Chapter 9. Factoring

Ex. 9.4

Answer 1CU.

In factoring ax^2+bx+c choose the values for m and n.

In choosing the values of m and n, m and n must satisfies,

$$m+n=b,$$
$$mn=ac$$

That is by adding m and n gives b multiplying m and n gives $a \cdot c$.

Therefore, $\boxed{m \,\&\, n}$ are factors of $\boxed{ac}$ that add to $\boxed{b}$.

Answer 1PQ.

Consider the trinomial $x^2-14x-72$.

The objective is to find factors of given trinomial.

Compare $x^2-14x-72$ with ax^2+bx+c.

Here $a=1$,

$b=-14$,

$c=-72$

Find two numbers m,n such that whose sum is

$b=-14$ and whose product

$ac=-72$.

Since $m+n=-14$ negative and

$mn=-72$ also negative.

So, either m (or) n negative but not both.

Now list all factors of -72, choose in those one pair whose sum is -14.

Factors of -72	Sum of factors
$1.-72$	-71
-1.72	71
$2.-36$	-34
-2.36	34
$3.-24$	-21
-3.24	21
$4.-18$	-14
-4.18	14
$6.-12$	-6
-6.12	6
$8.-9$	-1
-8.9	1

The correct factors are $4,-18$.

Therefore,

$x^2 - 14x - 72 = x^2 + 4x - 18x - 72$

(Because $-14 = 4 - 18$)

$= x \cdot x + 2 \cdot 2 \cdot x - 2 \cdot 3 \cdot 3x - 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

(Simplify)

$= x(x+4) - 18(x+4)$

(Group all terms with common factors)

$= (x-18)(x+4)$

(By distributive)

Therefore, $x^2 - 14x - 72 = (x-18)(x+4)$

<u>Check</u>:- To check the result, multiplying two factors using $FOIL$ method.

$(x-18)(x+4) = \overset{F}{x \cdot x} + \overset{O}{4 \cdot x} - \overset{I}{18 \cdot x} - \overset{L}{18 \cdot 4}$

($FOIL$ method)

$= x^2 + 4x - 18x - 72$

(Simplify)

$= x^2 - 14x - 72$ True

<u>Therefore, the factorized form of</u> $\boxed{x^2 - 14x - 72}$ is $\boxed{(x-18)(x+4)}$.

Answer 2CU.

Consider that sum of the pair of numbers

$=9$

Product of pair $=14$

The objective is to write the trinomial.

Let ax^2+bx+c be the trinomial.

Since sum $\begin{aligned} m+n &= b \\ &= 9 \end{aligned}$

Also product $\begin{aligned} m\cdot n &= ac \\ &= 14 \end{aligned}$

Since $m+n, mm$ are positive list all the positive factors of 14 whose sum is 9.

Factors of 14	Sum of factors
1.14	15
2.7	9

The correct factors are $2,7$.

Therefore,

$ax^2+bx+c = ax^2+9x+c$

$= ax^2+2x+7x+c$ (Because $ac=14$)

$= 2x^2+2x+7x+7$ (Since $ac=14,$ if $a=2, c=7$)

$= 2x^2+9x+7$

$ax^2+bx+c = ax^2+9x+c \quad (b=9)$

$= ax^2+7x+2x+c$

$= 7x^2+7x+2x+2$ (Since $ac=14$, of $a=7, c=2$)

$= 7x^2+9x+2$

There the required trinomials are $\boxed{2x^2+9x+7}$ or $\boxed{7x^2+9x+2}$.

Answer 2PQ.

Consider the trinomial $8p^2-6p-35$.

The objective is to find factors of given trinomial.

Compare $8p^2-6p-35$ with ax^2+bx+c.

Here $a=8,$

$b=-6,$

$c=-35$

Find two numbers m,n such that whose sum is

$b=-6$ and whose product is

$$ac=8\cdot-35$$
$$=-280$$

Since $m+n=-6$ negative and

$mn=-280$ also negative.

So, either m (or) n negative, but not both.

Now list all the factors of -280, choose one pair whose sum is -6.

Factors of -280	Sum of factors
-1.280	279
$1.-280$	-279
$2.-140$	-138
-2.140	138
$4.-70$	-66
-4.70	66

$5.-56$	-51
-5.56	51
$7.-40$	-33
-7.40	33
$8.-35$	-27
-8.35	27
$10.-28$	-18
-10.28	18
$14.-20$	-6
-14.20	6

The correct factors are $14,-20$.

$8p^2-6p-35=8p^2+14p-20p-35$

(Because $-6=14-20$)

$=2\cdot2\cdot2\cdot p\cdot p+2\cdot7\cdot p-2\cdot2\cdot5p-7\cdot5$

(Simplify)

$=2p(4p+7)-5(4p+7)$

(Group all terms common factors)

$=(2p-5)(4p+7)$

(By distributive)

Check:- To check the result, multiplying two factors using $FOIL$ method.

$$(2p-5)(4p+7)=2p\cdot\overset{F}{4}p+\overset{O}{2}p\cdot7-\overset{I}{5}\cdot4p-\overset{L}{5}\cdot7$$

($FOIL$ method)

$=8p^2+14p-20p-35$

(Simplify)

$=8p^2-6p-35$

True

Therefore, the factorized form of $\boxed{8p^2-6p-35}$ is $\boxed{(2p-5)(4p+7)}$.

Answer 3CU.

Consider the trinomial $2x^2+11x+18$

In factoring $2x^2+11x+18$, Dasan factoring as

Factors of 18	Sum
$1,18$	19
$3,6$	9
$9,2$	11

$$\begin{aligned}2x^2+11x+18 &= 2\left(x^2+11x+18\right)\\ &= 2(x+9)(x+2)\end{aligned}$$

Craig factoring as

Factors of 36	Sum
$1,36$	37
$2,18$	20
$3,12$	15
$4,9$	13
$6,6$	12

$2x^2+11x+18$ is prime.

In factoring ax^2+bx+c, we have to factor $a\cdot c$ of

$a \neq 1$, add factor c of

$a = 1$.

So Dasan is wrong, because he factor

$c = 18$, when

$a = 2$

$\neq 1$

Therefore, Craig is correct, when factoring trinomial of the form $\boxed{ax^2 + bx + c}$, where $\boxed{a \neq 1}$, must find the factors of $\boxed{ac}$ not of $\boxed{c}$.

Answer 9PQ.

Consider the trinomial $16a^2 - 24a + 5$.

The objective is to find the factors of given trinomial.

Compare $16a^2 - 24a + 5$ with $ax^2 + bx + c$.

Here $a = 16$,

$b = -24,$

$c = 5$

Now find two numbers m, n such that whose sum is

$b = -24$ and whose product

$ac = 16 \cdot 5$

$= 80$

Since $m + n = -24$ negative and

$mn = 80$ positive.

So, m and n must be both negative.

Now list all factors of 80, choose one pair whose sum is -24.

Factors of 80	Sum of factors
$-1, -80$	-81
$-2, -40$	-42
$-4, -20$	-24
$-5, -16$	-21
$-8, -10$	-18

The correct factors are $-4, -20$.

$16a^2 - 24a + 5 = 16a^2 - 4a - 20a + 5$

(Because $-24 = -4 - 20$)

$$= 4 \cdot 4 \cdot a \cdot a - 4 \cdot a - 5 \cdot 4 \cdot a + 5$$

(Simplify)

$$= 4a(4a - 1) - 5(4a - 1)$$

(Group all terms with common factors)

$$= (4a - 5)(4a - 1)$$

Check:- To check the result, multiplying two factors using $FOIL$ method.

$$(4a - 5)(4a - 1) = \overset{F}{4}a \cdot 4a - 1 \cdot \overset{O}{4}a - \overset{I}{5} \cdot 4a + \overset{L}{5} \cdot 1$$

($FOIL$ method)

$$= 16a^2 - 4a - 20a + 5$$

(Simplify)

$$= 16a^2 - 24a + 5$$

(True)

Therefore, the factorized form of $\boxed{16a^2 - 24a + 5}$ is $\boxed{(4a - 5)(4a - 1)}$.

Answer 4CU.

The objective is to factor the given trinomial.

Compare $3a^2+8a+4$ with ax^2+bx+c.

Here $a=3,$

$b=8,$
$c=4$

Now find two numbers m,n such that whose sum is

$b=8$ and product is

$ac=3\cdot 4$
$=12$

For this list all the factors of 12 and choose a pair of factors with sum 8.

Factors of 12	Sum of factors
1.12	13
2.6	8
2.4	7

The correct factors are $2,6$.

Now,

$3a^2+8a+4=3a^2+6a+2a+4$ (Because $8=6+2$)

$=3\cdot a\cdot a+3\cdot 2\cdot a+2a+2\cdot 2$

(Simplify)

$=3a(a+2)+2(a+2)$ (Group the terms with common factors)

$=(3a+2)(a+2)$ (By distributive)

Therefore,

$3a^2+8a+4=(3a+2)(a+2)$

Check: Check the result by multiplying two factors using $FOIL$ method.

$$\left(3a+2\right)\left(a+2\right)=\overset{F}{3a}\cdot a+2\cdot\overset{O}{3a}+\overset{I}{2}\cdot a+\overset{L}{2}\cdot 2$$ ($FOIL$ method)

$=3a^2+6a+2a+4$ (Simplify)

$=3a^2+8a+4$ True

The factorization of $\boxed{3a^2+8a+4}$ is $\boxed{\left(3a+2\right)\left(a+2\right)}$.

Answer 4PQ.

Consider the trinomial $n^2-17n+52$.

The objective is to find factors of given trinomial.

Compare $n^2-17n+52$ with ax^2+bx+c.

Here $a=1$,

$b=-17$,

$c=52$

Now find two numbers m,n such that whose sum is

$b=-17$ and whose product is

$ac=1\cdot 52$

$=52$

Since $m+n=-17$ and

$mn=52$ positive.

So, m and n both must be negative.

Now list all the factors of 52, choose one pair whose sum is -17.

Factors of 52	Sum of factors
$-1.-52$	-53
$-2.-26$	-28
$-4.-13$	-17

The correct factors are $-4,-13$.

$n^2-17n+52=n^2-4n-13n+52$ (Because $-17=-4-13$)

$=n\cdot n-2\cdot 2\cdot n-13\cdot n+13\cdot 4$

(Simplify)

$=n\left(n-4\right)-13\left(n-4\right)$ (Group all terms with common factors)

$=(n-13)(n-4)$ (By distributive)

Therefore, $n^2-17n+52=(n-13)(n-4)$

Check:- To check the result, multiplying two factors using $FOIL$ method.

$(n-13)(n-4)=\overset{F}{n}\cdot n-\overset{O}{4}\cdot n-\overset{I}{13}\cdot n+\overset{L}{13}\cdot 4$ ($FOIL$ method)

$=n^2-4n+13n+52$ (Simplify)

$=n^2-17n+52$ True

Therefore, the factorized form of $\boxed{n^2-17n+52}$ is $\boxed{(n-13)(n-4)}$.

Answer 5CU.

Consider the trinomial $2a^2-11a+7$.

The objective is to factor the given trinomial.

Compare $2a^2-11a+7$ with ax^2+bx+c.

$a=2,$
$b=-11,$
$c=7$

Now find two number m,n such that whose sum is

$b=-11$ and product is

$ac=2\cdot 7$
$=14$

Also $m+n=-11$ is negative add

$mn=14$ is positive, so m and n must both be negative. So list all negative factors of $14,$ in those choose a pair of factors whose sum is -11.

Factors of 14	Sum of factors
$-1,-14$	-15
$-2,-7$	-9

There are no prime factors whose sum is -11.

Therefore $2a^2-11a+7$ cannot be factored using integers.

Since a polynomial that cannot be written as a product of two polynomials with integral coefficients is called a prime polynomial.

Therefore, $\boxed{2a^2-11a+7}$ is a prime polynomial.

Answer 5PQ.

Consider the trinomial $24c^2+62c+18$.

The objective is to find the factors of given trinomial.

Compare $24c^2+62c+18$ with ax^2+bx+c.

Here $a=24,$

$b=62,$

$c=18$

Now find the two numbers m,n such that whose sum is 62 and whose product is

$ac=24\cdot18$

$=432$

Since $m+n=62$ positive and

$mn=432$ also positive.

Now list all the factors of 432, choose one pair whose sum is 62.

Factors of 432	Sum of factors
1.432	433
2.216	218
3.144	147
4.108	112
6.72	78
8.54	62
9.48	57
12.36	48
16.27	43
18.24	42

The correct factors are $8,54$.

$24c^2+62c+18=24c^2+8c+54c+18$ (Because $62=8+54$)

$=2\cdot2\cdot2\cdot3\cdot c\cdot c+2\cdot2\cdot2\cdot c+3\cdot3\cdot3\cdot2c+3\cdot3\cdot2$

(Simplify)

$= 8c(3c+1)+18(3c+1)$ (Group all terms with common factors)

$= (8c+18)(3c+1)$

Therefore,

$24c^2+62c+18=(8c+18)(3c+1)$

Check:- To check the result, multiplying two factors using $FOIL$ method.

$(8c+18)(3c+1)=\overset{F}{8c\cdot 3c}+\overset{O}{8c\cdot 1}+\overset{I}{18\cdot 3c}+\overset{L}{18\cdot 1}$

($FOIL$ method)

$= 24c^2+8c+54c+18$ (Simplify)

$= 24c^2+62c+18$ True

Therefore, the factorized form of $\boxed{24c^2+62c+18}$ is $\boxed{(8c+18)(3c+1)}$.

Answer 6CU.

Consider the trinomial $2p^2+14p+24$.

The objective is to factor the given trinomial.

Compare $2p^2+14p+24$ with ax^2+bx+c.

Here

$a=2,$
$b=14,$
$c=24$

Now find two numbers m,n such that whose sum is

$b=14$ and product is

$ac=2\cdot 24$
$=48$

For this list all the factors of 48 and choose a pair of factors with sum is 14.

Factors of 48	Sum of factors
1.48	49
6.8	14

2.24	26
3.16	19
4.12	16

The correct factors are $6,8$.

Now,

$$2p^2+14p+24=2p^2+6p+8p+24$$

(Because $14=6+8$)

$$=2\cdot p\cdot p+3\cdot 2\cdot p+2\cdot 4\cdot p+2\cdot 12$$

(Simplify)

$$=2\cdot p\cdot p+3\cdot 2\cdot p+2\cdot 2\cdot 2\cdot p+2\cdot 2\cdot 2\cdot 3$$

(Simplify)

$$=2p(p+3)+8(p+3)$$

(Group of terms with common factors)

$$=(2p+8)(p+3)$$

(By distribution)

Therefore,

$$2p^2+14p+24=(2p+8)(p+3)$$

Check:- Check the result by multiplying two factors using $FOIL$ method.

$$(2p+8)(p+3)=\overset{F}{2p\cdot p}+\overset{O}{3\cdot 2p}+\overset{I}{8\cdot p}+\overset{L}{8\cdot 3}$$

($FOIL$ metod)

$$=2p^2+6p+8p+24$$

(Simplify)

$$=2p^2+14p+24 \text{ True}$$

The factorization of $\boxed{2p^2+14p+24}$ is $\boxed{(2p+8)(p+3)}$.

Answer 6PQ.

Consider the trinomial $3y^2 + 33y + 54$.

The objective is to find factors of given trinomial.

Compare $3y^2 + 33y + 54$ with $ax^2 + bx + c$.

Here $a = 3$,

$b = 33,$

$c = 54$

Now find two numbers m, n such that whose sum is

$b = 33$, whose product is

$ac = 3 \cdot 54$

$= 162$

Since $m + n = 33$ positive and

$mn = 162$ also positive.

So, list all the factors of 162, choose one pair whose sum is 3.

Factors of 162	Sum of factors
1.162	163
2.81	83
3.54	57
6.27	33
9.18	27

The correct factors are $6, 27$.

$3y^2 + 33y + 54 = 3y^2 + 6y + 27y + 54$

(Because $33 = 6 + 27$)

Answer 7CU.

Consider the trinomial $2x^2+18x+20$

The objective is to factor the given trinomial.

Compare $2x^2+18x+20$ with ax^2+bx+c.

Here $a=2$,

$b=13,$

$c=20$

Now find two numbers m,n such that whose sum is

$b=13$ and product is

$ac=2\cdot 20$

$=40$

For this list all factors of 40 and choose a pair of factors with sum is 13.

Factors of 40	Sum of factors
1.40	41
2.20	22
4.10	14
5.8	13

The correct factors are $5,8$.

Now

$2x^2+13x+20=2\cdot x\cdot x+5\cdot x+8\cdot x+20$

(Because $13=5+8$)

$=2\cdot x\cdot x+5\cdot x+2\cdot 4\cdot x+4\cdot 5$

(Simplify).

$=x(2x+5)+4(2x+5)$

(Group the terms with common factors)

$=(x+4)(2x+5)$

$=(x+4)(2x+5)$

(By distributive)

Therefore,

$2x^2+13x+20=(x+4)(2x+5)$

Check:- To check the result by multiplying two factors using $FOIL$ method.

$(x+4)(2x+5)=\overset{F}{x\cdot 2x}+\overset{O}{5\cdot x}+\overset{I}{4\cdot 2x}+\overset{L}{4\cdot 5}$

($FOIL$ Method)

$=2x^2+5x+8x+20$

(Simplify)

$=2x^2+13x+20$ True

Therefore, the factorization of $\boxed{2x^2+13x+20}$ is $\boxed{(x+4)(2x+5)}$.

Answer 7PQ.

Consider the equation

$b^2+14b-32=0$

The objective is to solve the given equation. For this first find factors of given equation and then use zero product property.

Compare $b^2+14b-32$ with ax^2+bx+c.

Here $a=1$,

$b=14,$

$c=-32$

Now find two numbers m,n such that whose sum is

$b=14$ and whose product is

$ac=1\cdot -32$

$=-32$

Since $m+n=14$ positive and

$mn=-32$ negative.

So, either m (or) n negative but not both.

Factors of -32	Sum of factors

$1.-32$	-31
-1.32	31
$2.-16$	-14
-2.16	14
$4.-8$	-4
-4.8	4

The correct factors are $-2,16$.

$b^2+14b-32=b^2-2b+16b-32$

(Because $14=-2+16$)

$=b\cdot b-2\cdot b+4\cdot 2\cdot 2\cdot b-4\cdot 2\cdot 2\cdot 2$

(Simplify)

$=b(b-2)+16(b-2)$

(Group all terms with common factors)

$=(b+16)(b-2)$

(Factors)

Now $b^2+14b-32=0$

$\Rightarrow \quad (b+16)(b-2)=0$ (Factors)

$\Rightarrow \quad b+16=0$

Or, $b-2=0$ (Use zero product property)

Now solve each equation separately.

$b+16=0$

$b+16-16=0-16$ (Subtract 16 on both sides)

$b=-16$

$b-2=0$

$b-2+2=0+2$ (Add 2 on each side)

$b=2$

The solution set is $\{-16,2\}$.

Check:- To check the result, substitute b by $-16,2$ in given equation.

For $x=-16$,

$b^2+14b-32=0$

$(-16)^2+14(-16)-32=0$ (Put $b=-16$)

$256-224-32=0$ (Simplify)

$0=0$ True

For $x=2$,

$b^2+14b-32=0$

$(2)^2+14(2)-32=0$ (Put $b=2$)

$4+28-32=0$

$0=0$ True

Therefore, the solution set is $\boxed{\{-16,2\}}$.

Answer 8CU.

Consider the trinomial $6x^2+15x-9$.

The objective is to factor the given trinomial.

Compare $6x^2+15x-9$ with ax^2+bx+c

$$a=6,$$
$$b=15,$$
$$c=-9$$

Now find two numbers m,n such that whose sum is

$b=15$ and products

$$a\cdot c=6\cdot -9$$
$$=-54$$

Since $m+n=15$ is positive and

$m\cdot n=-54$ is negative

So either m or n is negative but not both list all the factors to -54, where one factor in each pair is negative.

Factors of -54	Sum of factors
-1.54	53
$1.-54$	-53
-2.27	25
$2.-27$	-25
$3.-18$	-15
-3.18	15
-6.9	3

$6,-9$	-3

The correct factors are $-3,18$.

$6x^2+15x-9=6x^2+mx+nx-9$

$=6x^2-3x+18x-9$ $(m=-3,n=18)$

$=(6x^2-3x)+(18x-9)$ (Group the terms with common factors)

$=3x(x-1)+9(2x-1)$ (Factor the GCF)

$=(3x+9)(2x-1)$ (By distributive $(b+c)a=ba+ca$)

Check: Check the result by multiplying the factors by using $FOIL$ method.

$$(3x+9)(2x-1)=3x\cdot\overset{F}{2}x+(\overset{O}{-1})\cdot 3x+9\cdot\overset{I}{2}x+(\overset{L}{-1})\cdot 9$$

($FOIL$ Mthod)

$=6x^2-3x+18x-9$ (Simplify)

$=6x^2+15x-9$ True (Combine like terms)

Therefore, the factorization of $\boxed{6x^2+15x-9}$ is $\boxed{(3x+9)(2x-1)}$.

Answer 8PQ.

Consider the equation

$x^2+45=18x$

$x^2+45-18x=18x-18x$ (Subtract $18x$ on each side)

$x^2-18x+45=0$

The objective is to find solution of given equation. For this, first find the factors of given equation and then use zero product property.

Compare $x^2-18x+45$ with ax^2+bx+c.

Here $a=1$,

$b=-18$,

$c=45$

Now find two numbers m,n such that whose sum is

$b=-18$ and whose product

$$ac = 1 \cdot 45$$
$$= 45$$

Since $m + n = -18$ negative and

$mn = 45$ positive.

So, m and n must be both negative.

Now list all factors of 45 and choose one pair whose sum is -18.

Factors of 45	Sum of factors
$-1, -45$	-46
$-3, -15$	-18
$-5, -9$	-14

The correct factors are $-3, -15$.

$$x^2 - 18x + 45 = x^2 - 3x - 15x + 45$$

(Because $-18 = -3 - 15$)

$$= x \cdot x - 3 \cdot x - 3 \cdot 5 \cdot x + 3 \cdot 3 \cdot 5$$

(Simplify)

$$= x(x-3) - 15(x-3)$$

(Group all terms with common factors)

$=(x-15)(x-3)$ (By distributive)

Now $x^2-18x+45=0$

$(x-15)(x-3)=0$ (Factors)

$x-15=0$

Or, $x-3=0$ (Using zero product property)

Now solve each equation separately.

$x-15=0$

$x-15+15=0+15$ (Add 15 on each side)

$x=15$

$x-3=0$

$x-3+3=0+3$ (Add 3 on both sides)

$x=3$

The solution set is $\{15,3\}$.

Check:- To check the solution, substitute x by $15,3$ in given equation.

For $x=15$,

$x^2-18x+45=0$

$(15)^2-18(15)+45=0$ (Put $x=15$)

$225-270+45=0$ (Simplify)

$0=0$ True

For $x=3$,

$x^2-18x+45=0$

$(3)^2-18(3)+45=0$ (Put $x=3$)

$9-54+45=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{15,3\}}$.

Answer 9CU.

Consider the Trinomial $4n^2-4n-35$.

The objective is to factor the given trinomial

Compare $4n^2-4n-35$ with ax^2+bx+c.

Here $a=4$,

$b=-4,$

$c=-35$

Now find two numbers m,n such that whose sum is -4 and product is

$$a\cdot c=4\cdot(-35)$$
$$=-140$$

Since $m+n=-4$ is negative and

$m\cdot n=-140$ is negative.

So either m or n is negative but not both.

Now list all the factors of -140 where one factor in each pair is negative. In those a pair whose sum is -4.

Factors of -140	Sum of factors
-1.140	139
$1.-140$	-139
$2.-70$	-68
-2.70	68
-4.35	31
$4.-35$	-31
-5.28	23

$5,-28$	-23
$-7,20$	13
$7,-20$	-13
$-10,14$	4
$10,-14$	-4

The correct factors are $10,-14$.

$4n^2-4n-35=4n^2+10n-14n-35 \quad (m=10, n=-14)$

$=\left(4n^2+10n\right)+\left(-14n-35\right)$

(Group terms with common factors)

$=\left(2n\cdot 2n+2n\cdot 5\right)+\left(-7\cdot 2n-7\cdot 5\right)$

$=2n\left(2n+5\right)-7\left(2n+5\right)$ (Factor GCF)

$=\left(2n-7\right)\left(2n+5\right)$ (By distributive $\left(b+c\right)a=ba+ca$)

Check:- Check the result by multiplying the factors by using $FOIL$ method.

$\left(2n-7\right)\left(2n+5\right)=\overset{F}{2n\cdot 2n}+\overset{O}{5\cdot 2n}+\overset{I}{-7\cdot 2n}-\overset{L}{7\cdot 5}$

($FOIL$ method)

$=4n^2+10n-14n-35$ (Simplify)

$=4n^2-4n-35$ True

Therefore, the factorization of $\boxed{4n^2-4n-35}$ is $\boxed{\left(2n-7\right)\left(2n+5\right)}$.

Answer 9PQ.

Consider the equation

$$12y^2 - 7y - 12 = 0$$

The objective is to solve the given equation. For this, first find the factors of given equation, then use zero product property.

Here $a = 12$,

$b = -7,$

$c = -12$

Now find two numbers m, n such that whose sum is

$b = -7$ and whose product is

$$ac = 12 \cdot -12$$
$$= -144$$

Since $m + n = -7$ negative and

$mn = -144$ also negative.

So, either m (or) n negative but not both.

Now list all factors -144 and choose one pair in those whose sum is -7.

Factors of -144	Sum of factors
$-1.-144$	-143
$-2.-72$	-70
-1.144	143
-2.72	70
-3.48	45
$3.-48$	-45
$4.-36$	-32
-4.36	32

$6.-24$	-18
-6.24	18
$8.-18$	-10
-8.18	10
-16.9	-7
$16.-9$	7
-12.12	7
-12.12	0
$12.-12$	0

The correct factors are $-16,9$.

$12y^2-7y-12=12y^2-16y+9y-12$

(Because $-7=-16+9$)

$=2\cdot2\cdot3\cdot y\cdot y-2\cdot2\cdot2\cdot2y+3\cdot3\cdot y-3\cdot2\cdot2$

(Simplify)

$=4y(3y-4)+3(3y-4)$

(Group all terms with common factors)

$=(4y+3)(3y-4)$

(By distributive)

Now $12y^2-7y-12=0$

$(4y+3)(3y-4)=0$ (Factors)

$4y+3=0$

Or, $3y-4=0$ (Use zero product property)

Now solve each equation separately.

$4y+3=0$

$4y+3-3=0-3$ (Subtract $4y+3-3=0-3$ on each side)

$4y=-3$

$\frac{4y}{4}=\frac{-3}{4}$ (Divide by 4 on each side)

$y=\frac{-3}{4}$

$3y-4=0$

$3y-4+4=0+4$ (Add 4 on each side)

$3y=4$

$\frac{3y}{y}=\frac{4}{3}$ (Divide by 3 on both sides)

$y=\frac{4}{3}$

The solution set is $\left\{\frac{-3}{4},\frac{4}{3}\right\}$.

Check:- To check the result, substitute y by $\frac{-3}{4},\frac{4}{3}$ in given equation.

For $y=\frac{-3}{4}$,

$12y^2-7y-12=0$

$12\left(-\frac{3}{4}\right)^2-7\left(-\frac{3}{4}\right)-12=0$ (Put $y=-\frac{3}{4}$)

$12\left(\frac{9}{16}\right)+\frac{21}{4}-12=0$

$\frac{108}{16}+\frac{21}{4}\cdot\frac{4}{4}-12\cdot\frac{16}{16}=0$ (Equating the denominators)

$$\frac{108+84-192}{16}=0$$

$0=0$ True

For $y=\frac{4}{3}$,

$$12y^2-7y-12=0$$

$$12\left(\frac{4}{3}\right)^2-7\left(\frac{4}{3}\right)-12=0$$ (Put $y=\frac{4}{3}$)

$$12\left(\frac{16}{9}\right)-\frac{28}{3}-12=0$$

$$\frac{192}{9}-\frac{28}{3}\cdot\frac{3}{3}-12\cdot\frac{9}{9}=0$$ (Equating the denominators)

$$\frac{192-84-108}{16}=0$$

$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{-3}{4},\frac{4}{3}\right\}}$.

Answer 10CU.

Consider the equation $3x^2+11x+6=0$

The objective is to solve the given equation.

Compare $3x^2+11x+6$ with x^2+bx+c.

Here $a=3$,

$b=11$,

$c=6$

Now find two numbers m,n such that whose sum is

$b=11$ and product is

$$\begin{aligned}ac&=3\cdot 6\\&=18\end{aligned}$$

For this list all the factors of 18 and choose a pair of factors with sum 11.

Factors of 18	Sum of factors

1.18	19
2.9	11
3.6	9

The correct factors are $2,9$.

$$3x^2+11x+6=3\cdot x\cdot x+2\cdot x+9\cdot x+6$$

$$(11=2+9)$$

$$=3\cdot x\cdot x+2\cdot x+3\cdot 3\cdot x+2\cdot 3$$

(Simplify)

$$=x(3x+2)+3(3x+2)$$

(Group the terms with common factors)

$$=(x+3)(3x+2)$$

(By distributive)

Therefore,

$$3x^2+11x+6=(x+3)(3x+2)$$

Therefore,

$$3x^2+11x+6=0$$

Therefore,

$3x^2+11x+6=0$

$(x+3)(3x+2)=0$ (Factors)

$x+3=0$ (or)

$3x+2=0$ (By use zero product property)

Now solve each equation separately.

$x+3=0$

$x+3-3=0-3$ (Subtract 3 on each side)

$x=-3$ (Simplify)

$3x+2=0$

$3x+2-2=0-2$ (Subtract 2 on each side)

$3x=-2$ (Simplify)

$\frac{3x}{3}=\frac{-2}{3}$ (Divide with 3 on each side)

$x=\frac{-2}{3}$ (Simplify)

The solution set is $\left\{-3,\frac{-2}{3}\right\}$.

Check:- To check the proposed solution, substitute x by $-3,\frac{-2}{3}$ in the given equation.

For $x = -3$,

$3x^2 + 11x + 6 = 0$

$3(-3)^2 + 11(-3) + 6 = 0$ (Put $x = -3$)

$3(9) - 33 + 6 = 0$ (Simplify)

$27 - 33 + 6 = 0$ (Simplify)

$0 = 0$ True

For, $x = \frac{-2}{3}$,

$3x^2 + 11x + 6 = 0$

$3\left(-\frac{2}{3}\right)^2 + 11\left(-\frac{2}{3}\right) + 6 = 0$ (Put $x = -\frac{2}{3}$)

$3\left(\frac{4}{9}\right) - \frac{22}{3} + 6 = 0$ (Simplify)

$\frac{4}{3} - \frac{22}{3} + 6 = 0$ (Simplify)

$\frac{4+18}{3} - \frac{22}{3} = 0$ ($LCM(3,1) = 3$)

$\frac{22}{3} - \frac{22}{3} = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{-3, \frac{-2}{3}\right\}}$.

Answer 10PQ.

Consider the equation $6a^2 = 25a - 14$

The objective is to solve the equation. For first find the factors by given equation and then use zero product property.

$6a^2 = 25a - 14$

$6a^2 - 25a = 25a - 14 - 25a$ (Subtract $25w$ on each side)

$6a^2 - 25a = -14$

$6a^2 - 25a + 14 = -14 + 14$ (Add 14 on each side)

$6a^2 - 25a + 14 = 0$

Compare $6a^2 - 25a + 14$ with $ax^2 + bx + c$.

Here $a = 6$,

$b = -25,$

$c = 14$

Now find two numbers m, n such that whose sum is

$b = -25$ and whose product

$ac = 6.14$

$= 84$

Since $m + n = -25$ negative and

$mn = 84$ positive.

So, m and n must be both negative.

Now list all the factors of 84, choose one pair in those whose sum is -25.

Factors of 84	Sum of factors
$-1.-84$	-85
$-2.-42$	-44
$-3.-28$	-31
$-4.-21$	-25

$-6,-14$	-20
$-7,-12$	-19

The correct factors are $-4,-21$.

$$6a^2-25a+14=6a^2-4a-21a+14$$

(Because $-25=-4-21$)

$$=2\cdot3\cdot a\cdot a-2\cdot2\cdot a-3\cdot7\cdot a+2\cdot7$$

(Simplify)

$$=2a(3a-2)-7(3a-2)$$

(Group all terms with common factors)

$=(2a-7)(3a-2)$ (BY distributive)

Now $6a^2-25a+14=0$

$(2a-7)(3a-2)=0$ (Factors)

$2a-7=0$

Or, $3a-2=0$ (Using zero product property)

Now solve each equation separately.

$2a-7=0$

$2a-7+7=0+7$ (Add 7 on each side)

$2a=7$

$\frac{2a}{2}=\frac{7}{2}$ (Divide by 2 on both sides)

$a=\frac{7}{2}$

$3a-2=0$

$3a-2+2=0+2$ (Add 2 on each side)

$3a=2$

$\frac{3a}{\ }=\frac{2}{\ }$ (Divide by 3 on both sides)

$$a = \frac{2}{3}$$

The solution set is $\left\{\frac{7}{2}, \frac{2}{3}\right\}$.

Check:- To check the result, substitute a by $\frac{7}{2}, \frac{2}{3}$ in given equation.

For $a = \frac{7}{2}$,

$$6a^2 - 25a + 14 = 0$$

$$6\left(\frac{7}{2}\right)^2 - 25\left(\frac{7}{2}\right) + 14 = 0 \text{ (Put } a = \frac{7}{2}\text{)}$$

$$6\left(\frac{49}{4}\right) - \frac{175}{2} + 14 = 0$$

$$\frac{294}{4} - \frac{175}{2} \cdot \frac{2}{2} + 14 \cdot \frac{4}{4} = 0 \text{ (Equating the denominators)}$$

$$\frac{294 - 350 + 56}{4} = 0 \text{ (Simplify)}$$

$0 = 0$ True

For $a = \frac{2}{3}$,

$$6a^2 - 25a + 14 = 0$$

$$6\left(\frac{2}{3}\right)^2 - 25\left(\frac{2}{3}\right) + 14 = 0 \text{ (Put } a = \frac{2}{3}\text{)}$$

$$6\left(\frac{4}{9}\right) - \frac{50}{3} + 14 = 0$$

$$\frac{24}{9} - \frac{50}{3} \cdot \frac{3}{3} + 14 \cdot \frac{9}{9} = 0 \text{ (Equating the denominators)}$$

$$\frac{24 - 150 + 126}{9} = 0 \text{ (Simplify)}$$

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{\frac{7}{2}, \frac{2}{3}\right\}}$.

Answer 11CU.

Consider the equation

$10p^2 - 19p + 7 = 0$

The objective is to solve the given equation.

For this first factor $10p^2 - 19p + 7$ and then use zero product property.

Compare $10p^2 - 19p + 7$ with $ax^2 + bx + c$.

$a = 10,$
$b = -19,$
$c = 7$

Now find two numbers m, n such that whose sum is

$b = -19$ and product

$ac = 10 \cdot 7$
$= 70$

Since $m + n = -19$ negative and

$m \cdot n = 70$ is positive.

So, m and n must be both negative.

Now list all the factors of 70, whose pair of factors in those, the sum of factors sum is -19.

Factors of 70	Sum of factors
$-1. -70$	-71
$-2. -35$	-37
$-5. -14$	-19
$-7. -10$	-17

$-7,-10$	-17

The correct factors are $-5,-14$.

$10p^2-19p+7=10\cdot p\cdot p-5p-14p+7$

(Because $19=5+14$)

$=2\cdot 5\cdot p\cdot p-5\cdot p-2\cdot 7\cdot p+7$

(Simplify)

$=5p(2p-1)-7(2p-1)$

(Group the terms with common factors)

$=(5p-7)(2p-1)$

(By distributive)

Therefore,

$10p^2-19p+7=(5p-7)(2p-1)$

Now $10p^2-19p+7=0$

$\Rightarrow \quad (5p-7)(2p-1)=0$ (Factors)

$\Rightarrow \quad 5p-7=0$ (or)

$2p-1=0$ (By zero product property)

Now solve each equation separately.

$5p-7=0$

$5p-7+7=0+7$

(Add 7 on each side)

$5p=7$

$\frac{5p}{5}=\frac{7}{5}$ (Divide by 5 on both sides)

$p=\frac{7}{5}$

$2p-1=0$

$2p-1+1=0+1$ (Add 1 on each side)

$2p=1$

$\frac{2p}{2}=\frac{1}{2}$ (Divide by 2 on both sides)

$p=\frac{1}{2}$

The solution set is $\left\{\frac{7}{5},\frac{1}{2}\right\}$.

Check:- To check the proposed solution, substitute p by $\frac{7}{5},\frac{1}{2}$ in given equation.

For $p=\frac{7}{5}$,

$10p^2-19p+7=0$

$10\left(\frac{7}{5}\right)^2-19\left(\frac{7}{5}\right)+7=0$ (Put $p=\frac{7}{5}$)

$10\left(\frac{49}{25}\right)-\frac{133}{5}+7=0$ (Simplify)

$\frac{490}{25}+7-\frac{133}{5}=0$

$\frac{490}{25}+7\cdot\frac{25}{25}-\frac{133}{5}\cdot\frac{5}{5}=0$ (Equate the denominators)

$\frac{490+175-665}{25}=0$ (Simplify)

$0=0$ True

For $p=\frac{1}{2}$,

$10p^2-19p+7=0$

$10\left(\frac{1}{2}\right)^2-19\left(\frac{1}{2}\right)+7=0$ (Put $p=\frac{1}{2}$)

$\frac{10}{4}-\frac{19}{2}+7=0$

$\frac{10}{4}-\frac{19}{2}\cdot\frac{2}{2}+7\cdot\frac{4}{4}=0$ (Equate the denominators)

$$\frac{10-38+28}{4}=0$$

$\Rightarrow \quad 0=0$ True

Therefore, the solution set is $\boxed{\frac{7}{5},\frac{1}{2}}$.

Answer 12CU.

Consider the equation

$6n^2+7n=20$

$6n^2+7n-20=20-20$ (Subtract 20 on each side)

$6n^2+7n-20=0$ (Simplify)

The objective is to solve the given equation.

For this first factor $6n^2+7n-20$ and then use zero product property.

Compare $6n^2+7n-20$ with ax^2+bx+c.

$a=6,$
$b=7,$
$c=-20$

Now find two numbers x, y such that whose sum is

$b=7$ and product is

$ac=6\cdot -120$
$=-120$

Since $x+y=7$ positive and

$xy=-120$ is negative.

So either x (or) y negative, but not both.

Now list all factors of -120, whose pair of factors in those, the sum 7.

Factors of -120	Sum of factors
-1.120	119
[illegible]	[illegible]

$1.-120$	-119
-2.60	58
$2.-60$	-58
$3.-40$	-37
-3.40	37
$4.-30$	-26
-4.30	26
$5.-24$	-19
-5.24	19
$6.-20$	-14
-6.20	14
-8.15	7
$8.-15$	-7

The correct factors are $-8,15$.

$$6n^2+7n-20=6\cdot n\cdot n-8\cdot n+15\cdot n-20$$

(Because $7=-8+15$)

$$=2\cdot 3\cdot n\cdot n-2\cdot 4\cdot n+3\cdot 5n-2\cdot 10$$

(Simplify)

$$=3n(2n+5)-4(2n+5)$$

(Group the terms with common factors)

$$(3n-4)(2n+5)$$

Now

$$6n^2+7n-20=0$$

$(3n-4)(2n+5)=0$ (Factors)

$3n-4=0$ (or)

$2n+5=0$ (By zero product property)

Now solve each equation separately.

$$3n-4=0$$

$3n+4+4=0+4$ (Add 4 on each side)

$$3n=4$$

$\frac{3n}{3}=\frac{4}{3}$ (Divide by 3 on both sides)

$$n=\frac{4}{3}$$

$$2n+5=0$$

$\Rightarrow \quad 2n+5-5=0-5$ (Substract 5 on each side)

$$2n=-5$$

$\frac{2n}{2}=\frac{-5}{2}$ (Divide by 2 on both sides)

$$n=\frac{-5}{2}$$

The solution set is $\left\{\frac{4}{3},\frac{-5}{2}\right\}$.

Check:- To check the proposed solution, substitute

$n=\frac{4}{3},\frac{-5}{2}$ in given equation.

For $n=\frac{4}{3}$,

3

$6n^2+7n-20=0$

$6\left(\frac{4}{3}\right)^2+7\left(\frac{4}{3}\right)-20$ (Put $n=\frac{4}{3}$)
$=0$

$6\left(\frac{16}{9}\right)+\frac{28}{3}\cdot\frac{3}{3}-20\cdot\frac{9}{9}$ (Simplify, equating the denominators)
$=0$

$\frac{96}{9}+\frac{84}{9}-\frac{180}{9}=0$ (Simplify)

$0=0$ True

For $n=\frac{-5}{2}$,

$6n^2+7n-20=0$

$6\left(\frac{-5}{2}\right)^2+7\left(\frac{-5}{2}\right)-20$ (Put $n=-\frac{5}{2}$)
$=0$

$6\left(\frac{25}{4}\right)-\frac{35}{2}\cdot\frac{2}{2}-20\cdot\frac{4}{4}$ (Equating the denominators)
$=0$

$\frac{150}{4}-\frac{70}{4}-\frac{80}{4}=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{4}{3},\frac{-5}{2}\right\}}$.

Answer 14PA.

Consider the trinomial $2x^2+7x+5$

The objective is to factor of the given trinomial.

Compare $2x^2+7x+5$ with ax^2+bx+c.

Here $a=2,$

$b=7,$

$c=5$

Now find two numbers m,n such that whose sum is

$b=7$ and product is

$ac=2\cdot 5$

$=10$

For this list all the factors of 10 and choose a pair of factors with sum 7.

Factors of 10	Sum of factors
1.10	11
2.5	7

The correct factors are $2,5$.

Now $2x^2+7x+5=2x^2+2x+5x+5$

(Because $10=2\cdot 5$)

$=2\cdot x\cdot x+2\cdot x+5\cdot x+5$

(Simplify)

$=2x(x+1)+5(x+1)$

(Group the terms with common factors)

$=(2x+5)(x+1)$

Therefore, $2x^2+7x+5=(2x+5)(x+1)$

Check:- To check the result by multiplying two factors using $FOIL$ method.

$$(2x+5)(x+1)=\overset{F}{2x}\cdot x+1\cdot\overset{O}{2x}+\overset{I}{5}\cdot x+\overset{L}{5}\cdot 1$$

($FOIL$ method)

$= 2x^2 + 2x + 5x + 5$

(Simplify)

$= 2x^2 + 7x + 5$

True

Therefore, the factorization of $\boxed{2x^2+7x+5}$ is $\boxed{(2x+5)(x+1)}$.

Answer 15PA.

Consider the trinomial $3x^2 + 5x + 2$.

The objective is to factor the given trinomial.

Compare $3x^2 + 5x + 2$ with $ax^2 + bx + c$.

Here $a = 3,$

$b = 5,$

$c = 2$

Now find two numbers m, n such that whose sum is

$b = 5$ and whose product is

$ac = 3 \cdot 2$

$= 6$

List all the factors of 6, choose one pair of those factors whose sum 5.

Factors of 6	Sum of factors
1.6	7
2.3	5

The correct factors are $2, 3$.

$3x^2 + 5x + 2 = 3x^2 + 3x + 2x + 2$ (Because $5 = 3 + 2$)

$= 3 \cdot x \cdot x + 3 \cdot x + 2 \cdot x + 2$

(Simplify)

$= 3x(x+1) + 2(x+1)$

(Group the terms with common factors)

$= (3x+2)(x+1)$ (By distributive)

Therefore, $3x^2 + 5x + 2 = (3x+2)(x+1)$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$$(3x+2)(x+1) = \overset{F}{3}x \cdot x + 1 \cdot \overset{O}{3}x + \overset{I}{2} \cdot x + \overset{L}{2} \cdot 1$$

($FOIL$ method)

$= 3x^2 + 3x + 2x + 2$

(Simplify)

$= 3x^2 + 5x + 2$ True

Therefore, the factorization of $\boxed{3x^2 + 5x + 2}$ is $\boxed{(3x+2)(x+1)}$.

Answer 16PA.

Consider the trinomial $6p^2+5p-6$.

The objective is to factor the given equation.

Compare $6p^2+5p-6$ with ax^2+bx+c.

Here $a=6$,

$b=5,$

$c=-6$

Since $m+n=5$ is positive and

$mn=-6$ is negative.

So either m (or) n is negative but not both.

Now list all the factors whose product is 5, choose a pair of factors whose sum is 5.

Factors of -6	Sum of factors
$1.-6$	-5
-1.6	5
-2.3	1
$2.-3$	-1

The correct factors are $-1,6$.

$6p^2+5p-6=6p^2-p+6p-6$ (Because $5=6-1$)

$=6\cdot p\cdot p-p+6\cdot p-6$ (Simplify)

There are no prime factors whose sum is 5.

Therefore, $6p^2+5p-6$ cannot be factored using integers.

Since a polynomial that cannot be written as a product of two polynomial with integral co-effacements is called a prime polynomial.

Therefore, $\boxed{6p^2+5p-6}$ is a prime polynomial.

Answer 17PA.

Consider the trinomial $5d^2+6d-8$.

The objective is to factor the given equation.

Compare $5d^2+6d-8$ with ax^2+bx+c.

Here $a=5$,

$b=6,$

$c=-8$

Now find two numbers m,n such that whose sum is

$b=6$ and product is

$$ac=5\cdot -8$$
$$=-40$$

Also $m+n=6$ positive and

$mn=-40$ is negative.

So, either m (or) n negative but not both.

List all the factors of -40, chose one pair of factors whose sum is b.

Factors of -40	Sum of factors
-1.40	39
$1.-40$	-39
$8.-5$	3
-8.5	-3
$2.-20$	-18
-2.20	18
[illegible]	[illegible]

-10.4	-6
-4.10	6

The correct factors are $-4,10$.

$5d^2+6d-8=5d^2+10d-4d-8$ (Because $6=10-4$)

$=5\cdot d\cdot d+2\cdot 5\cdot d-2\cdot 2\cdot d-2\cdot 2\cdot 2$

(Simplify)

$=5d(d+2)-4(d+2)$ (Group all the terms with common factors)

$=(5d-4)(d+2)$ (By destructive)

Therefore,

$5d^2+6d-8=(5d-4)(d+2)$

Check:- Check the result, by multiplying two factors using $FOIL$ method.

$(5d-4)(d+2)=\overset{F}{5}d\cdot d+2\cdot\overset{O}{5}d-\overset{I}{4}\cdot d-\overset{L}{4}\cdot 2$ ($FOIL$ method)

$=5d^2+10d-4d-8$ (Simplify)

$=5d(d+2)-4(d+2)$ (Group all the terms with common factors)

$=(5d-4)(d+2)$ (By distributive)

Therefore,

$5d^2+6d-8=(5d-4)(d+2)$

Check:- Check the result, by multiplying two factors using $FOIL$ method.

$(5d-4)(d+2)=\overset{F}{5}d\cdot 2\cdot\overset{O}{5}d-\overset{I}{4}\cdot d-\overset{L}{4}\cdot 2$ ($FOIL$ method)

$=5d^2+10d-4d-8$ (Simplify)

$=5d^2+6d-8$ True

Thereofe, the factorized form of $\boxed{5d^2+6d-8}$ is $\boxed{(5d-4)(d+2)}$.

Answer 18PA.

Consider the trinomial $8k^2-19k+9$.

The objective is to factor the given trinomial.

Compare $8k^2-19k+9$ with ax^2+bx+c.

$a=8,$
$b=-19,$
$c=9$

Now find the two numbers m,n such that whose sum is

$b=-19$ and the product is

$ac=8\cdot 9$
$=72$

Since $m+n=-19$ negative and

$mn=72$ is positive.

So, m,n must be negative.

Now list the factors of 72, choose one pair of factors whose sum is -19.

Factors of 72	Sum of factors
$-1.-72$	-73
$-8.-9$	-17
$-36.-2$	-38
$-24.-3$	-27
$-18.-4$	-22
$-6.-12$	-18

There are no prime factors whose sum is -19.

Therefore, $8k^2-19k+9$ cannot be factored using integers.

Since a polynomial that cannot be written as a product of two polynomials with integral co-efficient is called a prime polynomial efficient is called a prime polynomial.

Therefore, $\boxed{8k^2 - 19k + 9}$ is a prime polynomial.

Answer 19PA.

Consider the trinomial $9g^2 - 12g + 4$.

The objective is to factor the given trinomial.

Compare $9g^2 - 12g + 4$ with $ax^2 + bx + c$.

$a = 9,$
$b = -12,$
$c = 4$

Now find two numbers m, n such that whose sum is

$b = -12$ and product is

$ac = 9 \cdot 4$
$= 36$

Also $m + n = -12$ negative and

$mn = 36$ is positive. So m, n must both be negative. So list all negative factors of 36, in those choose a pair of factors whose sum is -12.

Factors of 36	Sum of factors
$-1, -36$	-37
$-9, -4$	-13
$-6, -6$	-12
$-12, -3$	-15
$-18, -2$	-20

The correct factors are $-6, -6$.

$9g^2 - 12g + 4 = 9g^2 - 6g - 6g + 4$ (Because $12 = 6 + 6$)

$=3\cdot3\cdot g\cdot g-2\cdot3\cdot g-2\cdot3g+2\cdot2$

(Simplify)

$=3g(3g-2)-2(3g-2)$ (Group all terms with common factors)

$=(3g-2)(3g-2)$ (By distributive)

Therefore,

$9g^2-12g+4=(3g-2)(3g-2)$

Check:- Check the results by multiplying two factors using $FOIL$ method.

$(3g-2)(3g-2)=3g\cdot\overset{F}{3}g-2\cdot\overset{O}{3}g-2\cdot\overset{I}{3}g+\overset{L}{4}$

($FOIL$ method)

$=9g^2-6g-6g+4$ (Simplify)

$=9g^2-12g+4$ True.

Therefore, the factorized form of $\boxed{9g^2-12g+4}$ is $\boxed{(3g-2)(3g-2)}$.

Answer 20PA.

Consider the trinomial $2a^2-9a-18$.

The objective is to factor the given trinomial.

Compare $2a^2-9a-18$ with ax^2+bx+c.

Here $a=2$,

$b=-9$,

$c=-18$

Now find two numbers m,n such that whose sum is

$b=-9$ and product is

$ac=2\cdot-18$

$=-36$

Also $m+n=-9$ negative and

$mn=-36$ positive. So, either m (or) n is negative but not both.

Now list all the factors of -36, choose one pair of factors whose sum is -9.

Factors of -36	Sum of factors
$1.-36$	-35
-1.36	35
-6.6	0
$6.-6$	0
-12.3	-9
$3.-12$	9
$9.-4$	5
-9.4	-5
-18.2	-16
-2.18	16

The correct factors are $-12,3$.

$2a^2-9a-18=2a^2-12a+3a-18$ (Because $9=-12+3$)

$=2\cdot a\cdot a-2\cdot 6\cdot a+3\cdot a-3\cdot 6$

(Simplify)

$=2a(a-6)+3(a-6)$ (Group the terms with common factors)

$=(2a+3)(a-6)$ (By distributive)

Therefore,

$2a^2-9a-18=(2a+3)(a-6)$

$2a^2-9a-18=(2a+3)(a-6)$

Check:- Check the result, by multiplying two factors using $FOIL$ method.

$$(2a+3)(a-6)=\overset{F}{2a\cdot a}-\overset{O}{6\cdot 2a}+\overset{I}{3\cdot a}-\overset{L}{3\cdot 6}$$ ($FOIL$ method)

$=2a^2-12a+3a-18$ (Simplify)

$=2a^2-9a-18$ True

Therefore, the factorized form of $\boxed{2a^2-9a-18}$ is $\boxed{(2a+3)(a-6)}$.

Answer 21PA.

Consider the trinomial $2x^2-3x-20$.

The objective is to factor the given trinomial.

Compare $2x^2-3x-20$ with ax^2+bx+c.

Here $a=2$,

$b=-3,$

$c=-20$

Now find two numbers m,n such that whose product is

$$\begin{aligned}ac&=2\cdot -20\\&=-40\end{aligned}$$ and sum is

$b=-3$.

Also $m+n=-3$ negative and

$mn=-40$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors of -40, choose one pair of those factors whose sum is -3.

Factors of -40	Sum of factors
-1.40	39
$1.-40$	-39
$2.-20$	-18

-2.20	18
$10.-4$	6
-10.4	-6
-8.5	-3
$8.-5$	3

The correct factors are $-8,5$.

$2x^2-3x-20=2x^2-8x+5x-20$ (Because $-3=-8+5$)

$=2\cdot x\cdot x-2\cdot 4\cdot x+5\cdot x-2\cdot 10$

(Simplify)

$=2x(x-4)+5(x-4)$ (Group all the terms with common factors)

$=(2x+5)(x-4)$ (By distributive)

Therefore,

$2x^2-3x-20=(2x+5)(x-4)$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$(2x+5)(x-4)=\overset{F}{2}x\cdot 4\cdot \overset{O}{2}x+\overset{I}{5}\cdot x-\overset{L}{5}\cdot 4$ ($FOIL$ method)

$=2x^2-8x+5x-20$ (Simplify)

$=2x^2-3x-20$ True

Therefore, the factorization form of $\boxed{2x^2-3x-20}$ is $\boxed{(2x+5)(x-4)}$.

Answer 22PA.

Consider the trinomial $5c^2 - 17c + 14$.

The objective is to factor the given trinomial.

Compare $5c^2 - 17c + 14$ with $ax^2 + bx + c$.

Here $a = 5$,

$b = -17,$

$c = 14$

Now find two numbers m, n such that whose sum is

$b = -17$ and product is

$$ac = 5 \cdot 14$$
$$= 70$$

Since $m + n = -17$ negative and

$mn = 70$ positive.

So m, n must both be negative.

Now list all the factors of 70, choose one pair of factors in those, whose sum is -17.

Factors of 70	Sum of factors
$-1. -70$	-71
$-2. -35$	-37
$-5. -14$	-19
$-7. -10$	-17

The correct factors are $-7, -10$

$5c^2 - 17c + 14 = 5c^2 - 7c - 10c + 14$ (Because $-17 = -7 - 10$)

$= 5 \cdot c \cdot c - 7 \cdot c - 2 \cdot 5 \cdot c + 2 \cdot 7$

(Simplify)

$= c(5c-7)-2(5c-7)$ (Group all the terms with common factors)

$= (c-2)(5c-7)$ (by distributive)

Therefore,

$5c^2-17c+14=(c-2)(5c-7)$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$(c-2)(5c-7)=c\cdot\overset{F}{5}c-7\cdot\overset{O}{c}-2\cdot\overset{I}{5}c+2\cdot\overset{L}{7}$ ($FOIL$ method)

$=5c^2-7c-10c+14$ (Simplify)

$=5c^2-17c+14$ True

Therefore, the factorization of $\boxed{5c^2-17c+14}$ is $\boxed{(c-2)(5c-7)}$.

Answer 23PA.

Consider the trinomial $3p^2-25p+16$.

The objective is to factor the given trinomial.

Compare $3p^2-25p+16$ with ax^2+bx+c.

Here $a=3$,

$b=-25,$

$c=16$

Now find two numbers m,n such that whose product is

$ac=3\cdot16$
$=48$ and sum is

$b=-25$.

Since $m+n=-25$ is negative and

$mn=48$ is positive.

So, m,n must both be negative.

Now list all the factors of 48, in those choose one pair of factors whose sum is -25.

Factors of -48	Sum of factors
$-1,-48$	-49

$-6, -8$	-14
$-12, -4$	-16
$-2, -24$	-26
$-3, -16$	-19

There are no prime factors whose sum is -25.

Therefore, $3p^2 - 25p + 16$ cannot be factored using integers.

Since a polynomial that cannot be written as a product of two polynomials with integral co-efficient is called a prime polynomial.

Therefore, $\boxed{3p^2 - 25 + 16}$ is a prime polynomial.

Answer 24PA.

Consider the trinomial $8y^2 - 6y - 9$.

The objective is to factor the given trinomial.

Compare $8y^2 - 6y - 9$ with $ax^2 + bx + c$.

Here $a = 8,$

$b = -6,$

$c = -9$

Now find two numbers m, n such that whose sum is

$b = -6$ and product is

$$ac = 8 \cdot -9$$
$$= -72$$

Since $m + n = -6$ negative but

$mn = -72$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors of -72, choose one pair of factors whose sum is -6.

Factors of -72	Sum of factors
-1.72	71
$1.-72$	-71
-8.9	1
$8.-9$	-1
-2.36	34
$2.-36$	-34
$3.-24$	-21
-3.24	21

-18.4	-14
$18.-4$	14
-12.6	-6
$12.-6$	6

The correct factors are $-12,6$.

$8y^2-6y-9=8y^2-12y+6y-9$ (Because $-6=-12+6$)

$=8\cdot y\cdot y-2\cdot 6\cdot y+2\cdot 3\cdot y-3\cdot 3$

(Simplify)

$=2\cdot 4\cdot y\cdot y-2\cdot 2\cdot 3\cdot y+2\cdot 3\cdot y-3\cdot 3$

(Simplify)

$=4y(2y-3)+3(2y-3)$ (Group the terms with common factors)

$=(4y+3)(2y-3)$

Therefore, $8y^2-6y-9=(4y+3)(2y-3)$.

Check: To check the result, by multiplying two factors using $FOIL$ method.

$(4y+3)(2y-3)=\overset{F}{4}y\cdot 2y-3\cdot\overset{O}{4}y+3\cdot\overset{I}{2}y-3\cdot\overset{L}{3}$

($FOIL$ method)

$=8y^2-12y+6y-9$ (Simplify)

$=8y^2-6y-9$ True

Therefore, the factorized form of $\boxed{8y^2-6y-9}$ is $\boxed{(4y+3)(2y-3)}$.

Answer 25PA.

Consider the trinomial $10n^2 - 11n - 6$.

The objective is to factor the given trinomial.

Compare $10n^2 - 11n - 6$ with $ax^2 + bx + c$.

Here $a = 10,$

$b = -11,$

$c = -6$

Now find two numbers m, n such that whose product is

$$ac = 10 \cdot -6$$
$$= -60$$

and sum is

$b = -11$.

Since $m + n = -11$ negative and

$mn = -60$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors of -60, choose one pair of factors whose sum is -11.

Factors of -60	Sum of factors
-1.60	59
$1.-60$	-59
-12.5	-7
-5.12	7
$10.-6$	4
-10.6	-4
-2.30	28

-2.30	28
$2.-30$	-28
$3.-20$	-17
-3.20	17
-15.4	-11
$15.-4$	11

The correct factors are $-15, 4$.

$10n^2 - 11n - 6 = 10n^2 - 15n + 4n - 6$ (Because $-11 = -15 + 4$)

$= 10 \cdot n \cdot n - 5 \cdot 3 \cdot n + 2 - 2 \cdot n - 2 - 3$

(Simplify)

$= 2 \cdot 5 \cdot n \cdot n - 5 \cdot 3 \cdot n + 2 \cdot 2 \cdot n - 2 \cdot 3$

(Simplify)

$= 5n(2n-3)+2(2n-3)$ (Group the terms with common factors)

$= (5n+2)(2n-3)$

Therefore,

$10n^2 - 11n - 6 = (5n+2)(2n-3)$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$(5n+2)(2n-3) = \overset{F}{5}n\cdot 2n - 3\cdot \overset{O}{5}n + 2\cdot \overset{I}{2}n - 2\cdot \overset{L}{3}$

($FOIL$ method)

$= 10n^2 - 15n + 4n - 6$ (Simplify)

$= 10n^2 - 11n - 6$ True

Therefore, the factorized form of $\boxed{10n^2 - 11n - 6}$ is $\boxed{(5n+2)(2n-3)}$.

Answer 26PA.

Consider the trinomial $15z^2 + 17z - 18$.

The objective is to factor the given trinomial.

Compare $15z^2 + 17z - 18$ with $ax^2 + bx + c$.

Here $a = 15$,

$b = 17,$

$c = -18$

Now find two numbers m, n such that whose sum is

$b = 17$ and whose product is

$ac = 15 \cdot -18$

$= -270$

Since $m + n = 17$ positive and

$mn = -270$ is negative.

So, either m (or) n negative but not both.

Now list all the factors of -270, choose one pair of those factors whose sum is 17.

Factor of -270	Sum of factors

-1.270	269
$1.-270$	-269
-2.135	133
$2.-135$	-133
$3.-90$	-87
-3.90	87
$5.-54$	-49
-5.54	49
$6.-45$	-39
-6.45	39
$9.-30$	-21
-9.30	21
-10.27	17
$10.-27$	-17

The correct factors are $-10, 27$.

$15z^2+17z-18=15z^2-10z+27z-18$ (Because $17=-10+27$)

$= 15\cdot z\cdot z - 2\cdot 5\cdot z + 3\cdot 9z - 3\cdot 6$

(Simplify)

$= 3\cdot 5\cdot z\cdot z - 2\cdot 5z + 3\cdot 3\cdot 3z - 3\cdot 6$

$= 3\cdot 5\cdot z\cdot z - 2\cdot 5\cdot z + 3\cdot 3\cdot 3\cdot z - 3\cdot 3\cdot 2$

(Simplify)

$= 5z(3z-2) + 9(3z-2)$ (Group all terms with common factors)

$= (5z+9)(3z-2)$ (By distributive)

Therefore,

$15z^2 + 17z - 18 = (5z+9)(3z-2)$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$(5z+9)(3z-2) = 5z\cdot \overset{F}{3}z - 2\cdot \overset{O}{5}z + 9\cdot \overset{I}{3}z - 9\cdot \overset{L}{2}$

($FOIL$ method)

$= 15z^2 - 10z + 27z - 18$ (Simplify)

$= 15z^2 + 17z - 18$ True

Therefore, the factorized form of $\boxed{15z^2 + 17z - 18}$ is $\boxed{(5z+9)(3z-2)}$.

Answer 27PA.

Consider the trinomial $14x^2 + 13x - 12$

The objective is to factor the given equation.

Compare $14x^2 + 13x - 12$ with $ax^2 + bx + c$

Here $a = 14,$

$b = 13,$

$c = -12$

Now find two numbers m, n such that whose sum is

$b = 13$ and whose product is

$ac = 14\cdot -12$

$= -168$

Since $m + n = 13$ positive and

$mn = -168$ is negative.

So, either m (or) n is negative.

Now list all the factors of -168, in those factors choose one pair of factors whose sum is 13.

Factors of -168	Sum of factors
$1.-168$	-167
-1.168	167
$2.-84$	-82
-2.84	82
$3.-56$	-53
-3.56	53
$4.-42$	-38
-4.42	38
$6.-28$	-22
-6.28	22
$7.-24$	-17
-7.24	17

$8.-21$	-13
-8.21	13
-12.14	2
$12.-14$	-2

The correct factors are $-8, 21$.

$$4x^2 + 13x - 12 = 14x^2 - 8x + 21x - 12$$

(Because $13 = 21 - 8$)

$$= 2\cdot 7\cdot x\cdot x - 2\cdot 2\cdot 2x + 3\cdot 7\cdot x - 3\cdot 2\cdot 2$$

(Simplify)

$$= 2x(7x-4) + 3(7x-4)$$

(Group all terms with common factors)

$$= (2x+3)(7x-4)$$

Therefore,

$$14x^2 + 13x - 12 = (2x+3)(7x-4)$$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$$(2x+3)(7x-4) = 2x\cdot \overset{F}{7}x - 4\cdot \overset{O}{2}x + 3\cdot \overset{I}{7}x - 3\cdot \overset{L}{4}$$

($FOIL$ method)

$$= 14x^2 - 8x + 21x - 12$$

(Simplify)

$$= 14x^2 + 13x - 12$$

True

Therefore, the factorized form of $\boxed{14x^2 + 13x - 12}$ is $\boxed{(2x+3)(7x-4)}$.

Answer 28PA.

Consider the trinomial $6r^2-14r-12$.

The objective is to factor of the given trinomial.

Compare $6r^2-14r-12$ with ax^2+6x+c.

Here $a=6$,

$b=-14$,

$c=-12$

Now find two numbers m,n such that whose sum is

$b=-14$ and product is

$$\begin{aligned} ac &= 6\cdot -12 \\ &= -72 \end{aligned}$$

Since $m+n=-14$ negative and

$mn=-72$ is also negative. So, either m (or) n negative but not both.

Now list all the factors of -72, choose one pair of factors whose sum is -14.

Factors of -72	Sum of factors
$1.-72$	-71
-1.72	71
$8.-9$	-1
-8.9	1
-2.36	34
$2.-36$	-34
$3.-24$	-21
-3.24	21

-18.4	-14
$18.-4$	14
-12.6	-6
-6.12	6

The correct factors are $-18, 4$.

$$6r^2 - 14r - 12 = 6r^2 - 18r + 4r - 12$$

(Because $-14 = -18 + 4$)

$$= 2 \cdot 3 \cdot r \cdot r - 2 \cdot 3 \cdot 3 \cdot r + 2 \cdot 2 \cdot r - 2 \cdot 2 \cdot 3$$

(Simplify)

$$= 6r(r-3) + 4(r-3)$$

(Group all the terms with the common factors)

$= (6r+4)(r-3)$ (By distributive)

Therefore, $6r^2 - 14r - 12 = (6r+4)(r-3)$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$$(6r+4)(r-3) = \overset{F}{6r \cdot r} - \overset{O}{3 \cdot 6r} + \overset{I}{4 \cdot r} - \overset{L}{3 \cdot 4}$$

($FOIL$ method)

$$= 6r^2 - 18r + 4r - 12$$

(Simplify)

$= 6r^2 - 14r - 12$ True

Therefore, the factorized form of $\boxed{6r^2 - 14r - 12}$ is $(6r+4)(r-3)$.

Answer 29PA.

Consider the trinomial $30x^2 - 25x - 30$.

The objective is to factor the given trinomial

The objective is to factor the given trinomial.

Compare $30x^2-25x-30$ with ax^2+bx+c.

Here $a=30$,

$b=-25,$

$c=-30$

Now find two numbers m,n such that whose sum is

$b=-25$ and whose product

$ac=30\cdot-30$

$\quad=-900$

Since $m+n=-25$ negative and

$mn=-900$ is also negative.

So, either m (or) n negative, but not both.

Now list all the factors of -900, choose one pair of factors whose sum is -25.

Factors of -900	Sum of factors
-1.900	899
-900.1	-899
-2.450	448
$2.-450$	-448
$3.-300$	-297
-3.300	297
-4.225	221
$4.-225$	-221

5. −180	−175
−5.180	175
−6.150	144
6. −150	−144
9. −100	−91
−9.100	91
10. −90	−80
−10.90	80
−12.75	63
12. −75	−63
15. −60	−45
−15.60	45
18. −50	−32
−18.50	32

20. − 45	−25
−20.45	25

$25, -36$	-11
$-25, 36$	11
$30, -30$	0
$-30, 30$	0

The correct factors are $20, -45$.

$30x^2 - 25x - 30 = 30x^2 - 45x + 20x - 30$

(Because $25 = -45 + 20$)

$= 2 \cdot 15 \cdot x \cdot x - 9 \cdot 5x + 4 \cdot 5x - 2 \cdot 15$

(Simplify)

$= 2 \cdot 3 \cdot 5 \cdot x \cdot x - 3 \cdot 3 \cdot 5 \cdot x + 2 \cdot 2 \cdot 5 \cdot x - 2 \cdot 5 \cdot 3$

(Simplify)

$= 15x(2x - 3) + 10(2x - 3)$

(Group the all terms with common factors)

$= (15x + 10)(2x - 3)$

Therefore,

$30x^2 - 25x - 30 = (15x + 10)(2x - 3)$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$(15x + 10)(2x - 3) = \overset{F}{15}x \cdot 2x - 3 \cdot \overset{O}{15}x + \overset{I}{10} \cdot 2x - \overset{L}{10} \cdot 3$

($FOIL$ method)

$= 30x^2 - 45x + 20x - 30$

(Simplify)

$= 30x^2 - 25x - 30$ True

Therefore, the factorized form of $\boxed{30x^2 - 25x - 30}$ is $\boxed{(15x + 10)(2x - 3)}$.

Answer 30PA.

Consider the polynomial $9x^2+30xy+25y^2$

The objective is to factor the given polynomial.

The first term $=9x^2$

$$=3\cdot 3\cdot x^2 \qquad (3\cdot 3=9)$$
$$=3^2\cdot x^2$$
$$=(3x)^2$$

Last term $=25y^2$

$$=5\cdot 5y^2 \qquad [5\cdot 5=25]$$
$$=5^2y^2$$
$$=(5y)^2 \qquad \left[(ab)^m=a^mb^m\right]$$

Here the first is perfect square

Last term is perfect square

Middle term is twice the product of square root of first and last term.

Thus, given polynomial is perfect square

Since $(a+b)^2=a^2+2ab+b^2$

$$9x^2+30xy+25y^2=(3x)^2+2\cdot(3x)(5y)+(5y)^2$$
$$=(3x+5y)^2$$

Therefore, the factorization of given polynomial is $\boxed{(3x+5y)^2}$

Answer 31PA.

Consider the polynomial $36a^2 + 9ab - 10b^2$

The objective is to factor the given polynomial.

$$
\begin{aligned}
&36a^2 + 9ab - 10b^2 \\
&\quad = 36a^2 + 24ab - 15ab - 10b^2 \quad [\text{Here } 24ab - 15ab = 9ab] \\
&\quad = 12 \cdot 3 \cdot a \cdot a + 12 \cdot 2 \cdot a \cdot 6 - 5 \cdot 3 \cdot a \cdot b + (-5) \cdot 2 \cdot b \cdot b \\
&\quad = 12a(3a + 2b) + (-5b)(3a + 2b) \\
&= (12a - 5b)(3a + 2b) \qquad [\text{By distributive; } (b + c)a = ba + ca]
\end{aligned}
$$

Therefore, the factor of given polynomial is $\boxed{(12a - 5b)(3a + 2b)}$

Answer 32PA.

$3 \cdot 8$	11
$-3 \cdot -8$	-11
$4 \cdot 6$	10
$-4 \cdot -6$	-10

Thus the possible values of *k* are $\pm 10, \pm 11, \pm 14, \pm 25$

Therefore, the values of *k* are $\boxed{\pm 10, \pm 11, \pm 14, \pm 25}$

Answer 33PA.

Consider the trinomial $2x^2 + kx + 15$

The objective is to find all values of *k* so that $2x^2 + 10x + 15$ can be factored using integers as binomials.

Compare $2x^2 + 10x + 15$ with $ax^2 + bx + c$

$$a = 2, b = k, c = 15$$

$$\begin{aligned} 2x^2 + kx + 15 &= 2x^2 + mx + nx + 15 \\ &= 2x^2 + (m+n)x + 15 \end{aligned}$$

$$k = m + n, mn = 2 \cdot 15 = 30$$

$2x^2 + kx + 15$ can be factored as two binomials only when there exists two number m, n such that $mn = 30$ and $m + n = k$

For this list all pair of factors of $mn = 30$, the sum of these factors nothing but the values of *k*

Factors of 30	Sum of factors or values of *k*
$-1 \cdot -30$	-31
$1 \cdot 30$	31

$2 \cdot 15$	17
$-2 \cdot -15$	-17
$3 \cdot 10$	13
$-3 \cdot -10$	-13
$5 \cdot 6$	11
$-5 \cdot -6$	-11

The possible values of *k* are $\pm 31, \pm 17, \pm 13, \pm 11$

Therefore, the values of *k* are $\boxed{\pm 31, \pm 17, \pm 13, \pm 11}$

Answer 34PA.

The equation is $2x^2 + 12x + k,\ \ k > 0$

The objective is to find all values of *k* so that $2x^2 + 12x + k,\ \ k > 0$ can be factored using integers as two binomials.

Compare $2x^2 + 12x + k$ with $ax^2 + bx + c$

Here $a = 2, b = 12, c = k$

$$\begin{aligned} 2x^2 + 12x + k &= 2x^2 + mx + nx + k \\ &= 2x^2 + (m+n)x + k \end{aligned}$$

$m+n, mn$ both are positive then m, n must be positive.

Now find the two numbers m, n so that $m+n=12$ and $mn = 2k$, must be even number

$m+n=12$	$mn=2k$
$1+11$	✓ 11

$2+10$	20
$3+9$	27
$4+8$	✓ 32
$5+7$	35
$6+6$	✓ 36

In the above, the even numbers are $20, 32, 36$

$$2k = 20 \quad \Rightarrow k = 10$$
$$2k = 32 \quad \Rightarrow k = 16$$
$$2k = 36 \quad \Rightarrow k = 18$$

The possible values of *k* are $10, 16, 18$

Therefore, the values of *k* are $\boxed{10,16,18}$

Answer 35PA.

Consider the equation

$$5x^2 + 27x + 10 = 0$$

The objective is to solve the given equation.

Compare $5x^2 + 27x + 10$ with $ax^2 + bx + c$.

Here $a = 5$,

$b = 27$,

$c = 10$

Now find two numbers m, n such that whose sum is

$b = 27$ and product is

$$ac = 5 \cdot 10$$
$$= 50$$

First find the factors and then use zero product property.

Since $m+n=27$ positive and

$mn=50$ is also positive.

Now list all the factors, choose one pair in those, whose sum is

Factors of 50	Sum of factors
1.50	51
2.25	27
10.5	15

The correct factors are $2,25$.

$$5x^2+27x+10=5x^2+2x+25x+10$$

(Because $27=2+25$)

$$=5x^2\cdot x+2\cdot x+5\cdot 5\cdot x+2\cdot 5$$

(Simplify)

$$=5x(x+5)+2(x+5)$$

(Group all terms with common factors)

$$=(5x+2)(x+5)$$

Therefore,

$$5x^2+27x+10=(5x+2)(x+5)$$

Now $5x^2+27x+10=0$

$(5x+2)(x+5)=0$ (Factors)

$5x+2=0$

Or, $x+5=0$ (By zero product property)

Now solve each equation separately.

$5x+2=0$

$5x+2-2=0-2$ (Subtract 2 on each side)

$5x=-2$

$\frac{5x}{5}=-\frac{2}{5}$ (Divide by 5 on each side)

$x=-\frac{2}{5}$

$x+5=0$

$x+5-5=0-5$ (Subtract 5 on each side)

$x=-5$

The solution set is $\left\{\frac{-2}{5},-5\right\}$.

Check: To check the result, x by $\frac{-2}{5},-5$ in given equation.

For $x=\frac{-2}{5}$,

$5x^2+27x+10=0$

$5\left(\frac{-2}{5}\right)^2+27\left(-\frac{2}{5}\right)+10=0$ (Put $x=-\frac{2}{5}$)

$5\left(\frac{4}{25}\right)-\frac{54}{5}+10=0$ (Simplify)

$\frac{20}{25}-\frac{54}{5}\cdot\frac{5}{5}+10\cdot\frac{25}{25}=0$ (Equating the denominators)

$\frac{20-270+250}{25}=0$ (Simplify)

$0=0$ True

For $x=-5$,

$5x^2+27x+10=0$

$5(-5)^2+27(-5)+10=0$ (Put $x=-5$)

$125-135+10=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{-2}{5},-5\right\}}$.

Answer 36PA.

Consider the equation

$3x^2-5x-12=0$

The objective is to solve the given equation.

First find the factors of $3x^2-5x-12$, and then use zero product property.

Compare $3x^2-5x-12$ with ax^2+bx+c.

Here $a=3$,

$b=-5,$

$c=-12$

Now find two numbers m,n such that whose product is

$$\begin{aligned} ac &= 3\cdot -12 \\ &= -36 \end{aligned}$$ and whose sum is

$b=-5$

Since $m+n=-5$ is negative and

$mn=-36$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors, choose in those one pair whose sum is -5.

Factors of -36	Sum of factors
$1,-36$	-35

-1.36	35
$2.-18$	-16
-2.18	16
$3.-12$	-9
-3.12	9
$4.-9$	-5
-4.9	5
$6.-6$	0
-6.6	0

The correct Factors are $4,-9$.

$3x^2-5x-12=3x^2-9x+4x-12$ (Because $-5=-9+4$)

$=3\cdot x\cdot x-3\cdot 3\cdot x+2\cdot 2\cdot x-2\cdot 2\cdot 3$

(Simplify)

$=3x(x-3)+4(x-3)$ (Group all the terms with common factors)

$=(3x+4)(x-3)$ (By distributive)

Now, $3x^2-5x-12=0$

$(3x+4)(x-3)=0$ (Factors)

$3x+4=0$

Or, $x-3=0$ (By zero product property)

Now solve each equation separately.

$3x+4=0$

$3x+4-4=0-4$ (Subtract 4 on each side)

$3x=-4$

$\frac{3x}{3}=\frac{-4}{3}$ (Divide by 3 on both sides)

$x=\frac{-4}{3}$

$x-3=0$

$x-3+3=0+3$ (Add 3 on each side)

$x=3$

The solution set is $\left\{\frac{-4}{3},3\right\}$.

Check:- To check the result, substitute x by $\frac{-4}{3},3$ in given equation.

For $x=-\frac{4}{3}$,

$3x^2-5x-12=0$

$3\left(\frac{-4}{3}\right)^2-5\left(\frac{-4}{3}\right)-12=0$ (Put $x=\frac{-4}{3}$)

$3\left(\frac{16}{9}\right)+\frac{20}{3}-12=0$ (Simplify)

$\frac{16}{3}+\frac{20}{3}-12\cdot\frac{3}{3}=0$ (Equating the denominators)

$\frac{16+20-36}{3}=0$ (Simplify)

$0=0$ True

For $x = 3$,

$3x^2 - 5x - 12 = 0$

$3(3)^2 - 5(3) - 12 = 0$ (Put $x = 3$)

$3(9) - 15 - 12 = 0$

$27 - 15 - 12 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{-\frac{4}{3}, 3\right\}}$.

Answer 37PA.

Consider the equation

$24x^2 - 11x - 3 = 3x$

The objective is to solve the given equation.

First find the factors of

$24x^2 - 11x - 3 = 3x$ and then use zero product property.

$24x^2 - 11x - 3 = 3x$

$\Rightarrow \quad 24x^2 - 11x - 3 - 3x = 3x - 3x$ (Subtract $3x$ on each side)

$\Rightarrow \quad 24x^2 - 14x - 3 = 0$ (Simplify)

Compare $24x^2 - 14x - 3$ with $ax^2 + bx + c$.

Here $a = 24$,

$b = -14$,

$c = -3$

Find two numbers m, n such that whose product

$ac = 24 \cdot -3 = -72$ and whose sum is

$b = -14$.

Since $m + n = -14$ negative and

$mn = -72$ is also negative.

So, either m (or) n negative, but not both.

Factors of -72	Sum of factors
-1.72	71
$1.-72$	-71
-2.36	34
$2.-36$	-34
-3.24	21
$3.-24$	-21
-4.18	14
$4.-18$	-14
-6.12	6
$6.-12$	-6
-8.9	1
$8.-9$	-1

The correct factors are $4,-18$.

$24x^2-14-3=24x^2+4x-18x-3$ (Because $-14=4-18$)

$=24\cdot x\cdot x+2\cdot 2\cdot x-2\cdot 3\cdot 3\cdot x-3$

(Simplify)

$=2\cdot 2\cdot 2\cdot 3\cdot x\cdot x+2\cdot 2\cdot x-2\cdot 3\cdot 3\cdot x-3$

$=4x(6x+1)-3(6x+1)$ (Group all terms with common factors)

$=(4x-3)(6x+1)$ (By distributive)

Therefore,

$24x^2-14x-3=(4x-3)(6x+1)$

Now, $24x^2-14x-3=0$

$(4x-3)(6x+1)=0$ (Factors)

$(4x-3)=0$

Or, $6x+1=0$ (By zero product property)

Now solve each equation separately.

$4x-3=0$

$4x-3+3=0+3$ (Add 3 on each side)

$4x=3$

$\frac{4x}{4}=\frac{3}{4}$ (Divide by 4 on each side)

$x=\frac{3}{4}$

$6x+1=0$

$6x+1-1=0-1$ (Subtract 1 on each side)

$6x=-1$

$\frac{6x}{6}=\frac{-1}{6}$ (Divide by 6 on both sides)

$x=\frac{-1}{6}$

The solution set is $\left\{\frac{3}{4},\frac{-1}{6}\right\}$.

Check:- To check the solution, substitute x by $\frac{3}{4},\frac{-1}{6}$ in given equation.

For $x=\frac{3}{4}$,

$$24x^2-14x-3=0$$

$$24\left(\frac{3}{4}\right)^2-14\left(\frac{3}{4}\right)-3=0$$ (Put $x=\frac{3}{4}$)

$$24\left(\frac{9}{16}\right)-\frac{42}{4}-3=0$$

$$\frac{216}{16}-\frac{42}{4}.\frac{4}{4}-3\cdot\frac{16}{16}=0$$ (Equating the denominators)

$$\frac{216-168-48}{16}=0$$

$0=0$ True

For $x=\frac{-1}{6}$,

$$24x^2-14x-3=0$$

$$24\left(\frac{-1}{6}\right)^2-14\left(\frac{-1}{6}\right)-3=0$$

$$\frac{24}{36}+\frac{14}{6}-3=0$$

$$\Rightarrow \quad \frac{24}{36}+\frac{14}{6}\cdot\frac{6}{6}-3\cdot\frac{36}{36}=0$$ (Equating the denominators)

$$\Rightarrow \quad \frac{24+84-108}{36}=0$$

$\Rightarrow \quad 0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{3}{4},\frac{-1}{6}\right\}}$.

Answer 38PA.

Consider the equation

$$17x^2-11x+2=2x^2$$

$$\Rightarrow \quad 17x^2-11x+2-2x^2$$ (Subtract $2x^2$ on both sides)

$= 2x^2 - 2x^2$

$\Rightarrow \quad 15x^2 - 11x + 2 = 0$

The objective is to solve the equation. For this, first find the factors and then use zero product property.

Compare $15x^2 - 11x + 2$ with $ax^2 + bx + c$.

Here $a = 15$,

$b = -11,$

$c = 2$

Since $(x+m)(x+n) = x^2 + (m+n)x + mn$

Here sum $b = -11$, product

$ac = 15 \cdot 2$

$= 30$

Here $m + n = -1$ negative and

$mn = 30$ is positive.

So, m and n both are negative.

Now list all the factors of 30, choose one pair in those whose sum -11.

Factors of 30	Sum of factors
$-1, -30$	-31
$-2, -15$	-17
$-3, -10$	-13
$-5, -6$	-11

The correct factors are $-5, -6$.

$15x^2 - 11x + 2 = 15x^2 - 5x - 6x + 2$

(Because $11 = 5 + 6$)

$= 3 \cdot 5 \cdot x \cdot x - 5 \cdot x - 2 \cdot 3 \cdot x + 2$

$= 5\cdot 3\cdot x\cdot x - 5\cdot x - 2\cdot 3\cdot x + 2$

(Simplify)

$= 5x(3x-1) - 2(3x-1)$

(Group all the terms with common factors)

$= (5x-2)(3x-1)$

Therefore,

$15x^2 - 11x + 2 = 0$

$(5x-2)(3x-1) = 0$ (Factors)

$5x - 2 = 0$

Or, $3x - 1 = 0$ (Using zero product property)

Now solve each equation separately.

$5x - 2 = 0$

$5x - 2 + 2 = 0 + 2$ (Add 2 on each side)

$5x = 2$

$\frac{5x}{5} = \frac{2}{5}$ (Divide by 5 on both sides)

$x = \frac{2}{5}$

$3x - 1 = 0$

$3x - 1 + 1 = 0 + 1$ (Divide by 3 on both sides)

$x = \frac{1}{3}$

The solution set is $\left\{\frac{2}{5}, \frac{1}{3}\right\}$.

Check:- to check the solution, substitute x by $\frac{2}{5}, \frac{1}{3}$ in the given equation.

For $x=\frac{2}{5}$,

$$15x^2-11x+2=0$$

$$15\left(\frac{2}{5}\right)^2-11\left(\frac{2}{5}\right)+2=0$$ (Put $x=\frac{2}{5}$)

$$15\left(\frac{4}{25}\right)-\frac{22}{5}+2=0$$ (Simplify)

$$\frac{60}{25}-\frac{22}{5}\cdot\frac{5}{5}+2\cdot\frac{25}{25}=0$$ (Simplify)

$$\frac{60-110+50}{25}=0$$ (Simplify)

$0=0$ True

For $x=\frac{1}{3}$,

$$15x^2-11x+2=0$$

$$15\left(\frac{1}{3}\right)^2-11\left(\frac{1}{3}\right)+2=0$$ (Put $x=\frac{1}{3}$)

$$\frac{15}{9}-\frac{11}{3}+2=0$$ (Simplify)

$$\frac{15}{9}-\frac{11}{3}\cdot\frac{3}{3}+2\cdot\frac{9}{9}=0$$ (Simplify)

$$\frac{15-33+18}{9}=0$$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{2}{5},\frac{1}{3}\right\}}$.

Answer 39PA.

Consider the equation $14n^2 = 25n + 25$

Now $14n^2 = 25n + 25$

$\Rightarrow \quad 14n^2 - 25n = 25n + 25 - 25n$ (Subtract $25n$ on both sides)

$\Rightarrow \quad 14n^2 - 25n = 25$

$\Rightarrow \quad 14n^2 - 25n - 25 = 25 - 25$ (Subtract 25 on each side)

$\Rightarrow \quad 14n^2 - 25n - 25 = 0$

The objective is to solve the given equation.

Compare $14n^2 - 25n - 25$ with $ax^2 + bx + c$.

Here, $a = 14$,

$b = -25,$

$c = -25$

Now first find the two numbers m, n such that whose sum is

$b = -25$ and whose product is

$$ac = 14 \cdot -25$$
$$= -350$$ and then use zero product property.

Since $m + n = -25$, negative and

$mn = -350$ also negative.

So, either m (or) n negative but not both.

Now list all factors of -350, choose one pair those whose sum is -25.

Factors of -350	Sum of factors
-1.350	349
$1.-350$	-349
$2.-175$	-173
-2.175	173

-5.70	65
$5.-70$	-65
-7.50	63
$7.-50$	-63
-10.35	25
$10.-35$	-25
-14.25	11
$14.-25$	-11

The correct factors are $10,-35$.

$14n^2-25n-25=14n^2+10n-35n-25$

(Because $25=10-35$)

$=2\cdot7\cdot n\cdot n+2\cdot5\cdot n-7\cdot5n-5\cdot5$

(Simplify)

$=2n(7n+5)-5(7n+5)$

(Group all the terms with common factors)

$=(2n-5)(7n+5)$ (By distributive)

Therefore,

$14n^2-25n-25=0$

$(2n-5)(7n+5)=0$ (Factors)

$2n-5=0$

Or, $7n+5=0$ (By zero product property)

Now solve each equation separately.

$2n-5=0$

$2n-5+5=0+5$ (Add 5 on each side)

$2n=5$ (Simplify)

$\frac{2n}{2}=\frac{5}{2}$ (Divide by 2 on both side)

$n=\frac{5}{2}$

$7n+5=0$

$7n+5-5=0-5$ (Subtract 5 on each side)

$7n=-5$

$\frac{7n}{7}=\frac{-5}{7}$ (Divide by 7 on both side)

$n=\frac{-5}{7}$

The solution set is $\left\{\frac{5}{2},\frac{-5}{7}\right\}$.

Check:- To check the solution, by substitute n by $\frac{5}{2},\frac{-5}{7}$ in given equation.

For $n=\frac{5}{2}$,

$$14n^2-25n-25=0$$

$$14\left(\frac{5}{2}\right)^2-25\left(\frac{5}{2}\right)-25=0$$ (Put $n=\frac{5}{2}$)

$$14\left(\frac{25}{4}\right)-\frac{125}{2}-25=0$$ (Simplify)

$$\frac{350}{4}-\frac{125}{2}\cdot\frac{2}{2}-25\cdot\frac{4}{4}=0$$ (Equating the denominators)

$$\frac{350-250-100}{4}=0$$ (Simplify)

$0=0$ True

For $n=\frac{-5}{7}$,

$$14n^2-25n-25=0$$

$$14\left(\frac{-5}{7}\right)^2-25\left(\frac{-5}{7}\right)-25=0$$ (Put $n=-\frac{5}{7}$)

$$14\left(\frac{25}{49}\right)+\frac{125}{7}-25=0$$ (Simplify)

$$\frac{350}{49}+\frac{125}{7}\cdot\frac{7}{7}-25\cdot\frac{49}{49}=0$$ (Equating the denominators)

$$\frac{350}{49}+\frac{875}{49}-\frac{1225}{49}=0$$

$$\frac{350+875-1225}{49}=0$$

$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{5}{2},\frac{-5}{2}\right\}}$.

Answer 40PA.

Consider the equation

$12a^2 - 13a = 35$

The objective is to solve the equation.

$12a^2 - 13a = 35$

$12a^2 - 13a - 35 = 35 - 35$ (Subtract 35 on both sides)

$12a^2 - 13a - 35 = 0$

First find the factors of $12a^2 - 13a - 35$ and then use zero product property.

Compare $12a^2 - 13a - 35$ with $ax^2 + bx + c$.

Here $a = -12$,

$b = -13,$

$c = -35$

Now find two numbers m, n such that whose sum is

$b = -13$ and whose product is

$ac = 12 \cdot -35$

$= -420$

Since $m + n = -13$ negative and

$mn = -420$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors of -420, choose one pair in those factors whose sum is -13.

Factors of -420	Sum of factors
-1.420	419
$1.-420$	-419
$2.-210$	-208
-2.210	208

3. −140	−137
−3.140	137
4. −105	−101
−4.105	101
5. −84	−79
−5.84	79
−6.70	64
6. −70	−64
−7.60	53
7. −60	−53
10. −42	−32
−10.42	32
−12.35	23
12. −35	−23
−14.30	16

$14.-30$	-16
-15.28	13
$15.-28$	-13
-20.21	1
$20.-21$	-1

The correct factors are $15,-28$.

$$12a^2-13a-35=12a^2+15a-28a-35$$

(Because $-13=15-28$)

$$=2\cdot2\cdot3\cdot a\cdot a+3\cdot5\cdot a-7\cdot2\cdot2\cdot a-7\cdot5$$

(Simplify)

$$=3a(4a+5)-7(4a+5)$$

(Group all the terms with common factors)

$=(3a-7)(4a+5)$ (By distributive)

Now $12a^2-13a-35=0$

$\Rightarrow \quad (3a-7)(4a+5)=0$ (Factors)

$$3a-7=0$$

Or, $4a+5=0$ (Use zero product property)

Now solve each equation separately.

$$3a-7=0$$

$3a-7+7=0+7$ (Add 7 on each side)

$3a=7$

$\frac{3a}{3}=\frac{7}{3}$ (Divide by 3 on both sides)

$a=\frac{7}{3}$

$4a+5=0$

$4a+5-5=0-5$ (Subtract 5 on each side)

$4a=-5$

$\frac{4a}{4}=\frac{-5}{4}$ (Divide by 4 on both side)

$a=\frac{-5}{4}$

The solution set is $\left\{\frac{7}{3},\frac{-5}{4}\right\}$.

Check:- to check result, substitute a by $\frac{7}{3},\frac{-5}{4}$ in given equation.

For $a=\frac{-5}{4}$,

$12a^2-13a-35=0$

$$\Rightarrow \quad 12\left(\frac{-5}{4}\right)^2-13\left(\frac{-5}{4}\right)-35 \text{ (Put } a=\frac{-5}{4}\text{)}$$

$$=0$$

$\Rightarrow \quad 12\left(\frac{25}{16}\right)+\frac{65}{4}-35$

$=0$

$\Rightarrow \quad \frac{300}{16}+\frac{65}{4}\cdot\frac{4}{4}-35\cdot\frac{16}{16}$ (Equating the denominators)

$=0$

$\Rightarrow \quad \frac{300+260-560}{16}=0$

$\Rightarrow \quad 0=0$ True

For $a=\frac{7}{3}$,

$12a^2-13a-35=0$

$\Rightarrow \quad 12\left(\frac{7}{3}\right)^2-13\left(\frac{7}{3}\right)-35$ (Put $a=\frac{7}{3}$)

$=0$

$\Rightarrow \quad \frac{588}{9}-\frac{91}{3}-35=0$

$\Rightarrow \quad \frac{588}{9}-\frac{91}{3}\cdot\frac{3}{3}-35\cdot\frac{9}{9}$ (Equating the denominators)

$=0$

$\Rightarrow \quad \frac{588-273-315}{9}=0$

$\Rightarrow \quad 0=0$ True

Therefore the solution set is $\boxed{\left\{\frac{7}{3},\frac{-5}{4}\right\}}$.

Answer 41PA.

Consider the equation

$6x^2 - 14x = 12$

$\Rightarrow \quad 6x^2 - 14x - 12 = 12 - 12$ (Subtract 12 on each side)

$\Rightarrow \quad 6x^2 - 14x - 12 = 0$

The objective is to solve the given equation. For this first factors and then use zero product property.

Compare $6x^2 - 14x - 12$ with $ax^2 + bx + c$.

Here $a = 6$,

$b = -14,$

$c = -12$

Find two numbers m, n such that whose sum is

$b = -14$ and product is

$ac = 6 \cdot -12$

$= -72$

Since $m + n = -14$, negative and

$mn = -72$ also negative.

So, either m (or) n negative but not both.

Now list all the factors of -72, choose one pair in those whose sum is -14.

Factors of -72	Sum of factors
$1. -72$	-71
-1.72	71
$2. -36$	-34
-2.36	34
$3. -24$	-21

-3.24	21
$4.-18$	-14
-4.18	14
$6.-12$	-6
-6.12	6
$8.-9$	-1
-8.9	1

The correct factors are $4,-18$.

$$6x^2-14x-12=6x^2+4x-18x-12$$

(Because $-14=4-18$)

$$=2\cdot3\cdot x\cdot x+2\cdot2\cdot x-2\cdot3\cdot3\cdot x-2\cdot2\cdot3$$

(Simplify)

$$=2x(3x+2)-6(3x+2)$$

(Group all terms with common factors)

$=(2x-6)(3x+2)$

(By distributive)

Therefore,

$6x^2-14x-12=0$

$\Rightarrow \quad (2x-6)(3x+2)=0$ (Factors)

$2x-6=0$

Or, $3x+2=0$ (Using zero product property)

Now solve each equation separately.

$2x-6=0$

$2x-6+6=0+6$ (Add 6 on each side)

$2x=6$

$\frac{2x}{2}=\frac{6}{2}$ (Divide 2 on both sides)

$x=\frac{6}{2}$ (Because $6=2\cdot 3 \Rightarrow \quad \frac{6}{2}=3$)

$=3$

$3x+2=0$

$3x+2-2=0-2$ (Subtract 2 on each side)

$3x=-2$

$\frac{3x}{3}=-\frac{2}{3}$ (Divide by 3 on both sides)

$x=\frac{-2}{3}$

The solution set is $\left\{3,\frac{-2}{3}\right\}$.

Check:- To check the result, substitute x by $3,\frac{-2}{3}$ in given equation.

For $x=3$,

$6x^2 - 14x - 12 = 0$

$6(3)^2 - 14(3) - 12 = 0$ (Put $x = 3$)

$6(9) - 42 - 12 = 0$ (Simplify)

$54 - 54 = 0$

$0 = 0$ True

For $x = \frac{-2}{3}$,

$6x^2 - 14x - 12 = 0$

$6\left(-\frac{2}{3}\right)^2 - 14\left(-\frac{2}{3}\right) - 12 = 0$ (Put $x = \frac{-2}{3}$)

$6\left(\frac{4}{9}\right) + \frac{28}{3} - 12 = 0$ (Simplify)

$\frac{24}{9} + \frac{28}{3} \cdot \frac{3}{3} - 12 \cdot \frac{9}{9} = 0$ (Equating the denominators)

$\frac{24 + 84 - 108}{9} = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{3, \frac{-2}{3}\right\}}$.

Answer 42PA.

Consider the equation

$21x^2 - 6 = 15x$

$21x^2 - 6 - 15x = 15x - 15x$ (Subtract $15x$ on each side)

$21x^2 - 15x - 6 = 0$ (Simplify)

The objective is to solve the equation. For this first find factors and then use zero product property.

Compare $21x^2 - 15x - 6$ with $ax^2 + bx + c$.

Here $a = 21,$

$b = -15,$

$c = -6$

Find two numbers m, n such that whose sum is

$b = -15$ and whose product is

$ac = 21 \cdot -6$

$= -126$

Since $m + n = -15$ negative and

$mn = -126$ also negative.

So, either m (or) n negative but not both.

Now list all the factors of -126, choose one pair in those whose sum is -15.

Factors of -126	Sum of factors
$1.-126$	-125
-1.126	125
$2.-63$	-61
-2.63	61
$3.-42$	-39

-3.42	39
$6.-21$	-15
-6.21	15
$7.-18$	-11
-7.18	11
$9.-14$	-5
-9.14	5

The correct factors are $6,-21$.

$21x^2-15x-6=21x^2+6x-21x-6$

(Because $-15=6-21$)

$=3\cdot 7\cdot x\cdot x+2\cdot 3\cdot x-3\cdot 7\cdot x-2\cdot 3$

(Simplify)

$=3x(7x+2)-3(7x+2)$

$=(3x-3)(7x+2)$

(By distributive)

Therefore,

$21x^2-15x-6=(3x-3)(7x+2)$

Now $21x^2-15x-6=0$

$(3x-3)(7x+2)=0$ (Factors)

$3x-3=0$

Or $7x+2=0$ (Using zero product property)

Or, $7x+2=0$ (Using zero product property)

Now solve each equation separately.

$3x-3=0$

$3x-3+3=0+3$ (Add 3 on each side)

$3x=3$

$\frac{3x}{3}=\frac{3}{3}$ (Divide by 3 on both sides)

$x=1$

$7x+2=0$

$7x+2-2=0-2$ (Subtract 2 on both sides)

$7x=-2$

$\frac{7x}{7}=\frac{-2}{7}$ (Divide by 7 on each side)

$x=\frac{-2}{7}$

The solution set is $\left\{1,\frac{-2}{7}\right\}$.

Check:- To check the result, substitute x by $1,\frac{-2}{7}$ in given equation.

For $x=-1$,

$21x^2-15x-6=0$

$21(1)^2-15(1)-6=0$ (Put $x=1$)

$21-15-6=0$ (Simplify)

$0=0$ True

For $x=\frac{-2}{7}$,

$$21x^2 - 15x - 6 = 0$$

$$21\left(-\frac{2}{7}\right)^2 - 15\left(-\frac{2}{7}\right) - 6 = 0$$ (Put $x = \frac{-2}{7}$)

$$21\left(\frac{4}{49}\right) + \frac{30}{7} - 6 = 0$$

$$\frac{84}{49} + \frac{30}{7}\cdot\frac{7}{7} - 6\cdot\frac{49}{49} = 0$$ (Equating the denominators)

$$\frac{84 + 210 - 294}{49} = 0$$ (Simplify)

Therefore, the solution set is $\boxed{\left\{1, \frac{-2}{7}\right\}}$.

Answer 43PA.

Consider the equation

$$24x^2 - 30x + 8 = -2x$$

$$\Rightarrow \quad 24x^2 - 30x + 8 + 2x = -2x + 2x$$ (Add $2x$ on each side)

$$\Rightarrow \quad 24x^2 - 28x + 8 = 0$$

The objective is to solve the given equation, for this first find factors of given equation and then use zero product property.

Compare $24x^2 - 28x + 8$ with $ax^2 + bx + c$

Here $a = 24$,

$b = -28,$

$c = 8$

Find two numbers m, n such that whose sum is

$b = -28$ and whose product is

$$ac = 24\cdot 8$$
$$= 192$$

Since $m + n = -28$ negative and

$mn = 192$ is positive.

So, m, n must be both negative.

Now list all the factors of 192, choose one pair in those whose sum is -28.

Factors of 192	Sum of factors
$-1.-192$	-193
$-2.-96$	-98
$-3.-64$	-67
$-4.-48$	-52
$-6.-32$	-38
$-8.-24$	-32
$-12.-16$	-28

The correct factors are $-12,-16$.

$24x^2-28x+8=24x^2-12x-16x+8$

(Because $-28=-12-16$)

$=2\cdot2\cdot2\cdot3\cdot x\cdot x-2\cdot2\cdot3\cdot x-2\cdot2\cdot2\cdot2\cdot x+2\cdot2\cdot2$

(Simplify)

$=12x(2x-1)-8(2x-1)$

(Group all terms with common factors)

$=(12x-8)(2x-1)$

(By distributive)

Therefore, $24x^2-28x+8=0$

$(12x-8)(2x-1)=0$ (Fctors)

$12x-8=0$

Or, $2x-1=0$ (Using zero product property)

Or, $2x-1=0$ (Using zero product property)

Now solve each equation separately.

$12x-8=0$

$12x-8+8=0+8$ (Add 8 on both sides)

$12x=8$

$\frac{12x}{12}=\frac{8}{12}$ (Divide by 12 on each side)

$x=\frac{8}{12}$ (Simplify)

$x=\frac{2}{3}$ (Simplify)

$2x-1=0$

$2x-1+1=0+1$ (Add 1 on each side)

$2x=1$

$\frac{2x}{2}=\frac{1}{2}$ (Divide by 2 on both sides)

$x=\frac{1}{2}$

The solution set is $\left\{\frac{2}{3},\frac{1}{2}\right\}$.

Check:- To check the result, substitute x by $\frac{2}{3},\frac{1}{2}$ in given equation.

For $x=\frac{2}{3}$,

$24x^2-28x+8=0$

$24\left(\frac{2}{3}\right)^2-28\left(\frac{2}{3}\right)+8=0$ (Put $x=\frac{2}{3}$)

$24\left(\frac{4}{9}\right)-\frac{56}{3}+8=0$

$\frac{96}{9}-\frac{56}{3}\cdot\frac{3}{3}+8\cdot\frac{9}{9}=0$ (Equating the denominators)

$\frac{96-168+72}{9}=0$

$0=0$ True

For $x=\frac{1}{2}$,

$24x^2-28x+8=0$

$24\left(\frac{1}{2}\right)^2-28\left(\frac{1}{2}\right)+8=0$ (Put $x=\frac{1}{2}$)

$\frac{24}{4}-\frac{28}{2}\cdot\frac{2}{2}+8\cdot\frac{4}{4}=0$ (Equating the denominators)

$\frac{24-56+32}{4}=0$

$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{2}{3},\frac{1}{2}\right\}}$.

Answer 44PA.

Consider the equation

$$24x^2 - 46x = 18$$

$\Rightarrow 24x^2 - 46x - 18 = 18 - 18$ (Subtract 18 on both sides)

$$24x^2 - 46x - 18 = 0$$

The objective is to solve the given equation, for this first find factors of given equation and then use zero product property.

Compare $24x^2 - 46x - 18$ with $ax^2 + bx + c$.

Here $a = 24$,

$b = -46,$

$c = -18$

Now find two numbers m, n such that whose sum is

$b = -46$ and product

$$ac = 24 \cdot -18$$
$$= -432$$

Since $m + n = -46$ negative and

$mn = -432$ is also negative.

So, m and n must both be negative.

Now list all the factors of -432, choose one of those whose sum is -46.

Factors of -432	Sum of factors
$-1.-432$	-433
$-2.-216$	-218
$-3.-144$	-147
$-4.-108$	-112
$-6.-72$	-78
$-8.-54$	-62
$-9.-48$	-57
$-12.-36$	-48
$-16.-27$	-43
$-18.-24$	-42

There are no prime factors whose sum is -46.

Therefore, $24x^2-46x-18$ cannot be factored using integers.

Since a polynomial that cannot be written as a product of two polynomials with integral co-efficient is called a prime polynomial.

Therefore, $\boxed{24x^2-46x-18}$ has no solution.

Answer 45PA.

Consider the equation

$$\frac{x^2}{12}-\frac{2x}{3}-4=0$$

The objective is to solve the equation.

$$\frac{x^2}{12}-\frac{2x}{3}-4=0$$

$$\frac{x^2}{12}-\frac{2x}{3}\cdot\frac{4}{4}-4\cdot\frac{12}{12}=0$$ (Equating the denominators)

$$\frac{x^2-8x-48}{12}=0$$ (Simplify)

$$\frac{\left(x^2-8x-48\right)12}{12}=0.12$$ (Multiplying 12 on both sides)

$$x^2-8x-48=0$$ (Simplify)

For this, first find the factors of given equation, and then use zero product property.

Compare $x^2-8x-48$ with ax^2+bx+c.

Here $a=1$,

$b=-8,$

$c=-48$

Find two numbers m,n such that whose sum is

$b=-8$ and whose product is

$$ac=1\cdot-48$$
$$=-48$$

Since $m+n=-8$ negative and $mn=-48$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors of -48, choose one pair of those, whose sum is -8.

Factors of -48	Sum of factors
$1.-48$	-47
[illegible]	[illegible]

-1.48	47
$2.-24$	-22
-2.24	22
$3.-16$	-13
-3.16	13
$4.-12$	-8
-4.12	8
$6.-8$	-2
-6.8	2

The correct factors are $4,-12$.

$x^2-8x-48=x^2+4x-12x+48$

(Because $-8=4-12$)

$=x\cdot x+2\cdot 2\cdot x-2\cdot 2\cdot 3\cdot x-2\cdot 2\cdot 2\cdot 2\cdot 3$

(Simplify)

$=x(x+4)-12(x+4)$

(Group all terms with common factors)

$=(x-12)(x+4)$ (By distributive)

Therefore, $x^2-8x-48=0$

$(x-12)(x+4)=0$ (Factors)

$x-12=0$

Or, $x+4=0$ (Using zero product property)

Now solve each equation separately.

$x-12=0$

$x-12+12=0+12$ (Add 12 on each side)

$x=12$

$x+4=0$

$x+4-4=0-4$ (Subtract 4 on each side)

$x=-4$

The solution set is $\{12,-4\}$.

Check:- To check the result, substitute x by $12,-4$ in give equation.

For $x=12$,

$x^2-8x-48=0$

$(12)^2-8(12)-48=0$ (Put $x=12$)

$144-96-48=0$

$0=0$ True

For $x=-4$,

$x^2-8x-48=0$

$(-4)^2-8(-4)-48=0$ (Put $x=-4$)

$16+32-48=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{12,-4\}}$.

Answer 46PA.

Consider the equation $t^2 - \frac{t}{6} = \frac{35}{6}$

$\Rightarrow \quad t^2 - \frac{t}{6} - \frac{35}{6} = \frac{35}{6} - \frac{35}{6}$ (Subtract $\frac{35}{6}$ on each side)

$\Rightarrow \quad t^2 - \frac{t}{6} - \frac{35}{6} = 0$

$\Rightarrow \quad \frac{6}{6} \cdot t^2 - \frac{t}{6} - \frac{35}{6} = 0$ (Equating the denominators)

$\Rightarrow \quad \frac{6t^2 - t - 35}{6} = 0$ (Simplify)

$\Rightarrow \quad 6t^2 - t - 35 = 0$

The objective is to solve the given equation. For this first find the factors and then use zero product property.

Compare $6t^2 - t - 35$ with $ax^2 + bx + c$.

Here $a = 6$,

$b = -1,$

$c = -35$

Find two numbers m, n such that whose sum is

$b = -1$ and whose product is

$$\begin{aligned} ac &= 6 \cdot -35 \\ &= -210 \end{aligned}$$

Since $m + n = -1$ negative and

$mn = -210$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors of -210, choose in those one pair, whose sum is -1.

Factors of -210	Sum of factors
-1.210	209
$1.-210$	-209

$2.-105$	-103
-2.105	103
$3.-70$	-67
-3.70	67
$5.-42$	-37
-5.42	37
$6.-35$	-29
-6.35	29
-7.30	23
$7.-30$	-23
-10.21	11
$10.-21$	-11
$14.-15$	-1
-14.15	1

The correct factors are $14,-15$.

$6t^2 - t - 35 = 6t^2 + 14t - 15t - 35$

(Because $-1 = 14 - 15$)

$= 2 \cdot 3 \cdot t \cdot t + 2 \cdot 7 \cdot t - 3 \cdot 5 \cdot t - 7 \cdot 5$

(Simplify)

$= 2t(3t + 7) - 5(3t + 7)$

(Group all the terms with common factors)

$= (2t - 5)(3t + 7)$ (By distributive)

Therefore, $6t^2 - t - 35 = 0$

$(2t - 5)(3t + 7) = 0$ (Factors)

$2t - 5 = 0$

Or, $3t + 7 = 0$ (Using zero product property)

Now solve each equation separately.

$2t - 5 = 0$

$2t - 5 + 5 = 0 + 5$ (Add 5 on each side)

$2t = 5$

$\frac{2t}{2} = \frac{5}{2}$ (Divide by 2 on each side)

$t = \frac{5}{2}$

$3t + 7 = 0$

$3t + 7 - 7 = 0 - 7$ (Subtract 7 on each side)

$3t = -7$

$\frac{3t}{3} = \frac{-7}{3}$ (Divide by 3 on both sides)

$t = \frac{-7}{3}$

The solution set is $\left\{\frac{5}{2}, \frac{-7}{3}\right\}$.

Check:- To check the result, substitute t by $\frac{5}{2}, \frac{-7}{3}$ in the given equation.

For $t = \frac{5}{2}$,

$$6t^2 - t - 35 = 0$$

$$6\left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right) - 35 = 0$$ (Put $t = \frac{5}{2}$)

$$6\left(\frac{25}{4}\right) - \frac{5}{2} - 35 = 0$$

$$\frac{150}{4} - \frac{5}{2} \cdot \frac{2}{2} - 35 \cdot \frac{4}{4} = 0$$ (Equating the denominators)

$$\frac{150 - 10 - 140}{4} = 0$$ (Simplify)

$$0 = 0$$

For $t = -\frac{7}{3}$,

$$6t^2 - t - 35 = 0$$

$$6\left(-\frac{7}{3}\right)^2 - \left(-\frac{7}{3}\right) - 35 = 0$$ (Put $t = -\frac{7}{3}$)

$$\frac{294}{9} + \frac{7}{3} \cdot \frac{3}{3} - 35 \cdot \frac{9}{9} = 0$$ (Equating the denominators)

$$\frac{294}{9} + \frac{21}{9} - \frac{315}{9} = 0$$

$\Rightarrow \quad 0 = 0$ True

Therefore, the solution set is $\boxed{\left\{\frac{5}{2}, \frac{-7}{3}\right\}}$.

Answer 47PA.

Consider the equation

$$(3y+2)(y+3)=y+14$$

The objective is to solve the equation.

$$(3y+2)(y+3)=y+14$$

$$3y\cdot y+3\cdot 3\cdot y+2\cdot y+2\cdot 3 = y+4$$ (Simplify and multiplying the two factors)

$$3y^2+9y+2y+6=y+14$$

$$3y^2+11y+6-y=y+14-y$$ (Subtract y on each side)

$$3y^2+10y+6=14$$

$$3y^2+10y+6-14=14-14$$ (Subtract 14 on each side)

$$3y^2+10y-8=0$$

Compare $3y^2+10y-8$ with ax^2+bx+c.

Here $a=3,$

$b=10,$

$c=-8$

Find two numbers m,n such that whose sum is

$b=10$ and whose product is

$$ac=3\cdot -8 = -24$$

Since $m+n=10$ positive and

$mn=-24$ is negative.

So, either m (or) n negative but not both.

Find list all the factors of -24 and choose one pair in those whose sum 10.

Factors of -24	Sum of factors
-1.24	23
$1.-24$	-23

$2.-12$	-10
-2.12	10
$3.-8$	-5
-3.8	5
$4.-6$	-2
-4.6	2

The correct factors are $-2,12$.

$3y^2+10y-8=3y^2-2y+12y-8$

(Because $10=-2+12$)

$=3\cdot y\cdot y-2\cdot y+2\cdot 2\cdot 3y-2\cdot 2\cdot 2$

(Simplify)

$=y(3y-2)+4(3y-2)$

(Group all the terms with common factors)

$=(y+4)(3y-2)$ (By distributive)

Therefore, $3y^2+10y-8=0$

$(y+4)(3y-2)=0$ (Factors)

$y+4=0$

Or, $3y-2=0$ (Using zero product property)

Now solve each equation separately.

$y+4=0$

$y+4-4=0-4$ (Subtract 4 on each side)

$y=-4$

$3y-2=0$

$3y-2+2=0+2$ (Add 2 on both sides)

$3y=2$

$\frac{3y}{3}=\frac{2}{3}$ (Divide by 3 on each side)

$y=\frac{2}{3}$

The solution set is $\left\{-4,\frac{2}{3}\right\}$.

Check:- To check the result, substitute y by $-4,\frac{2}{3}$ in the given equation.

For $y=-4$,

$3y^2+10y-8=0$

$3(-4)^2+10(-4)-8=0$ (Put $y=-4$)

$3(16)-40-8=0$ (Simplify)

$48-48=0$

$0=0$ True

For $y=\frac{2}{3}$,

$3y^2+10y-8=0$

$3\left(\frac{2}{3}\right)^2+10\left(\frac{2}{3}\right)-8=0$ (Put $y=\frac{2}{3}$)

$3\left(\frac{4}{9}\right)+\frac{20}{3}-8=0$

$\frac{12}{9}+\frac{20}{3}\cdot\frac{3}{3}-8\cdot\frac{9}{9}=0$ (Equating the denominators)

$\frac{12+60-72}{9}=0$

$0=0$ True

Therefore, the solution set is $\boxed{\left\{-4,\frac{2}{3}\right\}}$.

Answer 48PA.

Consider the equation

$$(4a-1)(a-2)=7a-5$$

The objective is to solve the equation.

$$(4a-1)(a-2)=7a-5$$

$4a\cdot a-2\cdot 4a-1\cdot a+1\cdot 2=7a-5$ (Multiplying the two factors)

$4a^2-8a-a+2=7a-5$

$4a^2-9a+2=7a-5$ (Simplify)

$4a^2-9a+2-7a=7a-5-7a$ (Subtract $7a$ on each side)

$4a^2-16a+2=-5$

$4a^2-16a+2+5=-5+5$ (Add 5 on each side)

$4a^2-16a+7=0$

Compare $4a^2-16a+7$ with ax^2+bx+c.

Here $a=4$,

$b=-16,$

$c=7$

For this first find the factors and then use zero product property.

Since find two numbers m,n such that whose sum is

$b=-16$ and whose product is

$$\begin{aligned}ac&=4\cdot 7\\&=28\end{aligned}$$

Since $m+n=-16$ negative and

$mn=28$ positive.

So, m,n must both be negative

Factors of 28	Sum of factors
$-1.-28$	-29

$-2,-14$	-16
$-4,-7$	-11

The correct factors are $-2,-14$.

$4a^2-16a+7=4a^2-2a-14a+7$

(Because $16=2+14$)

$=2\cdot2\cdot a\cdot a-2\cdot a-2\cdot7\cdot a+7$

(Simplify)

$=2a(2a-1)-7(2a-1)$

(Group all the terms with common factors)

$=(2a-7)(2a-1)$

(By distributive)

Therefore, $4a^2-16a+7=0$

$(2a-7)(2a-1)=0$ (Factors)

$2a-7=0$

Or, $2a-1=0$ (Using zero product property)

Now solve each equation separately.

$2a-7=0$

$2a-7+7=0+7$ (Add 7 on each side)

$2a=7$

$\frac{2a}{2}=\frac{7}{2}$ (Divide by 2 on each side)

$a=\frac{7}{2}$

$2a-1=0$

$2a-1+1=0+1$ (Add 1 on each side)

$2a=1$

$\frac{2a}{2}=\frac{1}{2}$ (Divide by 2 on each side)

$\frac{}{2} = \frac{}{2}$ (Divide by 2 on each side)

$a = \frac{1}{2}$

The solution set is $\left\{\frac{7}{2}, \frac{1}{2}\right\}$.

Check:- To check the result, substitute a by $\frac{7}{2}, \frac{1}{2}$ in given equation.

For $a = \frac{1}{2}$,

$4a^2 - 16a + 7 = 0$

$4\left(\frac{1}{2}\right)^2 - 16\left(\frac{1}{2}\right) + 7 = 0$ (Put $a = \frac{1}{2}$)

$\frac{4}{4} - \frac{16}{2} + 7 = 0$

$\frac{4}{4} - \frac{16}{2} \cdot \frac{2}{2} + 7 \cdot \frac{4}{4} = 0$ (Equating the denominators)

$\frac{4 - 32 + 28}{4} = 0$ (Simplify)

$0 = 0$ True

For $a = \frac{7}{2}$,

$4a^2 - 16a + 7 = 0$

$7\left(\frac{7}{2}\right)^2 - 16\left(\frac{7}{2}\right) + 7 = 0$ (Put $a = \frac{7}{2}$)

$4\left(\frac{49}{4}\right) - \frac{112}{2} + 7 = 0$

$\frac{196}{4} - \frac{112}{2} \cdot \frac{2}{2} + 7 \cdot \frac{4}{4} = 0$ (Equating the denominators)

$$\frac{196-224+28}{4}=0 \text{ (Simplify)}$$

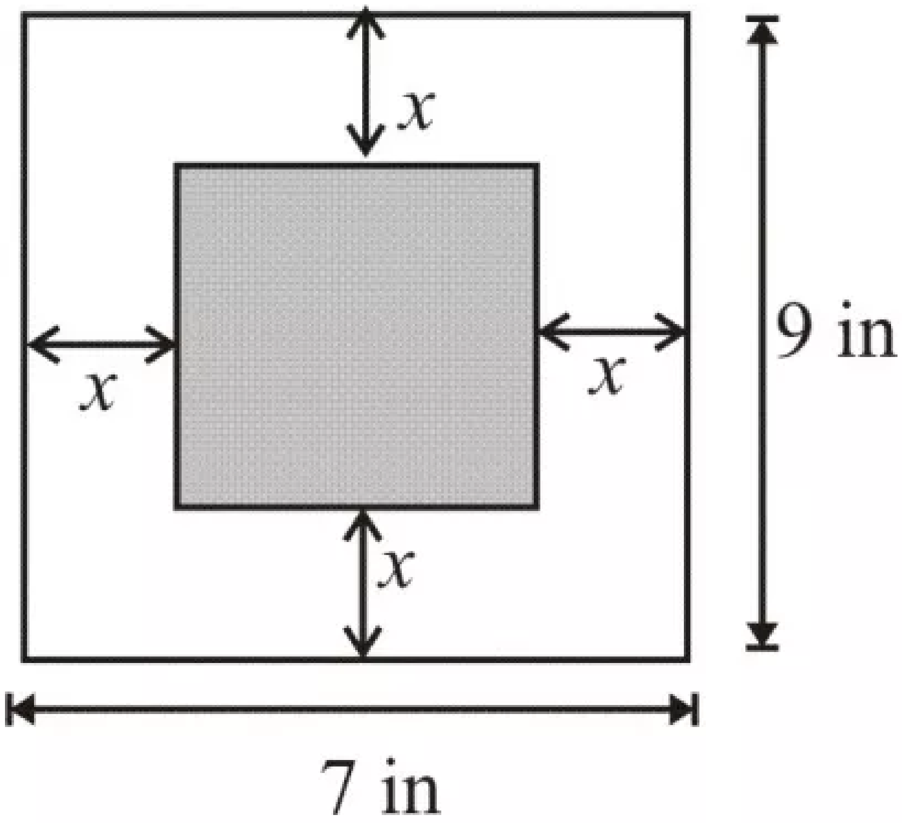
$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{7}{2},\frac{1}{2}\right\}}$.

Answer 49PA.

Consider that the rectangle with an area of 35 square inches is formed by cutting off strips of equal with from rectangular piece of paper.

The objective is finding the width of each strip.



Since the length of piece of paper = 9 in

Width piece of paper = 7 in.

A new rectangle is formed by cutting off strips of equal with of x from a rectangular piece of paper

Length of new rectangle = Length of piece of paper – 2 times width of strip

$$=9-2x$$

Width new rectangle = width piece of paper – 2 times width strip

$$=7-2x$$

Consider that Area of new rectangle = 35

Area of rectangle $= \text{length} \cdot \text{width} = 35$

$(9-2x)(7-2x)=35$

F O I L

$9\cdot 7+9(-2x)+(-2x)7+(-2x)(-2x)=36$ [By FOIL method]

$63-18x-14x+4x^2=35$

$63-18x-14x+4x^2=35$ [Simplify]

$4x^2-32x+63-35=35-35$ [Subtract 35 on both sides]

$4x^2-32x+28=0$

$4\cdot x^2+(-4)\cdot 8\cdot x+4\cdot 7=0$

$4(x^2-8x+7)=0$

$4(x^2-x-7x+7)=0$

$4[x(x-1)-7(x-1)]=0$

$4(x-7)(x-1)=0$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both.

$x-7=0$ or $x-1=0$

$x-7+7=0+7$ [Add 7on both sides]

$x=7$

Now, $x-1=0$

$x-1+1=0+1$ [Add 1 on both sides]

$x=1$

The solution set is $\{1,7\}$

Here $x=7$ is not suitable. Since width = $7-2x$

$=7-2(7)$

$=-7 \quad (-ve)$

The suitable solution for *x* is 1.

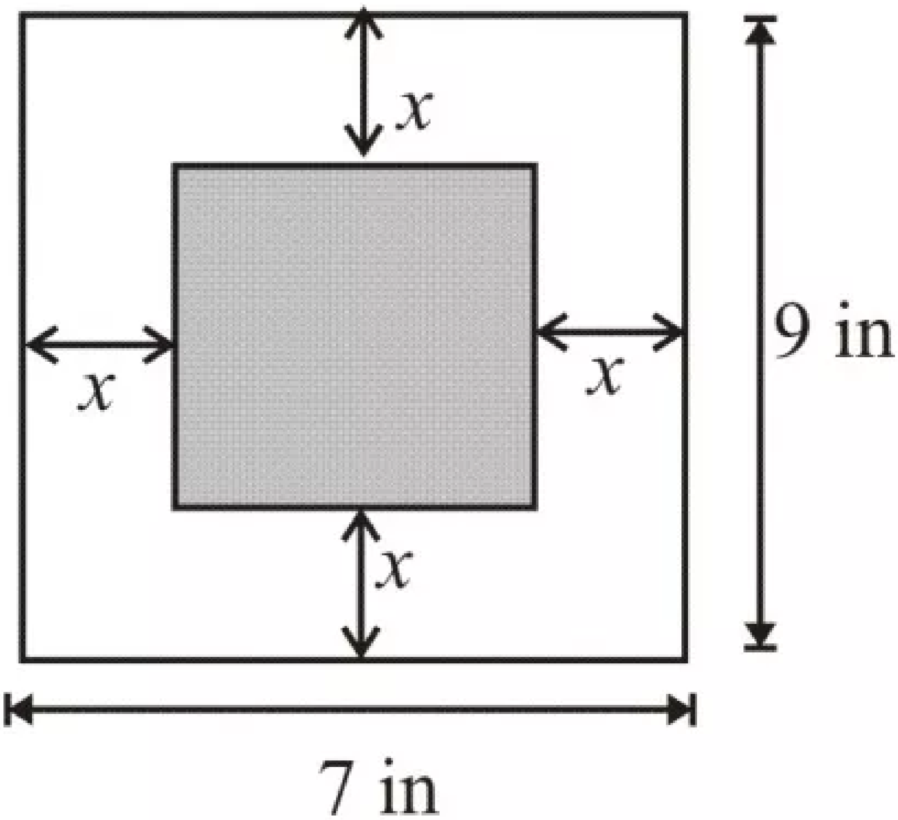
Width of strip is 1

Therefore, width of each strip is $\boxed{1\text{ in}}$

Answer 50PA.

Consider that the rectangle with an area of 35 square inches is formed by cutting off strips of equal with from rectangular piece of paper.

The objective is finding the width of each strip.



Since the length of piece of paper = 9 in

Width piece of paper = 7 in.

A new rectangle is formed by cutting off strips of equal with of *x* from a rectangular piece of paper

Length of new rectangle = Length of piece of paper – 2 times width of strip

$= 9 - 2x$

Width new rectangle = width piece of paper – 2 times width strip

$= 7 - 2x$

Consider that Area of new rectangle 35

$$(9-2x)(7-2x)=35$$

Area of rectangle $=\text{length}\cdot\text{width}=35$

$$(9-2x)(7-2x)=35$$

F O I L

$$9\cdot 7+9(-2x)+(-2x)7+(-2x)(-2x)=36 \quad [\text{By FOIL method}]$$

$$63-18x-14x+4x^2=35$$

$$63-18x-14x+4x^2=35 \quad [\text{Simplify}]$$

$$4x^2-32x+63-35=35-35 \quad [\text{Subtract 35 on both sides}]$$

$$4x^2-32x+28=0$$

$$4\cdot x^2+(-4)\cdot 8\cdot x+4\cdot 7=0$$

$$4(x^2-8x+7)=0$$

$$4(x^2-x-7x+7)=0$$

$$4[x(x-1)-7(x-1)]=0$$

$$4(x-7)(x-1)=0$$

The zero product property is if $ab = 0$ then $a = 0$ or $b = 0$ or both.

$x - 7 = 0$ or $x - 1 = 0$

$x - 7 + 7 = 0 + 7$ [Add 7on both sides]

$x = 7$

Now, $x - 1 = 0$

$x - 1 + 1 = 0 + 1$ [Add 1 on both sides]

$x = 1$

The solution set is $\{1, 7\}$

Here $x = 7$ is not suitable. Since width = $7 - 2x$

$= 7 - 2(7)$

$= -7 \quad (-ve)$

The suitable solution for *x* is 1.

Width of strip is 1

Therefore, width of each strip is $\boxed{1 \text{ in}}$

Thus, lengths of new rectangle $= 9 - 2x$

$= 9 - 2(1) \quad (x = 1)$

$= 7$

Width of new rectangle $= 7 - 2x$

$= 7 - 2(1) \quad (x = 1)$

$= 7 - 2$

$= 5$

Therefore, length of new triangle is 7 inches; width of new triangle is 5 inches.

Answer 54PA.

The equation is $2p^2 - p - 3 = 0$

The objective is to find the solution set of given equation

Compare $2p^2 - p - 3$ with $ax^2 + bx + c$

Here $a = 2, b = -1, c = -3$

$$\begin{aligned} 2p^2 - p - 3 &= 2p^2 + mp + np - 3 \\ &= 2p^2 + (m+n)p - 3 \end{aligned}$$

Now find two numbers m, n such that $m + n = -1$ and $mn = 2 \cdot -3 = -6$

Since $m + n, mn$ are negative then one of *m* or *n* must be negative but not both.

For this list all the factors of $mn = -6$ in those choose a pair whose sum is -1

Factors of -6	Sum of factors
$-1 \cdot 6$	5
$1 \cdot -6$	-5
$-2 \cdot 3$	1
$2 \cdot -3$	✓ -1

The connect factors are $2, -3$

$$\begin{aligned} 2p^2 - p - 3 &= 2p^2 + mp + np - 3 \\ &= 2p^2 + 2p - 3p - 3 \quad [m = 2, n = -3] \\ &= 2p(p+1) - 3(p+1) \\ &= (2p-3)(p+1) \quad [\text{By distributive } (b+c)a = ba + ca] \end{aligned}$$

Thus $2p^2 - p - 3 = 0$

$(2p-3)(p+1) = 0$

The zero product property is if $ab = 0$ then $a = 0$ or $b = 0$ or both

$$2p-3=0$$
$$2p-3+3=0+3 \quad [\text{add 3 on both sides}]$$
$$2p=3$$
$$\frac{2p}{2}=\frac{3}{2} \quad [\text{Divide with 2 on both sides}]$$
$$p=\frac{3}{2}$$
$$p+1=0$$
$$p+1-1=0-1 \quad [\text{Subtract 1 on both sides}]$$
$$p=-1$$

Therefore, the solution set of given equation is $\boxed{\left\{\frac{3}{2},-1\right\}}$

Answer 55PA.

Consider that a person standing atop a building 397 feet tall throws a ball upward

If the person release the ball 4 feet above the top of the building, the ball height *h* in feet, after *t* seconds is given by the equation $h=-16t^2+48t+402$

The objective is to find the time taken by the ball to reach 338 feet from the ground.

That is $h=338$

$$-16t^2+48t+402=h$$
$$-16t^2+48t+402=338 \quad [\text{Put } h=338]$$
$$-16t^2+48t+402-338=338-338 \quad [\text{Subtract 338 on both sides}]$$
$$-16t^2+48t+64=0$$
$$-16\cdot t^2+16\cdot 3t+16\cdot 4=0$$
$$-16\left[t^2-3t-4\right]=0 \quad [\text{Take factor GCF } -16]$$
$$-16\left[(t-4)(t+1)\right]=0 \quad [\text{Factor}]$$
$$(t+1)(t-4)=0 \quad [\text{Since; } -16\neq 0]$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$$t+1=0 \text{ or } t-4=0 \quad [\text{By zero product property}]$$
$$t+1=0$$
$$t+1-1=0-1 \quad [\text{Subtract 1 on both sides}]$$
$$t=4$$

Thus, $t=-1$ or 4

Since the time always positive,

Therefore, the ball time taken by the ball to reach 338 feet from the ground is $\boxed{4 \text{ seconds}}$

Answer 56MYS.

Consider the trinomial $a^2 - 4a - 21$.

The objective is to factor of the given trinomial.

Compare $a^2 - 4a - 21$ with $ax^2 + bx + c$.

Here $a = 1,$

$b = -4,$
$c = -21$

Now find two numbers m, n such that whose sum is

$b = -4$ and whose product

$$\begin{aligned} ac &= 1 \cdot -21 \\ &= -21 \end{aligned}$$

Since $m + n = -4$ negative and

$mn = -21$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors of -21, choose one pair in those whose sum is -4.

Factors of -21	Sum of factors
$1, -21$	-20
$-1, 21$	20
$-3, 7$	4
$3, -7$	-4

The correct factors are $3, -7$.

$a^2-4a-21=a^2+3a-7a-21$ (Because $-4=3-7$)

$=a\cdot a+3\cdot a-7\cdot a-3\cdot 7$ (Simplify)

$=a(a+3)-7(a+3)$ (Group all the terms with common factors)

$=(a-7)(a+3)$ (By distributive)

Therefore,

$a^2-4a-21=(a-7)(a+3)$

Check:- To check the result, by multiplying two factors using $FOIL$ method.

$(a-7)(a+3)=\overset{F}{a\cdot a}+\overset{O}{3\cdot a}-\overset{I}{7\cdot a}-\overset{L}{7\cdot 3}$ ($FOIL$ method)

$=a^2+3a+7a-21$ (Simplify)

$=a^2-4a-21$ True

Therefore, the factorized form of $\boxed{a^2-4a-21}$ is $\boxed{(a-7)(a+3)}$.

Answer 57MYS.

Consider the trinomial t^2+2t+2

The objective is to find the factors of the given trinomial.

Compare t^2+2t+2 with ax^2+bx+c

Here $a=1,$

$b=2,$
$c=2$

Now find two numbers m,n such that whose sum is 2 and whose product is

$$ac=1\cdot 2$$
$$=2$$

Since $m+n=2$ positive and

$mn=2$ also positive.

So, list all the factors of 2, choose one pair in those, whose sum is 2.

Factors of 2	Sum of factors
1.2	3

There is no prime factors whose sum is 2.

Therefore, t^2+2t+2 cannot be factored using integers.

Since, a polynomial that cannot be written as a product of two polynomials with integral co-efficient is called a prime polynomial.

Therefore, $\boxed{t^2+2t+2}$ is a prime polynomial.

Answer 58MYS.

Consider the trinomial $d^2+15d+44$.

The objective is to find the factors of given trinomial.

Compare $d^2+15d+44$ with ax^2+bx+c.

Here $a=1,$

$b=15,$
$c=44$

Now find two numbers m,n such that whose sum is

$b=15$ and whose product is

$ac=1\cdot 44$
$=44$

Since $m+n=15$ positive and

$mn=44$ is also positive.

Now for this list all the factors of 44, in those choose one pair with sum 15.

Factors of 44	Sum of factors
1.44	45
2.22	24
4.11	15

The correct factors are $4,11$.

$d^2+15d+44=d^2+4d+11d+44$ (Because $15=11+4$)

$=d\cdot d+2\cdot 2\cdot d+11\cdot 1d+11\cdot 4$

(Simplify)

$=d(d+4)+11(d+4)$ (Group all terms with common factors)

$=(d+11)(d+4)$

Therefore, $d^2+15d+44=(d+11)(d+4)$

Check:- To check the result, by multiplying two factors by using $FOIL$ method.

$(d+11)(d+4)=\overset{F}{d}\cdot d+\overset{O}{4}\cdot d+\overset{L}{11}\cdot 4$ ($FOIL$ method)

$=d^2+4d+11d+44$ (Simplify)

$=d^2+15d+44$ True

Therefore, the factorized form of $\boxed{d^2+15d+44}$ is $\boxed{(d+11)(d+4)}$.

Answer 59MYS.

Consider the equation

$(y-4)(5y+7)=0$

The objective is to solve the given equation.

$(y-4)(5y+7)=0$

$y\cdot 5y+7\cdot y-4\cdot 5y-4\cdot 7=0$ (Multiplying the two terms)

$5y^2+7y-20y-28=0$ (Simplify)

$5y^2-13y-28=0$

Now first find the factors of given equation, then use zero product property.

Compare $5y^2-13y-28$ with ax^2+bx+c.

Find two numbers m,n such that whose sum is

$b=-13$ and whose product is

$ac=5\cdot -28$
$=-140$

Since $m+n=-13$ negative and

$mn=-140$ is also negative.

So, either m (or) n negative but not both.

Now list all the factors of -140, choose in those one pair whose sum is -13.

Factors of -140	Sum of factors
-1.140	139

1.−140	−139
2.−70	−68
−2.70	68
4.−35	−31
−4.35	31
5.−28	−23
−5.28	23
7.−20	−13
−7.20	13
10.−14	−4
−10.14	4

The correct factors are $7,-20$.

$$5y^2-13y-28=5y^2+7y-20y-28$$

(Because $13=7+20$)

$$=5\cdot y\cdot y+7\cdot y-2\cdot 2\cdot 5\cdot y-2\cdot 2\cdot 7$$

(Simplify)

$$=y(5y+7)-4(5y+7)$$

(Group all terms with common factors)

(Group all terms with common factors)

$=(y-4)(5y+7)$

(By distributive)

Therefore, $5y^2-13y-28=0$

$(y-4)(5y+7)=0$ (Factors)

$y-4=0$

Or, $5y+7=0$ (By zero product property)

Now solve each equation separately.

$y-4=0$

$y-4+4=0+4$ (Add 4 on each side)

$y=4$

$5y+7=0$

$5y-7+7=0-7$ (Subtract 4 on each side)

$5y=-7$

$\frac{5y}{5}=-\frac{7}{5}$ (Divide by 5 on each side)

$y=\frac{-7}{5}$

The solution set is $\left\{4,\frac{-7}{5}\right\}$.

Check:- To check the result, substitute y by $4,\frac{7}{5}$ in given equation.

For $y=4$,

$5y^2-13y-28=0$

$5(4)^2-13(4)-28=0$ (Put $y=4$)

$5(16)-52-28=0$ (Simplify)

$\frac{245}{25}+\frac{91}{5}\cdot\frac{5}{5}-28\cdot\frac{25}{25}=0$ (Equation the denominators)

$\frac{245-455-210}{25}=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\left\{4,\frac{-7}{5}\right\}}$.

Answer 60MYS.

Consider the equation

$(2k+9)(3k+2)=0$

$2k\cdot3k+2\cdot2k+9\cdot3k+9\cdot2=0$ (Multiplying two factors)

$6k^2+4k+27k+18=0$ (Simplify)

$6k^2+31k+18=0$

The objective is to solve the given equation. For this first find factors and then use zero product property.

Compare $6k^2+3k+18$ with ax^2+bx+c.

Here $a=6,$

$b=31,$

$c=18$

Now find two numbers m,n such that whose sum is

$b=31$ and whose product

$ac=6\cdot18$

$=108$

Since $m+n=31$ positive and

$mn=108$ also positive.

So, list all the factors of 108, choose one pair in those whose sum is 31.

Factors of 108	Sum of factors
1.108	109
2.54	56
3.36	39

4.27	31
6.18	24
9.12	21

The correct factors are $4,27$.

$6k^2+31k+18=6k^2+4k+27k+18$

$(31=4+27)$

$=2\cdot3\cdot k\cdot+2\cdot2\cdot k+3\cdot3\cdot3k+3\cdot3\cdot2$

(Simplify)

$=2k(3k+2)+9(3k+2)$

(Group all the terms with common factors)

$=(2k+9)(3k+2)$

(By distributive)

Therefore, $6k^2+31k+18=0$

$(2k+9)(3k+2)=0$ (Factors)

$2k+9=0$

Or, $3k+2=0$ (Using zero product property)

Now solve each factor separately.

$2k+9=0$

$2k+9-9=0-9$ (Subtract 9 on each side)

$2k=-9$

$\frac{2k}{2}=\frac{-9}{2}$ (Divide by 2 on each side)

$k=\frac{-9}{2}$

$3k+2=0$

$3k+2-2=0-2$ (Subtract 2 on both sides)

$3k=-2$

$\frac{3k}{3}=\frac{-2}{3}$ (Divide by 3 on each side)

The solution set is $\left\{-\frac{9}{2},\frac{-2}{3}\right\}$.

Check:- To check the result, substitute k by $\frac{-9}{2},\frac{-2}{3}$ in given equation.

For $k = \frac{-9}{2}$,

$$6k^2 + 31k + 18 = 0$$

$$6\left(-\frac{9}{2}\right)^2 + 31\left(-\frac{9}{2}\right) + 18 = 0$$ (Put $k = \frac{-9}{2}$)

$$6\left(\frac{81}{4}\right) - \frac{279}{2} + 18 = 0$$ (Simplify)

$$\frac{486}{4} - \frac{279}{2}\cdot\frac{2}{2} + 18\cdot\frac{4}{4} = 0$$ (Simplify)

$0 = 0$ True

For $k = \frac{-2}{3}$,

$$6k^2 + 31k + 18 = 0$$

$$6\left(\frac{-2}{3}\right)^3 + 31\left(-\frac{2}{3}\right) + 18 = 0$$ (Put $k = \frac{-2}{3}$)

$$6\left(\frac{4}{9}\right) - \frac{62}{3} + 18 = 0$$ (Simplify)

$$\frac{24}{9} - \frac{62}{3}\cdot\frac{3}{3} + 18\cdot\frac{9}{9} = 0$$ (Equating the denominators)

$$\frac{24 - 186 + 162}{9} = 0$$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{\frac{-9}{2}, \frac{-2}{3}\right\}}$.

Answer 61MYS.

Consider the equation $124 = 4^2$

$\Rightarrow \quad 124 - 4^2 = 4^2 - 4^2$ (Subtract 4^2 on each side)

$\Rightarrow \quad 124 - 4^2 = 0$

$\Rightarrow \quad -\left(124 - 4^2\right) = 0$ (Take common ‘ −’)

$\Rightarrow \quad 4^2 - 124 = 0$

The objective is to solve the equation.

$4^2 - 124 = 0$

$4(4 - 12) = 0$ (Take common 4)

$4 = 0$

Or, $4 - 12 = 0$ (By zero product property)

Now solve each equation separately.

$4 = 0$

Therefore, $4 - 12 + 12 = 0 + 12$ (Add 12 on each side)

$4 = 12$

The solution set is $\{0, 12\}$.

Check:- To check the result, substitute 4 by $0, 12$ in given equation.

For $4 = 0$,

$4^2 - 124 = 0$

$(0)^2 - 12(0) = 0$ (Put $4 = 0$)

$0 - 0 = 0$

$0 = 0$ True

For $4 = 12$,

$4^2 - 124 = 0$

$(12)^2 - 12(12) = 0$ (Put $4 = 12$)

$144 - 144 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\{0, 12\}}$.

Answer 63MYS.

Consider the number 16

The objective is to find the principal square root of given numbers.

Resolve 16 into prime factors

$$\begin{aligned} 16 &= 2\cdot 8 && (2\cdot 8 = 16) \\ &= 2\cdot 2\cdot 4 && (2\cdot 4 = 8) \\ &= 2\cdot 2\cdot 2\cdot 2 && (2\cdot 2 = 4) \end{aligned}$$

Thus, $16 = 2\cdot 2\cdot 2\cdot 2$

Now make pair of prime factors such that both the factors in each pair are equal.

$$16 = 2\cdot 2 \quad 2\cdot 2$$

Here the factorization has four 2 make them as two groups.

$$(2\cdot 2),(2\cdot 2)$$

Take one factor from each pair, and product of these factors gives the square root

Those are 2, 2

Products of factors are $2\cdot 2 = 4$

Therefore, principal square root of 16 is $\boxed{4}$

Answer 64MYS.

Given number is 49

The objective is to find the principal square root of given numbers.

Resolve 49 into prime factors

$$49 = 7\cdot 7$$

Now make pair of prime factors such that both the factors in each pair are equal.

$$49 = 7\cdot 7$$

Here the factorization has two 7's make them as two groups.

$$7\cdot 7$$

Now take one factor from each group, and product of these factors gives the square root

Product of factors = 7

Therefore, principal square root of 49 is $\boxed{7}$

Answer 65MYS.

Consider the number 36

The objective is to find the principal square root of given numbers.

Resolve 36 into prime factors.

$$\begin{aligned} 36 &= 2\cdot 18 && (\text{Since; } 2\cdot 18 = 36) \\ &= 2\cdot 2\cdot 9 && (2\cdot 9 = 18) \\ &= 2\cdot 2\cdot 3\cdot 3 && (3\cdot 3 = 9) \end{aligned}$$

Thus, $36 = 2\cdot 2\cdot 3\cdot 3$

Now make pair of prime factors such that both the factors in each pair are equal.

$$36 = (2\cdot 2)(3\cdot 3)$$

Take one factor from each pair, and product of these factors gives the square root

Those are 2, 3

Products of factors are $2\cdot 3 = 6$

Therefore, principal square root of 36 is $\boxed{6}$

Answer 66MYS.

Consider the number 25

The objective is to find the principal square root of given numbers.

Resolve 25 into prime factors.

$$25 = 5\cdot 5 \qquad (\text{Since; } 5\cdot 5 = 25)$$

Thus $25 = 5\cdot 5$ is the prim factorization

Now make pair of prime factors such that both the factors in each pair are equal.

$$25 = 5\cdot 5$$

Take $5\cdot 5$ as group.

Take one factor from each group, and product of these factors gives the principal square root.

Those are 5

Product of factors = 5

Therefore, principal square root of 25 is $\boxed{5}$

Answer 67MYS.

Consider the number 100

The objective is to find the principal square root of given numbers.

For this first resolve 100 into prim factors

$$100 = 2\cdot 50 \qquad (2\cdot 50 = 100)$$
$$= 2\cdot 2\cdot 25 \qquad (50 = 2\cdot 25)$$
$$= 2\cdot 2\cdot 5\cdot 5 \qquad (5\cdot 5 = 25)$$

Thus, $100 = 2\cdot 2\cdot 5\cdot 5$

Now make pair of prime factors such that both the factors in each pair are equal.

$$100 = (2\cdot 2)(5\cdot 5)$$

The two pairs are $(2\cdot 2),(5\cdot 5)$

Take one factor from each pair, and product of these factors gives the square root

Those are 2, 5

Products of factors $2\cdot 5 = 10$

Therefore, principal square root of 100 is $\boxed{10}$

Answer 68MYS.

Consider the number 121

The objective is to find the principal square root of given numbers.

For this first resolve 121 into prim factors

$$121 = 11\cdot 11 \qquad (11\cdot 11 = 121)$$

Thus, $121 = 11\cdot 11$

Now make pair of prime factors such that both the factors in each pair are equal.

$$121 = (11\cdot 11)$$

The pairs is $(11\cdot 11)$

Take one factor from each pair, and product of these factors gives the square root

Those are 11

Products of factors = 11

Therefore, principal square root of 121 is $\boxed{11}$

Answer 69MYS.

Consider the number 169

The objective is to find the principal square root of given numbers.

For this first resolve 169 into prim factors

$$169 = 13 \cdot 13 \qquad (13 \cdot 13 = 169)$$

Thus, $169 = 13 \cdot 13$

Now make pair of prime factors such that both the factors in each pair are equal.

$$169 = (13 \cdot 13)$$

The pairs is $(13 \cdot 13)$

Take one factor from each pair, and product of these factors gives the square root

Those are 13

Products of factors = 13

Therefore, principal square root of 169 is $\boxed{13}$

Answer 70MYS.

Consider the number 225

The objective is to find the principal square root of given numbers.

For this first resolve 225 into prim factors

$$\begin{aligned} 225 &= 3 \cdot 75 && (3 \cdot 75 = 225) \\ &= 3 \cdot 3 \cdot 25 && (3 \cdot 25 = 75) \\ &= 3 \cdot 3 \cdot 5 \cdot 5 && (5 \cdot 5 = 25) \end{aligned}$$

Thus, $225 = 3 \cdot 3 \cdot 5 \cdot 5$

Now make pair of prime factors such that both the factors in each pair are equal.

$$225 = (3 \cdot 3)(5 \cdot 5)$$

Take one factor from each pair, and product of these factors gives the square root

Those are 3, 5

Products of factors are $3 \cdot 5 = 15$

Therefore, principal square root of 225 is $\boxed{15}$