**CBSE Test Paper 04**

**Chapter 8 Introduction to Trigonometry**

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1. $\sin^2 A + \sin^2 A \tan^2 A$ = **(1)**

   a. $tan^2 A$
   b. $cos^2 A$
   c. None of these
   d. $sin^2 A$

2. If $\sin\theta = \frac{5}{13}$ then $cos\theta$ = **(1)**

   1. $\frac{13}{12}$
   2. $\frac{\sqrt{5}}{13}$
   3. $\frac{12}{5}$
   4. $\frac{12}{13}$

3. If $\sin A + \sin^2 A = 1$, then the value of $\cos^2 A + \cos^4 A$ is **(1)**

   1. – 1
   2. 2
   3. 0
   4. 1

4. If $cot\ \theta$ = $\frac{7}{8}$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is **(1)**

   1. $\frac{64}{49}$
   2. $\frac{7}{8}$
   3. $\frac{8}{7}$
   4. $\frac{49}{64}$

5. If $\sin\alpha = \frac{1}{\sqrt{2}}$ and tan β = 1, then the value of $\cos(\alpha + \beta)$ is **(1)**

   1. 3
   2. 1
   3. 2
   4. 0

6. Evaluate: $\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ}$. **(1)**

7. Prove the trigonometric identity: $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$ **(1)**

8. In $\triangle$ABC, right angle at B, if AB = 12 cm and BC = 5 cm, find sin C and cot C. **(1)**

9. Evaluate: $\frac{\sec 49^\circ}{\cos ec 41^\circ}$ **(1)**

10. Without using trigonometric tables, prove that: $\cos 81^0 - \sin 9^0 = 0$ **(1)**

11. If $5 \sin \theta + 3 \cos \theta = 4$, find the value of $3 \sin \theta - 5\cos\theta$. **(2)**

12. Prove that: $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\sec\theta+1}{\sec\theta-1}$ **(2)**

13. Solve the equation for $\theta$ : $\frac{\cos^2\theta}{\cot^2\theta-\cos^2\theta} = 3$. **(2)**

14. If $\cos\theta = \frac{12}{13}$, show that $\sin\theta(1-\tan\theta) = \frac{35}{156}$ **(3)**

15. Find the acute angle $\theta$, when $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$. **(3)**

16. If $1 + \sin^2\theta = 3\sin\theta \cos\theta$ , then prove that $\tan\theta = 1$ or $\frac{1}{2}$. **(3)**

17. Evaluate: $\frac{\sec 41^\circ \sin 49^\circ + \cos 29^\circ \; cosec \; 61^\circ - \frac{2}{\sqrt{3}}(\tan 20^\circ \tan 60^\circ \tan 70^\circ)}{3\left(\sin^2 31^\circ + \sin^2 59^\circ\right)}$. **(3)**

18. The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top increases by 15°. Find the height of the tower. **(4)**

19. Prove the trigonometric identity:
$\left(\frac{1}{\sec^2\theta-\cos^2\theta} + \frac{1}{cosec^2\theta-\sin^2\theta}\right)\sin^2\theta\cos^2\theta = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$ **(4)**

20. Evaluate without using trigonometric tables:
$\frac{\cos ec^2(90^\circ-\theta)-\tan^2\theta}{4(\cos^2 48^\circ+\cos^2 42^\circ)} - \frac{2\tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\cos ec^2 70^\circ - \tan^2 20^\circ}$ **(4)**

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### Solution

1. a. $tan^2 A$

   **Explanation:** Given: $\sin^2 A + \sin^2 A \tan^2 A$

   = $\sin^2 A \left(1 + \tan^2 A\right)$

   = $\sin^2 A \left(\sec^2 A\right)$

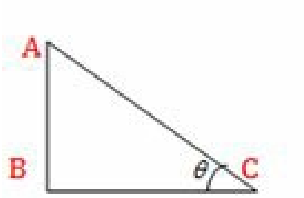
   = $\sin^2 A \times \frac{1}{\cos^2 A}$

   = $\frac{\sin^2 A}{\cos^2 A}$

   = $\tan^2 A$

2. d. $\frac{12}{13}$

   **Explanation:** Let AB = 5k and AC = 13k

   $\therefore$ BC = $\sqrt{(13k)^2 - (5k)^2}$ = $\sqrt{169k^2 - 25k^2}$

   $\Rightarrow$ BC = $\sqrt{144k^2}$ = 12k

   $\therefore \cos\theta = \frac{BC}{AC}$ = $\frac{12k}{13k} = \frac{12}{13}$

3. d. 1

   **Explanation:** Given: $\sin A + \sin^2 A = 1$

   $\Rightarrow \sin A = 1 - \sin^2 A$

   $\Rightarrow \sin A = \cos^2 A$

   Squaring both sides, we get

   $\Rightarrow \sin^2 A = \cos^4 A$

   $\Rightarrow 1 - \cos^2 A = \cos^4 A$

   $\Rightarrow \cos^2 A + \cos^4 A = 1$

4. d. $\frac{49}{64}$

   **Explanation:** Given: $\cot\theta = \frac{7}{8}$

   Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \left(\frac{7}{8}\right)^2$

$= \frac{49}{64}$

5. d. 0

**Explanation:** Given: $\sin\alpha = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\alpha = \sin 45^\circ$

$\Rightarrow \alpha = 45^\circ$

And tan β = 1

$\Rightarrow \tan\beta = \tan 45^\circ$

$\Rightarrow \beta = 45^\circ$

$\therefore \cos(\alpha+\beta) = \cos(45^\circ + 45^\circ) = \cos 90^\circ = 0$

6. We have,

$\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ}$

$= \frac{(\sin 45^\circ)^2 + (\cos 45^\circ)^2}{(\tan 60^\circ)^2}$

$= \frac{(1/\sqrt{2})^2 + (1/\sqrt{2})^2}{(\sqrt{3})^2}$

$= \frac{\frac{1}{2}+\frac{1}{2}}{3} = \frac{1}{3}$

7. LHS = $\sin^2A \cot^2A + \cos^2A \tan^2A$

$= \sin^2 A \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \frac{\sin^2 A}{\cos^2 A} \left[\because \cot^2 A = \frac{\cos^2 A}{\sin^2 A}, \tan^2 A = \frac{\sin^2 A}{\cos^2 A}\right]$

= $\cos^2A + \sin^2A$ [$\because \cos^2A + \sin^2A = 1$]

= 1

= RHS

Hence proved.

8. Sin C = $\frac{AB}{AC} = \frac{12}{13}$

and Cot C = $\frac{\text{BC}}{\text{AB}} = \frac{5}{12}$

9. $\frac{\sec 49^\circ}{\cos ec41^\circ} = \frac{\sec(90^\circ - 41^\circ)}{\cos ec41^\circ} = \frac{\cos ec41^\circ}{\cos ec41^\circ} = 1 \ [\because \sec(90^\circ - \theta) = \cos ec\theta].$

10. LHS = $\cos 81^\circ - \sin 9^\circ$

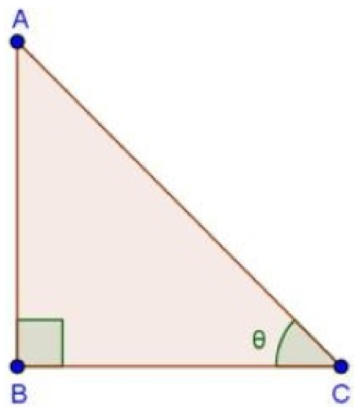
= $\cos(90^\circ - 9^\circ) - \sin 9^\circ$

= $\sin 9^\circ - \sin 9^\circ = 0$ = RHS

11. $5sin\theta + 3cos\theta = 4$
$\Rightarrow (5sin\theta + 3cos\theta)^2 = (4)^2$
$\Rightarrow 25 \sin^2\theta + 9\cos^2\theta + 30\sin\theta\cos\theta = 16$
$\Rightarrow 25(1 - \cos^2\theta) + 9(1 - \sin^2\theta) + 30\sin\theta\cos\theta = 16$ [
$\because sin^2\theta = 1 - cos^2\theta \ \ and \ \ cos^2\theta = 1 - sin^2\theta$]
$\Rightarrow (3\sin\theta - 5\cos\theta)^2 = 18$
$\Rightarrow 3sin\theta - 5cos\theta = \pm 3\sqrt{2}$

12. $\text{L.H.S.} = \frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$
$= \frac{\sin\theta\left(\frac{1}{\cos\theta} + 1\right)}{\sin\theta\left(\frac{1}{\cos\theta} - 1\right)} = \frac{\sec\theta + 1}{\sec\theta - 1} = R.HS$

13. $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$
$\Rightarrow \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$
$\Rightarrow 4\cos^2\theta = 3\cot^2\theta$
$\Rightarrow 4\cos^2\theta = \frac{3\cos^2\theta}{\sin^2\theta}$
$\Rightarrow 4\sin^2\theta = \frac{3\cos^2\theta}{\cos^2\theta}$
$\Rightarrow \sin^2\theta = \frac{3}{4}$
$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$
$\Rightarrow \quad \theta = 60^\circ$

14. 

Given $\cos\theta = \frac{12}{13} = \frac{BC}{AC}$

Let BC = 12K

and, AC = 13K

In $\Delta ABC$, By Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$AB^2 + (12K)^2 = (13K)^2$

$AB^2 + 144K^2 = 169K^2$

$AB^2 = 169K - 144K^2 = 25K^2$

$AB = \sqrt{25K^2} = 5K$

$\therefore \sin\theta = \frac{AB}{AC} = \frac{5K}{13K} = \frac{5}{13}$

$\tan\theta = \frac{AB}{BC} = \frac{5K}{12K} = \frac{5}{12}$

$\therefore\ LHS = \sin\theta(1-\tan\theta)$

$= \frac{5}{13}\left(1-\frac{5}{12}\right)$

$= \frac{5}{13}\left(\frac{12-5}{12}\right)$

$= \frac{5}{13} \times \frac{7}{12}$

$= \frac{35}{156} = RHS$

15. According to question

$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow \quad \frac{(\cos\theta-\sin\theta)+(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)-(\cos\theta+\sin\theta)} = \frac{(1-\sqrt{3})+(1+\sqrt{3})}{(1-\sqrt{3})-(1+\sqrt{3})}$

$[Applying\ componendo\ and\ dividendo]$

$\Rightarrow \quad \frac{2\cos\theta}{-2\sin\theta} = \frac{2}{-2\sqrt{3}}$

$\Rightarrow \quad \cot\theta = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$

16. Given,

$1 + \sin^2\theta = 3\sin\theta\cos\theta$

$\Rightarrow \sin^2\theta + \cos^2\theta + \sin^2\theta = 3\sin\theta\cos\theta$

$\Rightarrow 2\sin^2\theta + \cos^2\theta = 3\sin\theta\cos\theta$

$\Rightarrow 2\sin^2\theta - 3\sin\theta\cos\theta + \cos^2\theta = 0$

$\Rightarrow 2\sin^2\theta - 2\sin\theta\cos\theta - \sin\theta + \cos^2\theta = 0$

$\Rightarrow$ (sin$\theta$ - cos$\theta$)(2sin$\theta$ - cos$\theta$) = 0

either, sin$\theta$ - cos $\theta$ = 0 or, 2sin$\theta$ - cos$\theta$ = 0

$\Rightarrow$ sin$\theta$ = cos$\theta$. or, 2sin$\theta$ = cos$\theta$

$\Rightarrow$ tan$\theta$ =1 or, tan$\theta$ = $\frac{1}{2}$ . [$\because \frac{Sin\theta}{Cos\theta} = \tan\theta$ ]

Hence, proved.

17. We have,

$$\frac{\sec 41^\circ \sin 49^\circ + \cos 29^\circ \operatorname{cosec} 61^\circ - \frac{2}{\sqrt{3}}(\tan 20^\circ \tan 60^\circ \tan 70^\circ)}{3(\sin^2 31^\circ + \sin^2 59^\circ)}$$

$$= \frac{\sec(90^\circ - 41^\circ)\sin 49^\circ + \cos 29^\circ \operatorname{cosec}(90^\circ - 29^\circ) - \frac{2}{\sqrt{3}}[\tan 20^\circ \sqrt{3}\tan(90^\circ - 20^\circ)]}{3[\sin^2 31^\circ + \sin^2(90^\circ - 31^\circ)]}$$

$$= \frac{\operatorname{cosec} 49^\circ \sin 49^\circ + \cos 29^\circ \sec 29^\circ - \frac{2}{\sqrt{3}}[\tan 20^\circ \sqrt{3}\cot 20^\circ]}{3(\sin^2 31^\circ + \cos^2 31^\circ)}$$

$= \frac{1+1-2}{3} = \frac{2-2}{3}$

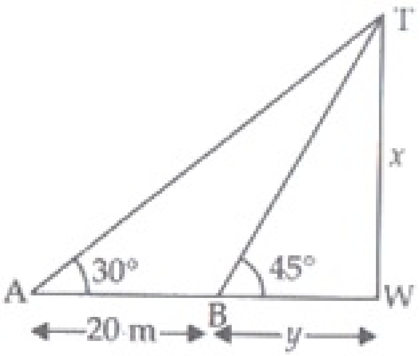
= 0

18. Let the height of the vertical tower (TW) = x m

When a observer at A, makes angle of elevation at the top of tower is 30°.

Now, angle of elevation of the top of tower is increased by 15° when observer moves 20m towards the tower.

i.e., it becomes 30° + 15° = 45°.




In $\triangle$TWB,

tan 45° $= \frac{P}{B} = \frac{x}{y}$

$\Rightarrow$ 1 = $\frac{x}{y}$

$\Rightarrow$ x = y ......(i)

Now, $\triangle$TWA, we have

tan 30° $= \frac{P}{B} = \frac{x}{20+y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{20+x}$ [From (i)]

$\Rightarrow \sqrt{3}x = 20 + x$

$\Rightarrow \sqrt{3}x - x$ = 20

$\Rightarrow x(\sqrt{3} - 1)$ = 20

$\Rightarrow x = \frac{20}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{(\sqrt{3}+1)}$

$\Rightarrow x = \frac{20(\sqrt{3}+1)}{3-1} = \frac{20(\sqrt{3}+1)}{2}$

$\Rightarrow x = 10(1.732 + 1)$

$\Rightarrow x = 10 \times 2.732 = 27.32m$

Hence, the height of the tower (TW) is 27.32 m.

19. LHS$= \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta(1-\sin^4\theta)+\sin^2\theta(1-\cos^4\theta)}{(1-\cos^4\theta)(1-\sin^4\theta)}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta(1-\sin^2\theta)(1+\sin^2\theta)+\sin^2\theta(1-\cos^2\theta)(1+\cos^2\theta)}{(1-\cos^2\theta)(1+\cos^2\theta)(1-\sin^2\theta)(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta\cdot\cos^2\theta(1+\sin^2\theta)+\sin^2\theta\sin^2\theta(1+\cos^2\theta)}{\sin^2\theta(1+\cos^2\theta)\cos^2\theta(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \left(\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{(1+\cos^2\theta)(1+\sin^2\theta)}\right)$

$= \left(\frac{\cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}\right)$

$= \left(\frac{\cos^4\theta+\sin^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}\right)$

$= \frac{(\cos^2\theta+\sin^2\theta)^2 - 2\cos^2\theta\sin^2\theta+\cos^2\theta\sin^2\theta(\cos^2\theta+\sin^2\theta)}{(1+\cos^2\theta)(1+\sin^2\theta)}$

$= \frac{1-2\cos^2\theta\sin^2\theta+\cos^2\theta\sin^2\theta\times 1}{1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta} \left[\sin^2\theta + \cos^2\theta = 1\right]$

$= \frac{1-\cos^2\theta\sin^2\theta}{1+1+\cos^2\theta\sin^2\theta}$

$= \frac{1-\cos^2\theta\sin^2\theta}{2+\cos^2\theta\sin^2\theta}$

= RHS

Hence proved

20. $\frac{\cos ec^2(90°-\theta)-\tan^2\theta}{4(\cos^2 48°+\cos^2 42°)} - \frac{2\tan^2 30°\sec^2 52°\sin^2 38°}{\cos ec^2 70°-\tan^2 20°}$

$= \frac{\sec^2\theta-\tan^2\theta}{4[\cos^2(90°-42°])+\cos^2 42°]} - \frac{2\left(\frac{1}{\sqrt{3}}\right)^2\sec^2(90°-38°)\sin^2 38°}{\cos ec^2(90°-20°)-\tan^2 20°}$

$= \frac{1}{4\left(\sin^2 42°+\cos^2 42°\right)} - \frac{\frac{2}{3}\cos ec^2 38°\sin^2 38°}{\sec^2 20°-\tan^2 20°}$

$\because \sec^2\theta - \tan^2\theta = 1$

$\cos(90° - \theta) = \sin\theta$

$\sec(90° - \theta) = \cos ec\theta$

$\cos ec(90° - \theta) = sec\theta$

$= \frac{1}{4(1)} - \frac{\frac{2}{3}\cdot\frac{1}{\sin^2 38}.\sin^2 38°}{1}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2 = 1$

$\cos ec\theta = \frac{1}{\sin\theta}$

$\frac{1}{4} - \frac{2}{3} = \frac{3-8}{12} = \frac{-5}{12}$