INDEFINITE INTEGRAL (XII, R. S. AGGARWAL)

EXERCISE 12 (Pg.No.: 597)

Very-Short-Answers Questions

Evaluate:

1. (i) $\int x^7 dx$ (ii) $\int x^{-7} dx$ (iii) $\int x^{-1} dx$ (iv) $\int x^{5/3} dx$

(v) $\int x^{-5/4} dx$ (vi) $\int 2^x dx$ (vii) $\int \sqrt[3]{x^2}\, dx$ (viii) $\int \frac{1}{\sqrt[4]{x^3}} dx$

(ix) $\int \frac{2}{x^2} dx$

Sol. (i) it is easy to see that $I = \frac{x^{7+1}}{7+1} + c \quad \therefore I = \frac{x^8}{8} + c$

(ii) $I = \frac{x^{-7+1}}{-7+1} \Rightarrow I = \frac{x^{-6}}{-6} + c \quad \therefore I = -\frac{x^{-6}}{-6} + c = -\frac{1}{6x^6} + c$

(iii) $I = \int x^{-1} dx \Rightarrow I = \int \frac{1}{x} dx \quad \therefore I = \log|x| + c$

(iv) $I = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c \Rightarrow I = \frac{x^{\frac{8}{3}}}{8/3} + c \quad \therefore I = \frac{3}{8} x^{8/3} + c$

(v) $I = \frac{x^{-\frac{5}{4}+1}}{\frac{-5}{4}+1} + c \Rightarrow I = \frac{x^{-\frac{1}{4}}}{-\frac{1}{4}} + c \quad \therefore I = \frac{-4}{x^{1/4}} + c$

(vi) $\int 2^x dx = \frac{2^x}{\log 2} + c$

(vii) $I = \int \left(x^2\right)^{1/3} dx \Rightarrow I = \int x^{2/3} dx \Rightarrow I = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c \Rightarrow I = \frac{x^{5/3}}{5/3} + c \quad \therefore I = \frac{3}{5} x^{5/3} + c$

(viii) $I = \int \frac{1}{\left(x^3\right)^{\frac{1}{4}}} dx \Rightarrow I = \int \frac{1}{x^{3/4}} dx \Rightarrow I = \int x^{\frac{-3}{4}} dx \Rightarrow I = \frac{x^{\frac{-3}{4}+1}}{\frac{-3}{4}+1} + c$

$\Rightarrow I = \frac{x^{1/4}}{1/4} + c \quad \therefore I = 4x^{1/4} + c$

(ix) $I = 2\int \frac{1}{x^2} dx \Rightarrow I = 2\int x^{-2} dx \Rightarrow I = 2.\frac{x^{-2+1}}{-2+1} + c$

$\Rightarrow I = 2\left(-\frac{1}{x}\right) + c \quad \therefore I = -\frac{2}{x} + c$

2. (i) $\int\left(6x^5-\frac{2}{x^4}-7x+\frac{3}{x}-5+4e^x+7^x\right)dx$ (ii) $\int\left(8-x+2x^3-\frac{6}{x^3}+2x^{-5}+5x^{-1}\right)dx$

(iii) $\int\left(\frac{x}{a}+\frac{a}{x}+x^a+a^x+ax\right)dx$

Sol. (i) Let $I=\int\left(6x^5-\frac{2}{x^4}-7x+\frac{3}{x}-5+4e^x+7^x\right)dx$

$\Rightarrow\ I=\int 6x^5dx-\int\frac{2}{x^4}dx-\int 7xdx+\int\frac{3}{x}dx-\int 5dx+\int 4e^xdx+\int 7^xdx$

$\Rightarrow\ I=6\int x^5dx-2\int x^{-4}dx-7\int x\,dx+3\int\frac{1}{x}dx-5\int dx+4\int e^xdx+\int 7^xdx$

$\Rightarrow I=\ 6.\frac{x^6}{6}-2\frac{x^{-3}}{-3}-7\frac{x^2}{2}+3\log|x|-5x+4e^x+\frac{7^x}{\log 7}+c$

$\therefore\ \ I=x^6+\frac{2}{3x^3}-\frac{7}{2}x^2+3\log|x|-5x+4e^x+\frac{7^x}{\log 7}+c$

(ii) Let $I=\int\left(8-x+2x^3-\frac{6}{x^3}+2x^{-5}+5x^{-1}\right)dx$

$\Rightarrow\ I=\int 8\,dx-\int x\,dx+\int 2x^3\,dx-\int\frac{6}{x^3}dx+\int 2x^{-5}dx+\int\frac{5}{x}dx$

$\Rightarrow\ I=8\int dx-\int x\,dx+2\int x^3dx-6\int\frac{1}{x^3}dx+2\int x^{-5}dx+5\int\frac{1}{x}dx$

$\Rightarrow\ I=8x-\frac{x^2}{2}+2\frac{x^4}{4}-6.\frac{x^{-2}}{-2}+2\ \frac{x^{-4}}{-4}+5\log|x|+c$

$\therefore\ \ I=8x-\frac{x^2}{2}+\frac{x^4}{2}+\frac{3}{x^2}-\frac{1}{2x^4}+5\log|x|+c$

(iii) Let $I=\int\left(\frac{x}{a}+\frac{a}{x}+x^a+a^x+ax\right)dx\ \ \Rightarrow I=\int\frac{x}{a}dx+\int\frac{a}{x}dx+\int x^adx+\int a^xdx+\int ax\,dx$

$\Rightarrow\ I=\frac{1}{a}\int xdx+a\int\frac{1}{x}dx+\frac{x^{a+1}}{a+1}+\frac{a^x}{\log a}+a\int x\,dx\quad\therefore\ I=\frac{x^2}{2a}+a\log|x|+\frac{x^{a+1}}{a+1}+\frac{a^x}{\log a}+a\frac{x^2}{2}+c$

3. (i) $\int(2-5x)(3+2x)(1-x)dx$ (ii) $\int\sqrt{x}\left(ax^2+bx+c\right)dx$

(iii) $\int\left(\sqrt{x}-\sqrt[3]{x^4}+\frac{7}{\sqrt[3]{x^2}}-6e^x+1\right)dx$

Sol. (i) Let $I=\int(2-5x)(3+2x)(1-x)dx\ \ \Rightarrow I=\int\left(6-17x+x^2+10x^3\right)dx$

$\Rightarrow\ I=\int 6\,dx-\int 17x\,dx+\int x^2dx+\int 10x^3dx\ \ \Rightarrow I=6\int dx-17\int x\,dx+\int x^2dx+10\int x^3dx$

$\Rightarrow\ I=6x-17\frac{x^2}{2}+\frac{x^3}{3}+10\frac{x^4}{4}+c\ \ \therefore\ I=6x-\frac{17}{2}x^2+\frac{x^3}{3}+\frac{5}{2}x^4+c$

(ii) Let $I=\int\sqrt{x}\left(ax^2+bx+c\right)dx\ \ \Rightarrow I=\int\left(a\sqrt{x}\,x^2+b\sqrt{x}\ x+c\sqrt{x}\right)dx$

$\Rightarrow\ I=a\int x^{1/2}x^2dx+b\int x^{1/2}x\,dx+c\int x^{1/2}dx\ \ \Rightarrow I=a\int x^{5/2}dx+b\int x^{3/2}dx+c\int x^{1/2}dx$

$$\Rightarrow\ I=a\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+b\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+c\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c \quad \Rightarrow I=a\frac{x^{7/2}}{7/2}+b\frac{x^{5/2}}{5/2}+c\frac{x^{3/2}}{3/2}+c$$

$$\therefore\quad I=\frac{2a}{7}x^{7/2}+\frac{2b}{5}x^{5/2}+\frac{2c}{3}x^{3/2}+c$$

(iii) Let $I=\int\left(\sqrt{x}-\sqrt[3]{x^4}+\frac{7}{\sqrt[3]{x^2}}-6e^x+1\right)dx \quad \Rightarrow I=\int\sqrt{x}\,dx-\int\sqrt[3]{x^4}dx+7\int\frac{1}{\sqrt[3]{x^2}}dx-6\int e^x dx$

$$\Rightarrow\ I=\int x^{1/2}dx-\int\left(x^4\right)^{\frac{1}{3}}dx+7\int\frac{1}{\left(x^2\right)^{\frac{1}{3}}}dx-6\int e^x+x$$

$$\Rightarrow\ I=\int x^{1/2}dx-\int x^{4/3}dx+7\int x^{\frac{-2}{3}}dx-6e^x+x \quad \Rightarrow I=\frac{x^{3/2}}{3/2}-\frac{x^{7/3}}{7/3}+7\frac{x^{1/3}}{1/3}-6e^x+x+c$$

$$\therefore\quad I=\frac{2}{3}x^{3/2}-\frac{3}{7}x^{7/3}+21x^{1/3}-6e^x+x+c$$

4. (i) $\int\left(x^2-\frac{1}{x^2}\right)^3 dx$ (ii) $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx$ (iii) $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

(iv) $\int\frac{(1+2x)^3}{x^4}dx$ (v) $\int\frac{(1+x)^3}{\sqrt{x}}dx$ (vi) $\int\frac{2x^2+x-2}{x-2}dx$

Sol. (i) $\int\left(x^2-\frac{1}{x^2}\right)^3 dx \quad \Rightarrow \int\left\{\left(x^2\right)^3-\left(\frac{1}{x^2}\right)^3-3\left(x^2\right)^2.\frac{1}{x^2}+3\left(\frac{1}{x^2}\right)^2.x^2\right\}dx$

$$\Rightarrow I=\int\left(x^6-\frac{1}{x^6}-3x^2+\frac{3}{x^2}\right)dx \quad \Rightarrow I=\int x^6dx-\int\frac{1}{x^6}dx-\int 3x^2dx+\int\frac{3}{x^2}dx$$

$$\Rightarrow I=\int x^6dx-\int x^{-6}dx-3\int x^2dx+3\int x^{-2}dx \quad \Rightarrow I=\frac{x^7}{7}-\frac{x^{-5}}{-5}-\frac{3x^3}{3}+3\frac{x^{-1}}{-1}+c$$

$$\therefore\ I=\frac{x^7}{7}+\frac{1}{5x^5}-x^3-\frac{3}{x}+c$$

(ii) Let $I=\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx \quad \Rightarrow I=\int\sqrt{x}\ dx-\int\frac{1}{\sqrt{x}}dx \quad \Rightarrow I=\int x^{1/2}dx-\int x^{-\frac{1}{2}}dx$

$$\Rightarrow\ I=\frac{x^{3/2}}{3/2}-\frac{x^{1/2}}{1/2}+c \quad \therefore\ I=\frac{2}{3}x^{3/2}-2x^{1/2}+c$$

(iii) Let $I=\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx \quad \Rightarrow I=\int\left\{\left(\sqrt{x}\right)^2+2\sqrt{x}.\frac{1}{\sqrt{x}}+\left(\frac{1}{\sqrt{x}}\right)^2\right\}dx$

$$\Rightarrow\ I=\int\left(x+2+\frac{1}{x}\right)dx \quad \Rightarrow I=\int x\ dx+2\int dx+\int\frac{1}{x}dx \quad \therefore\ I=\frac{x^2}{2}+2x+\log|x|+c$$

(iv) Let $I=\int\frac{(1+2x)^3}{x^4}dx \quad \Rightarrow I=\int\frac{(1)^2+(2x)^3+3(1)^2.2x+3(2x)^2.1}{x^4}dx$

$$\Rightarrow\ I=\int\frac{1+8x^3+6x+12x^2}{x^4}dx \quad \Rightarrow I=\int\frac{1}{x^4}dx+8\int\frac{x^3}{x^4}dx+6\int\frac{x}{x^4}dx+12\int\frac{x^2}{x^4}dx$$

$$\Rightarrow\ I=\int x^{-4}dx+8\int\frac{1}{x}dx+6\int\frac{1}{x^3}dx+12\int\frac{1}{x^2}dx\quad\Rightarrow I=\frac{x^{-3}}{-3}+8\log|x|+6\frac{x^{-2}}{-2}+12\left(\frac{-1}{x}\right)+c$$

$$\therefore\quad I=-\frac{1}{3x^3}+8\log|x|-\frac{3}{x^2}-\frac{12}{x}+c$$

(v) Let $I=\int\frac{(1+x)^3}{\sqrt{x}}dx\quad\Rightarrow I=\int\frac{(1)^3+(x)^3+3(1)^2.x+3(x)^2.1}{\sqrt{x}}dx$

$$\Rightarrow\ I=\int\frac{1+x^3+3x+3x^2}{\sqrt{x}}dx\quad\Rightarrow I=\int\frac{1}{\sqrt{x}}dx+\int\frac{x^3}{\sqrt{x}}dx+\int\frac{3x}{\sqrt{x}}dx+\int\frac{3x^2}{\sqrt{x}}dx$$

$$\Rightarrow\ I=\int x^{\frac{-1}{2}}dx+\int x^{5/2}dx+3\int x^{1/2}dx+3\int x^{3/2}dx$$

$$\Rightarrow\ I=\frac{x^{1/2}}{1/2}+\frac{x^{7/2}}{7/2}+\frac{3x^{3/2}}{3/2}+3\frac{x^{5/2}}{5/2}+c\quad\therefore\ I=2x^{1/2}+\frac{2}{7}x^{7/2}+2x^{3/2}+\frac{6}{5}x^{5/2}+c$$

(vi) Let $I=\int\frac{2x^2+x-2}{x-2}dx\quad\Rightarrow I=\int\left\{(2x+5)+\frac{8}{x-2}\right\}dx$

$$\Rightarrow\ I=\int(2x+5)dx+8\int\frac{1}{x-2}dx\quad\Rightarrow I=\frac{2x^2}{2}+5x+8\ \log|x-2|+c$$

$$\therefore\quad I=x^2+5x+8\log|x-2|+c$$

5. $\int\left[1+\frac{1}{(1+x^2)}-\frac{2}{\sqrt{1-x^2}}+\frac{5}{x\sqrt{x^2-1}}+a^x\right]dx$

Sol. Let $I=\int\left[1+\frac{1}{(1+x^2)}-\frac{2}{\sqrt{1-x^2}}+\frac{5}{x\sqrt{x^2-1}}+a^x\right]dx$

$$\Rightarrow\ I=\int dx+\int\frac{1}{1+x^2}dx-2\int\frac{1}{\sqrt{1-x^2}}dx+5\int\frac{1}{x\sqrt{x^2-1}}dx+\int a^x dx$$

$$\therefore\quad I=x+\tan^{-1}(x)-2\sin^{-1}(x)+5\sec^{-1}(x)+\frac{a^x}{\log a}+c$$

6. (i) $\int\left(\frac{x^2-1}{x^2+1}\right)dx$ (ii) $\int\left(\frac{x^6-1}{x^2+1}\right)dx$ (iii) $\int\left(\frac{x^4}{1+x^2}\right)dx$ (iv) $\int\left(\frac{x^2}{1+x^2}\right)dx$

Sol. (i) Let $I=\int\frac{x^2-1}{x^2+1}dx\quad\Rightarrow I=\int\left(\frac{x^2+1-2}{x^2+1}\right)dx\quad\Rightarrow I=\int\frac{x^2+1}{x^2+1}dx-2\int\frac{1}{x^2+1}dx$

$$\Rightarrow\ I=\int dx-2\int\frac{1}{x^2+1}dx\qquad\therefore\ I=x-2\tan^{-1}(x)+c$$

(ii) Let $I=\int\frac{x^6-1}{x^2+1}dx\quad\Rightarrow I=\int\frac{x^6+1-2}{x^2+1}dx\quad\Rightarrow I=\int\frac{x^6+1}{x^2+1}dx-2\int\frac{1}{x^2+1}dx$

$$\Rightarrow\ I=\int\frac{(x^2)^3+(1)^3}{x^2+1}dx-2\tan^{-1}(x)\quad\Rightarrow I=\int\frac{(x^2+1)(x^4-x^2+1)}{(x^2+1)}dx-2\tan^{-1}(x)$$

$$\Rightarrow\ I=\int(x^4-x^2+1)dx-2\tan^{-1}x\quad\therefore\ I=\frac{x^5}{5}-\frac{x^3}{3}+x-2\tan^{-1}(x)+c$$

(iii) Let $I=\int \frac{x^4}{1+x^2}dx \quad \Rightarrow I=\int \frac{x^4-1+1}{\left(1+x^2\right)}dx \quad \Rightarrow I=\int \frac{x^4-1}{1+x^2}dx+\int \frac{1}{1+x^2}dx$

$\Rightarrow \quad I=\int \frac{\left(x^2\right)^2-(1)^2}{1+x^2}dx+\tan^{-1}(x) \quad \Rightarrow I=\int \frac{\left(x^2-1\right)\left(x^2+1\right)}{\left(x^2+1\right)}dx+\tan^{-1}(x)$

$\Rightarrow \quad I=\int\left(x^2-1\right)dx+\tan^{-1}(x) \quad \therefore\ I=\frac{x^3}{3}-x+\tan^{-1}(x)+c$

(iv) Let $I=\int\left(\frac{x^2}{1+x^2}\right)dx \quad \Rightarrow I=\int \frac{x^2+1-1}{1+x^2}dx \quad \Rightarrow I=\int \frac{x^2+1}{1+x^2}dx-\int \frac{1}{1+x^2}dx$

$\Rightarrow \quad I=\int dx-\int \frac{1}{1+x^2}dx \quad \therefore\ I=x-\tan^{-1}(x)+c$

7. $\int\left(9\sin x-7\cos x-\frac{6}{\cos^2 x}+\frac{2}{\sin^2 x}+\cot^2 x\right)dx$

Sol. Let $I=\int\left(9\sin x-7\cos x-\frac{6}{\cos^2 x}+\frac{2}{\sin^2 x}+\cot^2 x\right)dx$

$\Rightarrow \quad I=9\int \sin x dx-7\int \cos x dx-6\int \frac{1}{\cos^2 x}dx+2\int \frac{1}{\sin^2 x}dx+\int \cot^2 x dx$

$\Rightarrow \quad I=9(-\cos x)-7\sin x-6\int \sec^2 x\ dx+2\int \operatorname{cosec}^2 x\ dx+\int\left(\operatorname{cosec}^2 x-1\right)dx$

$\Rightarrow \quad I=-9\cos x-7\sin x-6\tan x+2(-\cot x)-\cot x-x+c$

$\therefore \quad I=-9\cos x-7\sin x-6\tan x-3\cot x-x+c$

8. $\int\left(\frac{\cot x}{\sin x}-\tan^2 x-\frac{\tan x}{\cos x}+\frac{2}{\cos^2 x}\right)dx$

Sol. Let $I=\int\left(\frac{\cot x}{\sin x}-\tan^2 x-\frac{\tan x}{\cos x}+\frac{2}{\cos^2 x}\right)dx$

$\Rightarrow \quad I=\int \cot x \operatorname{cose} x\ dx-\int \tan^2 x\ dx-\int \tan x \sec x\ dx+2\int \sec^2 x\ dx$

$\Rightarrow \quad I=-\operatorname{cosec} x-\int\left(\sec^2 x-1\right)dx-\sec x+2\tan x$

$\Rightarrow \quad I=-\operatorname{cosec} x-\tan x+x-\sec x+2\tan x+c$

$\therefore \quad I=-\operatorname{cosec} x+\tan x+x-\sec x+c$

9. (i) $\int \sec x(\sec x+\tan x)dx$ (ii) $\int \operatorname{cosec} x(\operatorname{cosec} x-\cot x)dx$

Sol. (i) Let $I=\int \sec x(\sec x+\tan x)dx$

$\Rightarrow \quad I=\int \sec^2 x dx+\int \sec x \tan x\ dx \quad \therefore\ I=\tan x+\sec x+c$

(ii) Let $I=\int \operatorname{cosec} x\ (\operatorname{cosec} x-\cot x)dx \quad \Rightarrow I=\int \operatorname{cosec}^2 x\ dx-\int \operatorname{cosec} x\ \cot x\ dx$

$\Rightarrow \quad I=-\cot x+\operatorname{cosec}\ x+c \quad \therefore\ I=\operatorname{cosec} x-\cot x+c$

10. (i) $\int(\tan x+\cot x)^2 dx$ (ii) $\int\left(\frac{1+2\sin x}{\cos^2 x}\right)dx$ (iii) $\int\left(\frac{3\cos x+4}{\sin^2 x}\right)dx$

Sol. (i) Let $I=\int(\tan x+\cot x)^2 dx \quad \Rightarrow I=\int\left(\tan^2 x+\cot^2 x+2\tan x\cot x\right)dx$

$\Rightarrow \quad I=\int\left(\sec^2 x-1+\operatorname{cosec}^2 x-1+2\right)dx=\int\left(\sec^2 x+\operatorname{cosec}^2 x\right)dx=\tan x-\cot x+c$

(ii) Let $I=\int\left(\frac{1+2\sin x}{\cos^2 x}\right)dx \quad \Rightarrow I=\int\frac{1}{\cos^2 x}dx+2\int\frac{\sin x}{\cos^2 x}dx$

$\Rightarrow \quad I=\int\sec^2 x dx+2\int\tan x.\sec x\, dx \quad \therefore\ I=\tan x+2\sec x+c$

(iii) Let $I=\int\left(\frac{3\cos x+4}{\sin^2 x}\right)dx \quad \Rightarrow I=\int\frac{3\cos x}{\sin^2 x}dx+\int\frac{4}{\sin^2 x}dx$

$\Rightarrow \quad I=3\int\cot x\,\mathrm{cosec}\, x\, dx+4\int\mathrm{cosec}^2 x\, dx \quad \Rightarrow I=3(-\mathrm{cosec}\, x)+4(-\cot x)$

$\therefore \quad I=-3\,\mathrm{cosec}\, x-4\cot x+c$

11. (i) $\int\frac{1}{(1-\cos x)}dx$ (ii) $\int\frac{1}{(1-\sin x)}dx$

Sol. (i) Let $I=\int\frac{1}{(1-\cos x)}dx \quad \Rightarrow I=\int\frac{1}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}dx \quad \Rightarrow I=\int\frac{1+\cos x}{1-\cos^2 x}dx$

$\Rightarrow \quad I=\int\frac{1+\cos x}{\sin^2 x}dx \quad \Rightarrow I=\int\frac{1}{\sin^2 x}dx+\int\frac{\cos x}{\sin^2 x}dx$

$\Rightarrow \quad I=\int\mathrm{cosec}^2\, dx+\int\cot x\,\mathrm{cosec}\, x\, dx \qquad \therefore\ I=-\cot x-\mathrm{cosec}\, x+c$

(ii) Let $I=\int\frac{1}{(1-\sin x)}dx \quad \Rightarrow I=\int\frac{1}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}dx \quad \Rightarrow I=\int\frac{1+\sin x}{1-\sin^2 x}dx$

$\Rightarrow \quad I=\int\frac{1+\sin x}{\cos^2 x}dx \quad \Rightarrow I=\int\frac{1}{\cos^2 x}dx+\int\frac{\sin x}{\cos^2 x}dx$

$\Rightarrow \quad I=\int\sec^2 x\, dx+\int\tan x.\sec x\, dx \quad \therefore\ I=\tan x+\sec x+c$

12. (i) $\int\frac{\tan x}{(\sec x+\tan x)}dx$ (ii) $\int\frac{\mathrm{cosec}\, x}{(\mathrm{cosec}\, x-\cot x)}dx$

Sol. (i) Let $I=\int\frac{\tan x}{(\sec x+\tan x)}dx \quad \Rightarrow I=\int\frac{\tan x}{\sec x+\tan x}\times\frac{\sec x-\tan x}{\sec x-\tan x}$

$\Rightarrow \quad I=\int\frac{\tan x(\sec x-\tan x)}{\sec^2 x-\tan^2 x}dx \quad \Rightarrow I=\int\tan x(\sec x-\tan x)dx$

$\Rightarrow \quad I=\int\sec x\tan x\, dx-\int\tan^2 x\, dx \quad \Rightarrow I=\sec x-\int(\sec^2 x-1)dx$

$\therefore \quad I=\sec x-\tan x+x+c$

(ii) Let $I=\int\frac{\mathrm{cosec}\, x}{(\mathrm{cosec}\, x-\cot x)}dx \quad \Rightarrow I=\int\frac{\mathrm{cosec}\, x}{\mathrm{cosec}\, x-\cot x}\times\frac{\mathrm{cosec}\, x+\cot x}{\mathrm{cosec}\, x+\cot x}dx$

$\Rightarrow \quad I=\int\frac{\mathrm{cosec}\, x(\mathrm{cosec}\, x+\cot x)}{\mathrm{cosec}^2 x-\cot^2 x}dx \quad \Rightarrow I=\int\mathrm{cosec}\, x(\mathrm{cosec}\, x+\cot x)dx$

$\Rightarrow \quad I=\int\mathrm{cosec}^2 x\, dx+\int\mathrm{cosec}\, x\cot x\, dx \qquad \therefore\ I=-\cot x-\mathrm{cosec}\, x+c$

13. (i) $\int\frac{\cos x}{1+\cos x}dx$ (ii) $\int\frac{\sin x}{(1-\sin x)}dx$

Sol. (i) Let $I=\int\frac{\cos x}{1+\cos x}dx \quad \Rightarrow I=\int\frac{\cos x}{1+\cos x}\times\frac{1-\cos x}{1-\cos x}dx$

$\Rightarrow \quad I=\int\frac{\cos x(1-\cos x)}{1-\cos^2 x}dx \quad \Rightarrow I=\int\frac{\cos x(1-\cos x)}{\sin^2 x}dx$

$$\Rightarrow\ I=\int\frac{\cos x}{\sin^2 x}dx-\int\frac{\cos^2 x}{\sin^2 x}dx \quad \Rightarrow\ I=\int\cot x\operatorname{cosec} x\ dx-\int\cot^2 x\ dx$$

$$\Rightarrow\ I=-\operatorname{cosec} x-\int\left(\operatorname{cosec}^2 x-1\right)dx \qquad \therefore\ I=-\operatorname{cosec} x+\cot x+x+c$$

(ii) Let $I=\int\frac{\sin x}{(1-\sin x)}dx \ \Rightarrow\ I=\int\frac{\sin x}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}dx \quad \Rightarrow\ I=\int\frac{\sin x(1+\sin x)}{1-\sin^2 x}dx$

$$\Rightarrow\ I=\int\frac{\sin x+\sin^2 x}{\cos^2 x}dx \quad \Rightarrow\ I=\int\frac{\sin x}{\cos^2 x}dx+\int\frac{\sin^2 x}{\cos^2 x}dx$$

$$\Rightarrow\ I=\int\tan x\sec x\ dx+\int\tan^2 x\ dx \quad \Rightarrow\ I=\sec x+\int\left(\sec^2 x-1\right)dx$$

$$\therefore\ I=\sec x+\tan x-x+c$$

14. (i) $I=\int\sqrt{1+\cos 2x}\ dx$ (ii) $\int\sqrt{1-\cos 2x}\ dx$

Sol. (i) Let $I=\int\sqrt{1+\cos 2x}\ dx \ \Rightarrow\ I=\int\sqrt{2\cos^2 x}\ dx \ \Rightarrow\ I=\int\sqrt{2}\ \cos x\ dx$

$$\Rightarrow\ I=\sqrt{2}\int\cos x\ dx \qquad \therefore\ I=\sqrt{2}(\sin x)+c$$

(ii) Let $I=\int\sqrt{1-\cos 2x}\ dx \ \Rightarrow\ I=\int\sqrt{2\sin^2 x}\ dx \ \Rightarrow\ I=\int\sqrt{2}\ \sin x\ dx$

$$\Rightarrow\ I=\sqrt{2}\ (-\cos x)+c \qquad \therefore\ I=-\sqrt{2}\ \cos x+c$$

15. (i) $\int\frac{1}{(1+\cos 2x)}dx$ (ii) $\int\frac{1}{(1-\cos 2x)}dx$

Sol. (i) Let $I=\int\frac{1}{(1+\cos 2x)}dx \ \Rightarrow\ I=\int\frac{1}{2\cos^2 x}dx \ \Rightarrow\ I=\frac{1}{2}\int\sec^2 x\ dx \ \therefore\ I=\frac{1}{2}\tan x+c$

(ii) Let $I=\int\frac{1}{(1-\cos 2x)}dx \ \Rightarrow\ I=\int\frac{1}{2\sin^2 x}dx \ \Rightarrow\ I=\frac{1}{2}\int\operatorname{cosec}^2 x\ dx$

$$\Rightarrow\ I=\frac{1}{2}(-\cot x)+c \quad \therefore\ I=-\frac{1}{2}\cot x+c$$

16. $\int\sqrt{1+\sin 2x}\ dx$

Sol. Let $I=\int\sqrt{1+\sin 2x}\ dx \ \Rightarrow\ I=\int\sqrt{(\sin x+\cos x)^2}\ dx \ \Rightarrow\ I=\int(\sin x+\cos x)dx$

$$\Rightarrow\ I=-\cos x+\sin x+c \quad \therefore\ I=\sin x-\cos x+c$$

17. $\int\left(\frac{\sin^3 x+\cos^3 x}{\sin^2 x.\cos^2 x}\right)dx$

Sol. Let $I=\int\left(\frac{\sin^3 x+\cos^3 x}{\sin^2 x.\cos^2 x}\right)dx \ \Rightarrow\ I=\int\frac{\sin^3 x}{\sin^2 x.\cos^2 x}dx+\int\frac{\cos^3 x}{\sin^2 x.\cos^2 x}dx$

$$\Rightarrow\ I=\int\frac{\sin x}{\cos^2 x}dx+\int\frac{\cos x}{\sin^2 x}dx \quad \Rightarrow\ I=\int\tan x.\sec x dx+\int\cot x\operatorname{cosec} x\, dx$$

$$\therefore\ I=\sec x-\operatorname{cosec} x+c$$

18. $\int\tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)dx$

Sol. Let $I=\int\tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)dx \ \Rightarrow\ I=\int\tan^{-1}\left(\frac{2\sin x.\cos x}{2\cos^2 x}\right)dx$

$$\Rightarrow \; I=\int \tan^{-1}(\tan x)dx \quad \Rightarrow I=\int x\,dx \qquad \therefore \; I=\frac{x^2}{2}+c$$

19. $\int \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)dx$

Sol. Let $I=\int \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)dx$

$$\Rightarrow \; I=\int \cos^{-1}(\cos 2x)dx \;\Rightarrow I=\int 2x\,dx \qquad \Rightarrow I=2\frac{x^2}{2}+c \qquad \therefore \; I=x^2+c$$

20. $\int \cos^{-1}(\sin x)dx$

Sol. Let $I=\int \cos^{-1}(\sin x)dx \;\Rightarrow I=\int \cos^{-1}\left\{\cos\left(\frac{\pi}{2}-x\right)\right\}dx \;\Rightarrow I=\int\left(\frac{\pi}{2}-x\right)dx \;\therefore\; I=\frac{\pi}{2}x-\frac{x^2}{2}+c$

21. $\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}dx$

Sol. Let $I=\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}dx \quad \Rightarrow I=\int \tan^{-1}\left\{\sqrt{\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}}\right\}dx$

$$\Rightarrow \; I=\int \tan^{-1}\sqrt{\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)^2}\,dx \quad \Rightarrow I=\int \tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)dx$$

Now, dividing nominator & denominator by $\cos\frac{x}{2}$,

$$\Rightarrow \; I=\int \tan^{-1}\left(\frac{\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}}}\right)dx \quad \Rightarrow I=\int \tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)dx$$

$$\Rightarrow \; I=\int \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{2}\tan\frac{x}{2}}\right)dx \quad \Rightarrow I=\int \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}dx$$

$$\Rightarrow \; I=\int\left(\frac{\pi}{4}-\frac{x}{2}\right)dx \quad \Rightarrow I=\frac{\pi}{4}x-\frac{1}{2}\frac{x^2}{2}+c \quad \therefore \; I=\frac{\pi x}{4}-\frac{x^2}{4}+c$$

22. $\int(3\cot x-2\tan x)^2\,dx$

Sol. Let $I=\int(3\cot x-2\tan x)^2\,dx \quad \Rightarrow I=\int\left\{9\left(\operatorname{cosec}^2 x-1\right)-12+4\left(\sec^2 x-1\right)\right\}dx$

$$\Rightarrow \; I=\int\left(9\operatorname{cosec}^2 x-9-12+4\sec^2 x-4\right)dx \qquad \Rightarrow I=\int\left(9\operatorname{cosec}^2 x+4\sec^2 x-25\right)dx$$

$\therefore \quad I=-9\cot x+4\tan x-25x+c$

23. $\int (3\sin x+4\,\text{cosec}\,x)^2\,dx$

Sol. Let $I=\int (3\sin x+4\,\text{cosec}\,x)^2\,dx \quad \Rightarrow I=\int \left(9\sin^2 x+2.3.4\sin x\ \text{cosec}\ x+16\ \text{cosec}^2\ x\right)dx$

$\Rightarrow \quad I=9\int \sin^2 x\ dx+24\int \sin x\,\text{cosec}\,x\ dx+16\int \text{cosec}^2\ x\ dx$

$\Rightarrow \quad I=9\int \frac{1-\cos 2x}{2}dx+24\int dx+16(-\cot x) \quad \Rightarrow I=\frac{9}{2}\int (1-\cos 2x)dx+24x+16(-\cot x)+c$

$\Rightarrow \quad I=\frac{9}{2}\int (1-\cos 2x)dx+24x-16\cot x+c \quad \Rightarrow I=\frac{9}{2}\left(x-\frac{\sin 2x}{2}\right)+24x-16\cot x+c$

$\Rightarrow \quad I=\frac{9}{2}x-\frac{9}{4}\sin 2x+24x-16\cot x+c \quad \therefore I=\frac{57}{2}x-\frac{9}{4}\sin 2x-16\cot x+c$

24. $\int \frac{1}{\left(\sqrt{x+1}+\sqrt{x+2}\right)}dx$

Sol. Let $I=\int \frac{1}{\left(\sqrt{x+1}+\sqrt{x+2}\right)}dx \quad \Rightarrow I=\int \frac{1}{\sqrt{x+1}+\sqrt{x+2}}\times\frac{\sqrt{x+1}-\sqrt{x+2}}{\sqrt{x+1}-\sqrt{x+2}}dx$

$\Rightarrow \quad I=\int \frac{\sqrt{x+1}-\sqrt{x+2}}{\left(\sqrt{x+1}\right)^2-\left(\sqrt{x+2}\right)^2}dx \quad \Rightarrow I=\int \frac{\sqrt{x+1}-\sqrt{x+2}}{x+1-x-2}dx$

$\Rightarrow \quad I=\int \frac{\sqrt{x+1}-\sqrt{x+2}}{-1}dx \quad \Rightarrow I=-\int \sqrt{x+1}\ dx+\int \sqrt{x+2}\ dx$

$\Rightarrow \quad I=-\int (x+1)^{1/2}\ dx+\int (x+2)^{1/2}\,dx \quad \Rightarrow I=-\frac{(x+1)^{3/2}}{3/2}+\frac{(x+2)^{3/2}}{3/2}+c$

$\therefore \quad I=\frac{2}{3}(x+2)^{3/2}-\frac{2}{3}(x+1)^{3/2}+c$

25. $\int \frac{1}{\left(\sqrt{x+3}-\sqrt{x+2}\right)}dx$

Sol. Let $I=\int \frac{1}{\left(\sqrt{x+3}-\sqrt{x+2}\right)}dx \quad \Rightarrow I=\int \frac{1}{\sqrt{x+3}-\sqrt{x+2}}\times\frac{\sqrt{x+3}+\sqrt{x+2}}{\sqrt{x+3}+\sqrt{x+2}}\ dx$

$\Rightarrow \quad I=\int \frac{\sqrt{x+3}+\sqrt{x+2}}{\left(\sqrt{x+3}\right)^2-\left(\sqrt{x+2}\right)^2}dx \quad \Rightarrow I=\int \frac{\sqrt{x+3}+\sqrt{x+2}}{x+3-x-2}\ dx$

$\Rightarrow \quad I=\int \left(\sqrt{x+3}+\sqrt{x+2}\right)dx \quad \Rightarrow I=\int (x+3)^{1/2}\ dx+\int (x+2)^{1/2}\ dx$

$\Rightarrow \quad I=\frac{(x+3)^{3/2}}{3/2}+\frac{(x+2)^{3/2}}{3/2}+c \quad \therefore \ I=\frac{2}{3}(x+3)^{3/2}+\frac{2}{3}(x+2)^{3/2}+c$

26. $\int \left(\frac{1+\cos x}{1-\cos x}\right)dx$

Sol. Let $I=\int \left(\frac{1+\cos x}{1-\cos x}\right)dx \quad \Rightarrow I=\int \frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}\ dx \quad \Rightarrow I=\int \cot^2\frac{x}{2}\ dx \quad \Rightarrow I=\int \left(\text{cosec}^2\frac{x}{2}-1\right)dx$

$$\Rightarrow I=-\frac{\cot\frac{x}{2}}{1/2}-x+c \qquad \therefore\ I=-2\cot\frac{x}{2}-x+c$$

27. $\int\left(\frac{1+\tan x}{1-\tan x}\right)dx$

Sol. Let $I=\int\left(\frac{1+\tan x}{1-\tan x}\right)dx \quad \Rightarrow I=\int\frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx \quad \Rightarrow I=\int\frac{\cos x+\sin x}{\cos x-\sin x}dx$

Put, $\cos x-\sin x=t \quad \Rightarrow -\sin x-\cos x=\frac{dt}{dx} \quad \Rightarrow -(\sin x+\cos x)dx=dt$

$\Rightarrow (\sin x+\cos x)dx=-dt \quad \Rightarrow I=-\int\frac{dt}{t} \quad \Rightarrow I=-\log|t|+c \quad \therefore I=-\log|\cos x-\sin x|+c$

28. $\int\frac{\cos(x+a)}{\sin(x+b)}dx$

Sol. Let $I=\int\frac{\cos(x+a)}{\sin(x+b)}dx \Rightarrow I=\int\frac{\cos\{(x+b)+(a-b)\}}{\sin(x+b)}dx$

$\Rightarrow I=\int\frac{\cos(x+b)\cos(a-b)-\sin(x+b)\sin(a-b)}{\sin(x+b)}dx$

$\Rightarrow I=\int\frac{\cos(x+b)\cos(a-b)}{\sin(x+b)}dx-\int\frac{\sin(x+b)\sin(a-b)}{\sin(x+b)}dx$

$\Rightarrow I=\cos(a-b)\int\cot(x+b)dx-\sin(a-b)\int dx$

$\Rightarrow I=\cos(a-b)\log|\sin(x+b)|-\sin(a-b).\ x+c$

$\therefore\ I=\cos(a-b)\log|\sin(x+b)|-x\sin(a-b)+c$

29. $\int\frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$

Sol. Let $I=\int\frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx \Rightarrow I=\int\frac{\sin(x+\alpha-2\alpha)}{\sin(x+\alpha)}dx \quad \Rightarrow I=\int\frac{\sin\{(x+\alpha)-2\alpha\}}{\sin(x+\alpha)}dx$

$\Rightarrow I=\int\frac{\sin(x+\alpha)\cos 2\alpha-\cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}dx$

$\Rightarrow I=\int\frac{\sin(x+\alpha)\cos 2\alpha}{\sin(x+\alpha)}dx-\int\frac{\cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}dx$

$\Rightarrow I=\cos 2\alpha\int dx-\sin 2\alpha\int\cot(x+\alpha)dx \qquad \Rightarrow I=\cos 2\alpha.x-\sin 2\alpha\log|\sin(x+\alpha)|+c$

$\therefore\ I=x\cos 2\alpha-\sin 2\alpha\log|\sin(x+\alpha)|+c$

30. $\int(1-x)\sqrt{x}dx$

Sol. Let $I=\int(1-x)\sqrt{x}dx$

$I=\int\left(\sqrt{x}-x\sqrt{x}\right)dx$

$I=\int\left(x^{1/2}-x^{3/2}\right)dx$

$$I = \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^{5/2}}{\frac{5}{2}} + C$$

$$I = \frac{3}{2}x^{3/2} - \frac{2}{5}x^{5/2} + C$$

$$I = \frac{10x^{3/2} - 6x^{5/2}}{15} + C$$

$$I = \frac{2}{15}\left(5x^{3/2} - 3x^{5/2}\right) + C$$

31. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

Sol. Let $I = \int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx, \; I = \int \left(\frac{1}{\cos^2 x} \times \frac{\sin^2 x}{1} \right) dx, \; I = \int \left(\frac{\sin^2 x}{\cos^2 x} \right) dx, \; I = \int \tan^2 x dx$$

$$I = \int \left(\sec^2 x - 1\right) dx, \quad I = \int \sec^2 x dx - \int dx, \quad I = \tan x - x + C$$

32. $\int \left\{ \frac{2 - 3\sin x}{\cos^2 x} \right\} dx$

Sol. Let $I = \int \left\{ \frac{2 - 3\sin x}{\cos^2 x} \right\} dx$

$$I = \int \left\{ \frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right\} dx, \; I = \int \left\{ 2\sec^2 x - 3\sec x \tan x \right\} dx$$

$$I = \int \left\{ 2\sec^2 x \right\} dx - \int \left\{ 3\sec x \tan x \right\} dx, \; I = 2\int \sec^2 x dx - 3\int \sec x \tan x dx$$

$$I = 2\tan x - 3\sec x + c,$$