47. The Special Theory of Relativity

Short Answer

1. Question

The speed of light in glass is 2.0×10^8 m s^{-1}. Does it violate the second postulate of special relativity?

Answer

The second postulates describe that the speed of light (for medium vacuum) has same value in all inertial frame. Speed of light in any other medium will be less than that of vacuum using Snell's law. Snell's law is given by,

$$v = \frac{c}{\mu}$$

Where,

v=Speed of light in glass

c=Speed of light in vacuum

μ=Refractive index

Thus, it doesn't violets second postulate of the refractive index because the speed of light in glass is not more than that of vacuum.

2. Question

A uniformly moving train passes by a long platform. Consider the events 'engine crossing the beginning of the platform' and 'engine crossing the end of the platform'. Which frame (train frame or the platform frame) is the proper frame for the pair of events?

Answer

As the platform is stationary it is considered as the proper frame for this event.

The train travels with a speed which is very less as compared with the speed of light. So, it does not violate the second postulate of relativity.

Thus, both train and platform are proper frame.

3. Question

An object may be regarded to be at rest or in motion depending on the frame of reference chosen to view the object. Because of length contraction it would mean that the same rod may have two different lengths depending on the state of the observer. Is this true?

Answer

Let the speed of rod is v, the length is given by

$$l = l_0\sqrt{1-\left(\frac{v^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

Now assume that the observer and the rod are moving with the speed v in same direction. The length of the rod is given by,

$$l = l_0\sqrt{1-\left(\frac{(v-[-v])^2}{c^2}\right)}$$

$$l = l_0\sqrt{1-\left(\frac{4v^2}{c^2}\right)}$$

Thus, it is true that an object may be regarded to be at rest or in motion depending on the frame of reference chosen to view the object.

4. Question

Mass of a particle depends on its speed. Does the attraction of the earth on the particle also depend on the particle's speed?

Answer

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

The gravitational force of Earth is given by,

$$F = \frac{GMm_0}{r^2}$$

$$F = \frac{GMm_o}{r^2\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

Thus, both mass and the gravitational attraction depends on the particle's speed.

5. Question

A person travelling in a fast spaceship measures the distance between the earth and the moon. Is it the same, smaller or larger than the value quoted in this book?

Answer

There are two ways to measure the distance between the Earth and the Moon which are using meter scale and using light pulse.

In the first method the distance is measured using meter scale, the length gets contracted in moving frame. Thus, the length measured is smaller.

In second method, the time difference between emission of light pulse and reception is noted. So, the distance measured in this case is larger.

Thus, the distance measured is either smaller or larger than the actual distance.

Objective I

1. Question

The magnitude of linear momentum of a particle moving at a relativistic speed v is proportional to

A. v

B. $1 - v^2/c^2$

C. $\sqrt{1 - v^2 / c^2}$

D. none of these

Answer

The magnitude of linear momentum is given by,

P=mv

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

Thus, magnitude of linear momentum is,

$$p = \frac{m_o v}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

This value of the linear momentum does no match with any option.

Thus, correct option is D

2. Question

As the speed of a particle increases, its rest mass

A. increases

B. decreases

C. remains the same

D. changes.

Answer

As the mass is at rest, its value is not changed when observed with the frame moving with the velocity v.

Thus, correct option is C.

3. Question

An experimenter measures the length of a rod. Initially the experimenter and the rod are at rest with respect to the lab. Consider the following statements.

(A) If the rod starts moving parallel to its length but the observer stays at rest, the measured length will be reduced.

(B) If the rod stays at rest but the observer starts moving parallel to the measured length of the rod, the length will be reduced.

A. A is true but B is false.

B. B is true but A is false.

C. Both A and B are true

D. Both A and B are false.

Answer

The new length is given by,

$$l = l_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

As v << C

$\therefore \sqrt{1 - \left(\frac{v^2}{c^2}\right)} < 1$

Thus, $l < l_0$

Consider the rod at rest and observer moves with the speed of v then the new length is given by,

$$l = l_0\sqrt{1-\left(\frac{(-v)^2}{c^2}\right)}$$

$$l = l_0\sqrt{1-\left(\frac{(v)^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

As v << C

$\therefore \sqrt{1-\left(\frac{v^2}{c^2}\right)} < 1$

Thus, $l<l_0$

Thus, the length will be reduced in both the cases.

So, correct option is C.

4. Question

An experimenter measures the length of a rod. In the cases listed, all motions are with respect to the lab and parallel to the length of the rod. In which of the cases the measured length will be minimum?

A. The rod and the experimenter move with the same v in the same direction.

B. The rod and the experimenter move with the same speed v in opposite directions.

C. The rod moves at speed v but the experimenter stays at rest.

D. The rod stays at rest but the experimenter moves with the speed v.

Answer

The new length is given by,

$$l = l_0\sqrt{1-\left(\frac{v^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

Consider the rod at rest and observer moves with the speed of v parallel to the measured length then the new length is given by,

$$l = l_0\sqrt{1-\left(\frac{(-v)^2}{c^2}\right)}$$

$$l = l_0\sqrt{1-\left(\frac{(v)^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

Consider the rod at rest and observer moves with the speed of v opposite to the measured length then the new length is given by,

$$l = l_0\sqrt{1-\left(\frac{(v-[-v])^2}{c^2}\right)}$$

$$l = l_0\sqrt{1-\left(\frac{4v^2}{c^2}\right)}$$

As v << C

$\therefore \sqrt{1-\left(\frac{v^2}{c^2}\right)} < 1$

Thus, $l < l_0$

Thus, the length will be minimum for the rod moving in the opposite direction.

So, correct option is B.

5. Question

If the speed of a particle moving at a relativistic speed is doubled, its linear momentum will

A. become double

B. become more than double

C. remain equal

D. become less than double

Answer

The magnitude of linear momentum is given by, P=mv

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

Thus, magnitude of linear momentum is,

$$P = \frac{m_o v}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

$$P = m_0 v \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Using binomial expansion, we get,

$$P = m_0 v + \frac{m_0 v^3}{2c^2}$$

When the speed of the particle is doubled then,

$$P' = \frac{m_o 2v}{\sqrt{1 - \left(\frac{4v^2}{c^2}\right)}}$$

$$P = 2m_0 v + \frac{4m_0 v^3}{c^2}$$

Thus, the new value of linear momentum is more than double.

So, correct option is B.

6. Question

If a constant force acts on a particle, its acceleration will

A. remain constant

B. gradually decrease

C. gradually increase

D. be undefined

Answer

When the force acting on the particle is constant, the speed of the particle increases due to the force.

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

So, the mass of the particle increases, this results in decreasing the acceleration gradually.

Thus, correct option is B.

7. Question

A charged particle is projected at a very high speed perpendicular to a uniform magnetic field. The particle will

A. move along a circle

B. move along a curve with increasing radius of curvature

C. move along a curve with decreasing radius of curvature

D. move along a straight line.

Answer

The charged particle is projected at a very high speed perpendicular to the uniform magnetic field. The mass will increase using the relativistic relation.

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

So, its radius will increase.

Thus, correct option is B.

Objective II

1. Question

Mark the correct statements:

A. Equations of special relativity are not applicable for small speeds.

B. Equations of special relativity are applicable for all speeds.

C. Nonrelativistic equations give exact result for small speeds.

D. Nonrelativistic equations never give exact result.

Answer

Statement (a) is correct because the equation of special relativity is only applicable when the speed of object is comparable to the speed of light. Newton's equations of motion are used for small speeds.

When the body is travelling with the relativistic speed, then

mass becomes $m' = \gamma m$, length becomes $L' = \frac{L}{\gamma}$, change in time becomes $\Delta T' = \gamma \Delta T$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{v^2}{2c^2} + \dots\dots = \text{sum} << 1 \text{ when } v << c.$$

Still the sum of the series will be greater than 0

Hence, non-relativistic equations in which γ factor is taken to be exactly 1 never give exact results.

Option (B) and (D) are correct

2. Question

If the speed of a rod moving at a relativistic speed parallel to its length is doubled,

A. the length will become half of the original value

B. the mass will become double of the original value

C. the length will decrease

D. the mass will increase

Answer

If the speed of rod is relativistic speed then, its mas will be

$$m' = \gamma m = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{v^2}{2c^2} + \dots\dots > 1 \text{ as } v < c$$

And its length will be,

$$L' = \frac{L}{\gamma} = L\sqrt{1-\frac{v^2}{c^2}}$$

When speed v is doubled,

$$\gamma' = \frac{1}{\sqrt{1 - \frac{4v^2}{c^2}}}$$

$$\Rightarrow \gamma' = \left(1 - \frac{4v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{2v^2}{c^2} + \ldots\ldots\ldots > 2\gamma$$

$$and\ m' = \gamma' m > 2\gamma m, L' = \frac{l}{\gamma'} < \frac{l}{2\gamma}$$

Hence, mass will be increased and it will be more than double. Length will decrease but not exactly half of the original length.

Option (C) and (D) are correct.

3. Question

Two events take place simultaneously at points A and B as seen in the lab frame. They also occur simultaneously in a frame moving with respect to the lab in a direction.

A. parallel to AB

B. perpendicular to AB

C. making an angle of 45° with AB

D. making an angle of 135° with AB.

Answer

The two events occur at the same time at points A and B in rest frame. If they occur simultaneously in a frame moving with respect to the lab, the frame must be moving in the direction perpendicular to AB.

Option (b) is correct.

4. Question

Which of the following quantities related to an electron has a finite upper limit?

A. Mass

B. Momentum

C. Speed

D. Kinetic energy

Answer

If the electron is moving with very high-speed v, then its mass will increase by the factor γ i.e. $m' = \gamma m$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$,

Momentum, $p = m'v = \gamma mv$

Kinetic energy $k = \frac{1}{2}m'v^2 = \frac{1}{2}\gamma mv^2$

When the speed of electron is maximum i.e. v=c, then $\gamma = \infty$, also $m', p, k = \infty$

Therefore, the upper limit of speed is always less than c but mass, momentum and kinetic energy do not have any upper limit.

Option (c) is correct.

5. Question

A rod of rest length L moves at a relativistic speed. Let L' = L/γ. Its length

A. must be equal to L'

B. may be equal to L

C. may be more than L' but less than L

D. may be more than L.

Answer

the rest length of the rod= L

Relativistic speed = v

Length contraction= L'

$$L' = \frac{L}{\gamma} = L\sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

If the rod is moving with relativistic speed, then γ will be approximately equal to 1. This means that the length of the rod L' will also be almost equal to L. however, the length of rod in rest frame L, can be more than L' depending upon the frame of observer.

Option (B) and (c) are correct

6. Question

When a rod moves at a relativistic speed v, its mass

A. must increase by a factor of γ

B. may remain unchanged

C. may increase by a factor other than γ

D. may decrease

Answer

if the rod is moving with relativistic speed v, then mass will be given by,

$$m' = \gamma m = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{here, } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

thus, the mass will increase by the factor of γ

option (A) is correct

Exercises

1. Question

The *guru* of a *yogi* lives in a Himalayan cave, 1000 km away from the house of the yogi. The yogi claims that whenever he thinks about his guru, the guru immediately knows about it. Calculate the minimum possible time interval between the yogi thinking about the guru and the guru knowing about it.

Answer

The house of yogi is 1000km away from his guru in Himalayan cave. The velocity of travelling of the thoughts should be maximum so that it can reach to his guru in minimum time. The maximum velocity that can be attained is speed of light i.e. 3 x 10^8 m/sec.

Distance= 1000 km = 10^6 m

Velocity= 3 x 10^8 m/sec

$$\text{Time} = \frac{\text{Distance}}{\text{velocity}} = \frac{10^6}{3 \text{ x} 10^8} = \frac{1}{300} \text{ sec}$$

2. Question

A suitcase kept on a shop's rack is measured 50 cm × 25 cm × 10 cm by the shop's owner. A traveler takes this suitcase in a train moving with velocity 0.6c. If the suitcase is placed with its length along the train's velocity, find the dimensions measured by

(a) the traveler and

(b) a ground observer

Answer

(a) The traveler is traveling in the same frame in which suitcase is travelling. So, the dimensions of the suitcase for the traveler will remain same.

(b) The observer who is on the ground will not see the same dimensions of the suitcase. It is because the frame of reference of observer and the suitcase is different. The suitcase is in the frame of reference which is moving with the speed of 0.6c. Since, the suitcase is placed with its length along the train's velocity, the changes will be observed in the length only.

Length of the suitcase= 50cm

Breadth = 25cm

Height = 10cm

Velocity= 0.6c

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 50\sqrt{1-\frac{(0.6\ c)^2}{c^2}} = 50\sqrt{1-0.36} = 50 \text{ x } 0.8 = 40\text{cm}$$

Therefore, the length observed by the observer on the ground will be less than the original length of the suitcase. The dimensions will be 40cm x 25cm x 10cm.

3. Question

The length of a rod is exactly 1m when measured at rest. What will be its length when it moves at a speed of

(a) 3×10^5 m s^{-1}

(b) 3×10^6 m s^{-1} and

(c) 3×10^7 m $s^{-1?}$

Answer

Whenever the object moves with the speed comparable to the speed of light, the length of the object contracts. So, let us consider the following cases when the speed of rod is comparable to the speed of light.

(a) length of the rod= 1m

Speed of the rod= 3×10^5 m s^{-1}

Using the relativistic relation of length contraction

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 1\sqrt{1-\frac{9 \times 10^{10}}{9 \times 10^{16}}} = \sqrt{1-10^{-6}} = 0.99999995m$$

The length observed will be 0.99999995m

(b) length = 1m

Speed= 3×10^6 m s^{-1}

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 1\sqrt{1-\frac{9 \times 10^{12}}{9 \times 10^{16}}} = \sqrt{1-10^{-4}} = 0.99995m$$

(c) length= 1m

Speed = 3×10^7 m s^{-1}

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 1\sqrt{1-\frac{9 \times 10^{14}}{9 \times 10^{16}}} = \sqrt{1-10^{-2}} = 0.9949m$$

4. Question

A person standing on a platform finds that a train moving with velocity 0.6c takes one second to pass by him. Find

(a) the length of the train as seen by the person and

(b) the rest length of the train.

Answer

The velocity of the train, v = 0.6c

Time taken by the train to pass the man standing on platform, t= 1sec

(a) the length of the train as seen by the person, L' = vt = 0.6 x 3×10^8 = 1.8 x 10^8m

(b) the rest length of the train L

$$L' = L\sqrt{1-\frac{v^2}{c^2}}$$

$$1.8 \times 10^8 = L\sqrt{1-\frac{(0.6\ c)^2}{c^2}}$$

$$L = \frac{1.8 \times 10^8}{0.8} = 2.25 \times 10^8\ m/sec$$

5. Question

An Airplane travels over a rectangular field 100 m × 50m, parallel to its length. What should be the speed of the plane so that the field becomes square in the plane frame?

Answer

The field becomes square in the plane frame only when its length and breadth become equal. Originally, the field is in rectangular shape, with length = 100m and breadth = 50 m. In order to make the rectangular field into square field, either the breadth can be increased or length can be reduced. But we know that with the speed comparable to the speed of light, the object's length contracts. Hence, the only possible way to make the rectangular field into square field is that the plane must travel along the length of the field with the speed comparable to the speed of light.

$$L' = L\sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow 50 = 100\sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow \frac{1}{2} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow v = 0.866c$$

6. Question

The rest distance between Patna and Delhi is 1000 km. A nonstop train travels at 360 km h^{-1}.

(a) What is the distance between Patna and Delhi in the train frame?

(b) How much time elapses in the train frame between Patna and Delhi?

Answer

The distance between Patna and Delhi = 1000km

Speed of the train= 360 km h^{-1} =100 m/s

(a) Distance between the two cities in the train frame d

$$D' = D\sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow D' = 10^6\sqrt{1-\frac{10^4}{9 \text{ x } 10^{16}}}$$

$\Rightarrow D' = 10^9$

(b) time elapses in the train frame

$$\Delta t = \frac{\Delta L}{v}$$

To solve for ΔL

$$L' = L\sqrt{1 - \frac{v^2}{c^2}}$$

$$L' = 10^6 \sqrt{1 - \frac{100^2}{c^2}}$$

$\Rightarrow L' = 56 \text{ x } 10^{-9}\text{ m}$

Change in length ΔL = 56 nm

therefore, the distance between Patna and Delhi in the train frame is 56 nm less than 1000 km.

now, actual time taken by train= time = distance/ speed

$$t = \frac{1000 \text{ x } 10^3}{100} = 10^4 \text{ sec}$$

change in time, $\Delta T = \frac{\Delta L}{v} = \frac{56 \text{ x } 10^{-9}}{100} = 0.56\text{ns}$

So, the time lapse in the train between Patna and Delhi will be 0.56ns less than 10^4 sec

7. Question

A person travels by a car at a speed of 180 km h^{-1}. It takes exactly 10 hours by his wristwatch to go from the station A to the station B.

(a) What is the rest distance between the two stations?

(b) How much time is taken in the road frame by the car to go from the station A to the station B?

Answer

Speed of the car= 180 km h^{-1}= 50 m/sec

Time taken by car to reach from station A to station B = 10 hours

The distance travelled by person in a car L'= speed X time= 180 x 10= 1800km

$$L' = L\sqrt{1 - \frac{v^2}{c^2}}$$

$$1800 \; = L\sqrt{1 - \frac{(180)^2}{9 \text{ x } 10^{16}}}$$

$$\Rightarrow L = 1800 + 25 \text{ x } 10^{-9}$$

So, the rest distance between the two station is 25nm more than 1800km.

(b) time taken by car to cover the distance in the road= distance/ speed

$$\Rightarrow \frac{1.8 \text{ X } 10^6 + \; 25 \text{ x } 10^{-9}}{50}$$

$$\Rightarrow 0.36 \text{ x } 10^5 + 5\text{x}10^{-8} \; = \; 10 \text{ hours } 0.5 \text{ ns}$$

8. Question

A person travels on a spaceship moving at a speed of 5c/13.

(a) Find the time interval calculated by him between the consecutive birthday celebrations of his friend on the earth.

(b) Find the time interval calculated by the friend on the earth between the consecutive birthday celebrations of the traveler.

Answer

The speed of spaceship =5c/13

(a) consecutive birthday celebration means the time period t= 1 year.

But the time interval observed by the person is spacecraft will be

$$t' \; = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow t' \; = \frac{1}{\sqrt{1 - \frac{(5c/13\,)^2}{c^2}}}$$

$$\Rightarrow t' \; = \frac{1}{\sqrt{1 - \frac{25}{169}}}$$

$$\Rightarrow t' \; = \frac{13}{12} \text{years}$$

Thus, the time interval observed by the person will be more than one year i.e. 1.08 years (approx.)

(b) The person on Earth will also calculate the same time interval because for him also, the person in spacecraft is travelling with same speed.

9. Question

According to the station clocks, two babies are born at the same instant, one in Howrah and other in Delhi.

(a) Who is elder in the frame of 2301 UP Rajdhani Express going from Howrah to Delhi?

(b) Who is elder in the frame of 2302 Dn Rajdhani Express going from Delhi to Howrah.

Answer

Both the stations are in ground frame; hence the station clocks will record the proper time interval. But the clocks in train will record improper time because they are at different places travelling with different speed. The proper time interval ΔT is less than improper i.e. $\Delta T' = v\Delta T$. Hence, in case (a) In the train going from Howrah to Delhi, the baby born in Delhi will be elder. (b) In the train going from Delhi to Howrah, Howrah baby will be elder.

10. Question

Two babies are born in a moving train, one in the compartment adjacent to the engine and other in the compartment adjacent to the guard. According to the train frame, the babies are born at the same instant of time. Who is elder according to the ground frame?

Answer

Since, the frame is moving, the clocks will not record the synchronized time. The clock at the rear end leads the clock which is at the other end by $L\,v/c^2$ where, L is the rest separation between the clocks and v is the speed of the moving frame i.e. train. Thus, the baby born adjacent to the guard cell will be elder.

11. Question

Suppose Swarglok (heaven) is in constant motion at a speed of 0.9999c with respect to the earth. According to the earth's frame, how much time passes on the earth before one day passes on Swarglok?

Answer

velocity of Swarglok with respect to Earth = 0.9999c

According to the Earth' frame the velocity will be

$$v' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \ v' = \frac{1}{\sqrt{1 - \frac{(0.9999c)^2}{c^2}}} = \frac{1}{0.01414} = 70.712$$

Let one day on Earth = ΔT

One day on heaven= $\Delta T'$

$\Delta\, T' = v\Delta T$

$\Rightarrow \ \Delta T' = 70.71$ days in heaven

12. Question

If a person lives on the average 100 years in his rest frame, how long does he live in the earth frame if he spends all his life on a spaceship going at 60% of the speed of light.

Answer

As the Lorentz transformation suggests there will be time dilation for a moving object with respect to rest frame which is given by γT_o

Where $\gamma = \frac{1}{\sqrt{1 - V^2/c^2}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ m/s$

And T_o is time in rest frame.

ATQ, $V = 60\ \%$ of C i.e. $\frac{V}{C} = \frac{3}{5}$

$$\therefore \ \gamma = \frac{1}{\sqrt{1 - (\frac{3}{5})^2}} = \frac{5}{4}$$

$\therefore$ The person will live $\frac{5}{4} \times 100\ years = 125\ years$ in the spaceship with respect to earth frame.

13. Question

An electric bulb, connected to a make and break power supply, switches off and on every second in its rest frame. What is the frequency of its switching off and on as seen from a spaceship travelling at a speed 0.8c?

Answer

As the Lorentz transformation suggests there will be time dilation for a moving object with respect to rest frame which is given by γT_o

Where $\gamma = \frac{1}{\sqrt{1 - V^2/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8 \; {}^{m}/_{s}$

And T_o is time in rest frame.

ATQ. $\frac{V}{C} = \frac{4}{5}$

$$\therefore \; \gamma = \frac{1}{\sqrt{1 - (\frac{4}{5})^2}} = \frac{5}{3}$$

∴ Time taken by the bulb to switch off and switch on with respect to spaceship will be $\frac{5}{3} \times 1s$

∴ Frequency will be $\frac{1}{T} = \frac{3}{5} \; Hz$

14. Question

A person travelling by a car moving at 100 km h^{-1} finds that his wristwatch agrees with the clock on a tower A. By what amount will his wristwatch lag or lead the clock on another tower B, 1000 km (in the earth's frame) from the tower A when the car reaches there?

Answer

Time taken by the car to reach the tower B from tower A $= \frac{1000}{100} = 10 \; Hour$.

As the frame of wrist watch is moving ∴ the wrist watch will lag by $\gamma T_o - T_o$.

Where $\gamma = \frac{1}{\sqrt{1 - V^2/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8 \; {}^{m}/_{s}$

And T_o is time in rest frame .

V=$100 \; {}^{Km}/_{hr} = 100 \times \frac{5}{18} \, {}^{m}/_{s} = 27.78 \; m/s$

$\frac{V}{C} = \frac{27.78}{3 \times 10^8}$ as $\frac{V}{C} << 1$ using binomial expansion we know we can write

$(1 + x)^n = 1 + nx$ when $x << 1$

Similarly, $\gamma = \frac{1}{\sqrt{1 - (\frac{27.78}{3 \times 10^8})^2}} = (1 - (\frac{27.78}{3 \times 10^8})^2)^{-\frac{1}{2}} = 1 + \frac{1}{2} \times 86.0254 \times 10^{-16}$

$\therefore T_o(\gamma - 1) = 10\ hour \times 43 \times 10^{-16} = 1.548 \times 10^{-12} s$

$Since\ 1\ hour = 3600\ s$

And as we can see the calculated value is too small to make drastic affect that's why we don't face any trouble in real life situation.

15. Question

At what speed the volume of an object shrinks to half its rest value?

Answer

As we know the moving object appears to be contracted with the earth aur rest frame that's because of Lorentz contraction known as length contraction which is given by $l = \frac{l_o}{\gamma}$

Where $\gamma = \frac{1}{\sqrt{1-V^2/C^2}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ m/s$

And l_o is length at rest .

As volume always include power of 3 weather it is cube sphere we will take a simple case of cube where all of its length is given by l_o at rest

$\therefore$ Volume at rest is $= {l_o}^3$

ATQ Volume seems to be half at certain speed

$$\therefore l^3 = \frac{{l_o}^3}{2}$$

$$\Rightarrow l = \frac{l_o}{\sqrt[3]{2}} = 0.794 l_o$$

$$\therefore \frac{l_o}{\gamma} = 0.794 l_o$$

$$\Rightarrow \gamma = 1.26$$

And $1.26 = \frac{1}{\sqrt{1-V^2/C^2}}$

$$\Rightarrow \sqrt{1 - V^2/C^2} = 0.63$$

$$\Rightarrow 0.604 \times C^2 = V^2$$

$$\therefore V = 0.78C$$

16. Question

A particular particle created in a nuclear reactor leaves a 1 cm track before decaying. Assuming that the particle moved at 0.995c, calculate the life of the particle

(a) in the lab frame and

(b) in the frame of the particle.

Answer

a)We know time taken = $\frac{Distance\ traversed}{Speed\ of\ the\ particle}$

$$\therefore life\ of\ particle\ will\ be = \frac{1 \times 10^{-2}\ m}{0.995 \times 3 \times 10^{8}\ {}^{m}/_{s}} = 3.35 \times 10^{-11} s$$

b)In the frame of particle there will be length contraction or we can say time dilation we can solve the question with anyone of the approach.

As the Lorentz transformation suggests there will be time dilation for a moving object with respect to rest frame which is given by γT_o

Where $\gamma = \frac{1}{\sqrt{1-{}^{V^2}/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^{8}\ {}^{m}/_{s}$

And T_o is time in rest frame.

$\therefore T_o$ was found in previous case as $3.35 \times 10^{-11} s$

$$\therefore Life\ of\ particle\ in\ particle\ frame\ will\ be = T_o \times \frac{1}{\sqrt{1-{}^{V^2}/_{C^2}}}$$

$$\Rightarrow 3.35 \times 10^{-11} s \times \frac{1}{\sqrt{1-{}^{(0.995C)^2}/_{C^2}}} = 3.35 \times 10^{-11} s \times 10$$

$$= 3.35 \times 10^{-10} s$$

17. Question

By what fraction does the mass of a spring change when it is compressed by 1 cm? The mass of the spring is 200g at its natural length and the spring constant is 500 N m^{-1}.

Answer

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

∴ energy possessed by the compressed spring as potential energy is due to loss of the mass

$\therefore \frac{1}{2}kx^2 = mC^2$

Given:

$k = 500\ {}^N/_m$

$x = 1\ cm = 10^{-2}\ m$

$\Rightarrow \frac{1}{2} \times 500 \times (10^{-2}) = m \times 9 \times 10^{16}$

$\therefore m = 2.78 \times 10^{-19} Kg$

So, the change in mass of spring will be $2.78 \times 10^{-19} Kg$

18. Question

Find the increase in mass when 1 kg of water is heated from 0°C to 100°C. Specific heat capacity of water = 4200 J kg^{-1} K^{-1}.

Answer

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

∴ Heat energy given will be converted in the mass

$\Delta Q = mc\Delta T = \Delta mC^2$

Given:

c =Specific heat capacity of water = 4200 J kg^{-1} K^{-1}.

$m = 1kg$

$\Delta T = 373K - 273K = 100K$

$\therefore 1 \times 4200 \times 100 = \Delta m \times 9 \times 10^{16}$

$\Rightarrow \Delta m = \frac{4.2 \times 10^5}{9 \times 10^{16}} = 4.7 \times 10^{-12} Kg$

19. Question

Find the loss in the mass of 1 mole of an ideal monatomic gas kept in a rigid container as it cools down by 10°C. The gas constant R = 8.3 J K^{-1} mol^{-1}.

Answer

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

∴ Heat energy extracted will result in loss of the mass

$$\Delta Q = C_V \Delta T = \Delta m C^2$$

Given:

$$C_V = \frac{3}{2} R \ where \ R = 8.314 \ {}^{J}/_{K.mol}$$

$$\Delta T = 10K$$

$$\therefore \frac{3}{2} \times 8.314 \times 10 = \Delta m \times 9 \times 10^{16}$$

$$\Longrightarrow \Delta m = 13.86 \times 10^{-16} Kg$$

20. Question

By what fraction does the mass of a boy increase when he starts running at a speed of 12 km h^{-1}?

Answer

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

∴Gain in kinetic energy by the boy is possessed by the mass gain for the boy.

$$\therefore \frac{1}{2} \times m \times V^2 = \Delta m C^2$$

Given:

V=12 ${}^{km}/_{h} = 12 \times \frac{5}{18} {}^{m}/_{s}$

$$\therefore \frac{1}{2} \times m \times (12 \times \frac{5}{18})^2 = \Delta m \times 9 \times 10^{16}$$

$$\Longrightarrow \frac{\Delta m}{m} = \frac{1}{2} \times \frac{1}{9 \times 10^{16}} \times (12 \times \frac{5}{18})^2$$

$$\Longrightarrow \frac{\Delta m}{m} = 6.17 \times 10^{-17}$$

21. Question

A 100W bulb together with its power supply is suspended from a sensitive balance. Find the change in the mass recorded after the bulb remains on for 1 year.

Answer

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

∴ Loss in the mass due to continuous illumination of bulb

ATQ, the bulb illuminates for 1 year i.e $365 \times 24 \times 3600\ s$ with power of 1 W or 1 $^{J}/_{s}$.

Total energy dissipated by the bulb in the form of light energy will

be $100 \times 365 \times 24 \times 3600\ J = 3.15 \times 10^9 J$

∴ $3.15 \times 10^9 J = \Delta m \times 9 \times 10^{16}$

$\Rightarrow \Delta m = 3.5 \times 10^{-8} kg$

22. Question

The energy from the sun reaches just outside the earth's atmosphere at a rate of 1400 W m^{-2}. The distance between the sun and the earth is 1.5×10^{11} m.

(a) Calculate the rate at which the sun is losing its mass.

(b) How long will the sun last assuming a constant decay at this rate? The present mass of the sun is 2×10^{30} kg.

Answer

a)

The amount of energy radiated by the sun in $1m^2 = 1400\ W$

As the energy radiated is dependent on the surface area it covers which is given by $4 \times \pi \times r^2$

∴ Energy loosed by the sun till it reaches earth in 1 second is given by $4 \times \pi \times (1.5\ \times 10^{11}\ \text{m})^2 \times 1400 = 3.96 \times 10^{26}\ J$

So, the mass loosed by the sun in 1 second is $\frac{3.96\times10^{26}}{9\times10^{16}} = 4.4 \times 10^9 Kg$

b)

As the rate of decay of the mass is constant it will last up to $\frac{Total\ mass\ of\ sun}{Rate\ at\ which\ mass\ is\ decaying} = \frac{2\times10^{30}\ Kg}{4.4\times\frac{10^9 Kg}{s}} = 4.55 \times 10^{10}\ s$

In years $\frac{4.55\times10^{10}\ s}{365\times24\times3600\ s} = 1.44 \times 10^3\ Years$

23. Question

An electron and a positron moving at small speeds collide and annihilate each other. Find the energy of the resulting gamma photon.

Answer

As we know electron and proton has same mass i.e. $9.1 \times 10^{-31} Kg$

As they annihilate and produces gamma particle with some energy

$\Delta m = 2 \times 9.1 \times 10^{-31} Kg$

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E= $\Delta m C^2$

$$\therefore Energy\ of\ gamma\ particle = 2 \times 9.1 \times 10^{-31} Kg \times 9 \times 10^{16} \frac{m}{s}$$
$$= 1.66 \times 10^{-13} J$$

24. Question

Find the mass, the kinetic energy and the momentum of an electron moving at 0.8c.

Answer

As we know the moving object appears to be heavier in the moving frame that's because of Lorentz transformation known as apparent mass which is given by $m = m_o \gamma$

Where $\gamma = \frac{1}{\sqrt{1 - V^2/C^2}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ m/s$

And m_o is mass at rest

$$\therefore\ \gamma = \frac{1}{\sqrt{1 - (0.8C)^2/C^2}} = \frac{1}{0.6}$$

$$\therefore Apparent\ mass\ = \frac{9.1 \times 10^{-31} Kg}{0.6} = 1.52 \times 10^{-30}\ Kg$$

Kinetic energy gained by the electron will be due to change in mass of electron.

$(m - m_o)C^2 = (15.2 - 9.1) \times 10^{-31} \times 9 \times 10^{16} = 5.45 \times 10^{-15} J$

Momentum of electron will be is equal to = Velocity$\times$ apparent mass

$\Rightarrow 1.52 \times 10^{-30}\ Kg \times 0.8 \times 3 \times 10^8\ m/s = 3.65 \times 10^{-22}\ kg.m/s$

25. Question

Through what potential difference should an electron be accelerated to give it a speed of

(a) 0.6c, (b) 0.9c and (c) 0.99 ?

Answer

a) As we know the moving object appears to be heavier in the moving frame that's because of Lorentz transformation known as apparent mass which is given by $m = m_o\,\gamma$

Where $\gamma = \frac{1}{\sqrt{1 - V^2/C^2}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ m/s$

And m_o is mass at rest

$$\therefore\ \gamma = \frac{1}{\sqrt{1 - (0.6C)^2/C^2}} = \frac{1}{0.8}$$

$$\therefore Apparent\ mass\ = \frac{9.1 \times 10^{-31} Kg}{0.8} = 11.375 \times 10^{-31}\ Kg$$

Kinetic energy gained by the electron will be due to change in mass of electron.

$$(m - m_o)C^2 = (11.375 - 9.1) \times 10^{-31} \times 9 \times 10^{16} = 20.475 \times 10^{-15} J$$

As the gain in kinetic energy is given by charge $\times$ potential difference applied i.e. q $\times V$

As $q = 1.6 \times 10^{-19} c$

$$\therefore\ 1.6 \times 10^{-19} c \times V = 20.475 \times 10^{-15} J$$

$$\Longrightarrow V = 12.8 \times 10^4\ V$$

b)Similarly for 0.9c

$$\gamma = \frac{1}{\sqrt{1 - (0.9C)^2/C^2}} = \frac{1}{0.44}$$

$$\therefore Apparent\ mass\ = \frac{9.1 \times 10^{-31} Kg}{0.44} = 20.68 \times 10^{-31}\ Kg$$

Kinetic energy gained by the electron will be due to change in mass of electron.

$$(m - m_o)C^2 = (20.68 - 9.1) \times 10^{-31} \times 9 \times 10^{16} = 1 \times 10^{-13} J$$

As the gain in kinetic energy is given by charge $\times$ potential difference applied i.e. q $\times V$

As $q = 1.6 \times 10^{-19} c$

$$\therefore\ 1.6 \times 10^{-19} c \times V = 1 \times 10^{-13} J$$

$\Rightarrow V = 6.25 \times 10^5\ V$

c) Similarly for 0.99c

$$\gamma = \frac{1}{\sqrt{1 - {(0.99C)^2}/{C^2}}} = \frac{1}{0.1}$$

$$\therefore Apparent\ mass\ = \frac{9.1 \times 10^{-31} Kg}{0.1} = 9.1 \times 10^{-30}\ Kg$$

Kinetic energy gained by the electron will be due to change in mass of electron.

$$(m - m_o)C^2 = (91 - 9.1) \times 10^{-31} \times 9 \times 10^{16} = 7.37 \times 10^{-13} J$$

As the gain in kinetic energy is given by charge $\times$ potential difference applied i.e. q $\times V$

As $q = 1.6 \times 10^{-19} c$

$\therefore\ 1.6 \times 10^{-19} c \times V = 7.37 \times 10^{-13} J$

$\Rightarrow V = 4.6 \times 10^6\ V$

26. Question

Find the speed of an electron with kinetic energy

(a) 1 eV, (b) 10 keV and (c) 10 MeV.

Answer

We can't deal the question with classical mechanics approach as $\frac{1}{2} \times m_e \times v^2$ as when you calculate v with this approach it will come out to be more than speed of light in vacuum i.e. $3 \times 10^8\ {m}/{s}$ that violates the special theory of relativity.

So we have to solve these types of question taking relativistic approach.

As we know the gain in Kinetic energy is due to change in mass

As we know the moving object appears to be heavier in the moving frame that's because of Lorentz transformation known as apparent mass which is given by $m = m_o\, \gamma$

Where $\gamma = \frac{1}{\sqrt{1 - {V^2}/{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ {m}/{s}$

And m_o is mass at rest i.e.= $9.1 \times 10^{-31} Kg$

K.E= $m_o(\gamma - 1)C^2$

$\therefore m_o \times C^2 = 9.1 \times 10^{-31} \times 9 \times 10^{16} = 8.19 \times 10^{-14} kg\ {m^2}/{s^2}$

a) So 10for 1 eV i.e. $1.6 \times 10^{-19} J$

$8.19 \times 10^{-14} kg\ {m^2}/{s^2} \times (\gamma - 1) = 1.6 \times 10^{-19} J$

$\Longrightarrow\ \gamma - 1 = 1.95 \times 10^{-6}$

$\Longrightarrow\ \gamma = 1 + 1.95 \times 10^{-6}$

As $\gamma = \frac{1}{\sqrt{1 - {V^2}/{C^2}}}$

Squaring both side and using binomial as we know using binomial expansion we know we can write

$(1 + x)^n = 1 + nx$ when $x << 1$

$$\therefore 1 + 3.9 \times 10^{-6} = \frac{1}{1 - {V^2}/{C^2}}$$

$$\Longrightarrow 1 - {V^2}/{C^2} = \frac{1}{1 + 3.9 \times 10^{-6}}$$

$\Longrightarrow V^2 = C^2 \times 3.9 \times 10^{-6}$

V=$6 \times 10^5\ m/s$

b) Similarly for 10KeV i.e. $1.6 \times 10^{-15} J$

$8.19 \times 10^{-14} kg\ {m^2}/{s^2} \times (\gamma - 1) = 1.6 \times 10^{-15} J$

$\Longrightarrow\ \gamma - 1 = 1.95 \times 10^{-2}$

$\Longrightarrow\ \gamma = 1 + 1.95 \times 10^{-2}$

As $\gamma = \frac{1}{\sqrt{1 - {V^2}/{C^2}}}$

Squaring both side and using binomial as we know using binomial expansion we know we can write

$(1 + x)^n = 1 + nx$ when $x << 1$

$$\therefore 1 + = \frac{1}{1 - {V^2}/{C^2}}$$

$$\Longrightarrow 1 - {V^2}/{C^2} = \frac{1}{1 + 3.8 \times 10^{-2}}$$

$\Rightarrow V^2 = C^2 \times 3.9 \times 10^{-2}$

V=$6 \times 10^7\ m/s$

c) Similarly for 10KeV i.e. $1.6 \times 10^{-12} J$

$8.19 \times 10^{-14} kg\ m^2/_{s^2} \times (\gamma - 1) = 1.6 \times 10^{-12} J$

$\Rightarrow \gamma - 1 = 19.5$

$\Rightarrow \gamma = 20.5$

As $\gamma = \frac{1}{\sqrt{1 - V^2/_{C^2}}}$

$\therefore 420.5 = \frac{1}{1 - V^2/_{C^2}}$

$\Rightarrow 1 - V^2/_{C^2} = 0.0024$

$\Rightarrow V^2 = C^2 \times 0.9976$

$\therefore V = 0.998C$

27. Question

What is the kinetic energy of an electron in electron volts with mass equal to double its rest mass?

Answer

As we know that Kinetic energy gained by the electron is due to change in mass of electron.

ATQ mass becomes double $\therefore \Delta m = 2m_o - m_o$

And m_o is mass at rest i.e.= $9.1 \times 10^{-31} Kg$

As E=K.E=$\Delta m C^2$=$m_o \times 9 \times 10^{16} = 9.1 \times 10^{-31} \times 9 \times 10^{16} = 8.2 \times 10^{-14} J$

28. Question

Find the speed at which the kinetic energy of a particle will differ by 1% from its nonrelativistic value 1/2 m_0v^2.

Answer

As we know that Kinetic energy gained by the electron is due to change in mass of electron.

As E=K.E=$\Delta m C^2$

As we know relativistic Energy is always more than non-relativistic kinetic energy because Rest energy is always less then apparent mass energy

∴The relativistic value of Kinetic energy will be $\frac{101}{200} \times m_o \times V^2$

As we know the moving object appears to be heavier in the moving frame that's because of Lorentz transformation known as apparent mass which is given by $m = m_o\,\gamma$

Where $\gamma = \frac{1}{\sqrt{1-V^2/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ {}^{m}/_{s}$

K.E= $m_o(\gamma - 1)C^2 = \frac{101}{200} \times m_o \times V^2$

$$\Rightarrow \frac{1}{\sqrt{1 - V^2/_{C^2}}} - 1 = \frac{101}{200} \times m_o \times \frac{V^2}{C^2}$$

Let $\frac{V^2}{C^2} = K$

$$\therefore \frac{1}{\sqrt{1-K}} - 1 = \frac{101}{200} \times K$$

$$\Rightarrow \frac{1}{\sqrt{1-K}} = \frac{301}{200} \times K$$

Squaring both side we get

$$\Rightarrow \frac{1}{1-K} = 2.265 \times K^2$$

$$\Rightarrow 1 = 2.265K^2 - 2.265K^3$$

As K=$\frac{V^2}{C^2}$, $K^3 = \frac{V^6}{C^6}$ which is $<< 1$ ∴we can neglect K^3 *term.*

$$\Rightarrow K^2 = \frac{1}{2.265} \Rightarrow K = 0.441$$

$$\Rightarrow \frac{V^2}{C^2} = 0.441$$

$$\therefore V = 0.66\,C$$