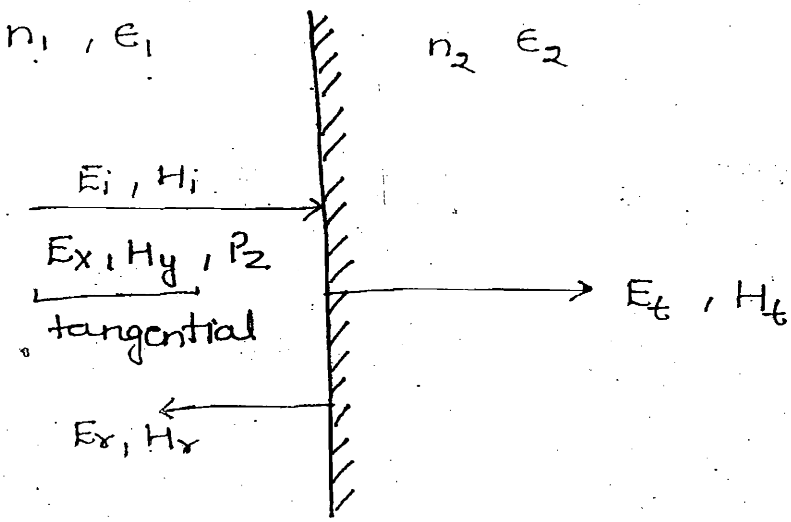
Lecture-11

Reflections / Transmissions of EM Waves.

(I) Normal Incidence (II) Dielectric - Dielectric

$Z<0$ $Z=0$ (XY Plane)



(I) $E_t > E_i$ or $E_t < E_i$ $\longrightarrow$ It depends on ϵ_1 & ϵ_2

(II) $H_t > H_i$ if $E_t < E_i$

~~(III)~~ $H_t < H_i$ if $E_t > E_i$

(III) $P_t < P_i$ $\longrightarrow$ Always

E_x	H_y	Prop. Z	
a_x	a_y	a_z	$\longrightarrow$ Incidence
a_x	$-a_y$	$-a_z$	$\longrightarrow$ Reflection
$-a_x$	a_y	$-a_z$	

$\rightarrow$ When reflections occur either E or H negates along with propogation direction but not both

$$E_i = \eta_1 H_i$$

$$E_t = \eta_2 H_t$$

$$E_r = -\eta_1 H_r$$

$\rightarrow$ For normal incidence the E/H fields are tangential.

$$E_{t_1} = E_{t_2} \longrightarrow E_i + E_r = E_t$$

$$H_{t_1} = H_{t_2}$$

$$E_i + E_r = E_t$$

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\Rightarrow \boxed{1 + \Gamma = \tau} \quad \text{——— (1)}$$

where $\Gamma = \frac{E_r}{E_i}$

= Reflection coefficient for the E fields

$\tau = \frac{E_t}{E_i}$ = Transmission coefficient for the E fields

$$H_i + H_r = H_t$$

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{H_t}{\eta_2}$$

$$E_i - E_r = \frac{\eta_1}{\eta_2} E_t$$

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

Note :-

$$\Gamma = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = -\Gamma_E$$

$$\Gamma_P = \Gamma_E \cdot \Gamma_H = -\Gamma_E^2 = -\Gamma_H^2$$

$$\boxed{1 + \Gamma = \tau}$$ Always - Either E or H or Power

(IV) If $\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$ & $\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$

$$\Gamma_E = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

If $\epsilon_1 > \epsilon_2 \Rightarrow \Gamma_E = +ve \quad \tau_E > 1$

$E_t > E_i$

$\Rightarrow \Gamma_H = -ve , \quad \tau_H < 1$

$H_t < H_i$

$\Gamma_P = -ve , \quad \tau_P < 1 \longrightarrow$ Always

Workbook :-

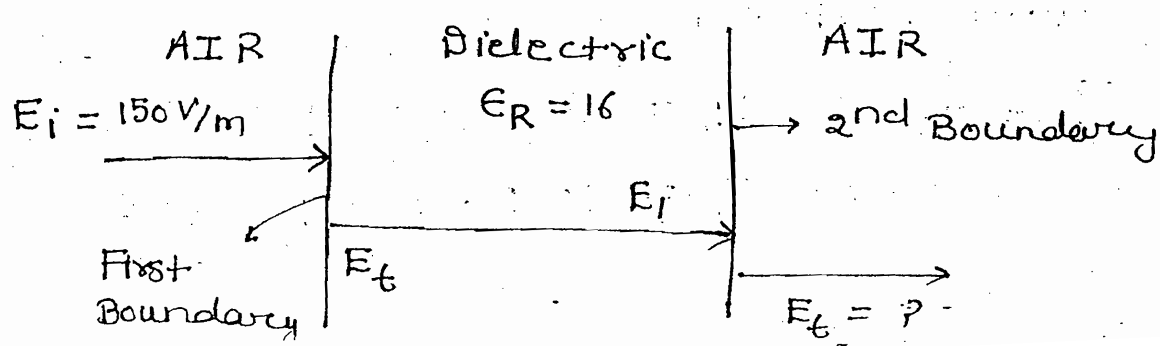
12. $\frac{\text{Air}}{\epsilon_0}$ - $\frac{\text{Dielectric}}{4\epsilon_0}$ $\quad \tau_P = ?$

$$\Gamma_E = \frac{\sqrt{\epsilon_0} - \sqrt{4\epsilon_0}}{\sqrt{\epsilon_0} + \sqrt{4\epsilon_0}} = \frac{1-2}{1+2} = -\frac{1}{3}$$

$$\Gamma_H = +\frac{1}{3} \qquad \Gamma_P = -\frac{1}{9}$$

$$\Rightarrow \tau_P = 1 - \frac{1}{9} = \frac{8}{9}$$

13.



At the 1st boundary

$$\Gamma_E = \frac{1-\sqrt{16}}{1+\sqrt{16}} = \frac{-3}{5}$$

$$\tau_E = 1-\frac{3}{5} = \frac{2}{5} = \frac{E_t}{E_i} \Rightarrow E_t = \frac{2}{5} \times 150 \text{ V/m}$$

$$= 60 \text{ V/m}$$

E_i at the 2nd Boundary = 60 V/m

$$\Gamma_E = \frac{\sqrt{16}-\sqrt{1}}{\sqrt{16}+\sqrt{1}} = \frac{3}{5}$$

$$\tau_E = 1+\frac{3}{5} = \frac{8}{5} = \frac{E_t}{E_i}$$

$$E_t = \frac{8}{5} \times 60 = 96 \text{ V/m}$$

14. $$\Gamma_E = \frac{1-\sqrt{4}}{1+\sqrt{4}} = \cancel{-\frac{1}{3}} \; -\frac{1}{3}$$

$$\tau_E = 1-\frac{1}{3} = \frac{2}{3} = \frac{E_t}{E_i}$$

$$E_t = \frac{2}{3} E_0 \cos(\omega t - 2\beta z)\, a_y$$

$$\left(\beta' = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_R} = 2\beta\right)$$

Note:—

$\alpha, \beta, \gamma, \eta, V_p, \lambda \rightarrow$ Propogation aspects & material aspects and change as σ, ϵ, μ changes

$\omega \rightarrow$ source aspect / time aspect and is same for every material.

15. $\Gamma_E = -\frac{1}{3}$ $\qquad H_i = \cos(10^8 t - \beta z)\, a_y$

$$E_i = 120\pi \cdot \cos(10^8 t - \beta z)\, a_x$$

$$\frac{10^8}{\beta} = 3\times10^8 \Rightarrow \beta = \frac{1}{3}$$

$$E_r = \left(-\frac{1}{3}\cdot 120\pi\right)\cos\left(10^8 t + \frac{z}{3}\right) a_x$$

46. $$\frac{E_t}{E_r} = \frac{E_t}{E_i \frac{E_r}{E_i}} = \frac{\tau}{\Gamma} = \frac{1+\Gamma}{\Gamma} = -2$$

$$\Gamma = -\frac{1}{3} = \frac{1-\sqrt{\epsilon_2/\epsilon_1}}{1+\sqrt{\epsilon_2/\epsilon_1}} \Rightarrow \frac{\epsilon_2}{\epsilon_1} = 4$$

48. EM Wave reflections at conductors :-

$$\Gamma_E = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \qquad \eta_1 = 120\pi = 377$$

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma}} = 0$$

→ EM wave power is completely reflected at conductors

$$\Gamma_E = \frac{E_r}{E_i} = 1$$

$$E_i + E_r = E_t = 0$$

$$\tau_E = 0$$

$$\Gamma_H = \frac{H_r}{H_i} = 1 \qquad \text{Ans-(c)}$$

$$H_i + H_r = H_t = H_{max}$$

$$\tau_H = 2$$

47. $$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \quad (\text{if } \sigma = 0) \text{ real}$$

$$= \sqrt{\frac{\mu^*}{\epsilon^*}} \quad (\text{if } \sigma \neq 0) \text{ complex}$$

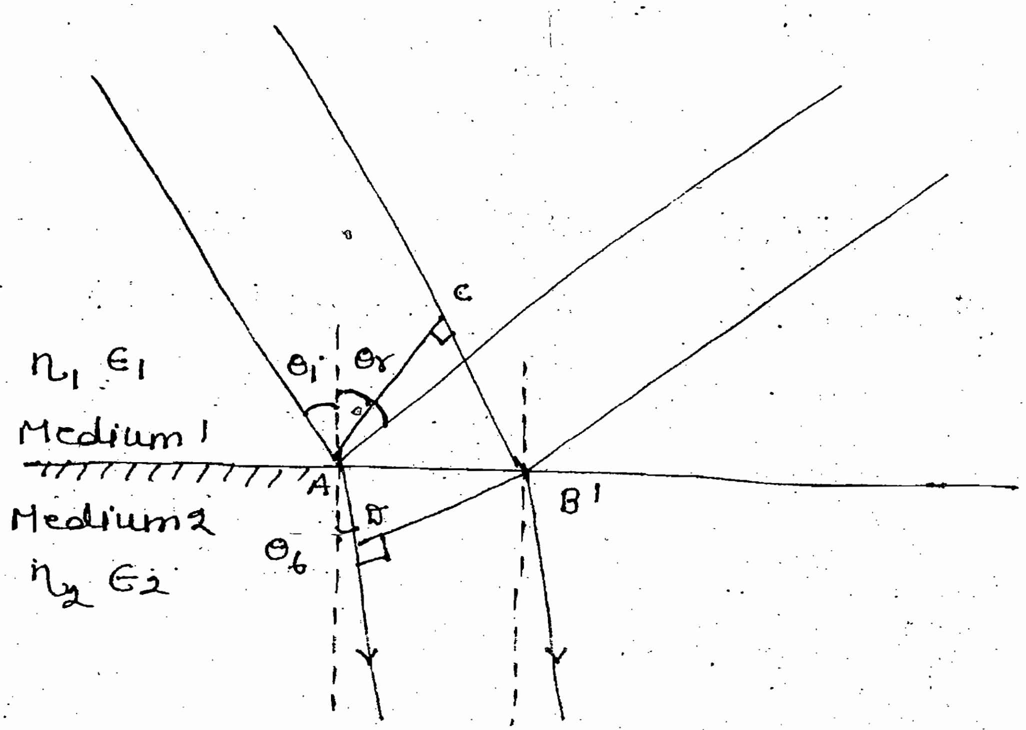
As $\mu^* = \epsilon^*$

$$\eta = \sqrt{\frac{\mu^*}{\epsilon^*}} = 1$$

$$\Gamma = \frac{1-377}{1+377} \Rightarrow \Gamma \cong -1$$

Ans – (c)

Oblique (Inclined) Incidence :–



→ A/C are inphase points in medium 1

→ CB is the extra distance travelled in medium 1

→ B/D are inphase points in medium 2

→ AD is the extra distance travelled in medium 2

$$\frac{CB}{AD} = \frac{V_1}{V_2} = \text{Refracting Index of med. 2 w.r.t med 1}$$

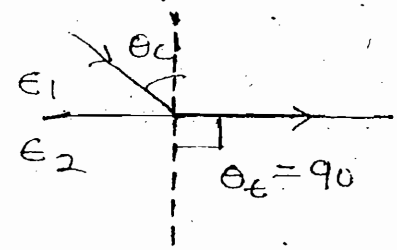
$$\frac{AB\cos(90-\theta_i)}{AB\cos(90-\theta_t)} = \frac{1}{\sqrt{\mu_0\epsilon_1}}\sqrt{\mu_0\epsilon_2}$$

$$\boxed{\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}} \longrightarrow \text{Refracting Index}$$

Snell's Ist law $\rightarrow$ $\boxed{\theta_i = \theta_r}$

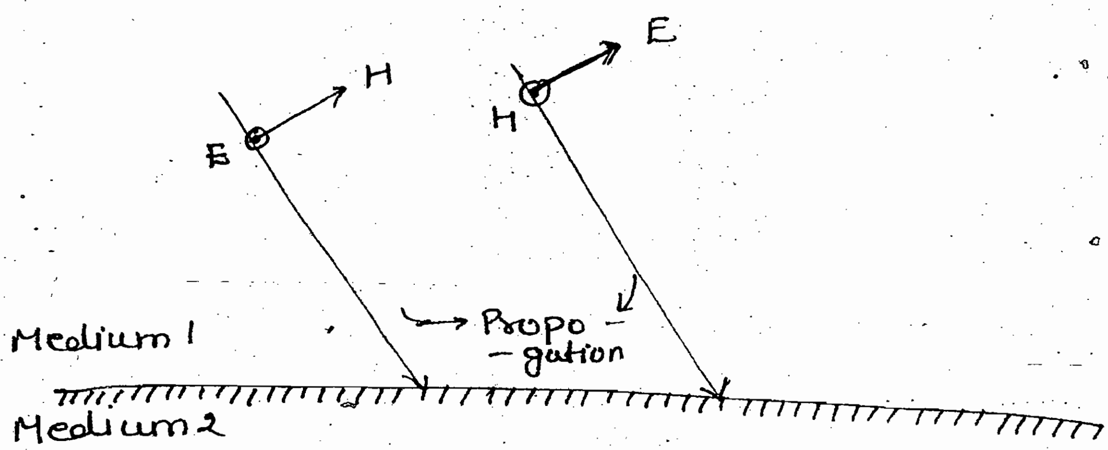
If $\theta_t = 90^\circ$ then $\theta_i = \theta_c$ = critical angle

For all $\theta_i > \theta_c$ complete reflections and zero transmission takes place



$$\boxed{\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

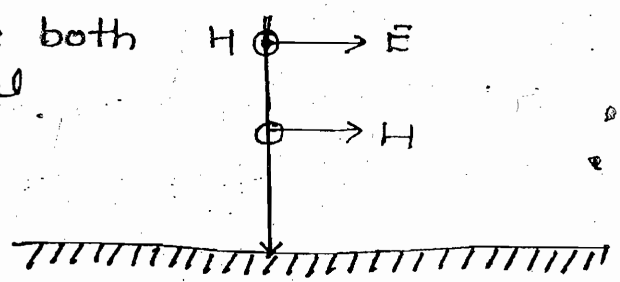
S & P Polarized Waves :-



S-Polarized Horizontal Polarization

P- Polarized vertical polarization.

In normal incidence both fields are Horizontal



Applying the Boundary conditions,

$$E_{t_1} = E_{t_2}$$

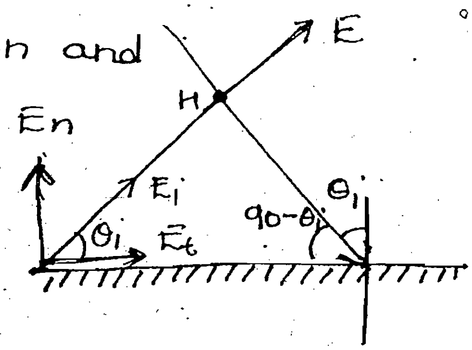
For s- polarized waves

$$E_i + E_r = E_t$$

$$\boxed{1 + \Gamma_s = \tau_s} \quad \text{—— (1s)}$$

Note:-

The angle b/w propogation and normals is the same angle b/w fields and surface



For p-polarised waves

$$E_i \cos\theta_i + E_r \cos\theta_r = E_t \cos\theta_t$$

$$E_i + E_r = E_t \frac{\cos\theta_t}{\cos\theta_i}$$

$$\boxed{1 + \Gamma = \tau \frac{\cos\theta_t}{\cos\theta_i}} \quad \text{— } 1p$$

similarly using $H_{t_1} = H_{t_2}$

$$\Gamma_s = \frac{\eta_2 \sec\theta_t - \eta_1 \sec\theta_i}{\eta_2 \sec\theta_t + \eta_1 \sec\theta_1}$$

$$\Gamma_p = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Note:-

$\Gamma_s \neq \Gamma_p$ in oblique incidence

but when $\theta_i = \theta_r = \theta_t = 0$ then

$$\Gamma_s = \Gamma_p = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

As $\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$ $\quad \eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$

$$\Gamma_s = \frac{\sqrt{\epsilon_1} \cos\theta_i - \sqrt{\epsilon_2} \cos\theta_t}{\sqrt{\epsilon_1} \cos\theta_i + \sqrt{\epsilon_2} \cos\theta_t}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\,\sin\theta_i$$

$$\cos\theta_t = \sqrt{1 - \frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i}$$

$$\Gamma_s = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}$$

$$\Gamma_p = \frac{\frac{\epsilon_2}{\epsilon_1}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\frac{\epsilon_2}{\epsilon_1}\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}$$

If $\Gamma_p = 0$ i.e. zero reflections & complete transmission

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 (1 - \sin^2\theta_i) = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i$$

$$\sin^2\theta_i \left(1 - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2\right) = \frac{\epsilon_2}{\epsilon_1} - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2$$

$$\sin^2\theta_i = \frac{\frac{\epsilon_2}{\epsilon_1}\left(1 - \frac{\epsilon_2}{\epsilon_1}\right)}{\left(1 + \frac{\epsilon_2}{\epsilon_1}\right)\left(1 - \frac{\epsilon_2}{\epsilon_1}\right)}$$

$$\sin\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$\tan\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

This $\theta_i = \theta_B$ is called as Brewster angle for p-polarised wave

If $\Gamma_s = 0$

$$\cos^2\theta_i = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i$$

$$\Rightarrow \epsilon_2 = \epsilon_1$$

Brewster angle and zero reflections cannot occur for s-polarized waves.

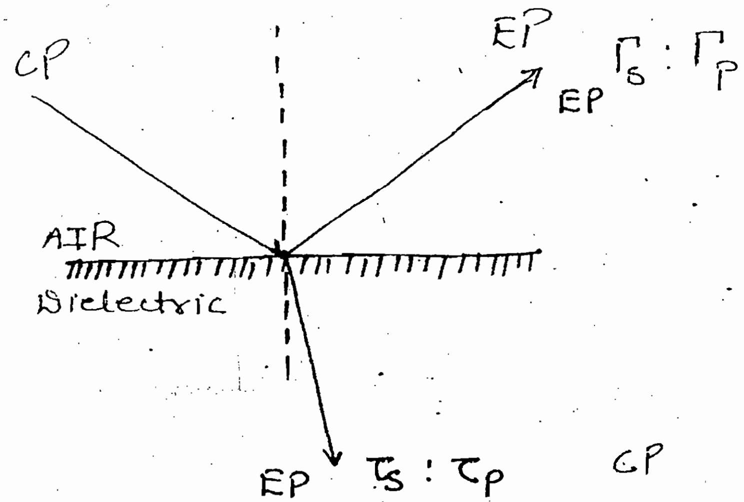
Summary :-

zero transmission, complete reflections θ_c = critical angle	zero reflections, complete transmission θ_B = Brewster angle
$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$,	2. $\tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$
$\epsilon_1 > \epsilon_2$ is a required condition	3. No such medium restriction
No such polarization restriction	4. Exists only for p-polarized waves
For all $\theta_i > \theta_c$ → the same effect	5. At $\theta_i = \theta_B$ only this occurs

Workbook :-

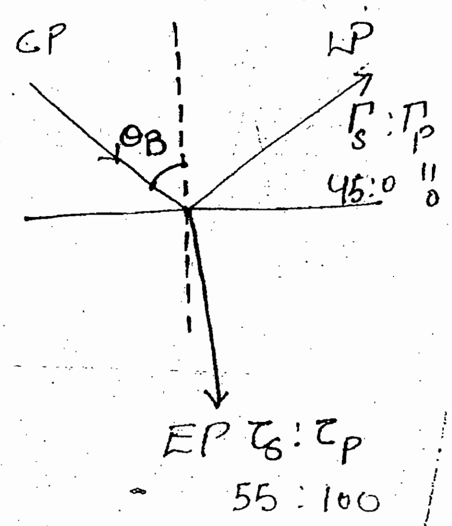
19.

RCP → S + P



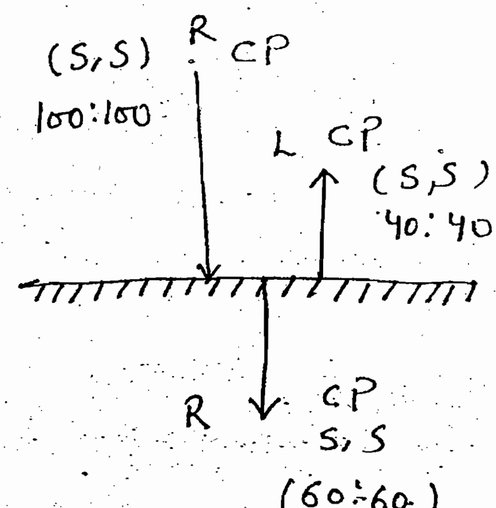
$$\tan\theta_B = \tan 60^\circ = \sqrt{\frac{\epsilon_R \epsilon_0}{\epsilon_0}}$$

$$\Rightarrow \epsilon_R = 3$$

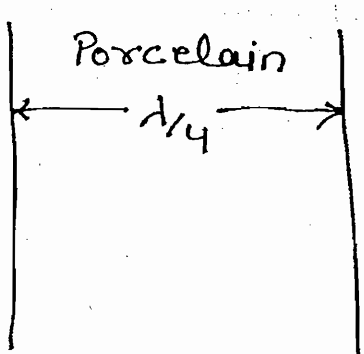


50 Linearly Polarised

51. a b



52.



$$\text{thickness} = \frac{\lambda}{4} = \left(\frac{3\times10^8}{\sqrt{\epsilon_R}\times 10\times 10^9\times 4}\right)$$

53. μ_R [$J_b \rightarrow$ Bound current Density

$\rightarrow$ Due to spinning / revolving electrons constituting a magnetic dipole.

σ [$J_c \rightarrow$ conduction current – moving electron

ϵ [$J_d \rightarrow$ displacement current $\rightarrow$ changing Electric flux

41. $E = 3\sin(\omega t - \beta z)\,a_x + 6\sin(\omega t - \beta z + 75°)\,a_y$

$E_0 = \sqrt{3^2 + 6^2} = 3\sqrt{5}$

$$P_{avg} = \frac{1}{2}\frac{E_0^2}{\eta}\,a_z$$

$$= \frac{1}{2}\frac{(3\sqrt{5})^2}{120\pi} \cong 58\text{ mW}$$

OR

$$E = 3\sin(\omega t - \beta z)\,(a_x + 2e^{j75}a_y)$$

$$H = \left(\frac{3}{120\pi}\right)\sin(\omega t - \beta z)\,(a_y - 2e^{j75}a_x)$$

$$P_{avg} = \frac{1}{2}\cdot 3\,(a_x + 2e^{j75}a_y) \times \left(\frac{3}{120\pi}(a_y - 2e^{-j75}a_x)\right)$$

$$= \frac{1}{2}\frac{3\cdot 3}{120\pi}(a_z + 4a_z)$$

$$= \frac{1}{2}\frac{(3\sqrt{5})^2}{120\pi}\,a_z$$

TRANSMISSION LINES

Energy Propogation

MHZ/GHz

V/I Waves

E/H Waves

V/I Waves	E/H Waves
→ $\sigma = \infty$	→ $\sigma = 0$
→ conductors	→ Free space
→ Wired or point to point	→ Broadcast wireless
→ Extremely low frequency $f < kHz$	→ Extremely high frequency $f > 10^{12} Hz$

V_s, I_s → ω source → R, L, G, C Primary constant → Transmission line → $V(x)$, $I(x)$ → Propogation

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

E_s $\longrightarrow$ ω source $\longrightarrow$ μ, σ, ϵ Material constant $\longrightarrow$ Medium $\longrightarrow$ $E(z)$
H_s $H(z)$

$\downarrow$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

<u>Primary constants</u>:-

<u>Resistance</u>:-

→ It is the resistance of the conducting material all along the length of the line.

→ $R = \rho \frac{l}{A}$ $\qquad R \propto l$.

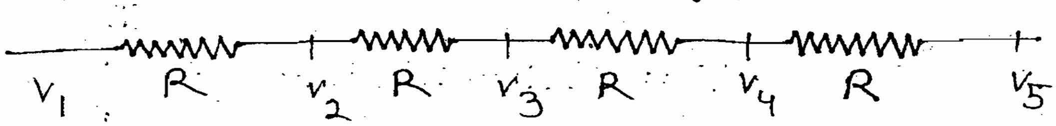
It is distributed all along the length

$$R = \frac{R}{l} = \text{ohm/m}$$

$\downarrow$
Primary constant

→ It is the resistance at any instance on the line

→ It appears in series along the line

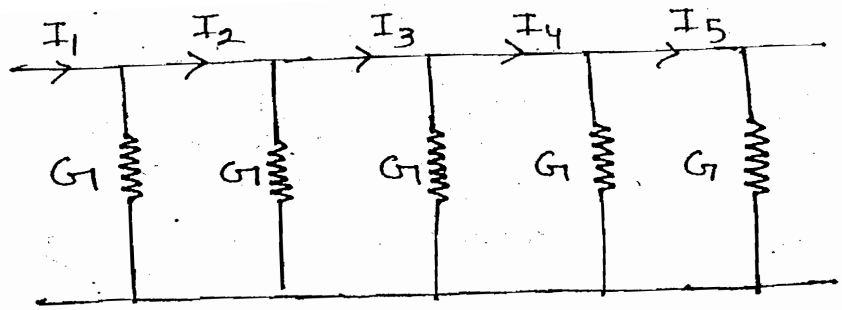


→ It causes a voltage decay all along the line

<u>Conductance</u>:-

→ It is the conductance of the insulating material b/w the lines

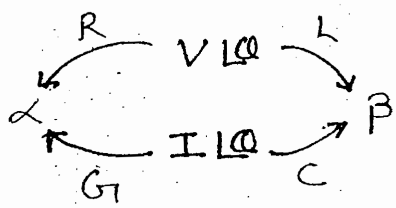
→ It appears in shunt b/w the lines

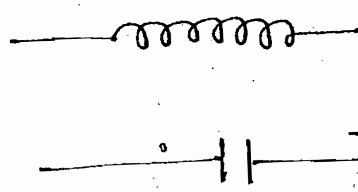


→ It causes current decay

→ It is distributed all along the length

$$G = \frac{G}{l} = \frac{mho}{m}$$



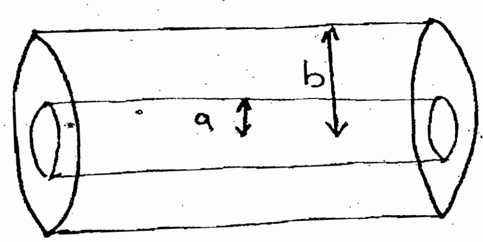


→ flow

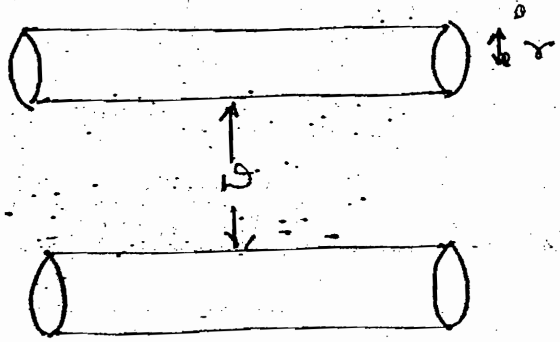
→ accumulation

Capacitance :-

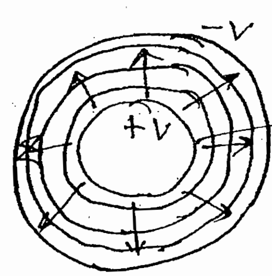
→ It is due to the voltages on the line there is a surrounded E field and hence capacitance



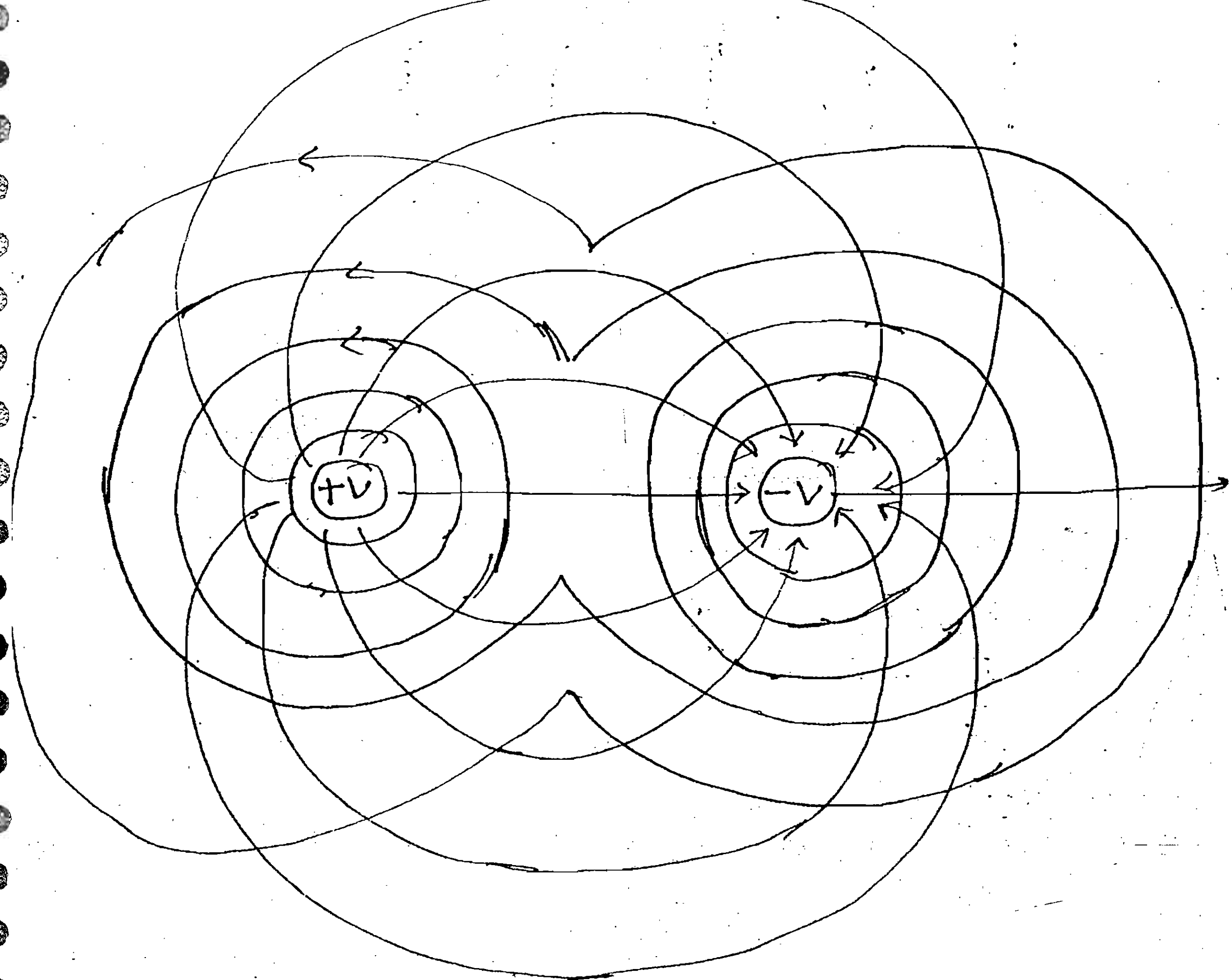
→ Co-axial cable



→ Parallel Wire.



→ Transmission line.



$$C = \frac{2\pi \epsilon l}{\ln(b/a)} \longrightarrow \text{co-axial cable}$$

$$C = \frac{\pi \epsilon l}{\ln(D/r)} \longrightarrow \text{Parallel wire}$$

$\longrightarrow$ $C \propto l$ $\longrightarrow$ capacitance is distributed

$\longrightarrow$ $c = \frac{C}{l} = \frac{\text{Farads}}{m}$

$\longrightarrow$ It appears in shunt b/w the cables

Inductance :-

→ It is due to the currents on the line there is a surrounded H-field and hence inductance

→ $$L = \frac{\mu l \ln(b/a)}{2\pi} \longrightarrow \text{co-axial cable}$$

$$L = \frac{\mu l}{\pi} \cdot \ln(D/r) \longrightarrow \text{parallel wire}$$

→ L is distributed i.e. $L \propto l$

$$\boxed{L' = \frac{L}{l} \quad \frac{\text{Henry}}{m}}$$

Note :-

$$\boxed{LC = \underset{L}{\text{Distributed}} \times \underset{C}{\text{Distributed}} = \mu\epsilon}$$

For any transmission line and for any geometry.