Polynomials

04

Digit and Numbers

Consider the following statements:

You are reading chapter no. 4. Gomti's age is 14 years. The night temperature was 10° celsius. The population of Aamgaav is 6000. There is 30 *kg* of rice in the sack. The distance to mars from Earth is 54.6 million kilometers. The speed of light is 186000 miles per second etc. Several such statements are part of our conversations.

In all these statements 4, 14, 10, 6000, etc. are numbers that are written using the digits 0, 1, 2, ..., 5, ..., 9. These can also be written using other kind of digits as symbols (I, II, ..., IX, X, ...) or written in some other number system. Can you give some other examples of numbers written using a different system?

Some more Mathematical Statements

In all the above statements we expressed distance, age, temperature, population, speed etc. using digits and numeral systems.

We make use of some other types of statements as well. For instance-

- If the side of a square is a, then its perimeter will be $4a$.
- The length of a rectangle is l and its breadth b, find its area and perimeter.
- The age of the father is 6 years more than twice the age of his son's.This statement can be written as $x = 2y + 6$.
- The relation between principal, interest etc. is simple interest = $\frac{p \times r \times t}{100}$
- Two lines make an angle θ with each other.

In all these examples a, l, b, x, y, r, t, θ etc. all signify some number. These are known as 'literal numbers.'

Do you see any difference between the two types of numbers mentioned above, i.e. numbers written using digits and literal numbers?

Are there any such literal numbers whose numerical values is fixed?

One such number is π.

If the diameter of a circle is D, than its circumference P will be written as P = πD. Here the values of D will be different for different circles and hence the values of their circumference P. But the values of π will remain $\frac{22}{7}$ or 3.14.

Do you know any other such literal number?

One difference is that 5, 14, 10 etc. are written using digits while $a, l, b, x, y, r, t, \theta$ etc. are expressed using alphabets.

Another difference is that the first type of numbers are a fixed value while the literal numbers have different values in different situation.

Do they also have some similarity?

Think about the situations where you use literal numbers.

Can we perform operations like additon, subtraction, multiplication or division on literal numbers as well?

You can discuss such question with your friends.

Algebraic Expression and their Terms

In previous classes, you may have seen several examples of numbers in which literal numbers were also used:-

like, $4a,\ \frac{\sqrt{3}a^2}{4},\ a+b+c,\ \frac{4}{3}\pi r^3,\ x^2+2x+3,\ (x+3),\ \sqrt{\frac{x}{y}},\ m-9,\ 2p+q$

We know each one of these as an algebraic expression.

In these algebraic expressions, some have only one term, some have two terms while others may have three or more terms.

For example, $4a,\ \frac{\sqrt{3}a^2}{4},\ \frac{4}{3}\pi r^3,\ 5ax^2yz$ etc.

are all algebraic expressions having only one term.

$(x + 3)$, $m - 9$, $2p + q$ etc. are algebraic expressions having two terms.

In $m - 9$, m and -9 are the two terms and in $2p + q$, $2p$ and q are the two terms, similarly $a + b + c$, $x^2 + 2x + 3$ are algebraic expressions having three terms. In $a + b + c$, a is the first term, b is second and c is the third term. In the second example x^2 is the first term, $2x$ is second and 3 is the third term.

Can you think of why these have two or three terms while several expressions that seem longer contain only one term?

The term of an expression is a number, or an literal number or the product of one or more numbers and literal numbers. The expression can be long with many numbers and literal numbers multiplied to each other. Any expression is made up of one or more than one term. The terms of an expression are decided based on '+' or '–' and not on '×' or '÷' signs.

Look at the following :

1. $3x$ — One term
2. $2x + 3y$ — Two terms
3. $-xy - 4x + 35$ — Three terms
4. $\frac{x}{yz}$ — One term
5. xy — One term

How many terms will be there in the expression 2P + P, $(x + 3)^2$, $(x-y)^2$? Can you say?

By just looking at all the three expressions, it may seem that each one of these has two terms since these terms are seperated by '+' or '–' signs, but it is not like that. We can replace $2p + p$ by $3p$ and thus, it is clear that $3p$ is just one term. Similarly observe the following:-

$(x+3)^2$ can be written as $x^2 + 6x + 9$, where we can see three terms.

$(x-y)^2$ can be written as $x^2 - 2xy + y^2$, here also there are three terms.

You saw that when we simplify the expressions we find that the number of terms can be different from what appears to be before simplification.

Therefore, we can say that if it is possible to write an expression in its simplified form then we must count its terms only after writing it in simplified form.

Polynomials (Special Kinds of Algebraic Expression)

Some algebraic expressions are given below:-

$x^2 + 5x$ $\qquad$ $p - 1$ $\qquad$ $x^3 - 2x^2 + 3x - 7$

All these are called polynomials. Is there any special property that is common to all these?

Look at some algebraic expressions that are not polynomial.

$x + \frac{1}{x}$, $\qquad$ $y^2 + y^{½} + 3$, $\qquad$ $p^3 - 2p + \sqrt[3]{p}$, $\qquad$ $3x^{-1}$

Can you find any property in these four examples that makes them different from the previous examples? What could be the reason for these not being polynomials?

Compare all the expressions those that are polynomials and those that are not polynomials. Discuss with your teacher and friends about when can an expression be called as a polynomial and when it cannot be?

0, 1, 2, 3, 4, ...
are whole numbers

You would have reached the conclusion that only these algebraic expressions that have non negative integral powers of literal numbers, are polynomials.

Try This

1. Which of the following are polynomials-

   (i) S-3, (ii) $5y^{-3}$, (iii) $p+\frac{1}{p}$,

   (iv) $ax^2 + b$, (v) $x^{\frac{1}{2}}+1$, (vi) $5p^2 + 2p + 1$

2. Make 5 new polynomials.

Terms of Polynomials

We need to learn how to count the numbers of terms of a polynomial, to find the coefficient of the terms of a polynomial and to find the powers of polynomials. We will need them later. In the polynomial $x^2 + 3x$, these are two terms, first term is x^2 and the second is $3x$. Similarly there are four terms in $m^3 - 2m^2 + 9m + 1$, three terms in $x^2 - x + 1$ and one term in $3y$.

The polynomials are named according to the number of terms in them. As polynomials are algebraic expressions therefore their terms are counted just as an algebraic expresions.

In the above examples:-

$3y$ is a monomial,

$x^2 + 3x$ is a binomial, $x^2 - x + 1$ is a trinomial,

$m^3 - 2m^2 + 9m + 1$ is a polynomial.

Think and Discuss

1. How many terms can be there in a polynomial? Would they be finite or infinite?
2. Is $2p + p$ a monomial or a binomial?
3. How many terms does the expression $(x + 2)^2$ has?

Try This

Pick out the following expressions based on the number of terms:-

(i) 9C (ii) $\frac{1}{2}t+\frac{a}{2}$ (iii) $a^2+2ab+b^2$ (iv) $\frac{p}{q}$

(v) $4x-y$ (vi) $2m+c$ (vii) x^4+3x^2+1

Degree of Polynomials

In a polynomial there are some other numbers that are written in the terms of a polynomial in the form of literal numbers. For example -

$3x^5 - 2x^4 + 3x^3 + 9$

In this polynomial, the powers of the literal number x are 5, 4 and 3 respectively. The largest power among these is 5. In this situation, the degree of polynomial $3x^5 - 2x^4 + 3x^3 + 9$ is five.

Polynomials of degree one are also called linear polynomials.

Look at some more examples.

In $x^2 + 3$, the degree of the polynomial is 2.

In $x^2 - 2x^7 + 3x - 1$, the degree of polynomial is 7.

In $5y$, the degree of polynomial is 1.

What will be the degree of the polynomial $x^2y + xy$?

In the polynomial $x^2y + xy$, 2 is the degree for x in first term and 1 is the degree for y. Therefore the degree for x^2y is 3 and similarly 2 is the degree for xy. So the degree of $x^2y + xy$ is 3.

What will be the degree of polynomial xy? In the polynomial xy, there are two variables; x and y. Both have degrees1. The degree of polynomial 'xy' is equal to the sum of these degree i.e., 2.

Sometimes, numbers are also represented by letters. In this case, literal number are fixed or are constant. For example in $ax + b$, if a and b are constant, then literal number x is a variable but a, b will be constants.

Try This

1. In the following polynomials find the degree of the polynomial.
   (i) 8 (ii) xyz (iii) $uv + uv^2 + v^3$
2. Write some polynomials, and with the help of discussions with your friends, find out their degrees.

Constant Polynomials

Now think about this question:-

Is 6 an algebraic expression?

Is 6 also a polynomial?

It seems that because there is no literal number with 6 and in the standard example of expressions we always find an literal number, so 6 not an algebraic expression. But can we not write 6 as $6x^\circ$? ($x^\circ = 1$).

In this the power of literal number x is zero. Zero is a whole number. Therefore $6x^0$ or 6 is also an algebraic expression and a mononomial too.

So, terms that are just a number are also algebraic expressions as well as a polynomial.

Such polynomials are called constant polynomials. That means 2, 7, –6, $\frac{3}{2}$, 122 are constant polynomials. Can you make a few more examples of constant polynomials?

Representation of Polynomials

Sometimes we require that a polynomial be written several times. In such situations we will have to write the big long polynomial several times. There is one more way to express a polynomial. In this we only know the literal number of the polynomial but nothing else. However in the context of the question, we know which polynomial does the expression relate to.

If the literal number of the polynomial is x then we express it as $p(x)$, $q(x)$, $r(x)$ etc. If the literal number of the polynomial is y then it is expressed as $p(y)$, $q(y)$, $s(y)$, $t(y)$. See few examples:-

$p(x) = 3x^5 - 2x^4 + 3x^3 + 9$ $\quad$ $t(x) = x^6 - 2x^7 + 3x - 1$

$q(y) = 5y$ $\quad$ $s(u) = u^2 + 3u^3$ $\quad$ $r(b) = b^4 - b^2 + 6$

In this, there is no basis for choice p, t, q. But if we choose them, we must use the same form for that particular question.

General form of Polynomials

Observe the polynomials of degree one given below carefully.

$p(x) = 2x + 3$ $\quad$ $q(x) = \sqrt{2} - x$ $\quad$ $s(x) = \frac{1}{2}x + \frac{3}{2}$

$r(x) = x$ $\quad$ $p(x) = \sqrt{7}\,x - 4$ $\quad$ $p(x) = 2(3x + 8)$

The maximum number of terms in these are two.

Think and Discuss

Can you make a polynomial of a single degree with more than two terms. Keep in mind that while making a polynomial, similar terms must be collected and written together.

In the above examples, every polynomial has atleast one literal number. In other words we can also say that in these polynomials the coefficient of the literal numbers is never zero. Besides, other real numbers (+3, –4, $\frac{1}{2}$, $\sqrt{2}$, 0 etc.) are also included in the polynomials.

Can we express these polynomials in the form of $ax + b$ where a and b are some real constant numbers and $a \neq 0$?

Compairing the second example to $ax + b$:-

$$\sqrt{2} - x = -x + \sqrt{2}$$
$$= (-1)\,x + \sqrt{2}$$
$$= a\,x + b \qquad \text{(Compare)}$$

Here, $a = (-1)$ where a is not zero.

$b = \sqrt{2}$ where $\sqrt{2}$ is a real number.

We can see that polynomial $\sqrt{2} - x$ is similar to $ax + b$.

See one more polynomial:-

$$x = 1 \times x$$
$$= 1 \times x + 0$$
$$= ax + b$$
$$a = 1,\ b = 0$$

The number that multiplies the literal number is called as the coefficient. For example:- $m^3 - 3m^2 + 1$, the coefficient of m^3 is 1 and coefficient of m^2 is –3. Think, what will be the coefficient of m^2 in $m^2 + 1$?

Real numbers comprise of all integers, rational and irrational numbers.

$-2, -1, 0, 1, 2, \ldots, -\frac{1}{2}, -\frac{4}{5}, -\frac{2}{7}, \sqrt{2}, \sqrt{7}, \ldots$

Therefore polynomial x can also be written in the form of $ax + b$. Now write the remaining four polynomials in the form of $ax + b$.

$ax + b$ is a linear polynomial of x with degree 1, in this a and b as real constant numbers and $a \neq 0$.

Now consider the following polynomials:-

$4x^2 + 3x$, $\quad -y^2 + 2$, $\quad x^2 - 4x - 9$, $\quad \sqrt{2}\,m^2 - \frac{3}{2}m - 9$

These polynomials are binomials each with one literal number. These are called as quadratic polynomials. We can write these in the form of $ax^2 + bx + c$. Where a, b and c are real constants and $a \neq 0$.

Try This

1. Make five new quadratic polynomials.
2. What is the largest number of terms possible in a quadratic polynomial?
3. What is the minimum number of terms possible in a quadratic polynomial?

General form of Polynomials with higher degree

Consider $4x^3+2x^2+5x-7$, $y^4+3y^3-5y^2+7y$, m^5-3m^2+2, $z^6-5z^5-3z^2+2z$

These are all polynomials. We can see that in these, the degree of polynomials is increasing and the number of maximum possible terms may also increase. In all these, apart from various powers of x, y, m, z etc. we have numbers that are real numbers. The degree of a polynomial depends upon the maximum degree of the literal numbers (x, y, m, z) contained in it. Therefore the coefficient of the literal with the maximum power can not be zero. Hence, we gave the condition of $a \neq 0$ for polynomials with degree two.

We will study more about polynomials and will also get to know about different uses of literal numbers. We will see the more generalized form of the polynomials for all the degrees. All this will however, be done in further classes. Can you write the polynomials with higher degree in a forms like $ax + b$, $ax^2 + bx + c$.

Zero Polynomials

If all the coefficients of a polynomial are zero; for example in $ax^2 + bx + c$ with $a = b = c = 0$ then we will get 0 as the result. This is called as zero polynomial. The degree of this is undefined. Therefore it can be written as a polynomial of any degree.

Exercise - 4.1

1. Which of the following expressions are polynomials and which are not? Give reasons for your answer.

   (i) $4x^2 - 3x + 5$ (ii) $z + \frac{3}{z}$ (iii) $\sqrt{y} + 2y + 3$

   (iv) $x^2 + \frac{3}{2}$ (v) $x^{10} + y^3 + t^{50}$

2. Write the coefficient of x^2 in the following polynomials :-

   (i) $3x^2 + 2x^2 + 3x + 2$ (ii) $3x^2 + 1$ (iii) $2 - 5x^2 + \frac{1}{2}x^3$

   (iv) $\frac{x^2}{2} + 1$ (v) $x^4 + x^3 + \frac{1}{4}x^2$

3. Write the coefficient of x and constant terms for the following:-

(i) $x^2 + \frac{1}{5}x + 5$ (ii) $\sqrt{2}\,x + 7$ (iii) $x^2 + 2$

4. Write an example for each of the following:-

(i) binomial of degree 4 (ii) trinomial of degree 6

(iii) monomial of degree 5

5. In the following polynomials write the degree of each.

(i) $x^3 - 6x^2 + x + 1$ (ii) $y^9 - 3y^7 + \frac{3}{2}y^2 + 4$ (iii) $3 - y^3z$

(iv) $x^2y - 2x + 1$ (v) $5t - \sqrt{11}$ (vi) 7

6. From the following, pick out the constant, linear, binomials and trinomials.

(i) $x^3 + x^2 + x + 1$ (ii) $9x^3$ (iii) $y + y^2 + \frac{3}{4}$

(iv) $t + 3$ (v) $y - y^3$ (vi) 8

(vii) $2x^2 + 3$ (viii) $P^2 - P + 5$ (ix) $x + \frac{2}{3}$

(x) 4 (xi) $-\frac{u}{4} + \frac{3}{2}$ (xii) $-\frac{3}{7}$

Zeroes of a Polynomial

By putting values $x = 1, 2, 0$ etc. in the polynomial $p(x) = x^2 + x - 6$ we get,

at $x = 1$, $p(1) = 1 + 1 - 6 = -4$

$p(1) = -4$

We say that, for $x = 1$, $p(x)$ has the value -4.

at $x = 2$

$p(2) = 4 + 2 - 6$

$p(2) = 0$

For $x = 2$, the value of $p(x)$ becomes 0. We say that 2 is a zero of the polynomial $p(x)$.

We can say that if the value of a polynomial is zero for some value of the variable than that value of the variable is a zero of the polynomial.

EXAMPLE-1. Find the zero of the polynomial $p(x) = 2x + 1$.

SOLUTION : Finding the zero of a polynomial is similar to finding the solution of an equation.

For $p(x) = 0$ we need the value of x which satisfies it.

Therefore, it is the value of x, when $2x + 1 = 0$

$$2x = -1 \qquad \text{or} \qquad x = \frac{-1}{2}$$

Clearly, putting the value $x = \frac{-1}{2}$ in $p(x)$ we get zero. Therefore $\frac{-1}{2}$ is a zero of the polynomial $p(x)$.

EXAMPLE-2. Check if 0 or 2 are zero of the polynomial $x^2 - 2x$.

SOLUTION : $p(x) = x^2 - 2x$

then putting $x = 0$

$p(0) = (0) - 2 \times 0 = 0$

that means 0, is a zero of $p(x)$.

putting $x=2$

$p(2) = 2^2 - 2 \times 2$

$= 4 - 4 = 0$

another zero of $p(x)$ is 2.

Exercise - 4.2

1. Find the value of polynomial $5x^3 - 2x^2 + 3x - 2$, for

   (i) $x = 0$ (ii) $x = 1$ (iii) $x = -2$

2. For all the following polynomials, find the values of $p(0)$, $p(-1)$, $p(2)$, $p(3)$.

   (i) $p(x) = 4x^3 + 2x^2 - 3x + 2$ (ii) $p(r) = (r - 1)(r + 1)$

   (iii) $p(t) = \frac{2}{3}t^2 - \frac{1}{3}t + \frac{1}{3}$ (iv) $p(y) = (y^2 - y + 1)(y + 1)$

   (v) $p(x) = x + 2$

3. Find out if the values written besides the polynomial, are their zeroes.

(i) $p(x) = 3x + 1 ; x = \frac{-1}{3}$ (ii) $p(x) = x + 2 ; x = -2$

(iii) $p(x) = 5x - 4 ; x = \frac{5}{4}$ (iv) $p(y) = y^2 - 1 ; y = 1, -1$

(v) $p(t) = (t + 1)(t - 2) ; t = +1, -2$ (vi) $p(x) = lx + m ; x = \frac{-m}{l}$

(vii) $p(r) = 3r^2 - 1 ; r = \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

4. Find the zeroes of the following polynomials

(i) $p(x) = x + 6$ (ii) $p(x) = x - 6$ (iii) $p(y) = 5y$

(iv) $p(t) = at$, $a \neq 0$, a is a real constant number.

(v) $p(r)=cr+d$, $c\neq0$, c, d are real constant numbers.

(vi) $p(u) = 3u - 6$ (vii) $r(s) = 2s + 3$

(viii) $p(x) = \sqrt{5} - x$ (ix) $q(t) = \frac{1}{2}t - \frac{2}{3}$

Addition and Subtraction of Polynomials

We have practiced the addition and subtraction of algebraic expressions. We have also learnt that all polynomials are algebraic expressions. Therefore the addition and subtraction of polynomials is similar to that of algebraic expressions.

We can perform addition/subtraction of the coefficients after observing all the terms and collecting similar ones.

Consider the following examples and tell what things you need to keep in mind while solving such questions.

EXAMPLE-3. Add the polymonial $3x^3 - x^2 + 5x - 4$ and $3x^2 - 7x + 8$.

SOLUTION : $3x^3 - x^2 + 5x - 4$ (We will write terms with same degrees together).

$$\begin{array}{l} \quad 3x^3 - x^2 + 5x - 4 \\ + \qquad\quad 3x^2 - 7x + 8 \\ \hline 3x^3 + (-1+3)x^2 + (5-7)x + (-4+8) \end{array} = 3x^3 + 2x^2 - 2x + 4$$

EXAMPLE-4. Find the sum of the polynomials $\frac{3}{2}y^3 + y^2 + y + 1$ and $y^4 - \frac{1}{2}y^3 - 3y + 1$.

SOLUTION :

$$\begin{array}{l} \quad\;\; \frac{3}{2}y^3 + y^2 + y + 1 \\ + y^4 - \frac{1}{2}y^3 \qquad\quad - 3y + 1 \\ \hline y^4 + \left(\frac{3}{2} - \frac{1}{2}\right)y^3 + y^2 + (1-3)y + (1+1) \\ \hline \end{array} = y^4 + y^3 + y^2 - 2y + 2$$

EXAMPLE-5. Subtract $4x^2 + 3x - 2$ from the polynomial $9x^2 - 3x - 7$.

SOLUTION :

$$\begin{array}{l} 9x^2 - 3x - 7 \\ 4x^2 + 3x - 2 \\ - \quad - \quad + \quad \text{(sign changes on subtracting)} \\ \hline (9-4)x^2 + (-3-3)x + (-7+2) \\ \hline \end{array} = 5x^2 - 6x - 5$$

EXAMPLE-6. Subtract $s(z) = 3z - 5z^2 + 7 + 3z^3$ from $x(z) = 2z^2 - 5 + 11z - z^3$

SOLUTION : First write both the polynomials in the decreasing order of the power of literal numbers.

$x(z) = 2z^2 - 5 + 11z - z^3 = -z^3 + 2z^2 + 11z - 5$

and $s(z) = 3z - 5z^2 + 7 + 3z^3 = 3z^3 - 5z^2 + 3z + 7$

now $x(z) - s(z) = -z^3 + 2z^2 + 11z - 5$

$$\begin{array}{l} 3z^3 - 5z^2 + 3z + 7 \\ - \quad + \quad - \quad - \\ \hline (-1-3)z^3 + (2+5)z^2 + (11-3)z + (-5-7) \\ \hline \end{array}$$

$= -4z^3 + 7z^2 + 8z - 12$

EXAMPLE-7. Find the sum of the polynomials $3x + 4 - 5x^2$, $5 + 9x$ and $4x - 17 - 5x^2$. What is the degree of the polynomial obtained after the addition.

SOLUTION : $-5x^2+3x+4$ (Writing all in decreasing degree of x).

$+9x+5$

$+\ -5x^2+4x-17$

$$(-5-5)x^2+(3+9+4)x+(4+5-17) = -10x^2+16x-8$$

The resultant, polynomial $-10x^2+16x-8$ has the degree 2.

EXAMPLE-8. Subtract $r(x)=2x^2-3x-1$ from the sum of $p(x)=3x^2-8x+11$ and $q(x)=-4x^2+15$. Find the degree of the resultant polynomial.

SOLUTION : First, we add $p(x)$ and $q(x)$.

$p(x)+q(x) = 3x^2-8x+11$

$-4x^2 \quad +15$

$$(3-4)x^2-8x+(11+15) = -x^2-8x+26$$

Now from this sum, subtract $r(x)$

$p(x)+q(x)-r(x) = -x^2-8x+26$

$2x^2-3x-1$

$- \quad + \quad +$ (On changing the signs)

$$(-1-2)x^2+(-8+3)x+(26+1) = -3x^2-5x+27$$

$p(x)+q(x)-r(x) = -3x^2-5x+27$

Therefore, the degree of resultant polynomial $-3x^2-5x+27$ is 2.

EXAMPLE-9. Find the sum and difference of $p(x)=4x^3+3x^2+2x-1$ and $q(x)=4x^3+2x^2-2x+5$.

SOLUTION : $p(x)+q(x) = (4x^3+3x^2+2x-1)+(4x^3+2x^2-2x+5)$

$= (4+4)x^3+(3+2)x^2+(2-2)x+(-1+5)$

$= 8x^3+5x^2+0x+4$

$= 8x^3+5x^2+4$

Similarly, the difference is

$p(x)-q(x) = (4x^3+3x^2+2x-1)-(4x^3+2x^2-2x+5)$

$= (4-4)x^3+(3-2)x^2+(2+2)x+(-1-5)$

$= x^2+4x-6$

Try This

Find the sum and difference of the following polynomials and give the degree of the resultant.

(i) $p(y) = y^2 + 5y$, $q(y) = 3y - 5$

(ii) $p(r) = 5r^2 - 9$, $s(r) = 9r^2 - 4$

(iii) $p(y) = 15y^4 - 5y^2 + 27$, $s(y) = 15y^4 - 9$

Sometimes when we know the sum or difference of two polynomials and know one of the polynomials, then we can easily find the polynomial.

EXAMPLE-10. What should be added to $2u^2 - 4u + 3$ so that we get the sum as $4u^3 - 5u^2 + 1$?

SOLUTION : Suppose that on adding $q(u)$ in $p(u) = 2u^2 - 4u + 3$ we get $r(u)=4u^3-5u^2+1$.

That means $p(u) + q(u) = r(u)$

$q(u) = r(u) - p(u)$

$$q(u) = (4u^3 - 5u^2 + 1) - (2u^2 - 4u + 3)$$
$$= 4u^3 - 5u^2 + 1 - 2u^2 + 4u - 3$$
$$= 4u^3 + (-5 - 2)u^2 + 4u + (1 - 3)$$
$$= 4u^3 - 7u^2 + 4u - 2$$

EXAMPLE-11. What should be subtracted from $2y^3 - 3y^2 + 4$ to get $y^3 - 1$ as the resultant.

SOULUTION : Suppose on subtracting $q(y)$ from $p(y)=2y^3-3y^2+4$ we get the difference $r(y)=y^3-1$.

That means $p(y) - q(y) = r(y)$

or $q(y) = p(y) - r(y)$

$$q(y) = (2y^3 - 3y^2 + 4) - (y^3 - 1)$$
$$= 2y^3 - 3y^2 + 4 - y^3 + 1$$
$$= (2 - 1)y^3 - 3y^2 + (4 + 1)$$
$$= y^3 - 3y^2 + 5$$

Try This

1. What should be subtracted from $5t^2 - 3t + 4$ to get $2t^3 - 4$.
2. What should be added to $6r^2 + 4r - 2$ to get $15r^2 + 4$.
3. Make 5 more questions of this kind and solve them.

Multiplication of Polynomials

Like addition and subtraction, the multiplication of polynomials is also similar to that for algebraic expressions;

EXAMPLE-12. Multiply the polynomial $p(x) = 2x^2 + 3x + 4$ by 3.

SOLUTION : $3\,p(x) = 3 \times (2x^2 + 3x + 4)$

$= 6x^2 + 9x + 12$

EXAMPLE-13. $(2x + 5) \times (4x + 3)$

SOLUTION : $= 2x(4x + 3) + 5(4x + 3)$

$= [(2x \times 4x) + (2x \times 3)] + [(5 \times 4x) + (5 \times 3)]$

$= 8x^2 + 6x + 20x + 15$

$= 8x^2 + 26x + 15$

EXAMPLE-14. $(2x + 5) \times (3x^2 + 4x + 6)$, find the degree of the polynomial?

SOLUTION : $= [2x \times (3x^2 + 4x + 6)] + [5 \times (3x^2 + 4x + 6)]$

$= 2x \times 3x^2 + 2x \times 4x + 2x \times 6 + 5 \times 3x^2 + 5 \times 4x + 5 \times 6$

$= 6x^3 + 8x^2 + 12x + 15x^2 + 20x + 30$

$= 6x^3 + (8 + 15)\,x^2 + (12 + 20)\,x + 30$

$= 6x^3 + 23x^2 + 32x + 30$

Here, the degree of the polynomial is 3.

EXAMPLE-15. If $p(x) = 2x + 3$

$q(x) = x^2 + x - 2$

SOLUTION : Then $p(x) \cdot q(x) = (2x + 3)(x^2 + x - 2)$

$= 2x(x^2 + x - 2) + 3(x^2 + x - 2)$

$= 2x \times x^2 + 2x \times x + 2x \times (-2) + 3 \times x^2 + 3x + 3 \times (-2)$

$= 2x^3 + 2x^2 - 4x + 3x^2 + 3x - 6$

$= 2x^3 + (2 + 3)\,x^2 + (-4 + 3)\,x - 6$

$= 2x^3 + 5x^2 - x - 6$

Try This

Multiply the following polynomials, find the degree of the product polynomial:-

(i) $p(x) = x^2 + 3x + 2$; $q(x) = x^2 + 3x + 1$

(ii) $p(v) = v^2 - 3v + 2$; $q(v) = v + 1$

(iii) $p(x) = 2x^2 + 7x + 3$; $q(x) = 5x^2 - 3x$

(iv) $p(y) = y^3 - y^2 + y - 1$; $q(y) = y + 1$

(v) $p(u) = 3u^2 - 12u + 4$; $q(u) = u^2 - 2u + 1$

Exercise - 4.3

1. Add the following polynomials:-

   (i) $2x^2 + x + 1$ and $3x^2 + 4x + 5$ (ii) $8p^2 - 3p + 4$ and $3p^3 - 4p + 7$

   (iii) $-5x^3 + 9x^2 - 5x + 7$ and $-2x^2 + 7x^3 - 3x - 8$

2. Add the following polynomials. Find the degree of the resultant polynomial.

   (i) $3y^2 + 2y - 5$; $2y^2 + 5 + 8y$ and $-y^2 - y$

   (ii) $5 + 7r - 3r^2$; $r^2 + 7$ and $r^2 - 3r + 5$

   (iii) $4x + 7 - 3x^2 + 5x^3$; $7x^2 - 2x + 1$ and $-2x^3 - 2x$

3. Subtract

   (i) $t^2 - 5t + 2$ from $7t^3 - 3t^2 + 2$

   (ii) $3p - 5p^2 + 7 + 3p^3$ from $2p^2 - 5 + 11p - p^3$

   (iii) $5z^3 + 7z^2 + 2z - 4$ from $-3z^2 + 11z + 12z^3 + 13$

4. From the sum of $x^4 + 3x^3 + 2x + 6$ and $x^4 - 3x^2 + 6x + 2$ subtract $x^3 - 3x + 4$.

5. If $p(u) = u^7 - u^5 + 2u^2 + 1$ and $q(u) = -u^7 + u - 2$, find the degree of $p(u) + q(u)$.

6. What should be added to $x^3 - 3x^2 + 6$ to get the sum as $x^2 - x + 4$?

7. What should be added to $u^7 - 3u^6 + 4u^2 + 2$ to get the sum as $u^6 - u - 4$?

8. What should be subtracted from $y^3 - 3y^2 + y + 2$ to get the difference as $y^3 + 2y + 1$?

9. What should be subtracted from $t^2 + t - 7$ to get the difference as $t^3 + t^2 + 3t + 4$?

10. Multiply the following

    (i) $3x + 4$ by $7x^2 + 2x + 1$ (ii) $5x^3 + 2x$ by $3x^2 - 9x + 6$

    (iii) $p^4 - 5p^2 + 3$ by $p^3 + 1$

11. If $p(x) = x^3 + 7x + 3$ and $q(x) = 2x^3 - 3$, then find the value of $p(x)\,q(x)$

12. If $p(u) = u^2 + 3u + 4$, $q(u) = u^2 + u - 12$ and $r(u) = u - 2$, then find the degree of $p(u)q(u)r(u)$.

What Have We Learnt

1. Any algebraic expression, in which the degree of literal number (variable) is an integer is called a polynomial.
2. The terms of a polynomial are identified by the '+' or '–' sign.
3. The coefficient of a polynomial is a number that multiplies the literal number or variable. Sometimes coefficients are expressed as literals and so the coefficients is not written as a number but as a literal. for example, the coefficient of x in $ax + b$ is a.
4. When the coefficients of all the literal number variables in a polynomial are zero, then it is called a zero polynomial.
5. Any number such as 3, 5, 6, ... etc. is an algebraic expression as well as a polynomial.
6. The polynomials with only numbers (having only numerals or constant literal numbers) are constant polynomials.
7. In a polynomial, the maximum degree of the variables (literal numbers) is the degree of the polynomial. Example- the degree of polynomial $x^5 + 3x^3 + 2x$ is 5.
8. Polynomials can be written as $p(x)$, $q(x)$, $r(x)$ etc., where the literal number written within the brackets represents the variable of the polynomial.
9. $ax + b$ is a one variable polynomial of degree 1 where a, b are constant real numbers and $a \neq 0$.
10. The polynomial in one variable having degree 2 is a quadratic polynomial.
11. When the value of a polynomial becomes zero for some value of the variable , then that value of the variable is a zero of the polynomial.
12. A general linear polynomial is $ax + b$, where a, b are constant real numbers and $a \neq 0$.
13. A general quadratic polynomial is $ax^2 + bx + c$ where a, b, c are constant real numbers and $a \neq 0$.
14. A general form of a cubic polynomial is $ax^3 + bx^2 + cx + d$ where a, b, c, d are constant real numbers and $a \neq 0$.