Chapter—14

MENSURATION - I

Akanksha and Ranu cut thick papers into different sizes of rectangles & parallelograms as shown below:

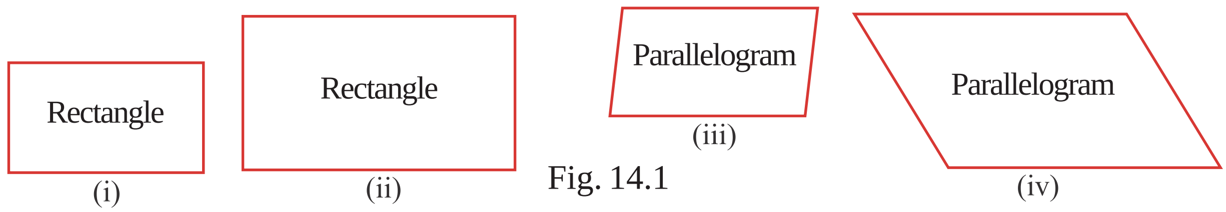
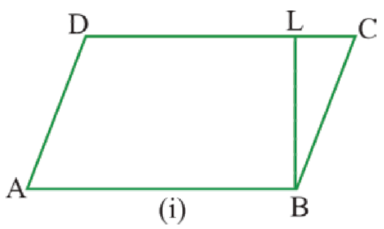


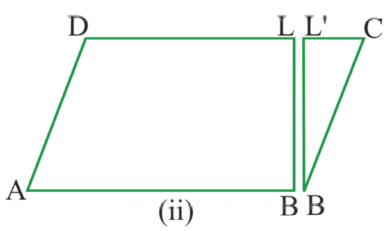
Fig. 14.1

Ranu asked Akaksha, to find out their areas. Akanksha multiplied the length and breadth of the rectangular pieces and found out the areas (Area of a rectangle = length x width) But she could not find the area of the parallelograms because she could not decide their length and breadths. Ranu said, “If we cut the pieces of parallelogram and make them rectangle, we will be able to get their areas, so they did the following activity.



(i) They took a figure of parallelogram and named it as ABCD. Considering AB as the base, draw a perpendicular on CD as BL. Then they could separate BLC and cut it out from the parallelogram figure 14.2.

(ii) When Ranu tried to join the triagular figure, the figure 14.2 (iii) was obtained After this, Akansha attached it to the figure and got the shape of fig. 14.2 (iv).



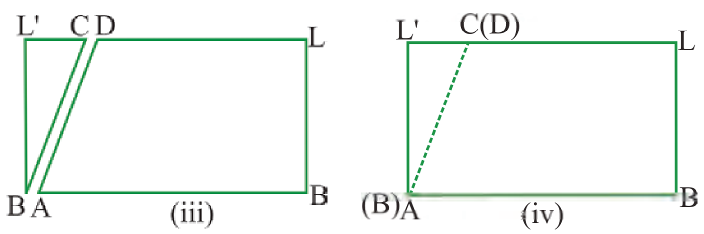


Fig. 14.2

Thus they got the shape of a rectangle. Akansha said figure 14.2(1) and figure 14.2 (iv) have equal areas, become fig. 14.2 (iv) is a changed form of fig. 14.2(1).

Rectangle ABLL' = AB × BL = Base of the parallelogram ABCD × height.

Therefore, area of the parallelogram ABCD = base × height

Thus Ranu & Akanksha found out area of parellegoram. So,

(i) Area of a parallegoram = *base* × *height*

(ii) Base for a parallelogram $= \dfrac{area}{height}$

(iii) Height of a parallelogram $= \dfrac{area}{base}$

Example 1 :

Find out the area of a parallelogram where base is 15 cm and height is 5cm.

Solution :

According to the question.

Base = 15 cm, height = 5 cm.

∴ Area of the parallelogram = base x height

= 15 cm x 5 cm

= 75 cm^2

Example 2 :

Find out the base of the parallelogram where area is 240 cm^2 and height is 8 cm.

Solution : We knew that base of a parallelogram $= \dfrac{\text{Area}}{\text{height}}$

area = 240 Cm^2

height = 8 cm

Base $= \dfrac{240cm^2}{8cm}$

= 30 cm.

Exercise 14.1

Q. 1. Find out the area of a parallelogram whose base and height are as follows:

(i) Base = 15 cm, Perpendicular (apex) = 10 cm

(ii) Base = 90 cm ,Perpendicular (apex) = 8 cm

(iii) Base = 120 cm, Perpendicular (apex) = 15 cm

Q.2. Find the area of the parallelogram where base is 26.5 cm and (apex) perpendicular is 7 cm.

Q.3 Find out the base of the parallelogram, whose area is 390 cm^2 and the perpendicular (apex) in 26 cm.

Q. 4. Find out the Perpendicular (apex) of parallelogram with area is 1200 m^2 and base is 60 m.

Let us do this activity:

Draw a triangle ABC taking A as centre and of BC as length and C as centre and of BA as length raw two arcs which intersects each other. Now join point A & C and name it by D. Thus parallelogram ABCD is obtained because AB = DC & AD = BC.

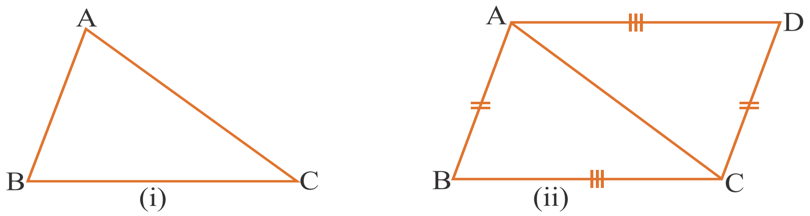


Fig.14.3

Activity

Cut a rectangular piece of hard board ABCD along its diagonal AC with the help of scissor.

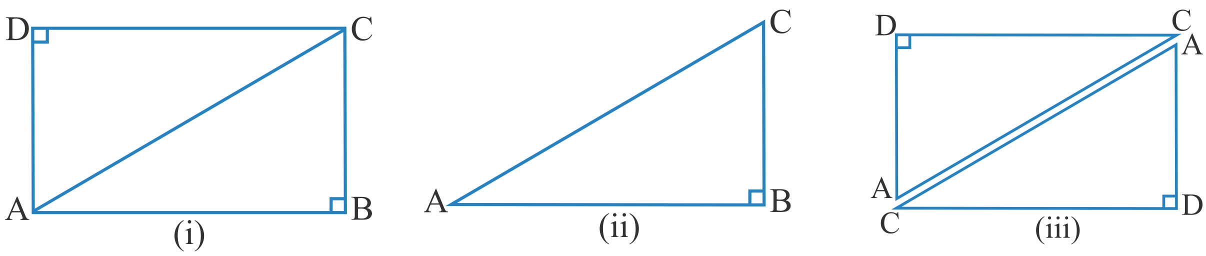


Fig.14.4

Thus, two triangles ABC and ADC were formed. Place the two triangles ABC on ADC on each other. Do they overlap each other completely? You will find that the two triangles are congruent and their areas are equal.

$\therefore$ Area of Δ ABC + area of Δ ADC = area of rectangle ABCD

Area of Δ ABC = area of Δ ADC

2 (Area of Δ ABC) = AB $\times$ BC

Area of Δ ABC = ½ × AB × BC

Practice 1

Make a parallelogram on a piece of hard board. Cut it out. Cut it along one of the diagonals. You will get two triangles. The areas of the two triangles are equal. Place them on each other & observe.

Area of a triangle:

We can construct a parallelogram by joining two triangles of equal measurement. On drawing a diagonal in a parallelogram we get two triangles of equal measurement. So when we draw a diagonal AC in the parallelogram ABCD we get two triangles ABC and ADC that are congruent. Their areas are also equal.

Therefore, the area of parallelogram ABCD = Area of Δ ABC + Area of Δ ADC

= 2 (Area of Δ ABC)

$\therefore$ Area of Δ ABC = ½ area of parallelogram gm ABCD

= ½ × base × height

= ½ × BC × AL

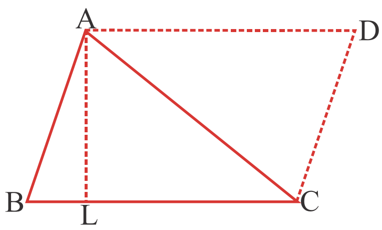


Fig.14.5

Therefore : **Area of a triangle A = ½ × b × h**

Where b = base of the triangle and h = height of the triangle

Remember : *The area of a triangle between two parallel lines is half the area of a parallelogram with the same base and height.*

Example 3 :

Find out the area of a triangle with base 28 cm and height 6 cm.

Solution :

According to the question.

Base of a triangle b = 28 cm

Height h = 6 cm.

So, the area of a triangle A = ½ × b × h

= ½ × 28 × 6 cm = 84 cm^2

Example 4 :

Find out of a triangle where base is 80 cm and area is 0 . 08 square m.

Solution:

Since base is given in Cms, the area also should be converted to cm.

Thus 1 $meter^2$ = 1 metre × 1 metre

= 100 cm × 100 cm

= 10000 cm^2 (∴ 1 m = 100 cm)

Therefore .08 $metre^2$ = 0.08 × 10000 Cm^2

= 800 Cm^2

Now the area of triangle A = ½ × b × h

$$\text{Height of a triangle} = h = \frac{2A}{b} = \frac{2\times800}{80}$$

⇒ h = 20 cm

Exercise 14.2

Q. 1. Find out the area of a triangle with base 12 cm and corresponding height is of 7 cm.

Q. 2 Find out the area of a triangle with base 25 cm and a Perpendicular from apex length is of 1.5 cm.

Q. 3. Find out the perpendicular (apex) length of a triangle where base is 6.5 cm and the area is 26 cm^2.

Q. 4. Find out the area of a triangle where base is 120 dm and height is 75 dm.

Area of a rhombus :

A rhombus is a type of parallelogram and so if its base and height are known, we can find its area, If the base of the fig. be ‘b’ and height be ‘h’ then area A = b x h

ABCD is a rhombus and d_1 and d_2 are its diagonals since they intersect each other at right angles, therefore, the four right angled triangles will have perpendicular sides $\frac{d_1}{2}$ and $\frac{d_2}{2}$.

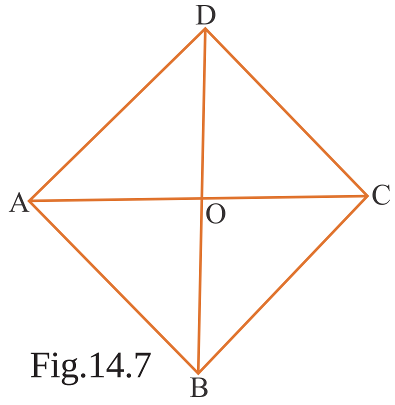


Fig.14.7

Area of the rhombus = 4 x area of a right angled triangle

$= 4 \times \frac{1}{2}$ base × height

$= 4 \times \frac{1}{2}\left(\frac{1}{2}d_1\right)\times\left(\frac{1}{2}d_2\right)$

Area of a rhombus	**$= \frac{1}{2} \times d_1 \times d_2$**

Area of the rhombus	**= ½ × 1st diagonal × 2nd diagonal**

Area of a Trapezium:

Trapezium is a quadrilateral whose two opposite sides are parallel to each other. A trapezium ABCD is shown in fig. 14.8. Side AB and DC are parallel. The perpendicular distance from the parallel sides have been shown as AM and CL.

If a diagonal AC of this triangle be drawn, we get two triangles ABC and ACD from this trapezium.

Therefore, the area of a trapezium ABCD = Area of Δ ABC + Area of Δ ACD

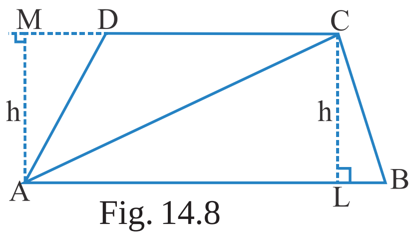


Fig. 14.8

Area of a trapezium ABCD =

= ½ × AB × CL + ½ × DC × AM

Since CL and AM indicate the height of the trapezium, therefore this will be equal so, we consider this as ‘h’.

Thus the Area of a Trapezium = $\frac{1}{2}$ AB × h + $\frac{1}{2}$ DC × h

If AB = b_1, and DC = b_2 then

Area of a Trapezium $= \frac{1}{2} b_1 \times h + \frac{1}{2} b_2 \times h$

$= \frac{1}{2}(b_1 + b_2) \times h$

= ½ × (Sum of parallel sides)× (Distance between parallel sides)

= ½ × (sum of parallel sides) × height

Area of a trapezium = ½ × (Sum of parallel sides) × height

or **Area of a Trapezium = ½ $(b_1 + b_2) \times h$**

Example 6 :

The sides of a rhombus are 7cm. and its height is 32 cm. find its area.

Solution :

According to the question

Base = 7 cm, height = 3.2 cm

Area of a rhombus = base × height

= 7 × 3.2 cm^2

= 22.4 cm^2

Example 7 :

The first diagonal of a rhombus is of 10 cm and the second diagonal of a rhombus is of 12 cm. find its area.

Solution :

According to the question

First diagonal of the rhombus = 10 cm

Second diagonal of the rhombus = 12 cm

Area of a rhombus = ½ × (1^{st} diagonal) × (2^{nd} diagonal)

= ½ × 10 × 12 cm

= 60 square cm

Example 8 :

The parallel sides of a trapezium are 25 m and 20 m respectively. The distance between the two parallel sides is 8 m. Find its area.

Solution :

Given :- b_1 = 25 m, b_2 = 20 m, h = 8 m

Therefore, Area of a trapezium A $= \frac{1}{2} \times h \times (b_1 + b_2)$

$= \frac{1}{2} \times 8 \times (25 + 20)$

$= \frac{1}{2} \times 8 \times (45)$

= 180 Square metres Ans.

Example 9 :

Area of a trapezium is 140 cm^2, if the lenght of one of its parallel sides is 25 cm and height is 7cm. then find out the length of the second parallel side.

Solution :

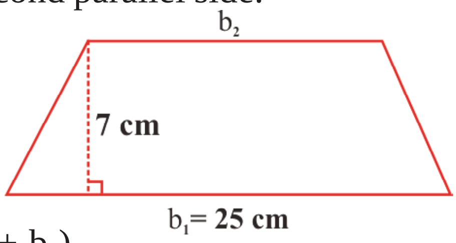


Fig.14.9

According to the question

A = 140 cm^2, b_1 = 25 cm, h = 7 cm

Area of a trapezium $= \frac{1}{2} \times h \times (b_1 + b_2)$

So, $140 = \frac{1}{2} \times 7 \times (25 + b_2)$

$$\frac{2 \times 140}{7} = 25 + b_2$$

$$40 = 25 + b_2$$

$$b_2 = 40 - 25$$

The second side b_2 = 15 cm

Exercise 14.3

Q. 1. Find the area of a rhombus where diagonals are of 24 cm & 10 cm.

Q. 2. Find the area of a rhombus where one side is 7.5 cm and the perpendicular from the apex is 4 cm.

Q. 3. Find the area of a trapezium where parallel sides are of 20 m and 8 m and the distance between there two sides are 12 cm.

Q. 4 What will be the area of the trapezium where base is 30 cm and 24.4 cm, if its perpendicular from apex is 1.5 cm

Q. 5. The area of a trapezium is 105 cm^2 and its height is 7 cm, if base of the one of the sides out of the two parallel arms is 6 cm more than the other, find the length of the two parallel sides.

Area of a Rectangular path:

Usually we need to find the area of the verandah around a school building, the road around a farm, the surrounding space of a play-ground etc., what do we do in these conditions?

In fig.14.10. we see a rectangular piece of farm that is surrounded by a road on all sides. What will you do, if you need to find the area of this road?

Fig.14.10

It is clear that we are getting two rectangles, so we can subtract the area of the smaller rectangle from the area of the bigger rectangle.

Area of the rectangular path = Area of the bigger rectangle – Area of the smaller rectangle

Example 10 :

A rectangular farm is 90 m. in length and 65 m. in width. On all the four sides there is path of 5 m. wide find the area of the path.

Solution :

It is clear from the picture that the area of the path =

Area of the rectangle ABCD - Area of the rectangle EFGH

Therefore , Area of the path = (AB × BC) – (EF × FG)

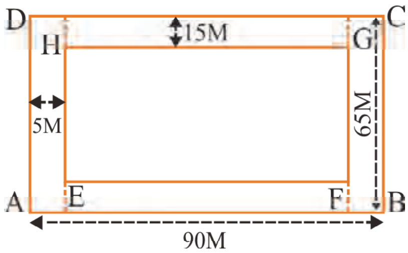


Fig. 14.11

Here, AB = AE + EF + FB

AB = 5 + 90 + 5

AB = 100 meter

Similarly, BC = 5 + 65 + 5

= 75 m.

Now the area of the path = (AB × BC) – (EF × FG)

= 100 × 75 – 90 × 65

= 7500 – 5850

= 1650 meter^2

Example 11 :

The length of a wall is 15.5 m and its width is 9m, There are two doors of 3m×1.5m size and two windows of 2m×1m size. What will be the cost of white washing this wall at the rate of Rs. 5 per square meter.

Solution :

First we need to find the area of the wall that is to be actually white washed.

Therefore, Area of the wall to be white washed

= Area of the complete wall – Area of (2 doors + 2 windows)

Area of the wall = Length × breadth

= 15.5×9

= $139.5\ m^2$

Area of one door = length × breadth

= 3×1.5

= $4.5\ m^2$

Area of two doors = $2 \times 4.5\ m^2$

= $9.0\ m^2$

Area of one window = length × breadth

= 2×1

= $2\ m^2$

Area of two windows = $2 \times 2\ m^2$

= $4\ m^2$

Area of the wall to be white washed

= $139.5 - (9.0 + 4)$

= $139.5 - 13.0$

= $126.5\ m^2$

Since the cost of white washing is Rs. 5/- square meter

Therefore, total cost = 126.5×5

= Rs. 632.50

Example 12 :

The length and breadth of a rectangular area is 35m and 24m respectively. There is a 2m wide road along its length and 1m wide road along its breadth. Find the area of the road.

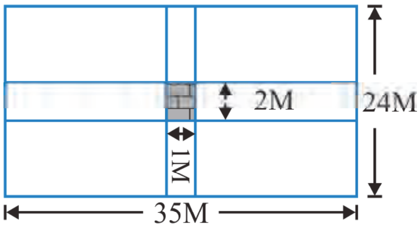


Fig. 14.12

Solution :

Area of the road along the length = $35 \times 2 = 70m^2$

Area of the road along the breadth = 24×1

= $24\ m^2$

Area of the shaded part = 2×1

= $2m^2$

(The shaded part comes in both the roads, So we will subtract that area once)

The area of the road = $70 + 24 - 2$

= $94 - 2$

= $92\ m^2$

Exercise 14.4

Q. 1. One 25 cm long and 10 cm wide picture is surrounded by a 2 cm wide frame on all the sides; find the area of the frame.

Q. 2. A rectangular playground measures 35m × 25m. A road 3m wide along the length and 2 m wide along the width goes through the middle of the ground. Find the area of the road.

Q.3 A basketball ground is 28 m long and 15 m wide. A 5 m wide gallery for spectators has to be built on all its sides of the ground. Find the area of the spectator gallery and the cost of making that gallery at Rs. 5.25 per square meter.

Area of a circular path:

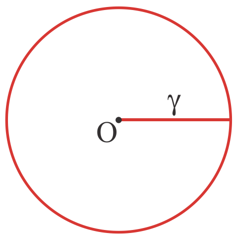


Fig. 14.13

In our previous classes, we have learnt about the circle. If the radius of the circle is ‘r’ then circumference C $=2\pi r$ and the area= πr^2

Since, π is a constant and its value is 22/7 or 3.14.

Example 13 :

Find the circumference and area of the wheel of a bicycle where radius is 21cm.

Solution :

The wheel of a bicycle is circular, therefore the circumference of a wheel

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 21 \text{ cm} = 132 \text{ cm}$

Area of the wheel $= \pi r^2$

$$= \frac{22}{7} \times (21)^2$$

$$= \frac{22}{7} \times 21 \times 21$$

$$= \frac{22}{\cancel{7}} \times \overset{3}{\cancel{21}} \times 21$$

$$= 1386 \text{ cm}^2$$

Concentric Circle:

In fig. 14.14 are given two concentric circles. If we need to find the area of the shaded portion, what wall we do? Obviously we shall subtract the smaller area from the bigger area.

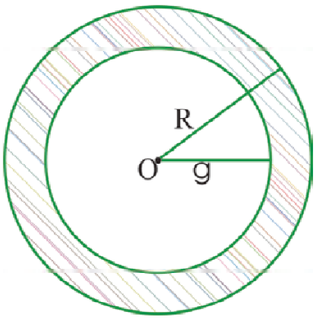


Fig. 14.14

Therefore:

Area of a circular path = Area of bigger circle – area of smaller circle

Example 14 :

A circular pond is 200 m in its radius. A circular path of 7 m width built along with the bank. Find the area of the path.

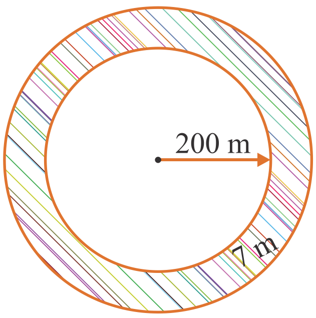


Fig. 14.15

Solution:

The area of a circular path =

Area of the bigger circle – area of the smaller circle

Radius of the smaller circle **r** = 200 m

Radius of the bigger circle **R** = 200 + 7 m

= 207 m

The area of the bank binding of the pond = $\pi R^2 - \pi r^2$

$= \pi[(207)^2 - (200)^2]$

$= \frac{22}{7}(207+200)(207-200)$ [$\because (a^2 - b^2) = (a+b)(a - b)$]

$= \frac{22}{7}(407)(7) = 22\ (407)\ m^2 = 8954\ m^2$

Exercise 14.5

Q. 1. The radius of two concentric circles are 9 cm and 12 cm respectively. Find the area of the circular path between the two circles.

Q.2. The area of a circle is 616 cm^2. There is a 2 m wide road on its edge. What will be area of that road?

Q.3 The radius of a circular cricket ground is 60 m. Around the ground a 7m circular gallery is to be built for spectators. What will be its area?

To find the Approximate Area of the Trapezium given on graph paper with the help of square grid method and verify the result with the formula method –

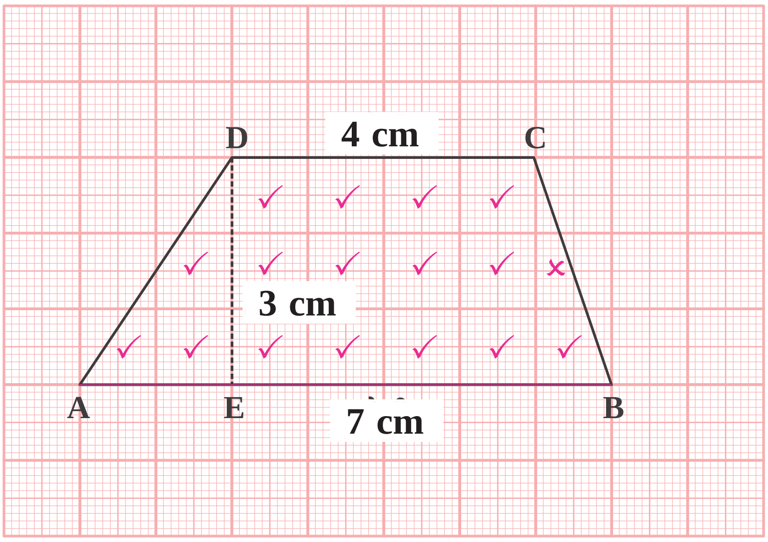


Fig. 14.16

For figure 14.16

Calculate the Approximate area of a Trapezium with the square grid.

In Trapezium ABCD

Number of complete squares = 14

Number of squares that are bigger than half in size = 2

Number of just half sized squares = 1

Area of the Trapezium = (No of Complete square + No of squares more than half in size) + $\frac{\text{No. of half sized squares}}{2}$

$= 14+2+\frac{1}{2}\times 1$

$= 16 + \frac{1}{2} \times 1$

$= 16 + \frac{1}{2}$

$= 16 + .5$

= 16.5 Sq. Cm.

Area of a Trapezium by formula method

In trapezium ABCD

Parallel Sides	AB	= 7cm.
and	CD	= 4cm.
Height of the Trapezium	DE	= 3cm.

Area of the Trapezium = ½ (Sum of the Parallel side) xHeight

$= \frac{1}{2}(AB + CD) \times DE$

$= \frac{1}{2}(7 + 4) \times 3$

$= \frac{1}{2} \times 11 \times 3$

$= \frac{33}{2}$

= 16.5 Sq. Cm.

It is clear that

Approximate Area of the Trapezium by square grid method = Area of the Trapezium by formula

To find Approximate Area of polygon given on graph paper by square grid –

In Polygon ABCDEFA

figure 14.17

The area of the figure 14.17 is done by this method.

- No. of complete squares = 21
- No. of squares that are bigger than half in size = 8
- No. of half sized square = 2

No. of squares for = (No of Complete square + No of squares more than half in size) + $\frac{\text{No. of half sized squares}}{2}$

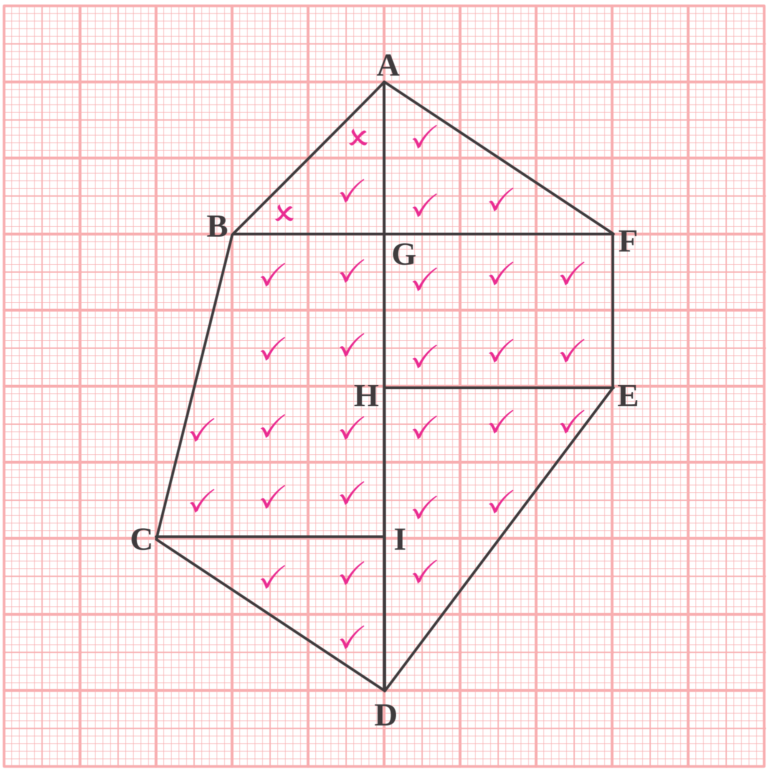


Fig. 14.17

$$= 21+8+\frac{1}{2}\times 2$$

$$= 29 + \frac{1}{2}\times 2$$

$$= 29 + 1$$

$$= 30$$

Therefore Area of the polygon ABCDEFA $= 30cm^2$

Calculation of Area of Trapezium by formula

Area of polygon ABCDEFA = Area of ΔAGB + Area of Trapezium BGIC + Area of ΔCID + Area of ΔDHE + Area of Rectangle HEFG + Area of ΔGFA

$$= \frac{1}{2} BG \times GA + \frac{1}{2}(BG + CI) \times GI + \frac{1}{2} CI \times ID + \frac{1}{2} HE \times HD + HE \times HG + \frac{1}{2} GF \times AG$$

$$= \frac{1}{2}\times 2 \times 2 + \frac{1}{2}(2 + 3)\times 4 + \frac{1}{2}\times 3\times 2 + \frac{1}{2}\times 3\times 4 + 3\times 2 + \frac{1}{2}\times 3\times 2$$

$$= 2 + (5 \times 2) + 3 + (3 \times 2) + 6 + 3$$

$$= 2 + 10 + 3 + 6 + 6 + 3$$

$$= 30cm^2$$

It is clear that

Approximate area of the polygon by square grid = Area of the polygon by formula method

WE HAVE LEARNT

1. Area of rhombus = base × height
2. The diagonal of a rhombus divides the rhombus into two equal triangles.
3. Area of triangle = $\frac{1}{2}$ × base × height

4. Area of equilateral triangle $= \frac{\sqrt{3}}{4}(side)^2$

5. Area of a rhombus $=$ Base × Height

$= \frac{1}{2} \times 1^{st}$ diagonal $\times 2^{nd}$ diagonal

$= \frac{1}{2} \times d_1 \times d_2$

6. Area of a trapezium $= \frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2} \times (b_1+b_2) \times h$

7. Area of a circle $= \pi \times (\text{radius})^2$

$= \pi r^2$ (here, π =constant $= \frac{22}{7} = 3.14$ approx.)

8. Circumference of a circle $= 2 \times \pi \times$ radius

$= 2\pi r$