

Topics : Simple Harmonic Motion, Friction, Rigid Body Dynamics, String Waves, Sound Waves, Geometrical Optics

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.5

(3 marks, 3 min.)

M.M., Min.

[15, 15]

Subjective Questions ('-1' negative marking) Q.6 to Q.7

(4 marks, 5 min.)

[8, 10]

Comprehension ('-1' negative marking) Q.8 to Q.10

(3 marks, 3 min.)

[9, 9]

1. A particle is executing SHM according to the equation $x = A \cos \omega t$. Average speed of the particle during the

interval $0 \leq t \leq \frac{\pi}{6\omega}$.

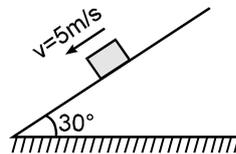
(A) $\frac{\sqrt{3}A\omega}{2}$

(B) $\frac{\sqrt{3}A\omega}{4}$

(C) $\frac{3A\omega}{\pi}$

(D) $\frac{3A\omega}{\pi} (2 - \sqrt{3})$

2. A particle of mass 5 kg is moving on rough fixed inclined plane (making an angle 30° with horizontal) with constant velocity of 5 m/s as shown in the figure. Find the friction force acting on a body by the inclined plane. (take $g = 10\text{m/s}^2$)



(A) 25 N

(B) 20 N

(C) 30 N

(D) none of these

3. A sphere rolls without sliding on a rough inclined plane (only mg and contact forces are acting on the body). The angular momentum of the body:

(A) about centre is conserved

(B) is conserved about the point of contact

(C) is conserved about a point whose distance from the inclined plane is greater than the radius of the sphere

(D) is not conserved about any point.

4. A string of length 1.5 m with its two ends clamped is vibrating in fundamental mode. Amplitude at the centre of the string is 4 mm. Distance between the two points having amplitude 2 mm is:

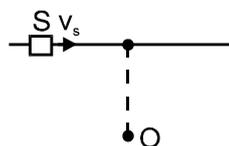
(A) 1 m

(B) 75 cm

(C) 60 cm

(D) 50 cm

5. The source (S) of sound is moving constant velocity v_0 as shown in diagram. An observer O listens to the sound emitted by the source. The observed frequency of the sound :



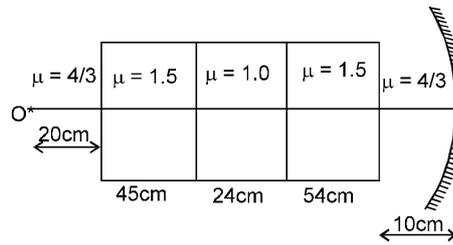
(A) continuously decreases

(B) continuously increases

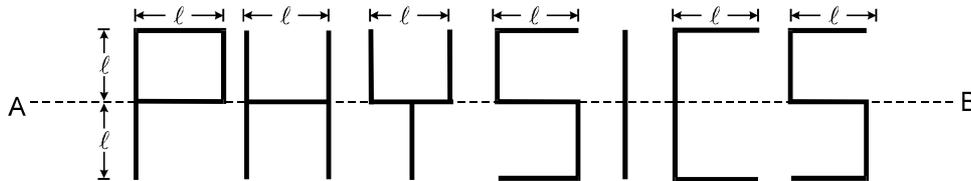
(C) first decreases then increases

(D) first increases then decreases.

6. A composite slab consisting of different media is placed in front of a concave mirror of radius of curvature 150 cm. The whole arrangement is placed in water. An object O is placed at a distance 20 cm from the slab. The R.I. of different media are given in the diagram. Find the position of the final image formed by the system.

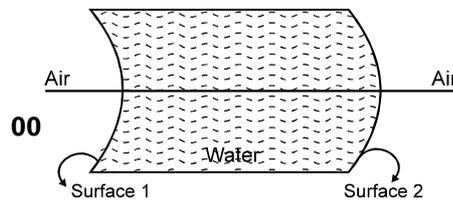


7. Find out the moment of inertia of the following structure (written as PHYSICS) about axis AB made of thin uniform rods of mass per unit length λ .



COMPREHENSION

All objects referred to the subsequent problems lie on the principle axis.



8. If light is incident on surface 1 from left, the image formed after the first refraction is definitely :
 (A) Real for a real object (B) Virtual for a real object
 (C) Real for a virtual object (D) Virtual for a virtual object
9. In above question if the object is real, then the final image formed after two refractions :
 (A) may be real (B) may be virtual (C) must be virtual (D) both A and B
10. If light is incident on surface 2 from right then which of the following is true for image formed after a single refraction.
 (A) Real object will result in a real image (B) Virtual object will result in a virtual image
 (C) Real object will result in a virtual image (D) Virtual object will result in a Real image

Answers Key

1. (D) 2. (A) 3. (C) 4. (A) 5. (A)
 6. On the object itself 7. $13\lambda\ell^3$
 8. (B) 9. (D) 10. (D)

Hints & Solutions

1. average speed $\bar{v} = \frac{\int_0^t \frac{dx}{dt} \cdot dt}{t} = \frac{\int_0^t dx}{t}$

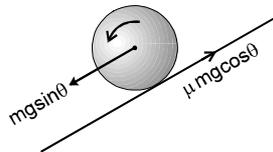
$$= \frac{x(t) - x(0)}{t} = \frac{A(\cos \pi/6 - 1)}{\pi/6\omega} = \frac{3A\omega}{\pi}(\sqrt{3} - 2)$$

since particle does not change its direction in the given interval, average speed

$$= |\bar{v}| = \frac{3A\omega}{\pi}(2 - \sqrt{3})$$

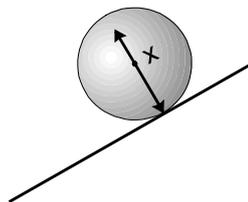
2. Since the block slides down the incline with uniform velocity, net force on it must be zero. Hence $mg \sin\theta$ must balance the frictional force 'f' on the block. Therefore $f = mg \sin\theta = 5 \times 10 \times \frac{1}{2} = 25 \text{ N}$.

3. (C) Angular momentum will be conserved if the net torque is zero. Now for the sphere to move down:
 $mg \sin\theta > \mu mg \cos\theta$



Let x be the perpendicular distance of the point (as shown in figure) about which torque remains zero.

for $\tau = 0$; $x > R$ as shown



Note: As $mg \sin\theta > \mu mg \cos\theta$, the point should be inside the sphere.

4. $\lambda = 2\ell = 3\text{m}$

Equation of standing wave

$$y = 2A \sin kx \cos \omega t$$

$y = A$ as amplitude is $2A$.

$$A = 2A \sin kx$$

$$\frac{2\pi}{\lambda} x = \frac{\pi}{6}$$

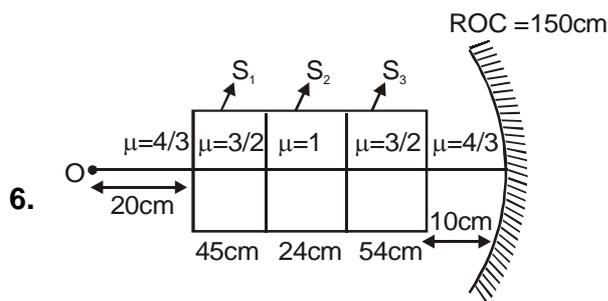
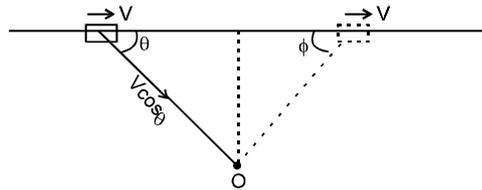
$$\Rightarrow x_1 = \frac{1}{4} \text{ m}$$

$$\text{and } \frac{2\pi}{\lambda} \cdot x = \frac{5\pi}{6}$$

$$\Rightarrow x_2 = 1.25 \text{ m} \Rightarrow x_2 - x_1 = 1 \text{ m}$$

5. From figure, the velocity of approach ($V \cos \theta$) decrease as the source comes closer (as θ increases). And the velocity of separation also increases as ϕ will decrease.

Hence the frequency of sound as heard by the observer decreases continuously.



Apparent shift in the object O due to three slabs S_1, S_2

and S_3 with respect to the medium of $\mu = \frac{4}{3}$ is given by

:

Shift

$$= 45 \left(1 - \frac{1}{\frac{3/2}{4/3}} \right) + 24 \left(1 - \frac{1}{\frac{1}{4/3}} \right) + 54 \left(1 - \frac{1}{\frac{3/2}{4/3}} \right)$$

$$\text{Shift} = 45 \left(1 - \frac{8}{9}\right) + 24 \left(1 - \frac{4}{3}\right) + 54 \left(1 - \frac{8}{9}\right).$$

$$\text{Shift} = 5 + (-8) + 6 = 3 \text{ cm}$$

$$\therefore U_{\text{net}} = 150 \text{ cm and ROC} = 150 \text{ cm.}$$

Hence image will be formed on the object itself.

7. The moment of inertia of all seven rods parallel to AB and not lying on AB is the moment of inertia of all five rods lying on AB = 0

The moment of inertia of all 18 rods perpendicular

$$\text{to AB is} = 18 (\lambda \ell) \frac{\ell^2}{3} = 6 \lambda \ell^3$$

Hence net MI of rod about

$$\text{AB} = 7 \lambda \ell^3 + 6 \lambda \ell^3 = \mathbf{13 \lambda \ell^3} \quad \text{Ans.}$$

$$8. \frac{\mu_2}{v} = \frac{\mu_1}{u} + \left(\frac{\mu_2 - \mu_1}{R}\right)$$

$(\mu_2 - \mu_1)$ is +ve and R is -ve if u is -ve, v will always be -ve

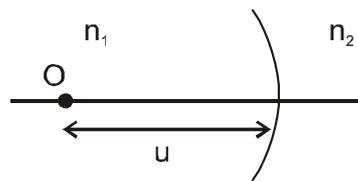
i.e. for real object image is always virtual.

Sol. 9. to 10.

Consider object on left side of spherical surface separating two media.

If real object is in rarer media i.e., $n_1 < n_2$

$$\text{Then } \frac{n_2}{v} = \frac{n_2 - n_1}{(-u)} + \frac{n_1}{(-R)} = -ve$$



Hence image shall be virtual for a real object lying

on concave side with rarer media(1)

If real object is in denser media i.e., $n_1 > n_2$

$$\frac{n_2}{v} = \frac{-(n_1 - n_2)}{(-u)} + \frac{n_1}{(-R)} = \frac{n_1 - n_2}{u} - \frac{n_1}{R}$$

\therefore Image is real if $\frac{n_1 - n_2}{u} > \frac{n_1}{R}$ or u

$$< \frac{(n_1 - n_2)R}{n_1} \quad \dots (2)$$

and image is virtual if $u > \left(\frac{n_1 - n_2}{n_1}\right)R$ (3)

From statements 1, 2 and 3 we can easily conclude the answers.