cannot be negative.

$\therefore$ n = 11

13. Given, 1 = a_1 = a_2

and a_n = a_{n-1} + a_{n-2}, n > 2

On putting n = 3, 4, 5, 6 respectively, we get

For n = 3, $a_3 = a_{3-1} + a_{3-2} = a_2 + a_1 = 1 + 1 = 2$

For n = 4, $a_4 = a_{4-1} + a_{4-2} = a_3 + a_2 = 2 + 1 = 3$

For n = 5, $a_5 = a_{5-1} + a_{5-2} = a_4 + a_3 = 3 + 2 = 5$

For n = 6, $a_6 = a_{6-1} + a_{6-2} = a_5 + a_4 = 5 + 3 = 8$

Now, $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5.

For n = 1, $\frac{a_2}{a_1} = \frac{1}{1}$ = 1

For n = 2, $\frac{a_3}{a_2} = \frac{2}{1}$ = 2

For n = 3, $\frac{a_4}{a_3} = \frac{3}{2}$

For n = 4, $\frac{a_5}{a_4} = \frac{5}{3}$

For n = 5, $\frac{a_6}{a_5} = \frac{8}{5}$

Hence, the terms are 1, 2, $\frac{3}{2}$, $\frac{5}{3}$ and $\frac{8}{5}$

14. Let a be the first term and d be the common differnce of the AP.

Given, $T_2 = 7\frac{3}{4} \Rightarrow a + d = \frac{31}{4}$ ...(i)

and $T_{31} = \frac{1}{2} \Rightarrow a + 30d = \frac{1}{2}$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

29 d = $\frac{1}{2} - \frac{31}{4} = \frac{-29}{4} \Rightarrow d = \frac{-1}{4}$

On putting value of d in Eq. (i), we get

a - $\frac{1}{4} = \frac{31}{4}$

$\Rightarrow a = \frac{31}{4} + \frac{1}{4} = \frac{32}{4} = 8$

Let the number of terms be n, so that $T_n = \frac{-13}{2}$

i.e., a + (n - 1)d = $\frac{-13}{2}$

$\Rightarrow 8 + (n - 1)\left(-\frac{1}{4}\right) = \frac{-13}{2}$

$\Rightarrow 8 - \frac{n}{4} + \frac{1}{4} = \frac{-13}{2}$

$\Rightarrow$ 32 - n + 1 = - 26

$\therefore$ n = 59

Hence, first term is 8 and number of terms is 59.

15. Let the four numbers in GP be

$\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar^3 ...(i)

Product of four numbers = 4096 [given]

$\Rightarrow \left(\frac{a}{r^3}\right)\left(\frac{a}{r}\right)$ (ar) (ar^3) = 4096

$\Rightarrow$ a^4 = 4096 $\Rightarrow$ a^4 = 8^4

On comparing the base of the power 4, we get

$\Rightarrow \frac{a}{r^3} + \frac{a}{r}$ + ar + ar^3 = 85

$\Rightarrow$ a $\left[\frac{1}{r^3} + \frac{1}{r} + r + r^3\right]$ = 85

$\Rightarrow$ 8 [r^3 + $\frac{1}{r^3}$] + 8 [r + $\frac{1}{r}$] = 85 [$\because$ a = 8]

$\Rightarrow$ 8 $\left[\left(r + \frac{1}{r}\right)^3 - 3\left(r + \frac{1}{r}\right)\right] + 8\left(r + \frac{1}{r}\right)$ = 85 [$\because$ a^3 + b^3 = (a + b)^3 - 3 (a +b)]

$\Rightarrow$ 8 $\left(r + \frac{1}{r}\right)^3$ - 16 $\left(r + \frac{1}{r}\right)$ - 85 = 0 ...(ii)

On putiing $\left(r + \frac{1}{r}\right)$ = x in Eq. (ii), we get

8x^3 - 16x - 85 = 0

$\Rightarrow$ (2x - 5) (4x^2 + 10x + 17) = 0

$\Rightarrow$ 2x - 5 = 0 [$\because$ 4x^2 + 10x + 177 = 0 has imaginary roots]

$\Rightarrow$ x $= \frac{5}{2} \Rightarrow$ r + $\frac{1}{r} = \frac{5}{2}$ [put x = r + $\frac{1}{r}$]

$\Rightarrow$ 2r^2 - 5r + 2 = 0

$\Rightarrow$ (r - 2) (2r - 1) = 0

$\Rightarrow$ r = 2 or r = $\frac{1}{2}$

On putting a = 8 and r = 2 or r = $\frac{1}{2}$ in Eq. (i), we obtain four numbers as

$\frac{8}{2^3}, \frac{8}{2}, 8 \times 2, 8 \times 2^3$

or $\frac{8}{(1/2)^3}, \frac{8}{(1/2)}, 8 \times \frac{1}{2}, 8 \times \left(\frac{1}{2}\right)^3$

$\Rightarrow$ 1, 4, 16, 64 or 64, 16, 4, 1.