Chapter 13. Similarity

Formulae

Similarities of triangles: When two triangles are similar, their corresponding angles are equal and corresponding sides are proportional.

For example :

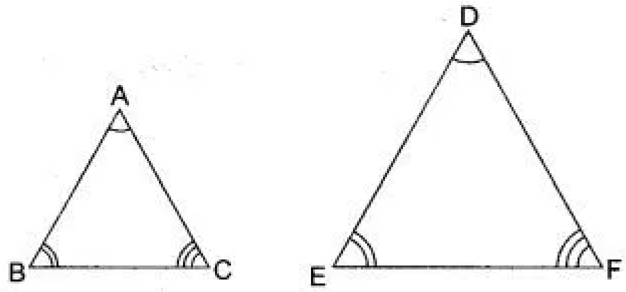
If Δ ABC is similar to Δ DEF,

i.e., $\Delta ABC \sim \Delta DEF$;

$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F,$

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

The sign '~' is read as, 'is similar to'.



Axioms of similarity of triangles: (i.e., three similarity postulates for triangle)

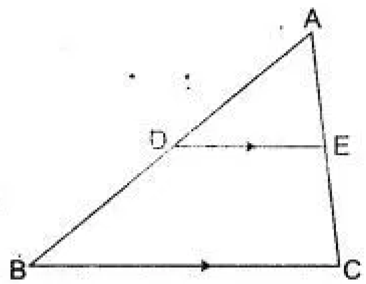
1. If two triangles have a pair of corresponding angles equal and the sides including them proportional; then the triangles are similar (SAS postulate).
2. If two triangles have two pairs of corresponding angles equal; the triangles are similar (AA or AAA postulate).
3. If two triangles have their three pairs of corresponding sides proportional, the triangles are similar (SSS postulate).

Basic Theorem of Proportionality:

1. A line drawn parallel to any side of a triangle, divides the other two sides proportionally. (Basic proportionality theorem).

In the given figure, DE // BC

$\Rightarrow \frac{AD}{BD} = \frac{AE}{CE}$

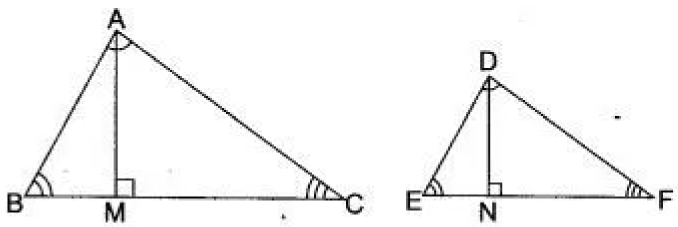


Conversely: If a line divides two sides of a triangle proportionally, the line is parallel to the third side.

$$i.e., \text{ if } \frac{AD}{BD} = \frac{AE}{CE} \Rightarrow DE \,//\, BC.$$

2. **Relation between the areas of two triangles: Theorem:** The areas of two similar triangles are proportional to the squares on their corresponding sides.

$$\text{If } \quad \Delta ABC \sim \Delta DEF$$
$$\text{such that } \quad \angle BAC = \angle EDF,$$
$$\angle B = \angle E \text{ and } \angle C = \angle F.$$



Then :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Determine the Following

Question 1. The model of a building is constructed with scale factor 1:30.
(i) If the height of the model is 80 cm, find the actual height of the building in metres.
(ii) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model.

Solution : (i) $\frac{\text{Height of model}}{\text{Height of actual building}} = \frac{1}{30}$

$$\frac{80}{H} = \frac{1}{30}$$

$\Rightarrow$ $H = 2,400 \text{ cm} = 24 \text{ m.}$ Ans.

(ii) $\frac{\text{Volume of model}}{\text{Volume of tank}} = \left(\frac{1}{30}\right)^3$

$$\frac{V}{27} = \frac{1}{27,000}$$

$$V = \frac{1}{1,000} m^3$$

$$= 1,000 \text{ cm}^3.$$ Ans.

Question 2. Triangles ABC and DEF are similar.

(i) If area (ΔABC) = 16 cm^2, area (ΔDEF) = 25 cm^2 and BC = 2.3 cm find EF.

(ii) If area (ΔABC) = 9 cm^2, area (ΔDEF) = 64 cm^2 and DE = 5.1 cm, find AB.

(iii) If AC = 19 cm and DF = 8 cm, find the ratio between the areas of two triangles.

(iv) If area (ΔABC) = 36 cm^2, area (ΔDEF) = 64 cm^2 and DE = 6.2 cm, find AB.

Solution : (i) We have

$$\text{area}(\Delta ABC) = 16 \text{ cm}^2$$
$$\text{area}(\Delta DEF) = 25 \text{ cm}^2$$

and $BC = 2{\cdot}3$ cm

Since, $\dfrac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \dfrac{BC^2}{EF^2}$

$$\Rightarrow \quad \frac{16}{25} = \frac{(2.3)^2}{EF^2}$$
$$\Rightarrow \quad \frac{2{\cdot}3}{EF} = \frac{4}{5}$$
$$\Rightarrow \quad 4\,EF = 5 \times 2{\cdot}3$$
$$\Rightarrow \quad EF = \frac{11{\cdot}5}{4}$$
$$\Rightarrow \quad EF = 2{\cdot}875 \text{ cm.} \qquad \text{Ans.}$$

(ii) We have

$$\text{area}(\Delta ABC) = 9 \text{ cm}^2$$
$$\text{area}(\Delta DEF) = 64 \text{ cm}^2$$

and $DE = 5{\cdot}1$ cm

Since, $\dfrac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \dfrac{AB^2}{DE^2}$

$$\Rightarrow \quad \frac{9}{64} = \frac{AB^2}{DE^2}$$
$$\Rightarrow \quad \frac{AB}{DE} = \frac{3}{8}$$
$$\Rightarrow \quad \frac{AB}{5{\cdot}1} = \frac{3}{8}$$
$$\Rightarrow \quad AB = \frac{3}{8} \times 5{\cdot}1 = \frac{15{\cdot}3}{8}$$
$$\Rightarrow \quad AB = 1{\cdot}9125 \text{ cm.} \qquad \text{Ans.}$$

(iii) In Δ ABC and Δ DEF, AC = 19 cm, DF = 8 cm.

Since, $\dfrac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \dfrac{AC^2}{DF^2} = \dfrac{(19)^2}{(8)^2} = \dfrac{361}{64}$

Hence, the required ratio is 361 : 64. Ans.

(iv) Area (ΔABC) = 36 cm^2

Area (ΔDEF) = 64 cm^2.

DE = 6·2 cm

AB = ?

We have

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{AB^2}{DE^2}$$
$$\Rightarrow \quad \frac{36}{64} = \frac{AB^2}{(6{\cdot}2)^2}$$
$$\Rightarrow \quad \frac{AB}{6.2} = \frac{6}{8}$$
$$\Rightarrow \quad AB = \frac{6 \times 6{\cdot}2}{8}$$
$$\Rightarrow \quad AB = 4{\cdot}65 \text{ cm.}$$

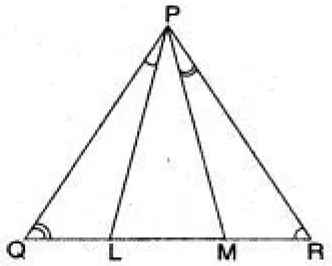
Prove the Following

Question 1. In ΔPQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$.

Prove that : (i) $\Delta PQL \sim \Delta RPM$

(ii) QL. RM = PL. PM

(iii) $PQ^2 = QR. QL$.



Solution : (i) Consider

ΔPQL and ΔRPM

Since $\angle PQL = \angle RPM$

and $\angle QPL = \angle RPM$ and

$\angle QPL = \angle PRM$

By A. A. Criterion

$\Delta PQL \sim \Delta RPM$. Hence proved.

(ii) In $\Delta PQL \sim \Delta RPM$

$$\frac{PQ}{RP} = \frac{QL}{PM} = \frac{PL}{MR}$$

then $$\frac{QL}{PM} = \frac{PL}{MR}$$

$QL \cdot MR = PL \cdot PM$. Hence Proved.

(iii) In ΔPQR and ΔLQP

$\angle PQR = \angle LQP$

$PQ = PQ$

Hence $\Delta PQR \sim \Delta LQP$

$$\frac{QR}{PQ} = \frac{PQ}{LQ}$$

$PQ^2 = QR \cdot QL$. Hence proved.

Question 2. D and E are points on the sides AB and AC respectively of a Δ ABC such that DE || BC and divides Δ ABC into two parts, equal in area. Find $\frac{BD}{AB}$.

Solution: We have

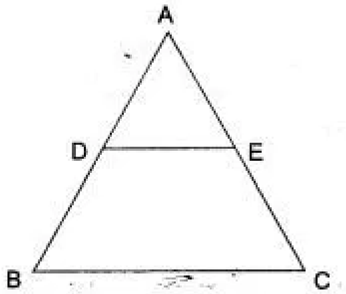
$$\text{area}(\Delta ADE) = \text{area (trapezium BCED)}$$
$$\Rightarrow \text{area}(\Delta ADE) + \text{area}(\Delta ADE) = \text{area (trapezium BCED)} + \text{area}(\Delta ADE)$$
$$\Rightarrow 2\,\text{area}(\Delta ADE) = \text{area}(\Delta ABC) \quad ...(i)$$

In Δ ADE and Δ ABC, we have

$\angle ADE = \angle B$, [$\because$ DE || BC]

$\therefore \angle AED = \angle C$ (corresponding angles)]

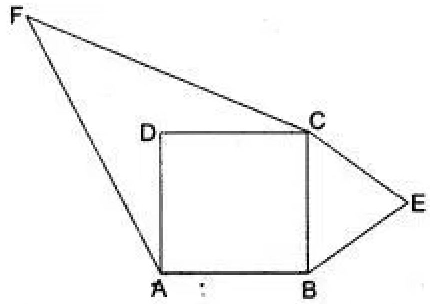
and $\angle A = \angle A$, [Common]



$$\therefore \quad \Delta ADE \sim \Delta ABC$$
$$\Rightarrow \frac{\text{area}(\Delta ADE)}{\text{area}(\Delta ABC)} = \frac{AD^2}{AB^2}$$
$$\Rightarrow \frac{\text{area}(\Delta ADE)}{2\,\text{area}(\Delta ADE)} = \frac{AD^2}{AB^2}$$
$$\Rightarrow \frac{1}{2} = \left(\frac{AD}{AB}\right)^2$$
$$\Rightarrow \frac{AD}{AB} = \frac{1}{\sqrt{2}}$$
$$\Rightarrow AB = \sqrt{2}\,AD$$
$$\Rightarrow AB = \sqrt{2}\,(AB - BD)$$
$$\Rightarrow (\sqrt{2}-1)\,AB = \sqrt{2}\,BD$$
$$\Rightarrow \frac{BD}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \left(\frac{2-\sqrt{2}}{2}\right).$$

Question 3. Prove that the area of the triangle BCE described on one side BC of a square ABCD as base is one half of the area of similar triangle ACF described on the diagonal AC as base.

Solution : ABCD is a square Δ BCE is described on side BC is similar to Δ ACF described on diagonal AC.



Since, ABCD is a square. Therefore

$$AB = BC = CD = DA$$

and $$AC = \sqrt{2}\,BC$$

[$\because$ Diagonal = $\sqrt{2}$ (side)]

Now, $\Delta BCE \sim \Delta ACF$

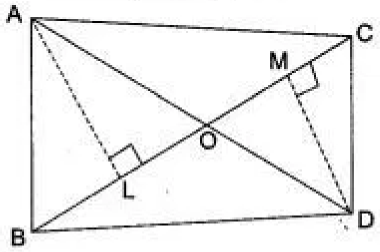
$$\Rightarrow \frac{\text{Area } (\Delta BCE)}{\text{Area } (\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area } (\Delta BCE)}{\text{Area } (\Delta ACF)} = \frac{BC^2}{(\sqrt{2}\,BC)^2} = \frac{1}{2}$$

$$\Rightarrow \text{Area } (\Delta BCE) = \frac{1}{2} \text{ area } (\Delta ACF).$$

Hence proved.

Question 4. In figure ABC and DBC are two triangles on the same base BC. Prove that $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DBC)} = \frac{AO}{DO}$.



Solution : In Δ AOL and Δ DOM

$\angle ALO = \angle DMO$, (90° each)

$\angle AOL = \angle DOM$,

(Vertically opposite 2 sides)

(Vert. opp-angles)

$\therefore$ $\Delta AOL \sim \Delta DOM$

$\therefore$ $\frac{AL}{DM} = \frac{AO}{DO}$ ...(1)

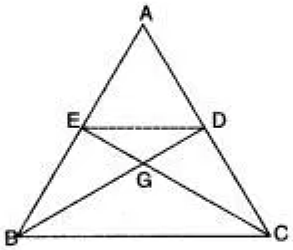
If two Δ's are similar the ratio between their corresponding sides is the same.

$$\text{Now, } \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM} = \frac{AL}{DM}$$

From (1), we get

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AO}{DO}.$$ Hence proved.

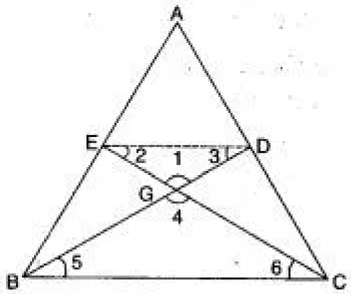
Question 5. In the adjoining figure, the medians BD and CE of a ΔABC meet at G. Prove that



(i) $\Delta EGD \sim \Delta CGB$ and

(ii) BG = 2GD for (i) above.

Solution : Since D and E are mid-point of AC and AB respectively in Δ ABC, ED is parallel to BC.



(i) In Δ s EGD and CGB,

$\angle EGD = \angle CGB$ (Vertically opp., angles)

$\angle EDG = \angle CBG$ (Alternate angles)

So, $\Delta EGD \sim \Delta CGB$. Hence proved.

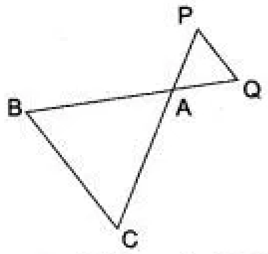
(ii) $\therefore \frac{BG}{GD} = \frac{BC}{DE}$

But $\frac{BC}{DE} = 2$, So $\frac{BG}{GD} = 2$

$\Rightarrow BG = 2GD$. Hence proved.

Figure Based Questions

Question 1. In the adjoining figure, $\Delta ACB \sim \Delta APQ$. If BC = 10 cm, PQ = 5 cm, BA = 6·5 cm and AP = 2·8 cm find the area (Δ ACB) : area (Δ APQ).



Solution : $\Delta ACB \sim \Delta APQ$.

Then, $\frac{\text{area } (\Delta ACB)}{\text{area } (\Delta APQ)} = \frac{BC^2}{PQ^2}$

$= \frac{(10)^2}{(5)^2}$

$= \frac{100}{25}$

$= \frac{4}{1}$

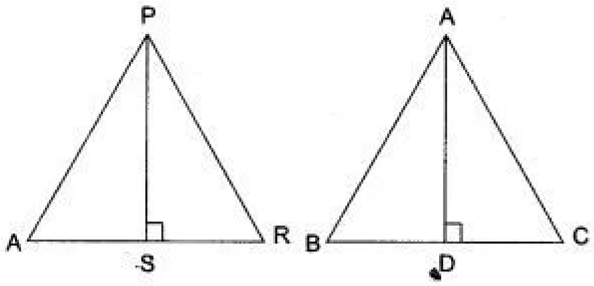
Required ratio is 4 : 1.

Question 2. Two isosceles triangle have equal vertical angles and their areas are in the ratio of 36 : 25. Find the ratio between their corresponding heights.

Solution : ΔABC and ΔPQR be the two isosceles triangles such that

$$\angle A = \angle P.$$

Then $\Delta ABC \sim \Delta PQR$.



Let AD and PS be their heights then

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \quad \frac{36}{25} = \frac{AD^2}{PS^2}$$

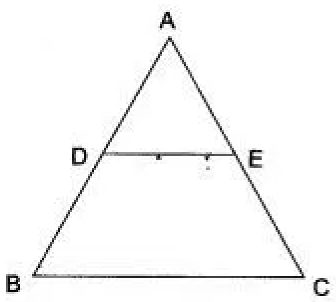
$$\Rightarrow \quad \frac{AD}{PS} = \frac{6}{5}$$

$$\Rightarrow \quad AD : PS = 6 : 5.$$ Ans.

Question 3. In ΔABC, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.

Solution : Since, D and E are the mid-points of AB and AC respectively.

Therefore, $DE \parallel BC$.



Consequently,

$$\Delta ADE \sim \Delta ABC$$

$$\Rightarrow \quad \frac{\text{area}(\Delta ADE)}{\text{area}(\Delta ABC)} = \frac{AD^2}{AB^2}$$

$$= \frac{AD^2}{(2AD)^2}$$

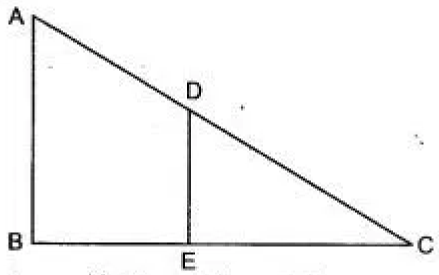
$$= \frac{1}{4} \qquad (\because AB = 2AD)$$

Question 4. In the given figure, AB and DE are perpendicular to BC.

(i) Prove that $\Delta ABC \sim \Delta DEC$

(ii) If AB = 6 cm; DE = 4 cm and AC = 15 cm. Calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC.



Solution : (i) Given $AB \perp BC$

$DE \perp BC$

To prove $\Delta ABC \sim \Delta DEC$

Proof : In ΔABC and ΔDEC

$\angle ABC = \angle DEC = 90°$ each (given)

$\angle C = \angle C$ {common}

$\therefore \Delta ABC \sim \Delta DEC$ {A.A criteria}

Hence proved.

(ii) AB = 6 cm, DE = 4 cm

AC = 15 cm, CD = ?

Since $\Delta ABC \sim \Delta DEC$

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{CD}$$

{Correspondings sides of similar Δ's are proportional}

$$\therefore \frac{6}{4} = \frac{15}{CD}$$

$$\Rightarrow CD = \frac{15 \times 4}{6} = 10. \quad \text{Ans.}$$

(iii) $$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEC} = \frac{AB^2}{DE^2} \text{ \{Area theorem\}}$$

$$= \frac{36}{16} = \frac{9}{4} \text{ or } 9 : 4. \quad \text{Ans.}$$

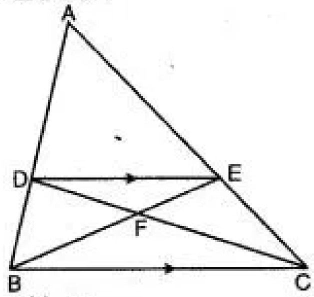
Question 5. In the given figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.

(i) Determine the ratios $\frac{AD}{AB}, \frac{DE}{BC}$.

(ii) Prove that Δ DEF is similar to Δ CBF.

Hence, find $\frac{EF}{FB}$.

(iii) What is the ratio of the areas of ΔDEF and Δ BFC ?



Solution : (i) Given

$$DE \parallel BC$$

and $$\frac{AD}{DB} = \frac{3}{2}$$

In Δ ADE and Δ ABC,

$\angle A = \angle A$, (Common Angles)

$\angle D = \angle B$ (Corresponding Angles)

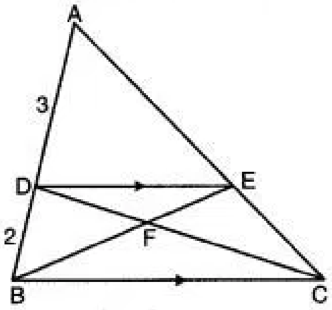
∴ $\Delta ADE \sim \Delta ABC$ (by A.A. criterion)

∴ $$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

Now $$\frac{AD}{AB} = \frac{AD}{AD+DB}$$

$$= \frac{3}{3+2} = \frac{3}{5}$$

∴ $$\frac{AD}{AB} = \frac{3}{5} = \frac{DE}{BC}.$$ Ans.



(ii) In Δ DEF and Δ CBF,

$\angle FDE = \angle FCB$(Alternate Angle)

$\angle DFE = \angle BFC$ (Vertically Opposite Angle)

∴ $\Delta DEF \sim \Delta CBF$ (by A.A. criterion)

Hence proved.

$$\frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5}$$

∴ $$\frac{EF}{FB} = \frac{3}{5}.$$ Ans.

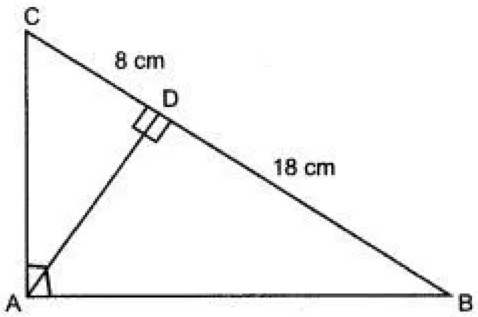
(iii) $$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CBF} = \frac{EF^2}{FB^2} = \frac{3^2}{5^2} = \frac{9}{25}.$$ Ans.

Question 6. In the adjoining figure ABC is a right angled triangle with $\angle BAC = 90°$, and $AD \perp BC$.

(i) Prove $\Delta ADB \sim \Delta CDA$.

(ii) If BD = 18 cm, CD = 8 cm find AD.

(iii) Find the ratio of the area of Δ ADB is to area of Δ CDA.



Solution : (i) In Δ ADB and Δ ADC

$AD = AD$ (self)

$\angle ADC = 90°$ $(AD \perp BC)$

$\angle ADB = 90°$ $(AD \perp BC)$

then $\angle ADC = \angle ADB$

so, $\Delta ADB \sim \Delta ADC$

(By AAA similarity)

or $\Delta ADB \sim \Delta CDA$ Hence proved.

(ii) $\because$ $\Delta ADB \sim \Delta CDA$

$\therefore$ $\dfrac{AD}{BD} = \dfrac{CD}{AD}$

(corresponding parts of similar Δ's are proportional)

or $AD^2 = BD \times CD$

$\Rightarrow$ $AD^2 = 18 \times 8 \left(\begin{matrix} BD = 18 \\ CD = 8 \end{matrix} \text{ (given)}\right)$

$\Rightarrow$ $AD^2 = 144$

$AD = 12$ cm

(iii) $\dfrac{\text{Area of } \Delta ADB}{\text{Area of } \Delta CDA} = \dfrac{BD^2}{AD^2}$

(Area theorem of similar triangles)

$= \dfrac{18^2}{12^2} = \dfrac{18 \times 18}{12 \times 12}$

$= \dfrac{3 \times 3}{2 \times 2} = \dfrac{9}{4}$

$\Rightarrow$ area (Δ ADB) : area (Δ CDA) = 9 : 4. Ans.

Question 7. Equilateral triangles are drawn on the sides of a right angled triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides.

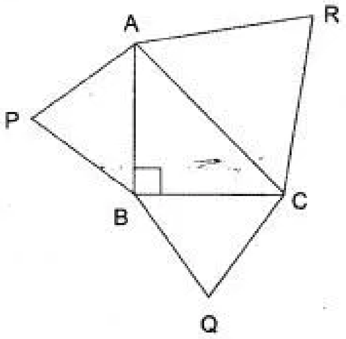
Solution :

Given. A right angled triangle ABC with right angle at B. Equilateral triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.

To Prove.

Area (Δ PAB) + Area (Δ QBC) = Area (Δ RAC).

Proof. Since, triangles PAB, QBC and RAC are equilateral. Therefore they are equiangular and hence similar.



$$\therefore \frac{\text{area}(\Delta PAB)}{\text{area}(\Delta RAC)} + \frac{\text{area}(\Delta QBC)}{\text{area}(\Delta RAC)} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AB^2 + BC^2}{AC^2} = \frac{AC^2}{AC^2} = 1,$$

$$\left[\begin{array}{l}\because \Delta ABC \text{ is a right angled triangle with } \angle B = 90^\circ \\ \therefore AC^2 = AB^2 + BC^2\end{array}\right]$$

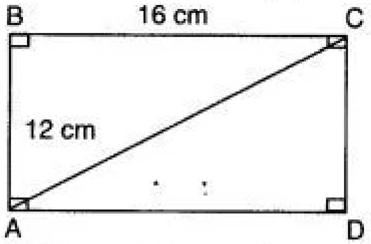
$$\Rightarrow \frac{\text{area}(\Delta PAB) + \text{area}(\Delta QBC)}{\text{area}(\Delta RAC)} = 1$$

$$\Rightarrow \text{area}(\Delta PAB) + \text{area}(\Delta QBC) = \text{area}(\Delta RAC).$$

Question 8: On a map drawn to scale of 1 : 2,50,000 a rectangular plot of land ABCD has the following measurement AB = 12 cm, BC = 16 cm angles A, B, C, and D are 900 each.
Calculate:

(i) The diagonal distance of the plot of land in
(ii) Actual length of diagonal.

Solution : (i) Here $k = \frac{1}{2,50,000}$



Length of diagonal (on map)

$$= \sqrt{AB^2 + BC^2}$$
$$= \sqrt{12^2 + 16^2}$$
$$= \sqrt{400} = 20 \text{ cm}$$

(ii) Length of diagonal on map.

$= k \times$ Actual length of the diagonal

$\Rightarrow 20 = \frac{1}{2,50,000} \times$ Actual length of the diagonal

$\Rightarrow$ Actual length of diagonal

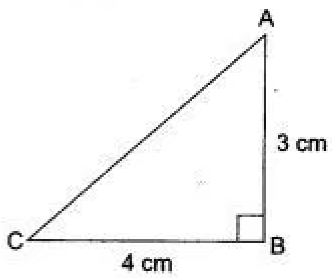
$= 20\ (2,50,000)$ cm
$= 5000000$ cm
$= 50$ km Ans.

Question 9. On a map drawn to a scale of 1 : 2,50,000, a triangular plot of land has the following measurements, AB = 3 cm, BC = 4 cm, $\angle ABC = 90°$. Calculate :

(i) The actual length of AB in kms.
(ii) The area of plot in sq. kms.

Solution : The scale of a map = 1 : 2,50,000 that is

$$1 \text{ cm} = \frac{2,50,000}{1,000 \times 100}$$
$$= 2\cdot5 \text{ kms}$$



(i) Actual length of AB

$= 3 \text{ cm} = 3 \times 2\cdot5$ kms
$= 7\cdot5$ kms Ans.

(ii) Area of triangular plot

$$= \frac{1}{2} AB \times BC$$

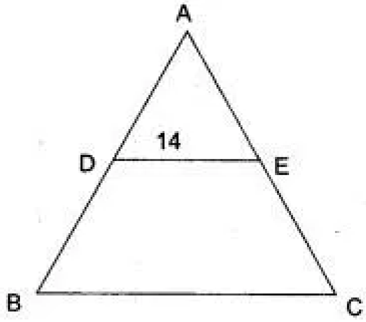
$$= \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ cm}^2$$

$$= 6 \times 2{\cdot}5 \times 2{\cdot}5 \text{ km}^2$$

$$= 37{\cdot}5 \text{ km}^2. \quad \text{Ans.}$$

Question 10. In the adjoining figure. BC is parallel to DE, area of ΔABC = 25 sq cm, area of trapezium BCED = 24 sq cm. DE = 14 cm. Calculate the length of BC.



Solution. Given : Area of Δ ABC = 25 cm²

Area of trapezium BCED = 24 cm²

and DE = 14 cm

∴ Area of Δ ADE = area of Δ ABC − area of trap. BCED

$$= 25 - 24$$

$$= 1 \text{ cm}^2$$

∵ Δ ABC ~ Δ ADE

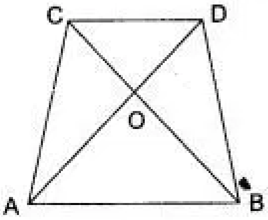
$$\therefore \quad \frac{\text{Area of } (\Delta \text{ ABC})}{\text{Area of } (\Delta \text{ ADE})} = \frac{BC^2}{DE^2}$$

$$\frac{25}{1} = \frac{BC^2}{14^2}$$

$$\frac{BC}{14} = \frac{5}{1}$$

$$BC = 5 \times 14 = 70 \text{ cm} \quad \text{Ans.}$$

Question 11. In fig. ABCD is a trapezium in which AB | | DC and AB = 2DC. Determine the ratio between the areas of Δ AOB and Δ COD.



Solution : In triangles AOB and COD, we have

$$\angle AOB = \angle COD,$$

[Vertically opposite angles]

and $$\angle OAB = \angle OCD,$$

[Corresponding angles]

So, by AA-criterion of similarity, we have

$$\Delta\, AOB \sim \Delta\, COD$$

$$\Rightarrow \frac{\text{Area }(\Delta\, AOB)}{\text{Area }(\Delta\, COD)} = \frac{AB^2}{DC^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta\, AOB)}{\text{Area }(\Delta\, COD)} = \frac{(2DC)^2}{(DC)^2}$$

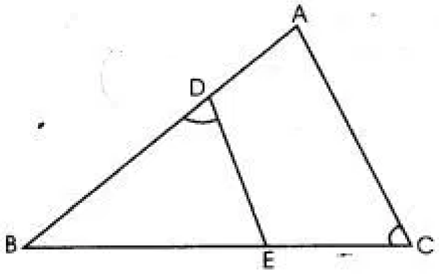
$$= \frac{4}{1}$$

Hence, area (Δ AOB) : area (Δ COD) = 4 : 1.

Question 12. In the given figure ABC is a triangle with ∠ EDB = ∠ ACB.

(i) Prove that Δ ABC ~ Δ EBD.

(ii) If BE = 6 cm, EC = 4 cm, BD = 5 cm and area of Δ BED = 9 cm^2. Calculate the length of AB and area of Δ ABC.



Solution : (i) In Δ ABC and Δ EBD,

$\angle ACB = \angle EDB$ (given)

$\angle ABC = \angle EBD$ (common)

$\therefore \quad \Delta ABC \sim \Delta EBD$

Hence Proved.

(ii) We have, $\frac{AB}{BE} = \frac{BC}{BD}$

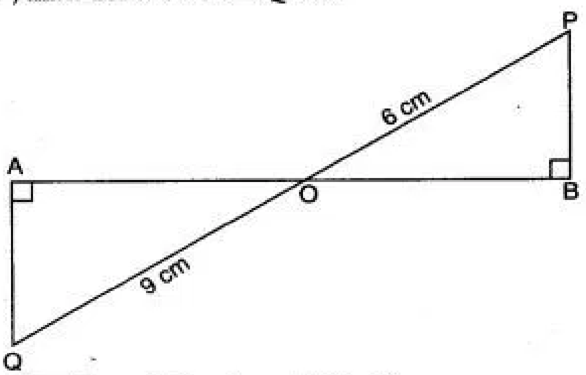
$\Rightarrow \quad AB = \frac{6 \times 10}{5} = 12$ cm. Ans.

(iii) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BED} = \left(\frac{AB}{BE}\right)^2$

$\Rightarrow$ Area of $\Delta ABC = \left(\frac{12}{6}\right)^2 \times 9 \text{ cm}^2$

$= 4 \times 9 \text{ cm}^2 = 36 \text{ cm}^2$.

Question 13. In the figure given below, PB and QA are perpendiculars to the line segment AB. If PO = 6 cm, QO = 9 cm and the area of ΔPOB = 120 cm^2, find the area of Δ QOA.



Solution : PO = 6 cm, QO = 9 cm.

Area of Δ POB = 120

In Δ POB and Δ QOB,

$\angle B = \angle A$ (each 90°)

$\angle POB = \angle QOA$

(Opposite vertical angle)

$\therefore \quad \Delta POB \sim \Delta QOA$

In similar Δ's $\frac{\text{Area of } \Delta QOA}{\text{Area of } \Delta POB} = \frac{OQ^2}{OP^2}$

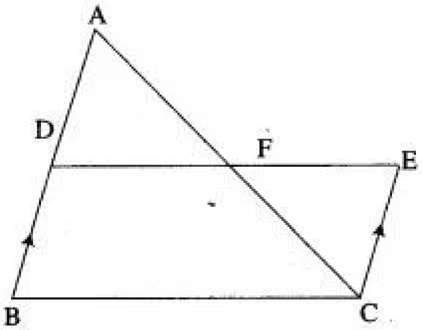
$\frac{\text{Area of } \Delta QOA}{120} = \frac{9^2}{6^2}$

Area of $\Delta QOA = \frac{81 \times 120}{36}$

$= 27 \times 10$

Area of Δ QOA = 270 cm^2 Ans.

Question 14. In the given figure ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8.



(i) Prove that $\Delta ADF \sim \Delta CEF$

(ii) Find AD if CE = 6 cm.

(iii) If DF is parallel to BC find area of ΔADF : area of ΔABC.

Solution : (i) In Δ ADF and Δ CFE

$$\angle DAF = \angle FCE$$

(alternate angles)

$$\angle AFD = \angle CFE$$

(vertically opp. angles)

$$\angle ADF = \angle CEF$$

$\therefore$ $\Delta ADF \sim \Delta CEF$ (by A.A.)

Hence Proved

(ii) $\Delta ADF \sim \Delta CEF$

$$\therefore \quad \frac{AD}{CE} = \frac{AF}{FC}$$

$$FC = AC - AF$$

$$= 8 - 5 = 3$$

$$\therefore \quad \frac{AD}{6} = \frac{5}{3}$$

$$\Rightarrow \quad AD = 10 \text{ cm.}$$

(iii) $DF \parallel BC \therefore \Delta ADF \sim \Delta ABC$

$\because \angle D = \angle B$ and $\angle F = \angle C$.

$$\therefore \quad \frac{\text{Ar. of } \Delta ADF}{\text{Ar. of } \Delta ABC} = \frac{AF^2}{AC^2}$$

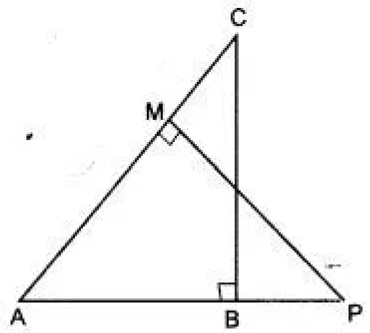
$$= \left(\frac{5}{8}\right)^2 = \frac{25}{64}$$

Question. 15. In the given figure Δ ABC and Δ AMP are right angled at B and M respectively.

Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

(i) Prove $\Delta ABC \sim \Delta AMP$.

(ii) Find AB and BC.



Solution. (i) In ΔABC and Δ AMP

$$\angle ABC = \angle AMP \quad \text{(90° each)}$$

$$\angle A = \angle A \quad \text{(common)}$$

$$\therefore \quad \Delta ABC \sim \Delta AMP$$

(by AA similarity)

Hence proved.

(ii) $\Delta ABC \sim \Delta AMP$

$$\Rightarrow \quad \frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \quad \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \quad \frac{BC}{12} = \frac{10}{15}$$

$$\Rightarrow \quad BC = \frac{10}{15} \times 12$$

$$BC = 8$$

Now, $AB^2 = AC^2 - BC^2$

$$= 10^2 - 8^2$$

$$= 100 - 64 = 36$$

$$AB = 6 \text{ cm.}$$

Question 16. Triangles ABC and DEF are similar.

(i) If area (ΔABC) = 16 cm^2, area (ΔDEF) = 25 cm^2 and BC = 2.3 cm find EF.

(ii) If area (ΔABC) = 9 cm^2, area (ΔDEF) = 64 cm^2 and DE = 5.1 cm, find AB.

(iii) If AC = 19 cm and DF = 8 cm, find the ratio between the areas of two triangles.

(iv) If area (ΔABC) = 36 cm^2, area (ΔDEF) = 64 cm^2 and DE = 6.2 cm, find AB.

Solution : (i) We have

$$\text{area}(\Delta\,ABC) = 16\text{ cm}^2$$

$$\text{area}(\Delta\,DEF) = 25\text{ cm}^2$$

and $BC = 2{\cdot}3$ cm

Since, $\frac{\text{area}(\Delta\,ABC)}{\text{area}(\Delta\,DEF)} = \frac{BC^2}{EF^2}$

$$\Rightarrow \quad \frac{16}{25} = \frac{(2.3)^2}{EF^2}$$

$$\Rightarrow \quad \frac{2{\cdot}3}{EF} = \frac{4}{5}$$

$$\Rightarrow \quad 4\,EF = 5 \times 2{\cdot}3$$

$$\Rightarrow \quad EF = \frac{11{\cdot}5}{4}$$

$$\Rightarrow \quad EF = 2{\cdot}875 \text{ cm.}$$

(ii) We have

$$\text{area}(\Delta\,ABC) = 9\text{ cm}^2$$

$$\text{area}(\Delta\,DEF) = 64\text{ cm}^2$$

and $DE = 5{\cdot}1$ cm

Since, $\frac{\text{area}(\Delta\,ABC)}{\text{area}(\Delta\,DEF)} = \frac{AB^2}{DE^2}$

$$\Rightarrow \quad \frac{9}{64} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{3}{8}$$

$$\Rightarrow \quad \frac{AB}{5{\cdot}1} = \frac{3}{8}$$

$$\Rightarrow \quad AB = \frac{3}{8} \times 5{\cdot}1 = \frac{15{\cdot}3}{8}$$

$$\Rightarrow \quad AB = 1{\cdot}9125 \text{ cm.}$$

(iii) In Δ ABC and Δ DEF, AC = 19 cm, DF = 8 cm.

Since, $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{(8)^2} = \frac{361}{64}$

Hence, the required ratio is 361 : 64.

(iv) Area (Δ ABC) = 36 cm^2

Area (Δ DEF) = 64 cm^2.

DE = 6·2 cm

AB = ?

We have

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{AB^2}{DE^2}$$

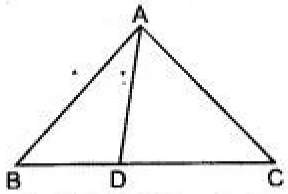
$$\Rightarrow \quad \frac{36}{64} = \frac{AB^2}{(6\cdot2)^2}$$

$$\Rightarrow \quad \frac{AB}{6.2} = \frac{6}{8}$$

$$\Rightarrow \quad AB = \frac{6 \times 6\cdot2}{8}$$

$$\Rightarrow \quad AB = 4\cdot65 \text{ cm.}$$

Question 17. In Δ ABC, $\angle$ABC = $\angle$ DAC. AB = 8 cm, AC = 4 cm, AD = 5 cm.



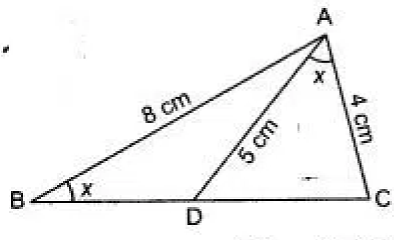
(i) Prove that Δ ACD is similar to Δ BCA.

(ii) Find BC and CD.

(iii) Find area of Δ ACD : area of Δ ABC.

Solution :

$\angle$ ABC = $\angle$ DAC = x (say)



AB = 8 cm,

AC = 4 cm, AD = 5 cm.

(i) In Δ ACD and Δ BCA

$\angle$ ABC = $\angle$ DAC (Given)

$\angle$ ACD = $\angle$ BCA

(Common)

$\Rightarrow$ Δ ACD ~ Δ BCA (AA axiom).

Hence Δ ACD is similar to Δ BCA.

Hence proved.

(ii) As we have,

$$\frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{BA}$$

$$\Rightarrow \quad \frac{4}{BC} = \frac{CD}{4} = \frac{5}{8}$$

$$\Rightarrow \quad \frac{4}{BC} = \frac{5}{8}$$

$$\Rightarrow \quad BC = \frac{8 \times 4}{5} = \frac{32}{5}$$

$$= 6{\cdot}4 \text{ cm.}$$

and

$$\frac{CD}{4} = \frac{5}{8}$$

$$\Rightarrow \quad CD = \frac{5 \times 4}{8}$$

$$\Rightarrow \quad CD = 2{\cdot}5 \text{ cm.}$$

Ans.

(iii)

$$\frac{\text{Area of } \Delta\, ACD}{\text{Area of } \Delta\, ABC} = \left(\frac{AC}{AB}\right)^2$$

$$= \left(\frac{4}{8}\right)^2$$

$$= \frac{1}{4}$$

Thus area of Δ ACD : area of Δ ABC = 1 : 4.

Ans.