GEOMETRY

UNIT - 5

Let us know something about the history of geometry.....

The moment human started distinguishing between shapes around him, it led to the creation of geometry. Since then there are many objects whose name is based on their geometrical shapes. To understand more about these shapes human started to draw these shapes in different ways and in this effort he created different lines and shapes.

In this process he also studied spatial relationship. This developed the understanding of angles and construction of shapes. In India geometry was predominantly used in construction of monuments and to locate and forecast the position of celestial bodies. There were many formulae to do this. Initial geometry was an endeavour based more on experience and extracting rules from these examples. Those rules were created to find and calculate length, breadth, height, angle, area, volume etc. easily. The aim of this was to find the use in daily needs like survey of land, construction of building ,bridges and for geological and other technical uses. But as often happens the scope of geometry was also to make further discoveries and it slowly got more extensive.

Geometry which means land measurement shows the reason for its creation. Many formulaes of those times were as complicated and deep as todays contemporary mathematics, 'which is not easy to find even today. This was the begininng of formal geometry. In the era of Harappan Civilization people were experts in measurement as well as creating geometrical shapes. Likewsie in Shulva-Sutra there is a description of how to construct and find area of triangles, squares, rectangles and other complicated geometrical shapes. These formulae could also be used more comprehensively. Thus, the formulae related to triangles and quadrilaterals can be used for all types of triangles and quadrilaterals.

Few examples of Shulva-Sutra:-

1. To construct a square equal to the sum of the area of two given squares.
2. To construct a square with double the area of the given square.

To construct a square with double the area of the given square, we need to find the side for that square, Katyayan and Apstambh had given the following Shulva-Sutras for this:-

$$\text{New side} = \text{old side}\left(1+\frac{1}{3}+\frac{1}{3\times 4}-\frac{1}{3\times 4\times 34}\right) = 1.4142156 \times \text{old side}$$

This value is the same as value of $\sqrt{2}$ correct up to five decimal places.

We find several examples of this kind of geometrical rules and formulae in Egyptian, Indian (Indus valley and Harappan), Babylonian, Arabian civilization.

This information has been collected from different books and presented. Teachers and students can collect more information about geometry from other sources.

Straight Line & Angle

09

There are lot of shapes hidden around us. In the picture given below we can see a door, frame of a window, top surface of a book, roof-top etc., all these shapes are rectangular.

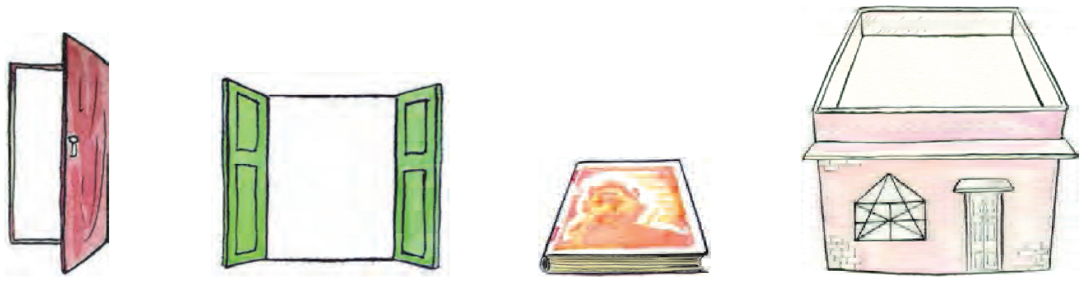


Fig. 1

Some other objects may have surfaces that are triangular, pentagonal or any other shape. Every angel rectangular object has all angles equal to 90° and has opposite side that are equal. Other shapes also have some line segments that are equal and their angles could be equal.

See the window grill in the picture. There are lots of line segments in the picture which intersect each other. In this grill and the other grid type we find several line segments which meet at different intersecting points. Is there any relation between the angles made at the intersecting points? In this chapter we will study the angles made by lines at their points of intersection.

Line Segment and End Points

Draw a line on your copy. What symbols are used to express it?

Now draw a ray. Which symbols are use for this?

Does a line have any end point? And in a ray? Discuss with your classmate.

See the picture given below:-

How many end points are there? This picture does not show a line or a ray. This is a line segment.

Fig. 2

This can also be marked on a line.

How many line segment can be there on a line?

Discuss with your friends.

Identifying a Line Segment

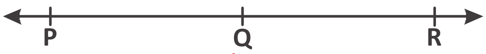


Fig. 3

1. How many points are marked on this line?
2. How many line segment are there on this line? Which are they?

Try to identify all these three things in figures 4, 5, 6 and 7.

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Number of points on the line	Name of the points	Name of line segment	Numbers of line segments
3	P, Q, R	PQ, PR, QR	3
4	P, Q, R, S		
5			
6			
7			

Here PQ and QP are the name for the same line segment.

If only two points are marked on a line then how many line segments would there be?

According to this table what relationship is there between the number of points and numbers of line segments.

Likewise if there are 8 points on the line then the number of line segments are

$$1 + 2 + 3 + 4 + 5 + 6 + 7.$$

If there are n points on the line then the number of line segment would be

$$1 + 2 + 3 + 4 + 5 + \dots\dots\dots\dots + (n - 1)$$

Do you agree with this? Discuss with your friends.

Collinear Points

In the above table P, Q, R, S etc. on a single line. These are collinear points. This means all the points which lie on the same line are called collinear points.

Here points A, B and C are collinear. (*Fig.*8)

Fig. 8

In *Fig.*9, are the points A, B, C and D collinear? Can we also say that B, C and D not collinear?

And are C and D also not collinear?

Can we draw a line on which both points C and D lie? Yes, line segment CD lies on this line.

Clearly any two points must have lie on any one line, and on this line they will be collinear.

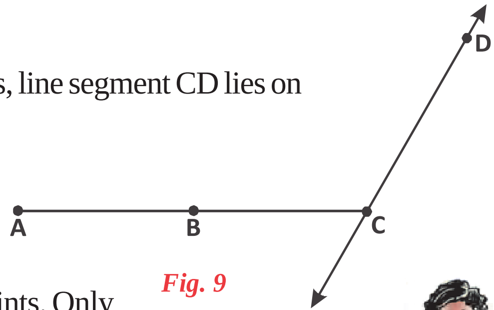


Fig. 9

So, whether the points are colinear or not, is a question which is relavant only when we are taking about at least three points. Only then would you ask the question whether the points are collinear or not?

Think and Discuss

Can three collinear points make a triangle?

Line and Angle

In *figure*-9 angle DCA at point 'C' is more than 90°, so this is an obtuse angle.

We know different kinds of angle like acute angle, right angle, obtuse angle, straight angle and reflex angle.

Try This

Draw each type of angle we have mentioned and write their names.

Adjacent, Complementary & Supplementary Angles

Here we will see which pair of angles comes into the category of adjacent angles, complementary angles and supplementary angles.

Adjacant Angles

See the given *Fig.*10(i) and (ii).

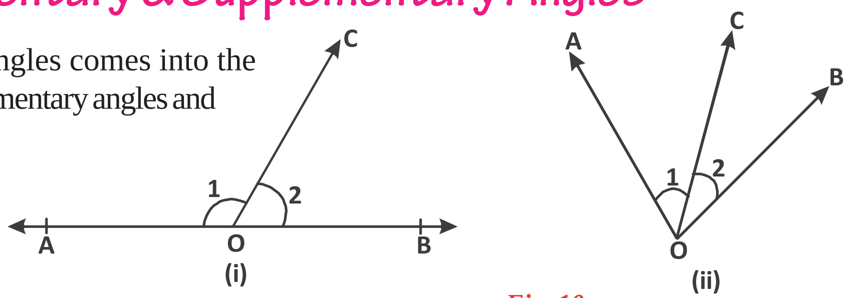


Fig. 10

There are two angles 1 and 2 both in *Fig.*10(i) and (ii). In which O is a vertex and side OC is in middle and is a common side.

Therefore in *Fig.*10 angles 1 and 2 are adjacent angles.

Now see *Fig.*11(i), (ii) and (iii):-

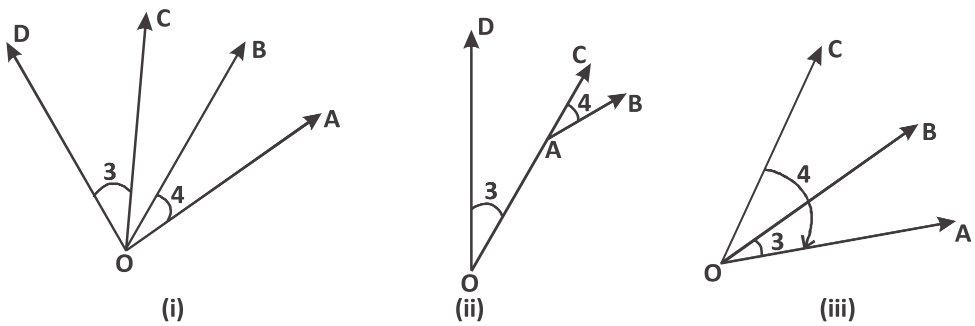


Fig. 11

In *Fig.*11(i), 3 and 4 have same vertex but there is no common side in these angles.

In *Fig.*(ii) angle 3 and 4 have different vertices, for angle 3 vertex is O, while for angle 4 it is A.

In *Fig.*11(iii) angle 3 and 4 have some common side OA, but angle 3 is a part of angle 4.

Therefore in all the *Fig.*11, angles 3 and 4, are not adjacent.

Two angles are adjacent when they have a common side, a common vertex and both angles do not overlap each other.

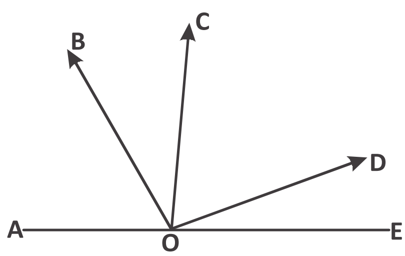
Try This

1. Draw two angle which are not adjacent.
2. See the figure and say whether the following are adjacent or not?

(i) $\angle AOB$ and $\angle BOC$

(ii) $\angle BOC$ and $\angle DOE$

(iii) $\angle EOD$ and $\angle DOC$



Think and Discuss

Where are two angles adjacent ?

(i) When both the angles are obtuse (ii) Both are acute

(iii) One angle is obtuse and the other is acute.

Complementary Angles

Look at the figures below, each figure has two angles. What is the sum of each of these angle pairs?

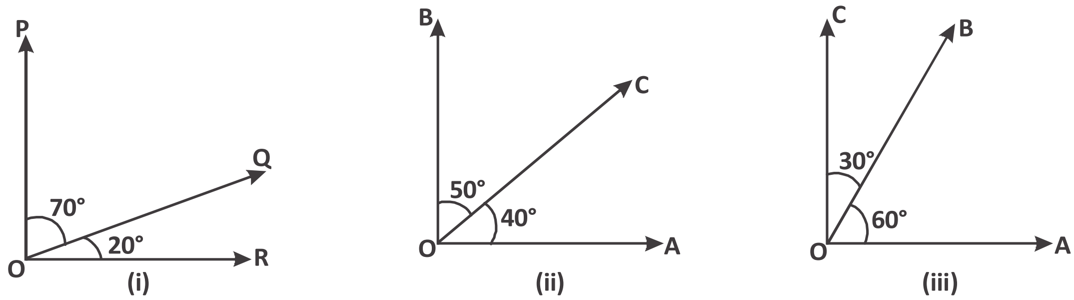


Fig. 12

When sum of a pair of angles is 90° , then each angle is complementary to the other.

See these complementary angles are also adjacent.

You too draw some more adjacent complementary angle like the ones given in *Fig.*12.

Supplementary Angle

What is the sum of $\angle 1$ and $\angle 2$ given in figure below?

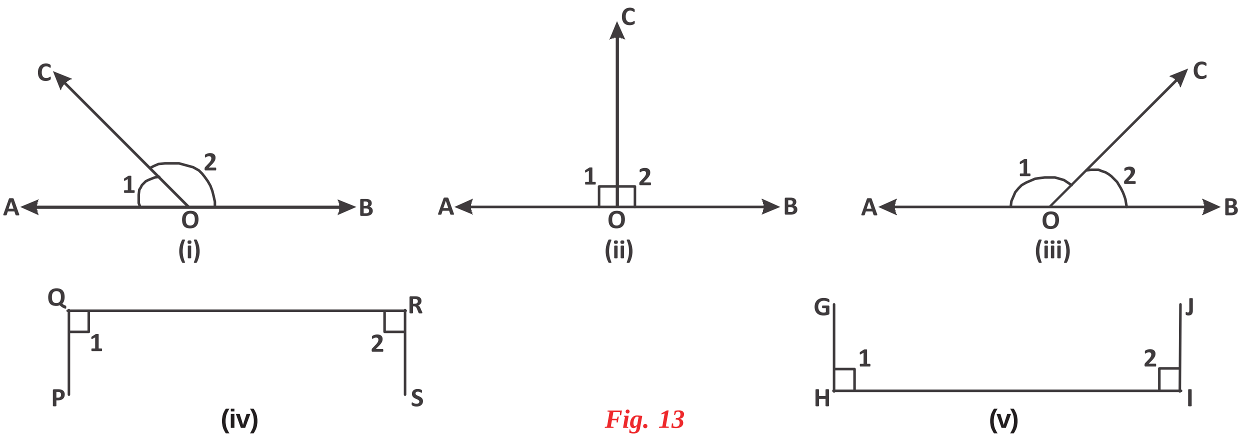


Fig. 13

Sum of all these pair angles is 180°, that means each angle is supplement to other.

Are the angles in *Fig.*13(i), (ii) and (iii) adjacent angles? Are the angles (iv) and (v) also adjacent?

Here in *Fig.*13(i), (ii) and (iii) the sum of pair of adjacent angles makes a straight line, this is also called straight angle. This kind of pair of angles is also called linear pair of angles.

Can we say each pair of supplementary angles is also a linear pair of angles?

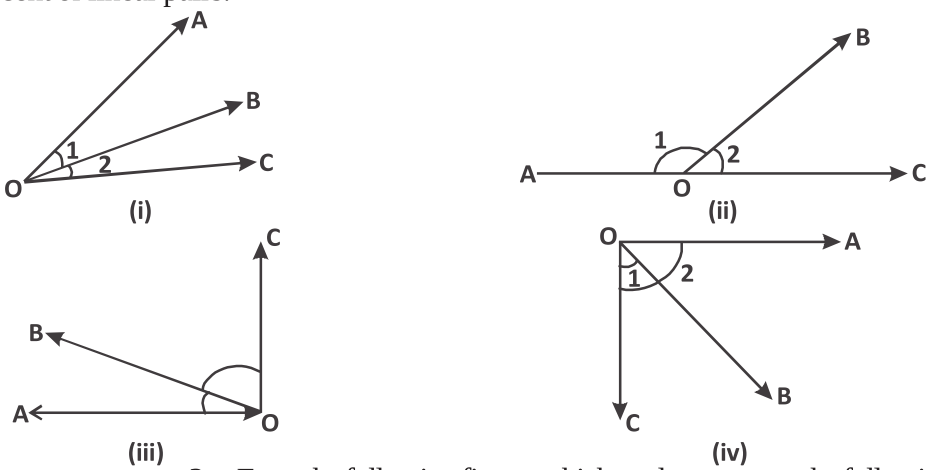
Are angles in*Fig.*13(iv) and (v) also a linear pair?

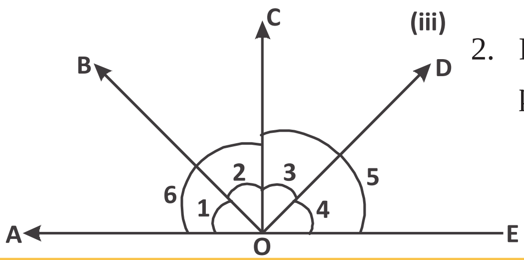
Think and Discuss

1. Can two right angles be a complementary angles?
2. Is each linear pair also supplementary angles?

Try This

1. Which of the following angles are complementary or supplementary? Which ones are adjacent or linear pairs.





2. From the following figure, which angles represent the following pair

(i) $\angle 1$ and $\angle 2$ (ii) $\angle 5$ and $\angle 6$ (iii) $\angle 6$ and $\angle 3$

(iv) $\angle 5$ and $\angle 2$ (v) $\angle 3$ and $\angle 4$

Intersecting and Non-intersecting Lines

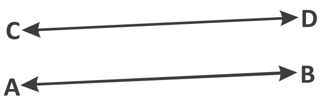


Fig. 14 (i)

In *Fig.*14(i) and (ii) if we increase the lines, then which pair of lines intersect each other?

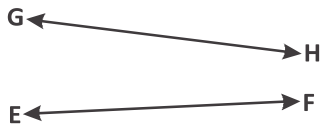


Fig. 14 **(ii)**

Here line AB and CD do not intersect each other. Where as lines GH and EF when extended to H and F meet at point O (*Fig.*15)

Fig. 15

Therefore AB and CD are non-intersecting lines. And EF or GH are intersecting lines.

Angle Made by Two Intersecting Lines

When tow lines intersect each other on any point, some angles are formed at the point of intersection .

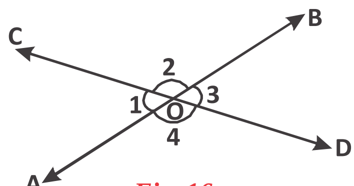


Fig. 16

Look at the *Fig.*16. Lines AB and CD intersect each other at point O. This forms $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. Is there any similarity between $\angle 1$ and $\angle 3$ or $\angle 2$ and $\angle 4$.

Here we can see that $\angle 1$ and $\angle 3$ meet at point O and are opposite to each other. Likewise $\angle 2$ and $\angle 4$. These angles are called vertrically opposite angles.

Propery of Vertically Opposite Angles

Here $\angle 1$, $\angle 3$ and $\angle 2$, $\angle 4$ are vertically opposite angles.

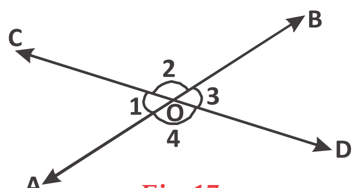


Fig. 17

Above the line CD

$\angle COB + \angle BOD = \angle COD$

What is $\angle COD$, here?

This is straight angle.

So, $\angle COB + \angle BOD = 180°$

Or $\angle 2 + \angle 3 = 180°$ ......(i)

Can you find the same type of relation above line AB.

$\angle AOC + \angle COB = \angle AOB$

Or $\angle 1 + \angle 2 = 180°$ .....(ii)

Here $\angle AOB$ is a straight line angle.

Now in (i) and (ii)

$\angle 2 + \angle 3 = \angle 1 + \angle 2$

Or $\angle 3 = \angle 1$

So $\angle 1 = \angle 3$ .....(iii)

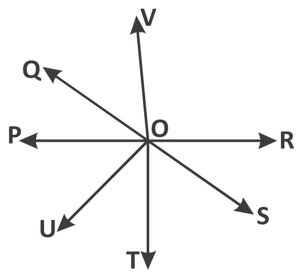
Similary, you can add the angles above line CD and add the angles below line AB and find the following relationship :

$\angle 2 = \angle 4$ .....(iv)

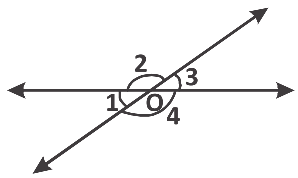
Here $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$ are vertically opposite angles. From the equation (iii) and (iv) we can say that these angles are equal. Therefore vertically opposite angles are equal.

Try This

1. Show all vertically opposite angles in the figure.



2. In figure, $\angle 2 = 110°$ then find the measurement of $\angle 1$ and $\angle 4$.



EXAMPLE-1. In *Fig.*18, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are opposite rays. What is the measurement of $\angle AOC$ and $\angle BOC$?

SOLUTION : $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are opposite rays.

which make $\angle AOC$ and $\angle BOC$ a linear pair.

Hence $\angle AOC + \angle BOC = 180°$

$(2x+20°)+2x=180°$

$4x+20°=180°$

$x=40°$

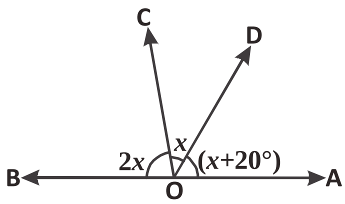


Fig. 18

Now $\angle AOC = 2x+20°$

$= 2(40°)+20°$

$= 100°$

$$\left\{\begin{aligned} \angle AOC &= \angle COD + \angle DOA \\ &= x+(x+20°) \\ &= 2x+20° \end{aligned}\right\}$$

And $\angle BOC = 2x$

$= 2(40°)$

$= 80°$

Hence $\angle AOC = 100°$ and $\angle BOC = 80°$

EXAMPLE-2. In the given *Fig.*19, lines AB and CD intersect at a point O. If $\angle AOC : \angle COB = 7:8$, find the measures of all angles.

SOLUTION : According to the questions $\angle AOC : \angle COB = 7:8$

So if $\angle AOC = 7x$ .....(i)

then $\angle COB = 8x$ .....(ii)

The ray OC stand on the line AB. (linear pair)

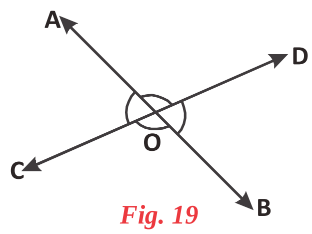


Fig. 19

$\angle AOC + \angle COB = 180^\circ$

$7x + 8x = 180^\circ$

$15x = 180^\circ$

$x = 12^\circ$ .....(iii)

Now $\angle AOC = 7x$

$= 7(12^\circ)$

$= 84^\circ$

And $\angle COB = 8x$

$= 8(12^\circ)$

$= 96^\circ$

Also $\angle BOD = \angle AOC = 84^\circ$ (Vertically opposite angles)

पुन: $\angle AOD = \angle COB = 96^\circ$ (Vertically opposite angles)

EXAMPLE-3. In the given *Fig.*20, $\angle COD = 90^\circ$, $\angle BOE = 72^\circ$ and if AOB is a straight line, find the measures of $\angle AOC$, $\angle BOD$ and $\angle AOE$.

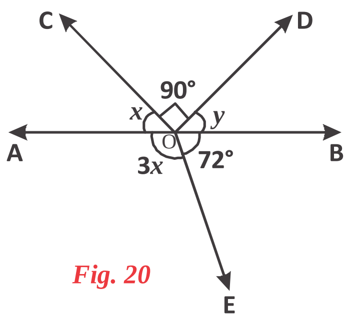


Fig. 20

SOLUTION : AOB is a straight line,

$\angle AOE + \angle BOE = 180^\circ$ (linear pair)

$\Rightarrow 3x + 72^\circ = 180^\circ$

$\Rightarrow 3x = 108^\circ$

$\Rightarrow x = 36^\circ$ .....(i)

Similarly, $\angle AOC + \angle COD + \angle DOB = 180^\circ$ (straight angle)

$\Rightarrow x + 90^\circ + y = 180^\circ$

$\Rightarrow 36^\circ + 90^\circ + y = 180^\circ$

$\Rightarrow 126^\circ + y = 180^\circ$

$\therefore\ y = 54^\circ$ .....(ii)

Hence $\angle AOC = x = 36^\circ$

$\angle BOD = y = 54^\circ$

And $\angle AOE = 3x = 3 \times 36^\circ = 108^\circ$

EXAMPLE-4 In the given *Fig.*21, ray OS starts from the point O on line PQ. Other rays OR and OT bisect angles $\angle$POS and $\angle$SOQ respectively. Find measures of $\angle$ROT.

SOLUTION : Ray OS, is on the line PQ, hence by the a linear pair axiom,

$\angle POS + \angle SOQ = 180^o$

If $\angle POS = x$

then $x + \angle SOQ = 180^o$

$\angle SOQ = 180^o - x$ .....(i)

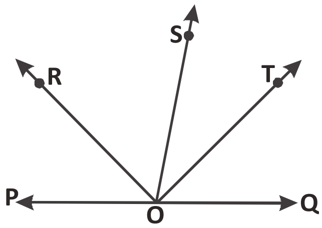


Fig. 21

But ray OR, besects $\angle$POS,

hence $\angle ROS = \frac{1}{2} \times \angle POS$

$= \frac{1}{2} \times x = \frac{x}{2}$ .....(ii)

Similarly Ray OT bisects $\angle$SOQ,

Hence $\angle SOT = \frac{1}{2}\angle SOQ$

$= \frac{1}{2}(180^o - x)$ (from (1))

$= 90^0 - \frac{x}{2}$ .....(iii)

It is clear from the figure,

$\angle ROT = \angle ROS + \angle SOT$

$= \frac{x}{2} + \left(90^o - \frac{x}{2}\right) \quad = \frac{x}{2} + 90^o - \frac{x}{2}$

Hence $\angle ROT = 90^o$

EXAMPLE-5. In the adjjacent figure, there are four rays OP, OQ, OR and OS, Prove that $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^o$

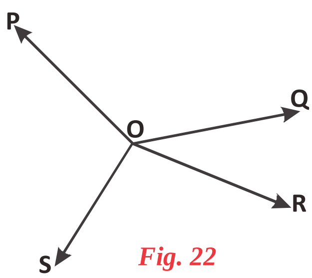


Fig. 22

SOLUTION : In the given *Fig.*22, We need to extend one of the rays OP, OQ, OR or OS to a point beyond O. (Why ?)

Extend ray OQ to a point T. (*Fig.*23) so that TOQ is a straight line.

Now it is clear from the figure that ray OP on TQ and so the linear pair axiom applies

$\angle TOP + \angle POQ = 180^o$ .....(i)

Similarly ray OS is on TQ and so by linear pair axiom,

$\angle TOS + \angle SOQ = 180^o$ .....(ii)

By adding equations (i) and (ii),

$\angle TOP + \angle POQ + \angle TOS + \angle SOQ = 360^o$ .....(iii)

It is clear from the figure that-

$\angle TOP + \angle TOS = \angle POS$

$\angle TOP = \angle POS - \angle TOS$ .....(iv)

And $\angle SOQ = \angle SOR + \angle QOR$ .....(v)

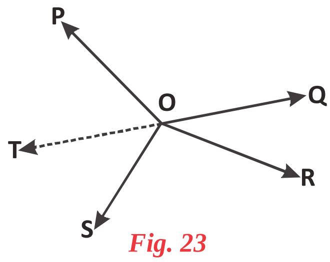


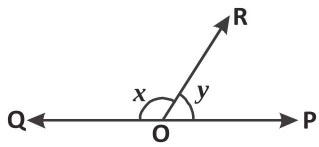
Fig. 23

Keeping values of equations (iv) and (v)in equation (iii)

$\angle POS - \angle TOS + \angle POQ + \angle TOS + \angle SOR + \angle QOR = 360^o$

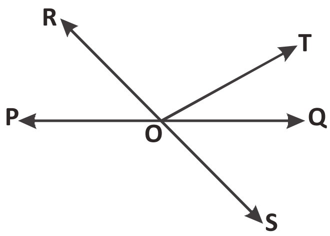
Hence, proved that $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^o$

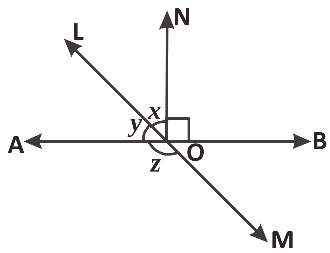
Exercise 9.1



1. In the figure alongside, $\angle POR$ and $\angle QOR$ are a linear pair. If $x - y = 80^o$ find the values of x and y.

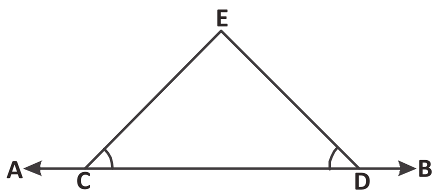
2. In the given figure lines PQ and RS intersect in point O. If $\angle POR + \angle QOT = 70^o$ and $\angle QOS = 40^o$, then find the measures of $\angle QOT$ and reflex $\angle ROT$.

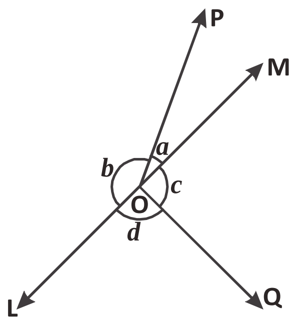




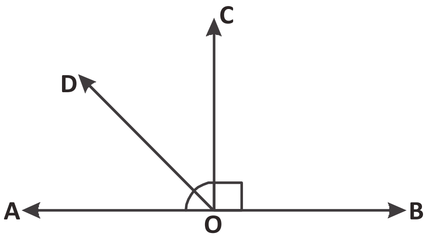
3. In the adjoining figure, lines AB and LM intersect in a point O. If $\angle NOB = 90^o$ and $x : y = 2 : 3$ then find the value of z.

4. In the given figure $\angle ECD = \angle EDC$ then prove that $\angle ECA = \angle EDB$





5. In the given figure $a + b = c + d$ then prove that LOM is a straight line.



6. In the given figure, AOB is a line Ray OC is perpendicular to AB. There is another ray OD between rays OA and OC. Prove that $\angle COD = \frac{1}{2}(\angle BOD - \angle AOD)$

7. If $\angle ABC = 64^{o}$ and AB is extended to a point X, Draw a figure to show this information. If ray BY biscects $\angle CBX$, then find measures of $\angle ABY$ and reflex $\angle YBX$.

Parallel Lines and Transversal Lines

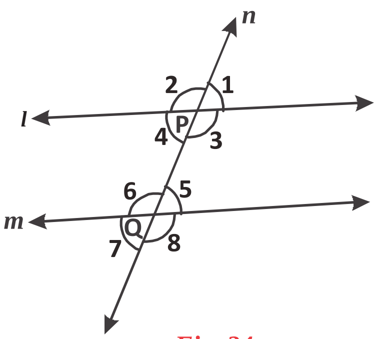


Fig. 24

What sort of lines ℓ, m can you see in *Fig.*24 and 25?

The lines ℓ, m in *Fig.*24 are non intersecting lines and in *Fig.*25 are interesecting lines.

In both the figures line *n* is intersecting both lines ℓ and *m* in points P and Q respectively. This line *n* is called a transversal.

Observe the *Fig.*24 and 25. In which of the two, is the distance same at all points of lines ℓ and *m*.

Here the lines ℓ, m in *Fig.*25 are intersecting and the distance between them is unequal and in *Fig.*24 they are non intersecting and the distance between them is the same. These are known as parallel lines.

When a transversal cuts two other lines, the following angles are formed in both *Fig.*24 and 25.

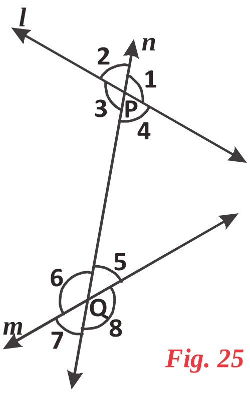


Fig. 25

(i) Corresponding angles

(a) $\angle 1$ and $\angle 5$ (b) $\angle 2$ and $\angle 6$
(c) $\angle 3$ and $\angle 7$ (d) $\angle 4$ and $\angle 8$

(ii) Alternate interior angles

(a) $\angle 4$ and $\angle 6$ (b) $\angle 3$ and $\angle 5$

(iii) Alternate exterior angle

(a) $\angle 1$ and $\angle 7$ (b) $\angle 2$ and $\angle 8$

(iv) The interior angles on the same side of the transversal

(a) $\angle 4$ and $\angle 5$ (b) $\angle 3$ and $\angle 6$

Interior angles on the same side of the transversal are also referred as consecutive interior angles or allied angles or co-interior angles. Quite often the alternate interior angles are simply refered as alternate angles.

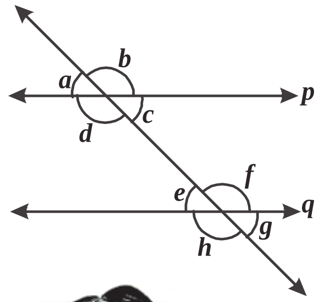
(v) The angles which are on the same side of transversal but on the exterior side of the two lines, are known as, consecutive exterior angles or allied exterior angles or coexterior angle.

(a) $\angle 1$ and $\angle 8$ (b) $\angle 2$ and $\angle 7$

Try This

Complete the table by observing the given figure-

S. No.	Pair of angles	Angles	Number of pairs of angles
1	Alternate Interior angle		2
2	Interior angles on same side of transversal		
3		$\angle a$ and $\angle g$ $\angle b$ and $\angle h$	
4		$\angle a$ and $\angle h$ $\angle b$ and $\angle g$	
5	Corresponding angles		4



Properties of Corresponding angle & Alternate Angle

Corresponding and alternate angles are formed when a transversal intersects two other lines. Is there a relation between the pairs so formed?

When a transversal intersects two intersecting lines, the pairs of corresponding and alternate angles are not equal, but if these two lines are parallel, then both pairs of corresponding angles and alternate angles are equal.

Think and Discuss

If a transversal intersect two other lines such that the corresponding pair of angles are equal, then can we say that the two lines are parallel?

Now the question is whether based on this property of corresponding angles, can we say something of the proportions of other angles pairs formed by the transversal with the parallel lines like the alternate interior angles or alternate exterior angles? Yes, in order to do this we take two parallel lines ℓ and m, which are cut by a transversal n at points P and Q. Look at *Fig.*26.

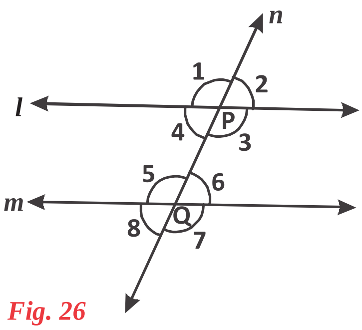


Fig. 26

Here $\angle 1 = \angle 5$ (Corresponding angles) – (i)

$\angle 1 = \angle 3$ (Vertically opposite angles) – (ii)

from (i) and (ii), $\angle 5 = \angle 3$

What angles are these? These are alternate interior angles.

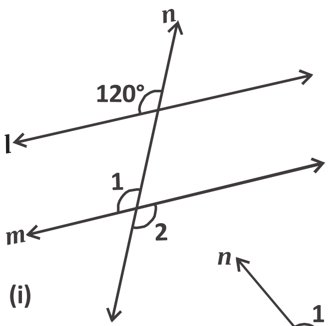
So we can say that when a transversal cuts two parallel lines, the alternate interior angles are equal.

Think and Discuss

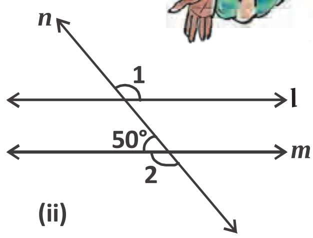
If a transversal intersects two lines such that the alternate interior angles are equal, can we say that the lines are parallel?

Try This

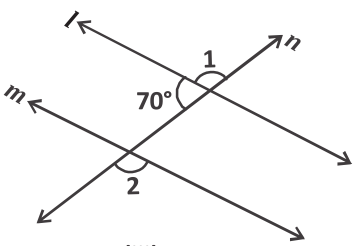
In the given figures, find the values of $\angle 1$ and $\angle 2$, if transversal n intersects two parellel lines l and m. Give reasons for your answers.



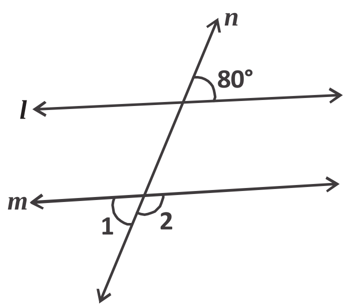
(i)



(ii)



(iii)



(iv)

Lines Parallel to the Same Line

If two lines are parallel to the same line, are they parallel to each other?

For inspecting this, draw three lines l, m, n as shown in adjoining figure, such that line m is parallel to line l and line n is parallel to line l.

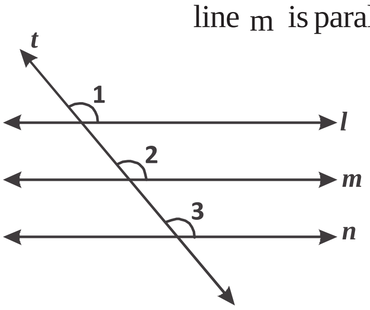


Fig. 27

Draw a transversal t intersecting lines l, m and n.

By the corresponding angles postulate,

$\angle 1 = \angle 2$ ....(1)

$\angle 1 = \angle 3$ ....(2)

Hence, from (1) and (2), We deduce

$\angle 2 = \angle 3$

But $\angle 2$ and $\angle 3$ are the corresponding angles formed by transversal *t* with lines m and n, hence we can say that line m and n are parallel.

This result can be written as a theorem as follows :

Theorem

It is that statement which can be proved logically using known facts and logic.

Theorem-1 : **The lines which are parallel to the same line, are parallel to each other.**

EXAMPLE-6. Find values of x, y, z and $\mathrm{a, b, c}$ from the given *Fig.* 28.

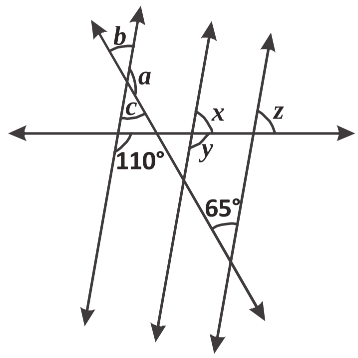


Fig. 28

SOLUTION : From the figure,

$y = 110^\circ$ $\quad$ ($\because$ Corresponding angles)

and $\quad x + y = 180^\circ$ (Linear pair)

$\Rightarrow \quad x + 110^\circ = 180^\circ$

$\Rightarrow \quad x = 70^\circ$

$z = x = 70^\circ$ $\quad$ ($\because$ Corresponding angles)

Again $c = 65^\circ$ $\quad$ ($\because$ Alternate angles)

And $\quad a + c = 180^\circ$ ($\because$ Linear pair)

$\Rightarrow \quad a + 65^\circ = 180^\circ$

$\Rightarrow \quad a = 115^\circ$

Also $\quad b = c = 65^\circ$ $\quad$ ($\because$ Vertically opposite angles)

Hence $\quad a = 115^\circ,\ b = 65^\circ,\ c = 65^\circ$

$x = 70^\circ,\ y = 110^\circ,\ z = 70^\circ$

EXAMPLE-7. In the given figure, if $\mathrm{PQ} \parallel \mathrm{RS}, \angle \mathrm{MXQ} = 135^\circ$ and $\angle \mathrm{MYR} = 40^\circ$ then find measure of $\angle \mathrm{XMY}$.

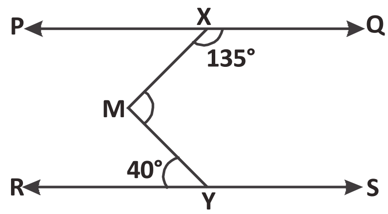


Fig. 29

SOLUTION : Construct a line AB parellel to PQ and passing through M.

Now, $AB \parallel RS$ and $\mathrm{PQ} \parallel \mathrm{RS}$.

Hence, $\angle \mathrm{QXM} + \angle \mathrm{XMB} = 180^\circ$ .....(1)

($\because$ AB || PQ, and these are cointerior angles on same side of transversal) fig–30

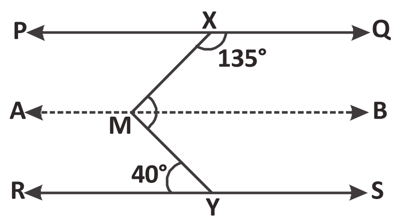


Fig. 30

According to the question, $\angle \mathrm{QXM} = 135^\circ$, so by equation (1)

$$135^\circ + \angle \mathrm{XMB} = 180^\circ$$

$$\angle \mathrm{XMB} = 180^\circ - 135^\circ$$

$\therefore \quad \angle \mathrm{XMB} = 45^\circ$ .....(2)

Again $\angle BMY = \angle MYR$ .....(3) $(\because AB \parallel RS,$ Alternate angles$)$

According to the questions, $\angle MYR = 40^\circ$, and by equaction (3)

$$\angle BMY = 40^\circ \quad .....(4)$$

Adding equation (2) and (4) we get,

$$\angle XMB + \angle BMY = 45^\circ + 40^\circ$$

That is $\angle XMY = 85^\circ$

EXAMPLE-8. In the given figure $AB \parallel CD$ find the value of x.

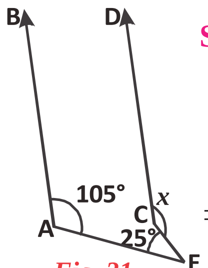


Fig. 31

SOLUTION : Draw $EF \parallel AB$ passing through point E. So obviously $EF \parallel CD$

Now $EF \parallel CD$ and CE is a transversal, So

$\angle DCE + \angle CEF = 180^\circ$

$\Rightarrow$ $x + \angle CEF = 180^\circ$ $\quad (\because \angle DCE = x)$

$\angle CEF = 180^\circ - x$ .....(1)

$EF \parallel AB$ and AE is a transversal

So $\angle BAE + \angle AEF = 180^\circ$ $(\because$ Cointerior angles$)$

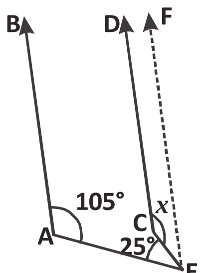


Fig. 32

$\Rightarrow$ $105^\circ + (\angle AEC + \angle CEF) = 180^\circ$

$(\because \angle BAE = 105^\circ)$

$\Rightarrow$ $105^\circ + 25^\circ + (180^\circ - x) = 180^\circ$

$(\because \angle AEC = 25^\circ$ and from equation (1)$)$

$\Rightarrow$ $310^\circ - x = 180^\circ$

Hence $x = 130^\circ$

EXAMPLE-9. If a transversal intersects two lines such that the bisectors of the pair of corresponding angles are parallel, then prove that the lines are parallel to each other.

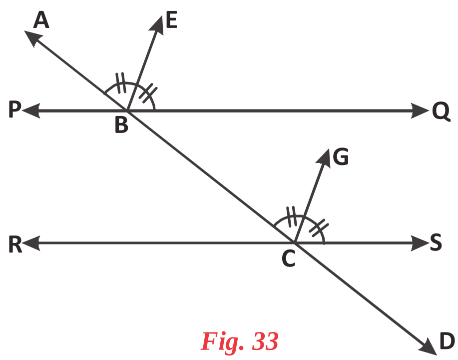


Fig. 33

SOLUTION : According to the *Fig.*33, AD is a transversal intersecting lines PQ and RS in points B and C respectively. Ray BE bisects $\angle ABQ$ and CG bisects $\angle BCS$. Also $BE \parallel CG$

To Prove that : $PQ \parallel RS$

We are given that BE bisects $\angle ABQ$,

Hence $\angle ABE = \frac{1}{2}\angle ABQ$ .....(1)

Similarly CG bisects $\angle BCS$,

Hence $\angle BCG = \frac{1}{2}\angle BCS$ .....(2)

But $BE \parallel CG$ and AD is a transversal, hence

$\angle ABE = \angle BCG$ .....(3)

From equctions (1), (2) and (3)

$$\frac{1}{2}\angle ABQ = \frac{1}{2}\angle BCS$$

Or $\angle ABQ = \angle BCS$

But these are cooespoding angles formed by the transveral AD on the lines PQ and RS .

Hence $PQ \parallel RS$

EXAMPLE-10. In the given figure, $AB \parallel CD$ and $CD \parallel EF$, Also $EA \perp AB$. If $\angle BEF = 55^o$, find the value of x, y and z.

SOLUTION : Given $AB \parallel CD$ and $CD \parallel EF$ Hence $AB \parallel EF$. In the *Fig.*34 BE is extended to a point G.

Now, $\angle DEF + \angle FEG = 180^o$ (Linear pair)

$55^o + \angle FEG = 180^o$ $(\because \angle DEF = 55^o)$

$\angle FEG = 125^o$

Thus $\angle FEG = y = x = 125^o$

(Corresponding angles)

Again $\angle CED + \angle DEF = 90^o$

$(\because EA \perp AB$ and $AB \parallel EF)$

Thus $z + 55^o = 90^o$

$\therefore$ $z = 35^o$

Hence $x = 125^o$, $y = 125^o$, $z = 35^o$

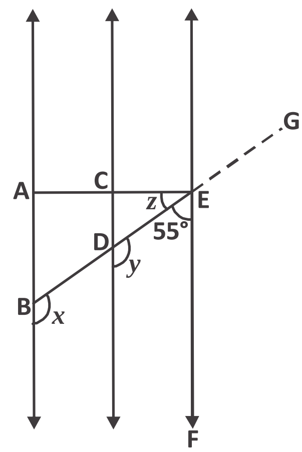
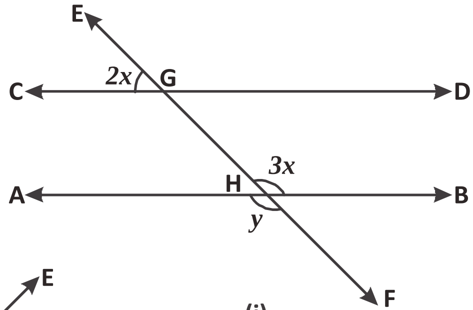


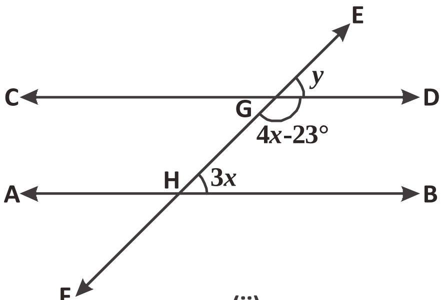
Fig. 34

Exercise-9.2

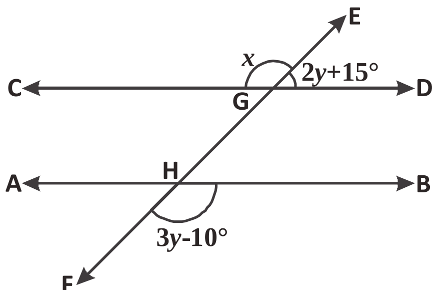
1. In the given figure $AB \parallel CD$ and EF is a transversal which intersects these line at H and G. Find the value of x and y.



(i)

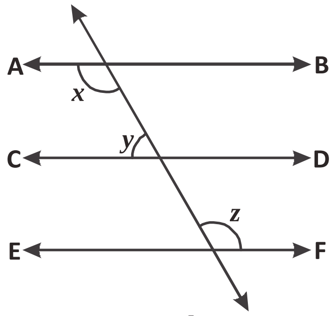


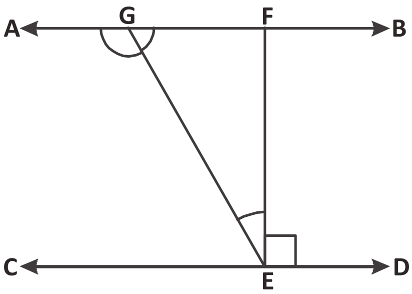
(ii)



(iii)

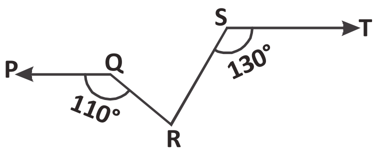
2. In the given figure, if $AB \parallel CD$, and $CD \parallel EF$. Further $y : z = 3 : 7$ then find the value of x.



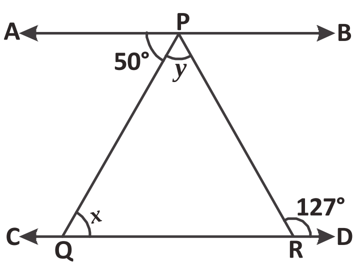


3. In the adjoining figure if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126°$ then find measures of $\angle AGE$, $\angle GEF$, and $\angle FGE$.

4. In the given figure, if $PQ \parallel ST$, $\angle PQR = 110°$ and $\angle RST = 130°$ then the measure of $\angle QRS$.

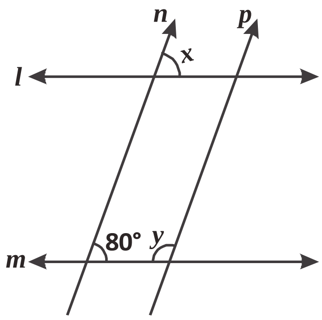


(**Hint**– Construct a line $\parallel$ to ST passing through R)

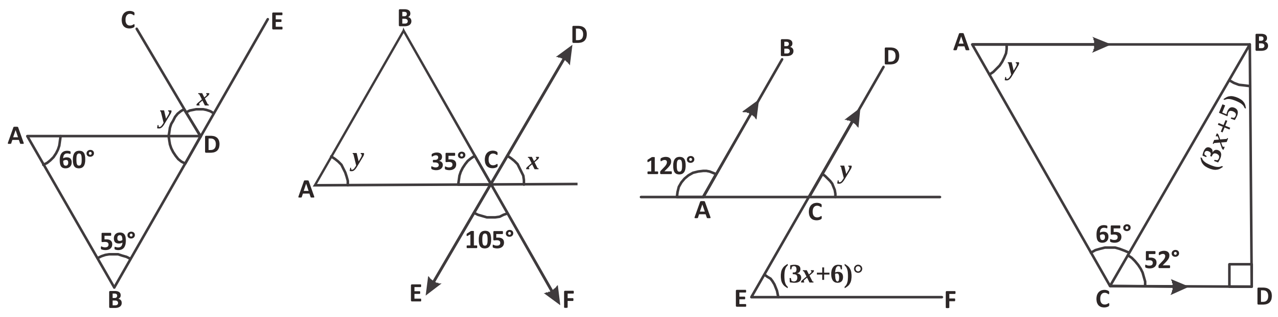


5. In the adjoining figure, if $AB \parallel CD$, $\angle APQ = 50°$ and $\angle PRD = 127°$ then find value of x and y.

6. Find value of x and y. (Given $\ell \parallel m$, $n \parallel p$)

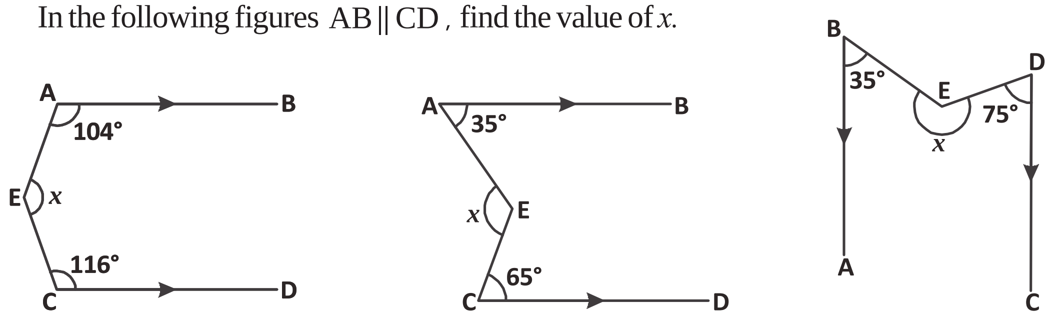


7. Find the values of x and y in the given figures here $AB \parallel CD$ है।



(i) (ii) (iii) (iv)

8. In the following figures $AB \parallel CD$, find the value of x.



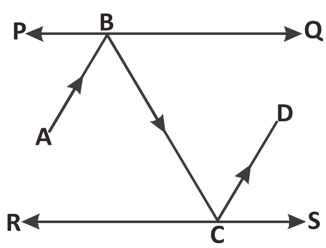
(i) (ii) (iii)

9. Complete the following table :

S.N.	Name of triangle	Measure of angles	Speciality and other properties
1.	Acute angled triangle		
2.		One angle 90°	
3.	Obtuse angled triangle		
4.		Each angle 60°	
5.			Two sides are equal
6.	Scalene triangle		

10. In the given figure PQ and RS are two mirrors kept parallel to each other. Incidental ray AB, strikes mirror PQ at B and reflects back along BC which strikes mirror RS at C and is reflected along CD. Show that $AB \parallel CD$

(**Hind :** Perpendicular to parallel lines are parallel to each other)



To Prove Mathematical Statement

We have proved using a protractor and paper cutting activity that sum of the internal angles of a triangle is $180°$. Now we will prove this statement using parallel lines and related postulates and theorems.

Theorem–2 : The sum of the internal angles of a triangle is 180^0 .

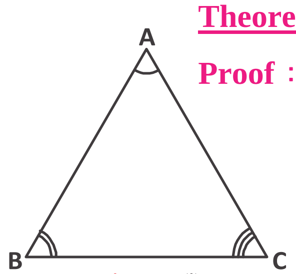


Fig. 35 (i)

Proof : Given Δ ABC with angle $\angle 1, \angle 2$ and $\angle 3$.

We have to prove that $\angle 1 + \angle 2 + \angle 3 = 180^o$ ।

To prove this draw a line PQ parallel to BC passing through A. (*Fig.*35(ii))

Now lines BC and PQ are parallel to each other. AB and AC are transversals. It is clear from the figure that $\angle 4$, $\angle 2$, and $\angle 5$, $\angle 3$ are alternate pairs of angles. Hence

hence $\angle 4 = \angle 2$ .....(1)

$\angle 5 = \angle 3$ .....(2)

But PAQ is a straight line, hence

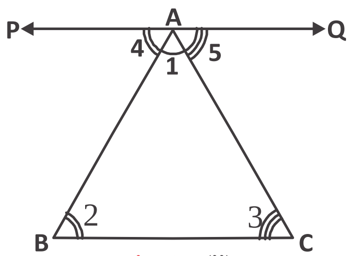


Fig. 35 (ii)

$$\angle 4 + \angle 1 + \angle 5 = 180^o \quad .....(3)$$

Replacing values of $\angle 4$ and $\angle 5$ from equations (1) and (2) in (3), We get

$$\angle 2 + \angle 1 + \angle 3 = 180^o$$

Or $$\angle 1 + \angle 2 + \angle 3 = 180^o$$

Hence, we can say that sum of interior angle of a triangle is 180^0.

Think and Discuss

Are the following statements true or false. Give reasons.

S.No.	Statement	True/False	Reason
1.	A triangle can have two right angles		
2.	A triangle can have two obtuse angles		
3.	Two angle of a triangle can be acute angles		
4.	A triangle can have all angles measuring less than 60^0		
5.	A triangle can be have all angles measuring more than 60^0		
6.	A triangle can have all angles that are exactly 60^0 .		

The Exterior Angles of a Triangle

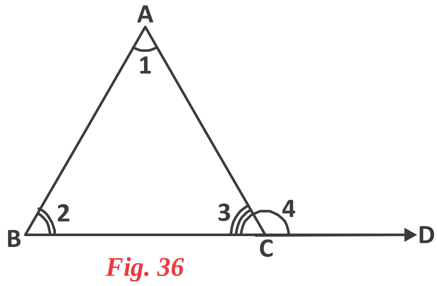


Fig. 36

In the given figure is a ΔABC whose side BC when extended to C forms an exterior angle $\angle ACD$

By the linear pair axiom we can say

$\angle 3 + \angle 4 = 180^o$ .....(1)

In Δ ABC, the sum of three interior angles is 180^0

Hence $\angle 1 + \angle 2 + \angle 3 = 180^o$ .....(2)

From equaction (1) and (2) we get,

$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

or $\angle 1 + \angle 2 = \angle 4$

This result can be written as the following theorem :

Theorem–3 : If the side of a triangle is extended, the exterior angle so formed is equal to the sum of the interior opposite angles.

This is known as the exterior angle theorem. It is also clear from this theorem that the exterior angle is greater than each of the interior opposite angles.

Statement-1 : Prove that the sum of the four interior angle of a quadrilateral ABC is 360^0

SOLUTION : Given that quadrilaterals ABCD has four interior angles $\angle A, \angle B, \angle C$ and $\angle D$ -

We have to prove

$\angle A + \angle B + \angle C + \angle D = 360^o$

As shown in the *Fig.*37 join A to C dividing the quadrilateral into two triangle ΔADC and ΔABC .

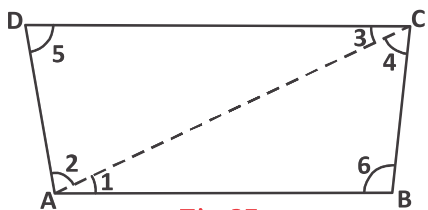


Fig. 37

By the angle sum property in ΔABC,

$\angle 1 + \angle 6 + \angle 4 = 180^o$ .....(1)

Similarly by angle sum property in ΔADC,

$\angle 2 + \angle 5 + \angle 3 = 180^o$ .....(2)

Adding equations (1) and (2) we get,

$\angle 1 + \angle 6 + \angle 4 + \angle 2 + \angle 5 + \angle 3 = 360^o$

or $(\angle 1 + \angle 2) + (\angle 3 + \angle 4) + \angle 5 + \angle 6 = 360^o$

or $\angle A + \angle C + \angle D + \angle B = 360^o$

That is $\angle A + \angle B + \angle C + \angle D = 360^o$

From the above example it is clear that you can find the sum of interior angle of any polygon by dividing it into triangles. For example :

Name of polygon	Number of triangles	Sum of interior angle
Quadrilateral	2	$2 \times 180^0 = 360^0$
Pentagon	3	$3 \times 180^0 = 540^0$
Hexagon	4	....
Octagon	....	....

We can now say that an *n* sides polygon can be divided into *n-2* triangles with common vertices, so the sum of the interior angles of a polygon with *n* sides

$= (n - 2) \times 180^o$

EXAMPLE-11. If the three angles of a triangle are $(2x+1)^\circ$, $(3x+6)^\circ$ and $(4x-16)^\circ$ respectively, find the measure of each angle.

SOLUTION : The sum of interior angles of a triangle is 180^0, hence

$$(2x+1)^\circ+(3x+6)^\circ+(4x-16)^\circ=180^0$$
$$\Rightarrow 9x-9=180^o$$
$$\Rightarrow 9x=189^o$$
$$\Rightarrow x=21^o$$

Hence $(2x+1)=(2\times 21^o+1)=43^o$

$(3x+6)=(3\times 21^o+6)=69^o$

$(4x-16)=(4\times 21^o-16)=68^o$

The angles are therefore 43^o, 69^o and 68^o respectively.

EXAMPLE-12. In the given figure $AB \parallel QR$, $\angle BAQ=142^o$ and $\angle ABP=100^o$ Find value of the following–

(i) $\angle APB$ (ii) $\angle AQR$ (iii) $\angle QRP$

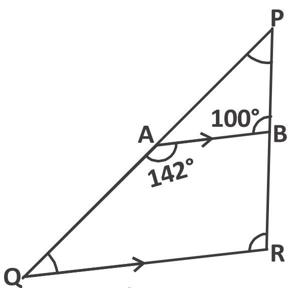


Fig. 38

SOLUTION : (i) The side PA of ΔAPB is extended till Q,

hence by exterior angle theorem,

$$\angle BAQ=\angle ABP+\angle APB$$
$$\Rightarrow 142^o=100^o+\angle APB$$
$$\Rightarrow \angle APB=142^o-100^o$$
$$\Rightarrow \angle APB=42^o$$

(ii) $\angle BAQ + \angle AQR = 180^o$

(Sum of cointerior angles is 180^0)

$\Rightarrow 142^o + \angle AQR = 180^o$

$\Rightarrow \angle AQR = 180^o - 142^o$

$\Rightarrow \angle AQR = 38^o$

(iii) As $AB \parallel QR$ and PR is a transversal, hence

$\angle QRP = \angle ABP$ (corresponding angles)

$\therefore \angle QRP = 100^o$

EXAMPLE-13. In the given figure if $BE \perp EC$, $\angle EBC = 40^o$, $\angle DAC = 30^o$ find the values of x and y.

SOLUTION : In ΔEBC

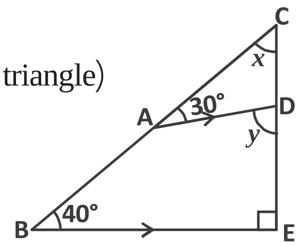


Fig. 39

$90^o + 40^o + x = 180^o$ (By the property sum of interior angles of a triangle)

$\Rightarrow 130^o + x = 180^o$

$\Rightarrow x = 50^o$ .....(1)

Now in ΔADC

$\angle ADE = \angle DAC + \angle ACD$ (By exterior angle theorem)

$\Rightarrow y = 30^o + x$

$\Rightarrow y = 30^o + 50^o$ (from equation (1))

$\therefore y = 80^o$

EXAMPLE-14. Find the value of x from the given *Fig.*40

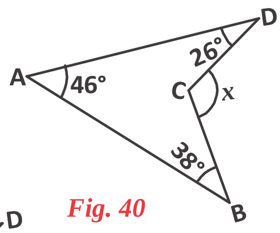


Fig. 40

SOLUTION : ABCD in the figure is a quadrilateral. Join AC and extend it to E as shown in *Fig.*41.

Assume $\angle DAE = p^o$

$\angle BAE = q^o, \angle DCE = z^o$ and

$\angle ECB = t^o$

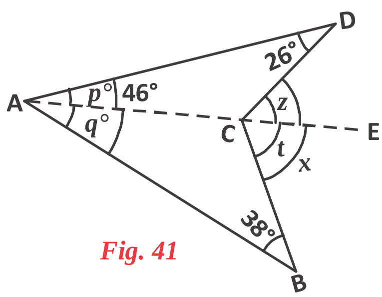


Fig. 41

By the exterior angle theorem in ΔACD

$$\angle DCE = \angle CAD + \angle ADC$$

$$z^o = p^o + 26^o \qquad .....(1)$$

Again in ΔABC

$$\angle BCE = \angle BAC + \angle ABC$$

$$t^o = q^o + 38^o \qquad .....(2)$$

Adding equations (1) and (2) we get

$$z^o + t^o = p^o + 26^o + q^o + 38^o$$

$$x = p + q + 64° \qquad \left[\begin{array}{l} \because z° + t° = x \\ p + q = 46° \end{array}\right]$$

$$x = 46° + 64°$$

$$x = 110°$$

EXAMPLE-15. In the given figure $\angle A = 40^o$. If BO and CO are the respective bisectors of $\angle B$ and $\angle C$ then find the measure of $\angle BOC$

SOLUTION : Say $\angle CBO = \angle ABO = x$ and $\angle BCO = \angle ACO = y$

($\because BO$ is bisector of $\angle B$ and CO is bisector of $\angle C$)

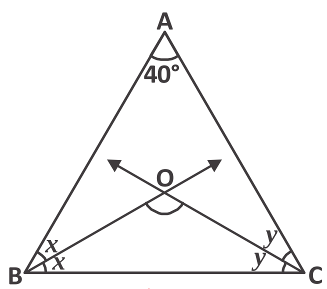


Fig. 42

Then , $\angle B = 2x, \angle C = 2y$

By angle sum property we can say,

$$\angle A + \angle B + \angle C = 180^o$$

$$\Rightarrow 40^o + 2x + 2y = 180^o \quad [\because \angle A = 40^o]$$

$$\Rightarrow 2(x + y) = 180^o - 40^o$$

$$\Rightarrow x + y = 70^o \qquad .....(1)$$

Again by angle sum property of ΔBOC,

$$x + \angle BOC + y = 180^o$$

$$\Rightarrow \angle BOC = 180^o - (x + y)$$

$$\Rightarrow \angle BOC = 180^o - 70^o \qquad \text{(From equation (1))}$$

$$\therefore \angle BOC = 110^o$$

EXAMPLE-16. In the given figure, the sides AB and AC of ΔABC are extended to E and D respectively. If the bisectores of $\angle CBE$ and $\angle BCD$, that is BO and CO respectively meet in point O, then prove that $\angle BOC = 90° - \frac{1}{2}\angle BAC$

SOLUTION : Ray BO bisects $\angle CBE$, hence

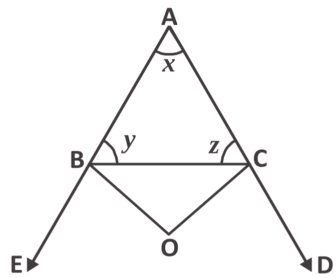


Fig. 43

$$\angle CBO = \frac{1}{2}\angle CBE$$

$$= \frac{1}{2}(180^\circ - y)$$

$$= 90^\circ - \frac{y}{2} \qquad .....(1)$$

Similarly CO bisects $\angle BCD$,

hence $\angle BCO = \frac{1}{2}\angle BCD$

$$= \frac{1}{2}(180^\circ - z) \quad = 90^\circ - \frac{z}{2} \qquad .....(2)$$

Now in ΔBOC, by angle sum property,

$$\angle BOC + \angle BCO + \angle CBO = 180^\circ \qquad .....(3)$$

Substituting (1) and (2) in (3) , we get

$$\angle BOC + 90^\circ - \frac{z}{2} + 90^\circ - \frac{y}{2} = 180^\circ$$

$$\Rightarrow \angle BOC + 180^\circ = 180^\circ + \frac{z}{2} + \frac{y}{2}$$

$$\therefore \angle BOC = \frac{1}{2}(z + y) \qquad .....(4)$$

Now in ΔABC by angle sum property,

$$x + y + z = 180^\circ$$

$$y + z = 180^\circ - x \qquad .....(5)$$

Substituting (5) in (4) we get,

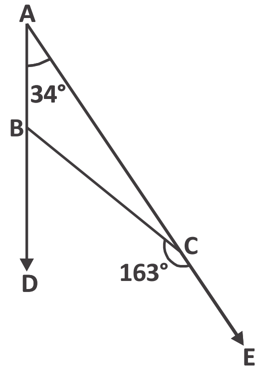
$$\angle BOC = \frac{1}{2}(180^\circ - x)$$

$$= 90^\circ - \frac{x}{2}$$

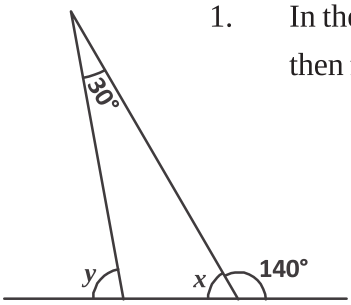
$$\angle BOC = 90^\circ - \frac{1}{2}\angle BAC$$

Exercise-9.3

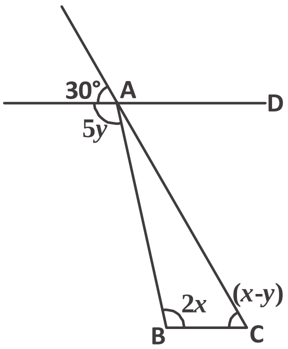
1. In the adjoining figure if $\angle BAC = 34^\circ$, $\angle BCE = 163^\circ$ then find measure of $\angle ACB$, $\angle ABC$ and $\angle DBC$.



2. Find the value of x and y from the given figures :

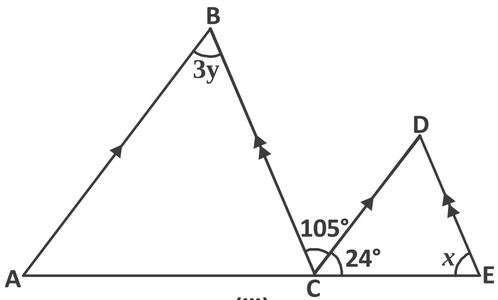


(i)



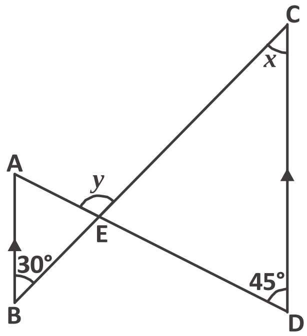
(ii)

(Hint AD || BC)



(iii)

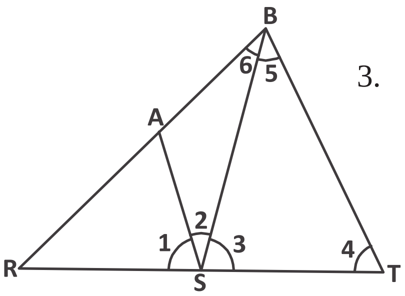
(Hint AB || CD, BC || DE)



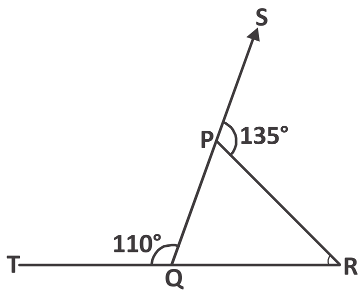
(iv)

(Hint AB || CD)

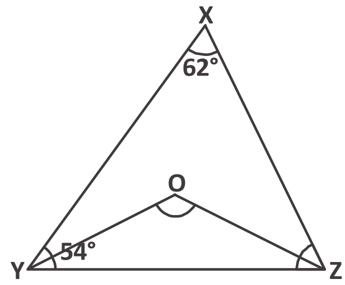
3. In the given figure $AS \parallel BT$, $\angle 4 = \angle 5$ and SB is an angle bisector of $\angle AST$, Find the value of $\angle 1$.

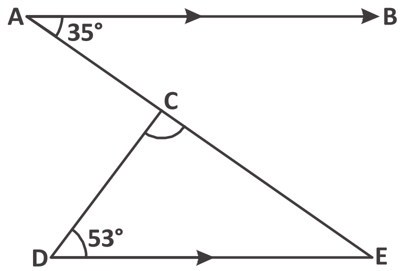


4. In the given figure, the side QP and RQ of the ΔPQR are extended to points S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find the measure of $\angle PRQ$.



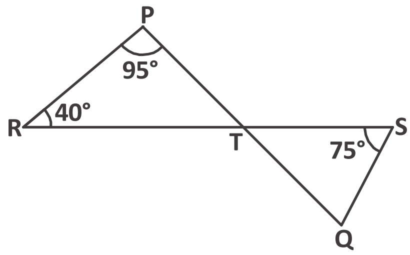
5. In the given figure $\angle ZXY = 62^o$ and $\angle XYZ = 54^o$. If YO of ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively in ΔXYZ, find value of $\angle OZY$ and $\angle YOZ$.

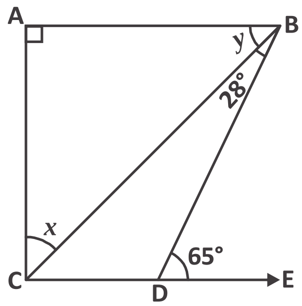




6. In the given figure $AB \parallel DE$, $\angle BAC = 35^o$ and $\angle CDE = 53^o$ find the measure of $\angle DCE$.

7. In the given figure lines PQ and RS intersect at a point T. If $\angle PRT = 40^o$, $\angle RPT = 95^o$ and $\angle TSQ = 75^o$, find value of $\angle SQT$

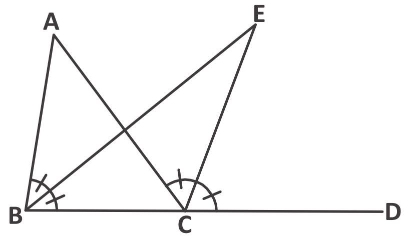




8. In the figure, $AB \perp AC$, and $AB \parallel CD$, if $\angle CBD = 28^o$, $\angle BDE = 65^o$, find the values of x and y

9. In the adjoining figure the side BC of ΔABC is extended upto D. If the bisectors of $\angle ABC$ and $\angle ACD$ meet in a point E then prove that

$$\angle BEC = \frac{1}{2}\angle BAC$$



(**Hint–** Angle sum of $\Delta ABC =$ Angles sum of ΔBEC and $\angle ACD = \angle BAC + \angle ABC$)

What Have We Learnt

1. If a ray stands on a straight line, the sum of the two adjacent angles so formed is 180^o and these are known as a linear pair.

2. If the sum of two adjacent angles is 180^o, then the sides which are not common, form a straight line.

(The above two axioms together are known as the linear pair axiom)

3. The vertically opposite angles formed by two intersecting lines are equal.
4. If a transversal cuts two parallel lines, then –
   (i) Each pair of corresponding angles are equal
   (ii) Each pair of alternate interior angles are equal.
   (iii) Each pair of cointerior angles on one side of the transversal are supplementary.
5. Two lines are parallel if the transversal which intersects is such that-
   (i) One pair of corresponding angles is equal or
   (ii) One pair of alternate interior angles is equal or
   (iii) One pair of cointerior angles on one side of the transversal is supplementary.
6. Those lines which are parallel to the same line, are parallel to each other.
7. The sum of the interior angles of a triangle is 180^{o}.
8. If any one side of a triangle is extended, the exterior angle so formed is equal to the sum of the interior opposite angles.