Introduction to Three Dimensional Geometry

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Pankaj and his father were walking in a large park. They saw a kite flying in the sky. The position of Kite, Pankaj and Pankaj's father are at (20, 30, 10), (4, 3, 7) and (5, 3, 7) respectively.

(A) The distance between Pankaj and Kite is:

(a) 41.32 units

(b) 31.52 units

(c) 38.32 units

(d) 40.39 units

(B) The distance between Pankaj's father and kite is:

(a) 31.30 units

(b) 38.43 units

(c) 31.03 units

(d) 29.00 units

(C) The co-ordinates of Pankaj lie in:

(a) IV quadrant

(b) III quadrant

(c) II quadrant

(d) I quadrant

(D) If co-ordinate of Kite, Pankaj and Pankaj's father form a triangle, then the centroid is:

(a) (9.67, 12, 8)

(b) (9.6, 8, 12)

(c) (12, 8, 10)

(d) (7,9, 7.2)

(E) The co-ordinates of points in the XY-plane are of the form:

(a) (0, 0, z)

(b) (x, y, 0)

(c) (x, 0, y)

(d) (0, x, y)

Ans. (A) (b) 31.52 units

Explanation: Required distance

$$= \sqrt{(20-4)^2+(30-3)^2+(10-7)^2}$$

$$= \sqrt{16^2+27^2+3^2}$$

$$= \sqrt{256+729+9}$$

$$= \sqrt{994}$$

= 31.52 units

(B) (c) 31.03 units

Explanation: Required distance

$$= \sqrt{(20-5)^2+(30-3)^2+(10-7)^2}$$

$$= \sqrt{15^2+27^2+3^2}$$

$$= \sqrt{255+729+9}$$

$$= \sqrt{963}$$

= 31.03 units

(C) (d) I quadrant

Explanation: Because in (4, 3, 7); all are positive.

Thus, the coordinate lies in the I quadrant.

(D) (a) (9.67, 12, 8)

Explanation: Centroid

$$= \left(\frac{20+4+5}{3}, \frac{30+3+3}{3}, \frac{10+7+7}{3}\right)$$

= (9.67, 12, 8)

(E) (b) (x, y, 0)

Explanation: For XY-plane, z = 0
⇒ The co-ordinates are of the form (x, y, 0).

2. Vikas and his friends went camping for 2 nights and 3 days. There they set up a tent which is triangular in shape. The vertices of the tent are A(4, 5, 9), B(3, 2, 5), C(5, 2, 5), D(-3, 2,-5) and E(- 4,5,-9) respectively.

The vertex A is tied up by the rope at the ends F and G and the vertex E is tied up at the ends I and H.

(A) If M denotes the position of their bags inside the tent and it is just in middle of the vertices B and D, then find the coordinates of M and the length AE.

(B) If the length of the rope by which E is tied up with H is 5√2 units, then find the equation denotes the set of point of H and the length BC.

(C) Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3,2,-1).

Ans. (A) As, M is the middle point of B(3, 2, 5) and D(-3, 2,-5)

:- The coordinates of M are

$$\left(\frac{3-3}{2}, \frac{2+2}{2}, \frac{5-5}{2}\right) = (0, 2, 0)$$

The length AE is

$$= \sqrt{(-4-4)^2 + (5-5)^2 + (-9-9)^2}$$

$$= \sqrt{64+0+324}$$

$$= \sqrt{388}$$

$$= 2\sqrt{97} \text{ units}$$

(B) As, the distance of H(x, y, z) from E(-4, 5, -9) is 5√2 units.

:- EH=5√2

$$\Rightarrow \quad \sqrt{(x+4)^2+(y-5)^2+(z+9)^2}=5\sqrt{2}$$

On squaring both sides, we get

$$(x+4)^2+(y-5)^2+(z+9)^2=25\times 2$$

$$x^2+y^2+z^2+8x-10y+18z+122=50$$

$$\Rightarrow \quad x^2+y^2+z^2+8x-10y+18z+72=0$$

The length BC is,

$$BC=\sqrt{(5-3)^2+(2-2)^2+(5-5)^2}$$

$$=\sqrt{4+0+0}$$

$$= 4 \text{ units}$$

(C) Assume that P(x, y, z) be the point that is equidistant from two points A(1, 2, 3) and B(3,2,-1).

Thus, we can say that, PA = PB

Take square on both the sides, we get

PA^2 = PB2

It means that,

$$(x-1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z+1)^2$$

$$\Rightarrow x^2-2x+1+y^2-4y+z^2-6z+9=x^2-6x+9+y^2-4y+4+z^2+2z+1$$

Now, simplify the above equation, we get

$$\Rightarrow \quad -2x-4y-6z+14=-6x-4y+2z+14$$

$$\Rightarrow \quad -2x-6z+6x-2z=0$$

$$\Rightarrow \quad 4x-8z=0$$

$$\Rightarrow \quad x-2z=0$$