CHAPTER 13

GEOMETRICAL CONSTRUCTIONS

Introduction

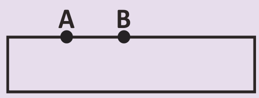
Geometrical construction implies drawing geometrical figures with the help of compass and ruler using exact measurements. Geometrical constructions help us experience and think about many geometrical concepts, relations and proofs. In this chapter, by applying geometrical concepts we will construct many geometrical shapes which we have studied in previous years. Along with constructions, we will also analyze them so that we can understand how these constructions takes place and why. To do so, while drawing the constructions according to the given problem, we will think about and discuss the different figures.

In mathematics, problems are solved keeping in mind logic and proof. Solving problems and observing whether it can be solved by more than one method; thinking which method is useful and easy; raising and thinking about such questions helps to build our logical and critical thinking ability.

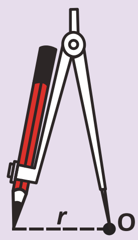
Let us do some constructions.

So far, we have made line-segments and angles of given measurements with the help of ruler or protractor. In this chapter, we will also learn how to use a compass during geometrical constructions.

Using ruler/scale in geometrical constructions: We know that given any two points A and B, only one straight line is possible such that it passes through A and B (axiom). We can use the ruler to draw a line AB or line segment AB or the ray AB.



Using compass in Constructions: We know from the definition of a circle that given a fixed point and fixed radius, only one circle can be drawn. Here, we will use the compass to draw a circle or an arc.



Construction-1: To Construct a Similar Angle

An angle is given and we have to construct another angle of equal value. What should we do? In one method, we can measure the angle with the help of a protractor and then construct an angle which is equal to it. But, if we do not have any instrument to measure the angle, then what can we do? Let us see-

Step–1. Before begin our construction, it will be helpful to think on the following questions:-

1. *What information is given in the question, how to solve it and which of the given information is useful?*

   An angle is given and we have to construct another angle which is equal to it.

   If the angle is ABC, then we have to construct another angle RPQ such that $\angle RPQ = \angle ABC$.

2. *Based on the given information, which geometrical concepts can be used during the construction?*

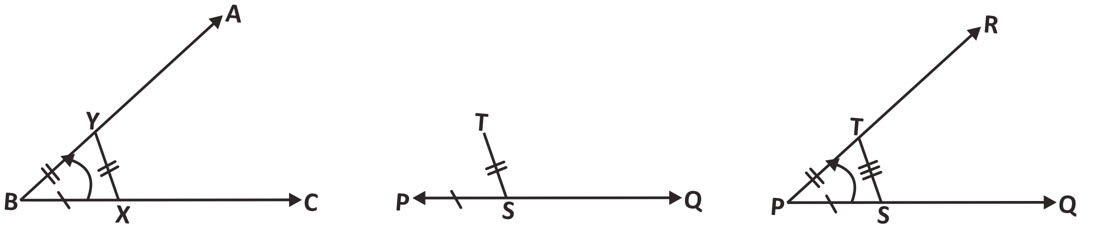


Figure - 1 *Figure - 2* *Figure - 3*

We know that if we rotate a ray from one position to another position then the measure of that rotation is known as angle. By rotating a ray BC to BA, we get angle ABC (Figure-1).

Suppose we construct a ray PQ and then rotate it equal to the rotation of BC to BA. But how do we do this?

Let us take two points X and Y on BC and BA respectively such that BX=BY and consider a point S, on PQ, such that BX=PS (Figure-2).

Now, if we consider a point (say T) such that the location of T with respect to P and S corresponds to the position of Y relative to B and X then the ray PT is congruent to BY (figure-3).

This point T, on the radius of the arc PS, will be located at a distance XY from S. If we draw a ray PR joining PT then $\angle TPS$ is equal to angle $\angle YBX$ (Or $\angle ABC$).

Step -2. After drawing the rough figure, geometrical construction can be done in a step by step manner.

Steps of construction:-

1. Consider a point P and draw a ray PQ from point P; this ray is one side of the new angle.
2. Now, in given angle ABC draw an arc of any measure from the vertex B which intersects BA at J and BC at K (see figure-4).
3. Now we draw an arc of same measure from point P which intersects ray PQ at M (Figure-5).
4. Now measure distance KJ from point K and cut an arc of the same length from point M intersecting the first arc. Let the intersection point be L.
5. Now draw ray PR joining P to L.

   ∠RPQ is the required angle.

   ∠RPQ = ∠ABC

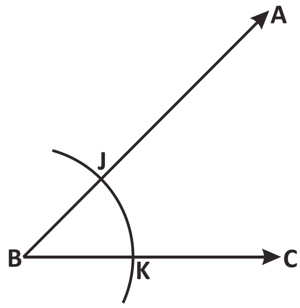


Figure - 4

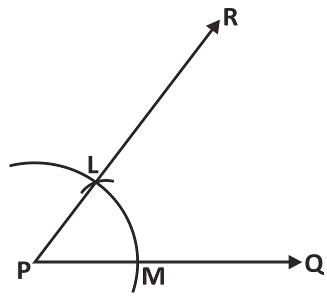


Figure - 5

<u>Step-3.</u> Checking the constructed figure- We can check whether the constructed figure is the same as per the information given in the problem. In addition to measurement, we can also check through proofs.

Let us see whether the constructed angle is equal to the obtained angle or not.

For this, we can construct triangles by using the angles from both the figures as references. Join M to L and K to J so that triangles PML and BKJ are formed.

If we look at triangles PML and BKJ then we find that –

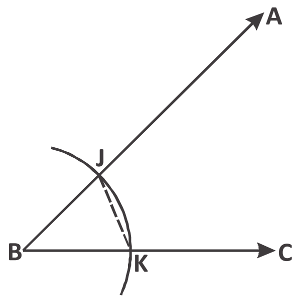


Figure - 6

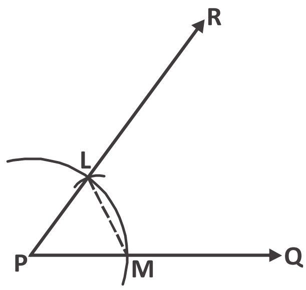


Figure - 7

PM = BK (from construction)

ML = KJ (from construction)

PL = BJ (from construction)

Therefore, triangle PML is ≅ to triangle BKJ (From SSS congruency)

Hence, ∠LPM = ∠JBK

Similarly, ∠RPQ = ∠ABC

Example-1. Construct a line segment which is equal to the given line segment.

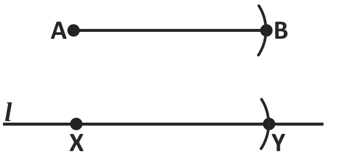


Figure - 8

Solution :

Step-1. A line segment AB is given. We have to construct a line segment which is equal to AB.

Step-2. **Steps of construction**

1. Draw a line, say *l*.
2. Choose any point X on line *l*.
3. Now open the compass so that the radius is equal to AB. Taking X as origin draw an arc on line *l* and let Y be the point of intersection.

   Line segment XY is congruent to line segment AB.

Step -3. **Proof**

Here, taking radius AB, we drew an arc from center X. Therefore XY=AB.

Example-2. Two angles are given. Construct an angle such that its measure is equal to the sum of the two angles.

Solution : Draw the angle $\angle LON$ which is congruent to $\angle A$ by using construction-1. Similarly, taking OL as one of the sides, draw $\angle MOL$ which is congruent to $\angle B$.

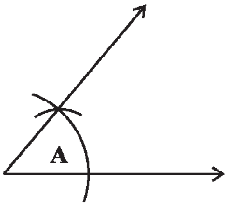


Figure - 9

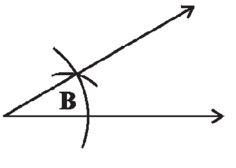


Figure - 10

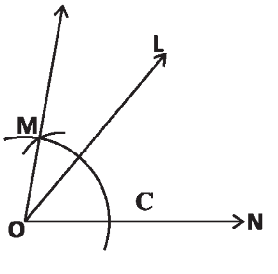


Figure - 11

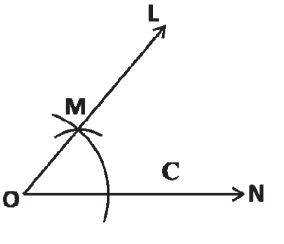


Figure - 12

Try These

1. Write detailed steps for the constructions given in example -2.
2. Draw angles measuring 30° and 90° and explain the steps.
3. Draw an isosceles triangle by taking any length for one side.
4. Draw an acute angle and construct another angle such that its value is twice the value of the previously constructed acute angle.

Construction-2 – Constructions of Parallel Lines.

Here, given a line and an external point we want to draw a parallel line passing through the point. Let us construct the parallel lines using the steps which we used in earlier constructions.

Step-1. Before the construction, think on the following questions:

1. *What is the information given in the question? In which sequence should we use it? What is to be constructed? In what order?*

   In the given information, what is useful and what is not useful?

   Here, a line and a point are given. This point is located outside the line. We have to construct a parallel line through the point (Figure-13).

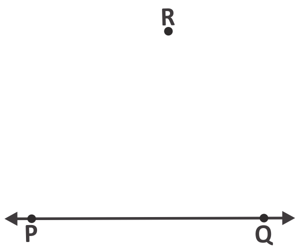


Figure - 13

2. *During the construction on the basis of the given information, what are the geometrical concepts that we will have to use?*

   We know that if a transversal line intersects two lines and if the corresponding angles made on them are equal then both the lines are parallel.

   So, from the given point we will construct a transversal line which will intersect the given line.

   If at the given point we construct an angle which is equal to the angle between the transversal line and the given line, then we can say that the obtained line is parallel to the given line.

Step-2. This involves drawing a rough diagram on the basis of the given information and determining which parts are known in the expected figure. What other things are required for the construction of figure? Finally we need to construct the figure, step by step.

Steps of construction:

A line PQ and a point R are given.

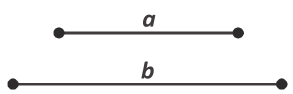


Figure - 14

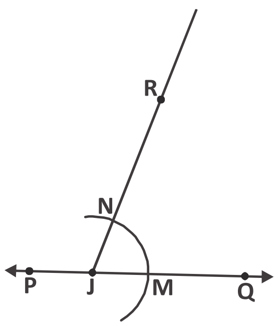


Figure - 15

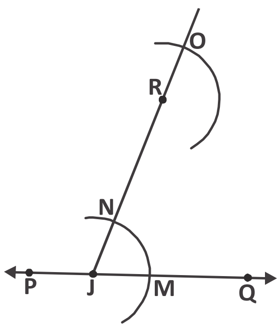


Figure - 16

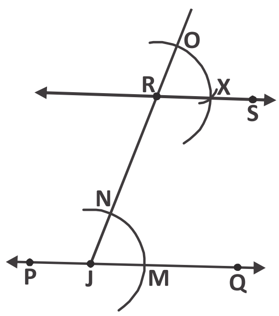


Figure - 17

We have to construct a line which passes through R and is parallel to PQ.

1. Draw a transversal line from R which intersects PQ at any point J. We know that if corresponding angles are equal then lines are parallel.
2. Now, we draw an arc of any measure from point J, which intersects PQ at M and JR at N (figure – 15).
3. Now, we will draw an arc of the same measure from the point R so that it intersects JR at Q (figure – 16).
4. Now, taking MN as length we will draw an arc from point O such that it intersects the arc which we drew earlier at point X.
5. Draw a line RS, by joining the point R to point X.

Thus, line PQ is parallel to line RS (figure-17)

Step -3. Checking the constructed figure. To see whether the constructed figure is the same as the required figure.

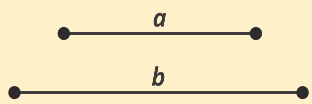
Proof : see the constructed figure.

Because $\angle ORX = \angle RJM$ (Corresponding Angles).

Then we can say that RS || PQ.

Exercise -1

1. Draw two line segments ‘a’ and ‘b’ in your notebook. Construct line segments according to the given information.

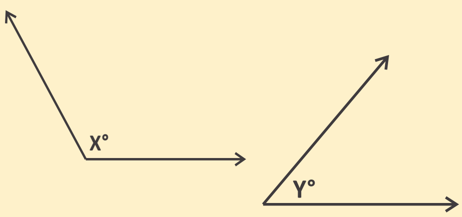


(a) a + b (b) b - a

(c) 2b + a (d) 3a - b

2. With the help of a protractor and a ruler draw the following angles: 15°, 45°, 105°, 75°.

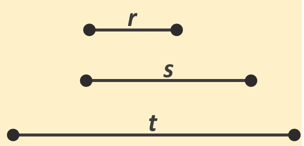
3. Two angles X (obtuse angle) and Y (acute angle) are given. Draw the angles having the following measures:



(a) $X^o - Y^o$ (b) $X^o + Y^o$

(c) $(180 - X)^o$ (d) $2Y^o$

4. Three line segments of fixed measure 'r', 's' and 't' are given.



   a) Is it possible to construct a triangle using these line segments? If yes then draw the triangle.

   b) Is it possible to construct a triangle using s, t and r+t ?

5. Draw a triangle ABC. Now from 'A' draw a line which is parallel to BC and check the sum of all the angles at vertex A as well as sum of all the angles of the triangle.

Construction - 3: Construction of a Line Segment in the Given Ratio

For this construction, we will use the Thales theorem. In the chapter on similar triangles you read about Thales theorem according to which "If in any triangle a line is drawn such that it is parallel to one of the sides of the triangle and also intersects the two remaining sides, then this parallel line will divide the two sides in the same ratio."

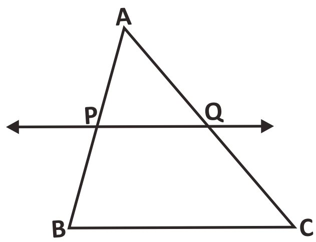


Figure - 18

If in given triangle ABC, PQ is parallel to BC, then from Thales theorem we can say that –

$$\frac{AP}{AB} = \frac{AQ}{AC} \quad \text{(Figure-18)}$$

If $AP = \frac{1}{3}AB$, then from the Thales theorem we can say that $AQ = \frac{1}{3}AC$.

Example-3. Find a point C on line segment AB such that AC:AB=2:3.

Solution :

Step-1. A line segment AB is given and on that line we have to obtain point C such that AC:AB =2:3.

This means that length of line segment AC is $\frac{2}{3}$ of the line segment AB (figure-19).

Think, how will we construct? Since AC:AB=2:3, so

Figure - 19

If we divide the line segment AB in three equal parts and select 2 parts out of the three, then this will be $\frac{2}{3}$ of the whole line segment.

We know that a line segment which is parallel to any side of a triangle divides the remaining sides of the triangle in the same ratio (Figure-21).

So why not draw a ray which makes an acute angle with AB on which three equal parts can be taken? Now keeping in mind the ratio 2:3, join the third point with B and draw a line parallel to it from the second point.

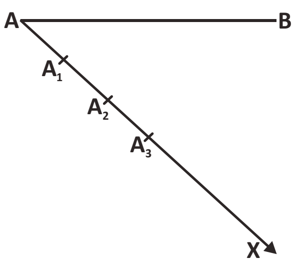


Figure - 20

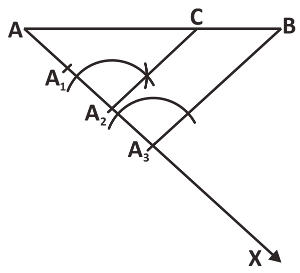


Figure - 21

Step-2. Steps of construction

1. Draw a ray AX making an acute angle at point A.
2. Cut three equal arcs on AX and name them AA_1, A_1A_2, A_2A_3.

   $AA_1 = A_1A_2 = A_2A_3$.
3. Now, join B with A_3 and draw a parallel line from A_2 to A_3B, which intersects AB at C.

   AC is the required line segment, because AC : AB = 2 : 3.

Step-3. Proof. On the basis of the geometrical construction how can we say that that

$$\frac{AC}{AB} = \frac{2}{3}$$

In ΔABA_3 or ΔACA_2, $A_2C \parallel A_3B$ (from construction)

From Thales theorem we can say that,

$$\frac{AC}{AB} = \frac{AA_2}{AA_3} \quad .....(1)$$

From the construction we know that,

$$\frac{AA_2}{AA_3} = \frac{2}{3}$$ (because AA_3 is divided into 3 equal parts)

Hence, $\frac{AC}{AB} = \frac{AA_2}{AA_3} = \frac{2}{3}$

Example-4. Draw a line segment which is $\frac{3}{2}$ of a given line segment.

Solution :

Step 1. A line segment AB is given, consider a point C on it such that

AC : AB = 3 : 2

In the previous example, point C was located between points A and B. In this example, a point C is such that AC:AB=3:2, therefore the point C is located outside of the line segment AB. When line AC is bigger than the line segment AB then only it will be $\frac{3}{2}$ times of AB.

Step 2: Steps of constructions

1. Draw a ray AY making an acute angle with point A and extend line segment AB upto X. (we extend AB upto X because we need to obtain a point C such that AC:AB=3:2).
2. Draw three equal arcs on AY and name them A_1, A_2, A_3.
3. Now join A_2 with B and draw a parallel line from A_3 to A_2B, which cuts AX at C.

   The required point C lies on AX such that $\frac{AC}{AB} = \frac{3}{2}$

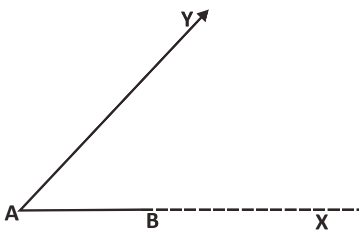


Figure - 22

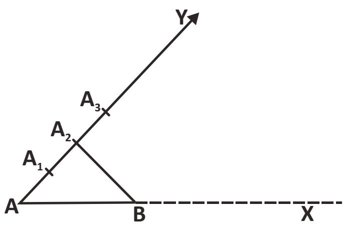


Figure - 23

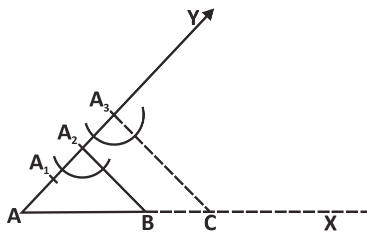


Figure - 24

Step-3. Proof. On the basis of the geometrical construction, can we say that $\frac{AC}{AB} = \frac{3}{2}$?

In ΔACA_3 and ΔABA_2

$A_2B \parallel A_3C$ (from construction)

$$\therefore \frac{AC}{AB} = \frac{AA_3}{AA_2} \quad \cdots\cdots(1) \text{ (from Thales theorem)}$$

From the construction, we know that

$$\frac{AA_3}{AA_2} = \frac{3}{2}$$

Thus, from equation (1) $\frac{AC}{AB} = \frac{3}{2}$

In this construction, we obtained line segment AC which is bigger than the given line segment by a fixed ratio. $AC = \frac{3}{2}AB$ or we can say that point C divides the line segment AB in the ratio 3:2.

Construction of the Similar Triangles

We know that in similar polygons corresponding angles are equal and corresponding sides are in the same ratio.

These two properties of similarity are also applicable for the similar triangles.

Construction – 4: Construct a triangle which is similar to given triangle ABC and whose sides are $\frac{3}{5}$ of the corresponding sides in triangle ABC.

Step – 1. We have to construct triangle which is similar to given triangle ABC. We know that in similar triangles, corresponding angles are equal and the corresponding sides are in the same ratio. This ratio $\frac{3}{5}$ is given. By using the previous construction, let us construct similar triangles.

Step - 2. Steps of construction

1. Draw a ray BX from B making an acute angle on the other side of A.
2. Cut 5 equal arcs on BX and name them B_1, B_2, B_3, B_4, B_5 respectively.

   From this we obtain $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

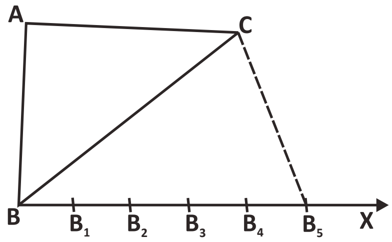


Figure - 25

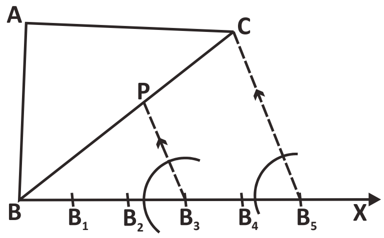


Figure - 26

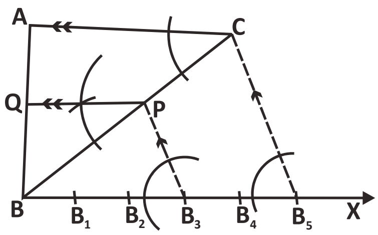


Figure - 27

3. Now join B_5 with C and draw a line parallel to B_5C from B_3 which intersects BC at P.

4. Now, draw a parallel line from P to AC, which intersects AB at Q.

QBP is the required triangle.

Step-3. Proof

How can we check that ΔQBP and ΔABC are similar triangles?

One of the ways in which we can do this is that we can measure the sides of both the triangles and see whether the corresponding sides are in the same ratio or not.

Another way of proving can be using the (Angle- Angle-Angle similarity)

In ΔQBP and ΔABC

$\angle QBP = \angle ABC$ (common angle)

$\angle PQB = \angle CAB$ (corresponding angles) (from the construction $PQ \parallel CA$)

$\angle BPQ = \angle BCA$ (corresponding angles) (from the construction $PQ \parallel CA$)

Hence, $\Delta QBP \sim \Delta ABC$ (Angle-Angle-Angle similarity)

That is $\frac{QB}{AB} = \frac{BP}{BC} = \frac{QP}{AC}$

$\frac{BP}{BC} = \frac{3}{5}$ (from the construction BC is equal to 5 parts and BP to 3 parts)

$BP = \frac{3}{5}BC$

$\therefore\ QP = \frac{3}{5}AC$ and $QB = \frac{3}{5}AB$

Example-5. Construct a triangle which is similar to the given triangle ABC and whose sides are $\frac{5}{3}$ times of the corresponding sides of triangle ABC.

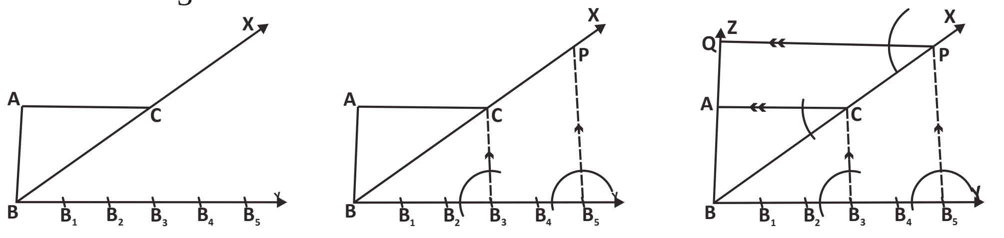


Figure - 28 *Figure - 29* *Figure - 30*

Step-1. A triangle ABC is given to us. We have to construct a triangle which is similar to the given triangle and whose sides are $\frac{5}{3}$ of the corresponding sides of the given triangle.

Step–2. Steps of construction

1. Draw a ray BY from point B making an acute angle on the other side of point A. Extend BC and BA to get rays BX and BZ respectively.
2. Now take 5 equal parts on BY and name them BB_1, B_1B_2, B_2B_3, B_3B_4, B_4B_5.
3. Now join B_3 to C. From B_5 draw a line parallel to B_3C which intersects BX at P.
4. Now draw a line parallel to AC from P which intersects BZ at Q.

QBP is the required triangle.

Construction of the similar quadrilaterals

Let us construct the similar quadrilateral in in the same way that we constructed similar triangles.

Quadrilateral ABCD is given to us. We have to construct a quadrilateral similar to ABCD such that each of its side is $\frac{2}{5}$ of the corresponding sides of quadrilateral ABCD.

Step-1. Quadrilateral ABCD is given to us. We have to construct a quadrilateral similar to ABCD such that each of its side is $\frac{2}{5}$ of the corresponding sides of quadrilateral ABCD. Here, construction is done in the same manner as in similar triangles. A point we need to remember is that first we have to construct the diagonal of given figure.

Step-2. Stages of construction

1. Construct a ray BX from point B making an acute angle CBX.
2. Now take 5 equal parts on BX and name them BB_1, B_1B_2, B_2B_3, B_3B_4, B_4B_5.
3. Join B_5 to D and from B_2 draw a line parallel to which intersects BD at R.
4. Now, from R draw a line parallel to AD which intersects AB at P.
5. Similarly, draw a line parallel to CD from R which intersects BC at Q.

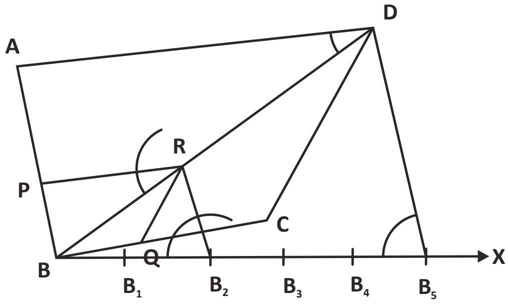


Figure - 31

Thus, we obtain the required quadrilateral PBQR.

Drawing Perpendiculars

Example-6. Draw a perpendicular at point C on line k.

Solution:

Step-1. We have to draw a perpendicular at point C on line k.

Figure - 32

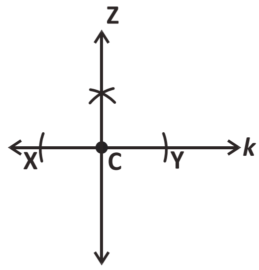


Figure - 33

Step-2. Steps of Constructions

1. Take point C as origin on k and on both of its sides cut arcs on k taking any radius. Name the points of intersection as X and Y.
2. Take a radius of more than CX and by taking X and Y as origins, draw arcs on one side of the line. The arcs cut each other at a certain point.
3. Draw a line CZ by joining C to this intersection point.

CZ is perpendicular to line k and passes through the point C.

Example-7. Point C is outside line k. Construct a perpendicular at line k which passes through point C.

Solution : **(Hint)** First consider the point C as origin and then mark points X and Y on k such that both points are at an equal distance from C. Then by taking the points X and Y as origins obtain the point Z.

Write the detailed steps of this construction on your own.

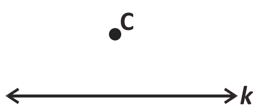


Figure - 34

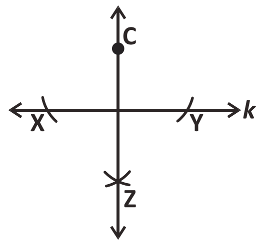


Figure - 35

Try these

1. Draw a line segment AB of length 5.8 cm and take a point C on it such that AC:CB =3:4. Check whether AC:CB is 3:4 or not.

2. Construct a line segment which is $\frac{7}{5}$ of any other line segment.

3. Draw a triangle PQR in which QR=6 *cm*, PQ=5 cm and $\angle PQR=60^{O}$. Draw a triangle ABC similar to triangle PQR, in which AB=$\frac{2}{5}$ PQ.

4. Construct a triangle ABC in which BC=5.5 *cm*, $\angle ABC=75^{O}$ and $\angle ACB= 45^{O}$. Draw a triangle XYZ similar to triangle ABC in which YZ = $\frac{5}{4}$ BC.

Exercise - 2

1. Construct a similar triangle which is $\frac{3}{5}$ of the given triangle.

2. Construct an isosceles triangle PQR. Also construct a triangle ABC in which PQ = $\frac{3}{4}$ AB.

3. Construct a triangle PQR. Also construct a triangle ABC in which AB=$\frac{2}{3}$ of PQ.

4. Construct two similar triangles. First triangle should be $\frac{4}{3}$ times the other triangle.

Till now you studied about construction of the similar triangles. Now, we will do some more constructions by using the properties which we studied in previous classes.

Perpendicular bisector

Perpendicular bisector is the line which divides a given line segment into two equal parts while making a right angle with it.

Construction of Perpendicular Bisector

1. Draw a line segment AB.

2. Extend both the sides of the compass such that its length is more than half of the given line segment.
3. Draw an arc on both the sides of the line segment by putting the tip of the compass at point A. Again repeat the same process by putting the compass at point B.
4. Join the intersection points of the arcs with the help of a scale.

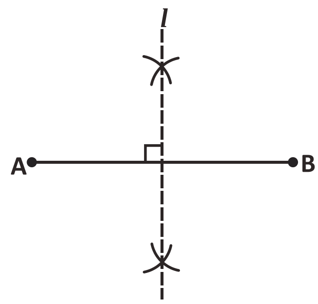


Figure - 36

This line "*l*" is the perpendicular bisector of line segment AB.

Is each point on the perpendicular bisector at an equal distance from point A and point B?

Let us see

Take a point O on the perpendicular bisector. Join this point to both the end points "A" and "B" of the line segment. Now, in triangle AOD or in triangle BOD,

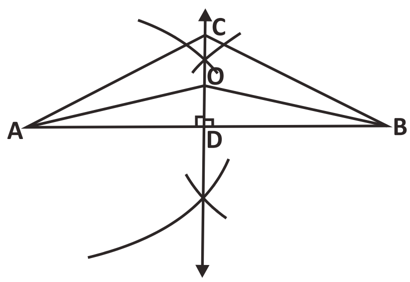


Figure - 37

AD=DB (D is the midpoint of AB)

$\angle ODA = \angle ODB$ (Right angle)

OD = OD (Common)

$\therefore\ \Delta AOD \cong \Delta BOD$ (SAS congruency)

Hence, OA = OB

Each point of the perpendicular bisector is at an equal distance from point A and point B.

Some more Constructions of triangles

<u>Step-1.</u> A triangle is given to us. Now we have to construct a circle such that it passes through all three vertices A,B and C of the triangle.

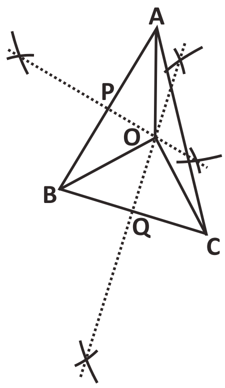


Figure - 38

Think about how to do this construction:

Because the required circle will pass through all the vertices of the triangle therefore we can say that the centre of the circle is at equal distance from all the vertices. We also know that any point on the perpendicular bisector is at equal distance from the end points of the sides. All the points on the perpendicular bisector of the side AB of triangle ABC will be at equal distance from vertex A and vertex B. Similarly all the point on the perpendicular bisector of side BC will be at equal distance from vertex B and vertex C.

Suppose that both the perpendicular bisectors intersect each other at a certain point and we name this point as "O". Because Point O is located on both the perpendicular bisectors hence OA=OB=OC. Now draw a circle by considering "O" as the center and OA as a radius. Is the circle which we have drawn passing through all the vertices?

Let us see -

Step–2. Steps of construction

1. Draw a triangle ABC.
2. Draw a perpendicular bisector on side AB and AC which intersects AB at P and BC at Q. The perpendicular bisectors intersect each other at "O".
3. Now draw a circle by taking O as center and with OA as radius. You can see that this circle is passing through all three vertices.

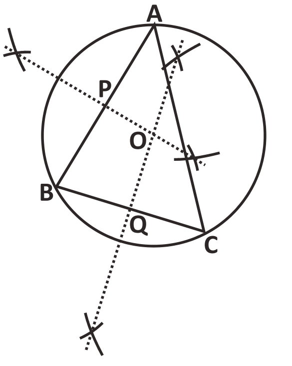


Figure - 39 (i)

Step-3. Proof

How do we check that the circle is passing through all three vertices A,B and C of the triangle?

In ΔAOP and ΔBOP (figure-39(ii))

AP = PB (Why?)

∠OPA = ∠OPB (Why?)

OP = OP (Common)

Hence, ΔAOP ≅ ΔBOP

From this we can say that,

OA = OB .... (i)

Figure - 39 (ii)

Similarly, ΔBOQ ≅ ΔCOQ

Therefore OB = OC .... (ii)

From (i) and (ii), we can say that

OA = OB = OC

This means that points A, B and C are at an equal distance from centre O. So the circle drawn by taking OA as radius will pass through B and C as well.

The point at which the perpendicular bisectors of the sides of any triangle meet is called the circumcenter of the triangle. Here point O is the circumcenter of triangle ABC and the circle which passes through ABC is called the circumcircle.

Angle Bisector

Divide the given angle into two equal parts.

Solution :

Step-1. A triangle ABC is given. We have to construct a ray which divides ∠ABC into two equal parts.

Think about how we can do this. We know that the angle bisector is a line which divides any angle into two equal parts, hence both the resulting angle are equal. If we construct two triangles in such a manner that angle bisector is a common side in both the triangles and the triangles are congruent (here BE=BF and DE=DF), then by the SSS congruency both the obtained triangles are congruent. For obtaining congruent triangles we require a point which is at an equal distance from the points on the sides BA and BC.

Step-2. Steps of constructions

1. By taking vertex B as centre, draw an arc of any radius which intersects BA and BC at points E and F respectively.

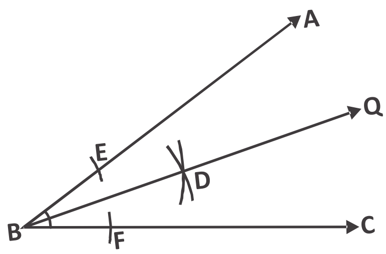


Figure - 40

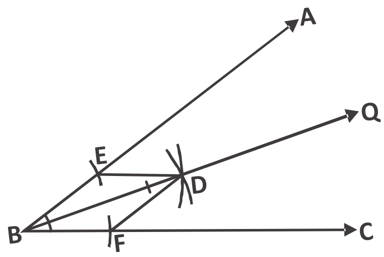


Figure - 41

2. Take E and F as center and radius slightly more than ½ of EF, draw arcs which intersects each other at D.
3. Now draw ray BD; this is the required angle bisector.

Step-3: Proof: How can we say that BD is the angle bisector of given angle. Let's see.

Join D to E and F, now in triangles BED and BFD,

BE=BF (radii of the same arc)

ED=FD (radius of the same arc)

BD=BD (common side)

Hence, $\Delta BED \cong \Delta BFD$ (from SSS)

From this we can say that $\angle ABD = \angle DBC$ (CPCT)

Distance of Angle Bisector from Sides

Take a point P on the angle bisector. To calculate the distance of point P from sides BA and BC, draw perpendiculars from point P on BA and BC.

Draw a perpendicular from point P on side AB which intersects AB at M. Similarly draw a perpendicular from point P on side BC which intersects at R. Now in triangle BMP and in triangle BRP,

$\angle BMP = \angle BRP$ (Right Angle)

$\angle MBP = \angle RBP$ (Because BP is Angle Bisector)

BP = BP (Common)

Hence, $\Delta BMP \cong \Delta BRP$ (AAS congruency)

From this we can say that PM = PR (CPCT))

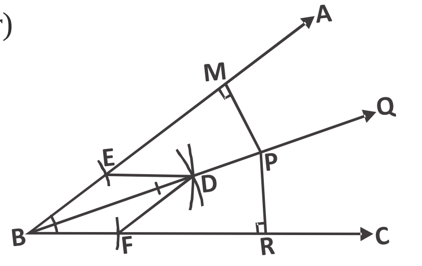


Figure - 42

Incircle

Step-1. A triangle ABC is given to us and we have to construct a circle which touches all the sides of the triangle.

How to construct

Because the circle touches all the sides of the triangle, therefore the centre of the circle is at equal distance from all the sides. We know that any point at the angle bisector is at an equal distance from the arms of the angle. There are many such points on the angle bisector of the angle ABC which are at an equal distance from BA and BC. Similarly, there are many such points on the angle bisector of angle BCA which are at an equal distance from CB and CA. If we take the point O as the point at which both the angle bisectors intersect each other then O is at an equal distance from AB, BC and CA. Take O as centre and with radius equal to the perpendicular distance between center and any side, draw a circle. Does the circle touch all the sides of triangle ABC?

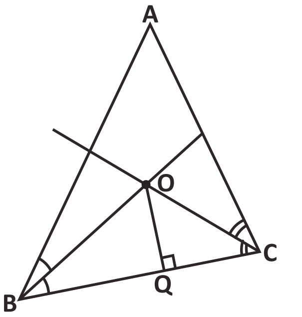


Figure - 43

Step 2. Steps of Construction

1. Construct a triangle ABC.
2. Draw angle bisectors of angle ABC and angle BCA. The point at which both cut each other is taken as the centre O.
3. Now draw a perpendicular from the point O to side BC which intersects BC at Q. Draw a circle taking O as centre and OQ as radius. In the figure you can see that this circle touches all the three sides. Hence, the perpendicular distances AB, AC and CA from O are same.

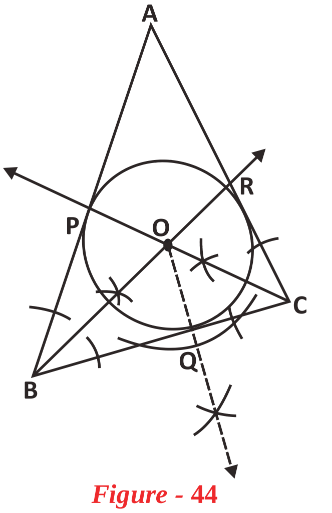


Figure - 44

Step-3. Proof

Let's see on the base of the mathematical arguments whether the obtained circle touches all the three sides?

In triangle POB and in triangle BOQ,

$\angle PBO = \angle QBO$

$\angle OPB = \angle OQB = 90^\circ$

OB = OB (Common)

Hence, $\Delta POB \cong \Delta QOB$

Therefore, OP = OQ....(i)

Similarly, $\Delta ROC \cong \Delta QOC$

Therefore, OR = OQ....(ii)

From (i) and (ii), we can say that

OP = OQ = OR

Now, taking O as origin and OP as radius make a circle touching all the three sides of the triangle.

The point at which the angle bisectors of any triangle meet each other is called incentre of the circle and that circle is called incircle.

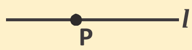
Try these

1. Construct incircle and circumcircle for triangle ABC when:
   (i) AB=*3cm*, BC=4*cm* and $\angle$B=90°. Also find the radius of the incircle and circumcircle.
   (ii) AB=BC=CA=6cm, where will the incentre and circumcentre be located?
   (iii) BC=7 *cm*, $\angle$B=45°, $\angle$A=105°, where will the incentre and circumcentre be located?

Exercise-3

1. Construct according to the given information (use a compass)

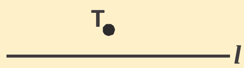
i) Draw a perpendicular at point p on line *l*.



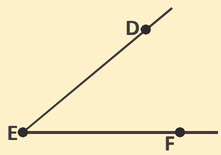
ii) Draw a perpendicular from point S on line *l*.

iii) Draw the perpendicular bisector of line segment JK.

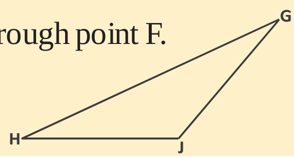
iv) Draw a line which is parallel to line *l* and passes through point T.



v) Draw a line which is parallel to ED and passes through point F.

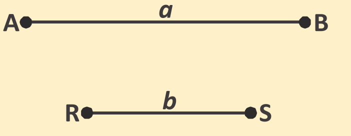


vi) draw a perpendicular from point G on HJ.



This can be done using measurements or proofs to check whether the constructed figure is as per requirements or not.

2. Two line segments AB=a and RS=b are given. Now, construct according to the given information.



(i) Draw a rectangle of sides 'a' and 'b'.

(ii) Draw a square of Perimeter 4b.

(iii) Draw a square where diagonal is equal to a.

(iv) Draw a cyclic quadrilateral having sides a and b and included angle is θ.

3. Construct a circumcircle on a right angle triangle. Find the value of the radius of the constructed circle.

4. Construct a incircle on a right angle triangle . find the value of the radius of the constructed circle.

5. Construct incircle and circumcircle on equilateral triangle, now calculate the incircle and circumcircle. Are both of them located at the same point?

6. Take three non collinear points and construct a circle which pass through them.

7. Mohan want to hoist a flag at the centre of the circular ground of the School. He has to take help from Rahul and Zoya to find the place on the field where the pole should be fixed. Think, now they all together found the place?

What We Have Learnt

1. Understanding a given question - Before we start working on a question, the first step is to read the question and to see what information is given and what is to be constructed. In some manner it is like to convert it into methematical form and context. . Here we have to understand which one of the given information is useful and which one is not. Which geometrical concept can be used on the basis of the given information, this kind of thinking is helpful in understanding the question.
2. On the basis of given information we can draw a rough diagram and by analyzing it we can think about ways to solve the problem. On the basis of the desired shape is known and calculating what more things are required to about the ways how to solve it. In this we have to think that which part of the desired shape is known now and what more should be required to construct the figure.
3. After analyzing and drawing the rough diagram, geometrical construction is possible in a stepwise manner.
4. At the end after the construction see whether the constructed figure meets the requirements of the question.