

A Brief Table of Integrals

1. $\int u \, dv = uv - \int v \, du$
2. $\int a^u \, du = \frac{a^u}{\ln a} + C, \quad a \neq 1, \quad a > 0$
3. $\int \cos u \, du = \sin u + C$
4. $\int \sin u \, du = -\cos u + C$
5. $\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$
6. $\int (ax + b)^{-1} \, dx = \frac{1}{a} \ln |ax + b| + C$
7. $\int x(ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a^2} \left[\frac{ax + b}{n+2} - \frac{b}{n+1} \right] + C, \quad n \neq -1, -2$
8. $\int x(ax + b)^{-1} \, dx = \frac{x}{a} - \frac{b}{a^2} \ln |ax + b| + C$
9. $\int x(ax + b)^{-2} \, dx = \frac{1}{a^2} \left[\ln |ax + b| + \frac{b}{ax + b} \right] + C$
10. $\int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln \left| \frac{x}{ax + b} \right| + C$
11. $\int (\sqrt{ax + b})^n \, dx = \frac{2(\sqrt{ax + b})^{n+2}}{a(n+2)} + C, \quad n \neq -2$
12. $\int \frac{\sqrt{ax + b}}{x} \, dx = 2\sqrt{ax + b} + b \int \frac{dx}{x\sqrt{ax + b}}$
13. a) $\int \frac{dx}{x\sqrt{ax - b}} = \frac{2}{\sqrt{b}} \tan^{-1} \sqrt{\frac{ax - b}{b}} + C$
 b) $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \right| + C$
14. $\int \frac{\sqrt{ax + b}}{x^2} \, dx = -\frac{\sqrt{ax + b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax + b}} + C$
15. $\int \frac{dx}{x^2\sqrt{ax + b}} = -\frac{\sqrt{ax + b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax + b}} + C$
16. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
17. $\int \frac{dx}{(a^2 + x^2)^2} = \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3} \tan^{-1} \frac{x}{a} + C$
18. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x + a}{x - a} \right| + C$
19. $\int \frac{dx}{(a^2 - x^2)^2} = \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{4a^3} \ln \left| \frac{x + a}{x - a} \right| + C$
20. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C = \ln(x + \sqrt{a^2 + x^2}) + C$
21. $\int \sqrt{a^2 + x^2} \, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + C$
22. $\int x^2 \sqrt{a^2 + x^2} \, dx = \frac{x}{8} (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2 + x^2}) + C$
23. $\int \frac{\sqrt{a^2 + x^2}}{x} \, dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$
24. $\int \frac{\sqrt{a^2 + x^2}}{x^2} \, dx = \ln(x + \sqrt{a^2 + x^2}) - \frac{\sqrt{a^2 + x^2}}{x} + C$
25. $\int \frac{x^2}{\sqrt{a^2 + x^2}} \, dx = -\frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + \frac{x\sqrt{a^2 + x^2}}{2} + C$
26. $\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$
27. $\int \frac{dx}{x^2\sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C$
28. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$
29. $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

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$$30. \int x^2 \sqrt{a^2 - x^2} dx = \frac{a^4}{8} \sin^{-1} \frac{x}{a} - \frac{1}{8} x \sqrt{a^2 - x^2} (a^2 - 2x^2) + C$$

$$31. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C \quad 32. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\sin^{-1} \frac{x}{a} - \frac{\sqrt{a^2 - x^2}}{x} + C$$

$$33. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \frac{1}{2} x \sqrt{a^2 - x^2} + C \quad 34. \int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$35. \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C \quad 36. \int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a} + C = \ln \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$37. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$38. \int (\sqrt{x^2 - a^2})^n dx = \frac{x(\sqrt{x^2 - a^2})^n}{n+1} - \frac{na^2}{n+1} \int (\sqrt{x^2 - a^2})^{n-2} dx, \quad n \neq -1$$

$$39. \int \frac{dx}{(\sqrt{x^2 - a^2})^n} = \frac{x(\sqrt{x^2 - a^2})^{2-n}}{(2-n)a^2} - \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{x^2 - a^2})^{n-2}}, \quad n \neq 2$$

$$40. \int x(\sqrt{x^2 - a^2})^n dx = \frac{(\sqrt{x^2 - a^2})^{n+2}}{n+2} + C, \quad n \neq -2$$

$$41. \int x^2 \sqrt{x^2 - a^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$42. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| + C \quad 43. \int \frac{\sqrt{x^2 - a^2}}{x^2} dx = \ln \left| x + \sqrt{x^2 - a^2} \right| - \frac{\sqrt{x^2 - a^2}}{x} + C$$

$$44. \int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right| + \frac{x}{2} \sqrt{x^2 - a^2} + C$$

$$45. \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C = \frac{1}{a} \cos^{-1} \left| \frac{a}{x} \right| + C \quad 46. \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$$

$$47. \int \frac{dx}{\sqrt{2ax - x^2}} = \sin^{-1} \left(\frac{x-a}{a} \right) + C$$

$$48. \int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + C$$

$$49. \int (\sqrt{2ax - x^2})^n dx = \frac{(x-a)(\sqrt{2ax - x^2})^n}{n+1} + \frac{na^2}{n+1} \int (\sqrt{2ax - x^2})^{n-2} dx$$

$$50. \int \frac{dx}{(\sqrt{2ax - x^2})^n} = \frac{(x-a)(\sqrt{2ax - x^2})^{2-n}}{(n-2)a^2} + \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{2ax - x^2})^{n-2}}$$

$$51. \int x \sqrt{2ax - x^2} dx = \frac{(x+a)(2x-3a)\sqrt{2ax - x^2}}{6} + \frac{a^3}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + C$$

$$52. \int \frac{\sqrt{2ax - x^2}}{x} dx = \sqrt{2ax - x^2} + a \sin^{-1} \left(\frac{x-a}{a} \right) + C \quad 53. \int \frac{\sqrt{2ax - x^2}}{x^2} dx = -2\sqrt{\frac{2a-x}{x}} - \sin^{-1} \left(\frac{x-a}{a} \right) + C$$

$$56. \int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$57. \int \cos ax \, dx = \frac{1}{a} \sin ax + C$$

$$58. \int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$59. \int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

$$60. \int \sin^n ax \, dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax \, dx$$

$$61. \int \cos^n ax \, dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$$

$$62. \text{ a) } \int \sin ax \cos bx \, dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C, \quad a^2 \neq b^2$$

$$\text{ b) } \int \sin ax \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\text{ c) } \int \cos ax \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$63. \int \sin ax \cos ax \, dx = -\frac{\cos 2ax}{4a} + C$$

$$64. \int \sin^n ax \cos ax \, dx = \frac{\sin^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$$

$$65. \int \frac{\cos ax}{\sin ax} \, dx = \frac{1}{a} \ln |\sin ax| + C$$

$$66. \int \cos^n ax \sin ax \, dx = -\frac{\cos^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$$

$$67. \int \frac{\sin ax}{\cos ax} \, dx = -\frac{1}{a} \ln |\cos ax| + C$$

$$68. \int \sin^n ax \cos^m ax \, dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax \, dx, \quad n \neq -m \quad (\text{reduces } \sin^n ax)$$

$$69. \int \sin^n ax \cos^m ax \, dx = \frac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n} \int \sin^n ax \cos^{m-2} ax \, dx, \quad m \neq -n \quad (\text{reduces } \cos^m ax)$$

$$70. \int \frac{dx}{b+c \sin ax} = \frac{-2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[\sqrt{\frac{b-c}{b+c}} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) \right] + C, \quad b^2 > c^2$$

$$71. \int \frac{dx}{b+c \sin ax} = \frac{-1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \sin ax + \sqrt{c^2-b^2} \cos ax}{b+c \sin ax} \right| + C, \quad b^2 < c^2$$

$$72. \int \frac{dx}{1+\sin ax} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) + C$$

$$73. \int \frac{dx}{1-\sin ax} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right) + C$$

$$74. \int \frac{dx}{b+c \cos ax} = \frac{2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[\sqrt{\frac{b-c}{b+c}} \tan \frac{ax}{2} \right] + C, \quad b^2 > c^2$$

$$75. \int \frac{dx}{b+c \cos ax} = \frac{1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \cos ax + \sqrt{c^2-b^2} \sin ax}{b+c \cos ax} \right| + C, \quad b^2 < c^2$$

$$76. \int \frac{dx}{1+\cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C$$

$$77. \int \frac{dx}{1-\cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C$$

$$78. \int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C$$

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80. $\int x^n \sin ax \, dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$
81. $\int x^n \cos ax \, dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$
82. $\int \tan ax \, dx = \frac{1}{a} \ln |\sec ax| + C$
83. $\int \cot ax \, dx = \frac{1}{a} \ln |\sin ax| + C$
84. $\int \tan^2 ax \, dx = \frac{1}{a} \tan ax - x + C$
85. $\int \cot^2 ax \, dx = -\frac{1}{a} \cot ax - x + C$
86. $\int \tan^n ax \, dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax \, dx, \quad n \neq 1$
87. $\int \cot^n ax \, dx = -\frac{\cot^{n-1} ax}{a(n-1)} - \int \cot^{n-2} ax \, dx, \quad n \neq 1$
88. $\int \sec ax \, dx = \frac{1}{a} \ln |\sec ax + \tan ax| + C$
89. $\int \csc ax \, dx = -\frac{1}{a} \ln |\csc ax + \cot ax| + C$
90. $\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$
91. $\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$
92. $\int \sec^n ax \, dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax \, dx, \quad n \neq 1$
93. $\int \csc^n ax \, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} ax \, dx, \quad n \neq 1$
94. $\int \sec^n ax \tan ax \, dx = \frac{\sec^n ax}{na} + C, \quad n \neq 0$
95. $\int \csc^n ax \cot ax \, dx = -\frac{\csc^n ax}{na} + C, \quad n \neq 0$
96. $\int \sin^{-1} ax \, dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1-a^2x^2} + C$
97. $\int \cos^{-1} ax \, dx = x \cos^{-1} ax - \frac{1}{a} \sqrt{1-a^2x^2} + C$
98. $\int \tan^{-1} ax \, dx = x \tan^{-1} ax - \frac{1}{2a} \ln(1+a^2x^2) + C$
99. $\int x^n \sin^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \sin^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} dx}{\sqrt{1-a^2x^2}}, \quad n \neq -1$
100. $\int x^n \cos^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cos^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1} dx}{\sqrt{1-a^2x^2}}, \quad n \neq -1$
101. $\int x^n \tan^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \tan^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} dx}{1+a^2x^2}, \quad n \neq -1$
102. $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$
103. $\int b^{ax} dx = \frac{1}{a} \frac{b^{ax}}{\ln b} + C, \quad b > 0, b \neq 1$
104. $\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) + C$
105. $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$
106. $\int x^n b^{ax} dx = \frac{x^n b^{ax}}{a \ln b} - \frac{n}{a \ln b} \int x^{n-1} b^{ax} dx, \quad b > 0, b \neq 1$
107. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$
108. $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$
109. $\int \ln ax \, dx = x \ln ax - x + C$
110. $\int x^n (\ln ax)^m dx = \frac{x^{n+1} (\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} dx, \quad n \neq -1$
111. $\int x^{-1} (\ln ax)^m dx = \frac{(\ln ax)^{m+1}}{m+1} + C, \quad m \neq -1$
112. $\int \frac{dx}{x \ln ax} = \ln |\ln ax| + C$

$$113. \int \sinh ax \, dx = \frac{1}{a} \cosh ax + C$$

$$114. \int \cosh ax \, dx = \frac{1}{a} \sinh ax + C$$

$$115. \int \sinh^2 ax \, dx = \frac{\sinh 2ax}{4a} - \frac{x}{2} + C$$

$$116. \int \cosh^2 ax \, dx = \frac{\sinh 2ax}{4a} + \frac{x}{2} + C$$

$$117. \int \sinh^n ax \, dx = \frac{\sinh^{n-1} ax \cosh ax}{na} - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx, \quad n \neq 0$$

$$118. \int \cosh^n ax \, dx = \frac{\cosh^{n-1} ax \sinh ax}{na} + \frac{n-1}{n} \int \cosh^{n-2} ax \, dx, \quad n \neq 0$$

$$119. \int x \sinh ax \, dx = \frac{x}{a} \cosh ax - \frac{1}{a^2} \sinh ax + C$$

$$120. \int x \cosh ax \, dx = \frac{x}{a} \sinh ax - \frac{1}{a^2} \cosh ax + C$$

$$121. \int x^n \sinh ax \, dx = \frac{x^n}{a} \cosh ax - \frac{n}{a} \int x^{n-1} \cosh ax \, dx$$

$$122. \int x^n \cosh ax \, dx = \frac{x^n}{a} \sinh ax - \frac{n}{a} \int x^{n-1} \sinh ax \, dx$$

$$123. \int \tanh ax \, dx = \frac{1}{a} \ln (\cosh ax) + C$$

$$124. \int \coth ax \, dx = \frac{1}{a} \ln |\sinh ax| + C$$

$$125. \int \tanh^2 ax \, dx = x - \frac{1}{a} \tanh ax + C$$

$$126. \int \coth^2 ax \, dx = x - \frac{1}{a} \coth ax + C$$

$$127. \int \tanh^n ax \, dx = -\frac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax \, dx, \quad n \neq 1$$

$$128. \int \coth^n ax \, dx = -\frac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax \, dx, \quad n \neq 1$$

$$129. \int \operatorname{sech} ax \, dx = \frac{1}{a} \sin^{-1} (\tanh ax) + C$$

$$130. \int \operatorname{csch} ax \, dx = \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right| + C$$

$$131. \int \operatorname{sech}^2 ax \, dx = \frac{1}{a} \tanh ax + C$$

$$132. \int \operatorname{csch}^2 ax \, dx = -\frac{1}{a} \coth ax + C$$

$$133. \int \operatorname{sech}^n ax \, dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{(n-1)a} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax \, dx, \quad n \neq 1$$

$$134. \int \operatorname{csch}^n ax \, dx = -\frac{\operatorname{csch}^{n-2} ax \coth ax}{(n-1)a} - \frac{n-2}{n-1} \int \operatorname{csch}^{n-2} ax \, dx, \quad n \neq 1$$

$$135. \int \operatorname{sech}^n ax \tanh ax \, dx = -\frac{\operatorname{sech}^n ax}{na} + C, \quad n \neq 0$$

$$136. \int \operatorname{csch}^n ax \coth ax \, dx = -\frac{\operatorname{csch}^n ax}{na} + C, \quad n \neq 0$$

$$137. \int e^{ax} \sinh bx \, dx = \frac{e^{ax}}{2} \left[\frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

$$138. \int e^{ax} \cosh bx \, dx = \frac{e^{ax}}{2} \left[\frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

