Chapter - 9
Rational Number

9.1 You are by now familiar with natural number, whole number and integers. Use of number is very essential in our day to day activities. All of you know that natural numbers are used to count objects in general. These numbers are 1, 2, 3, 4, ...You have come to know how it is expanded from natural numbers to whole numbers and from whole numbers to integers. In the discussion of antecedent of natural number you have seen that there is no antecedent of 1 in natural number. It means 1–1 = 0 is not a natural numbers. Including 0 with the natural numbers we get 0, 1, 2, 3.... and these are called whole numbers.

On the other hand, to express the opposite facts height-depth, upward-downward, profit-loss, increase-decrease etc in digit, we have to use '–' sign before the natural numbers to form –1, –2, –3, –4 ...... numbers.

The natural numbers including 0 along with the numbers which are created by 3 putting minus sign before natural numbers are together called integers.

Therefore, the integers are

.... – 4, –3, –2, –1, 0, 1, 2, 3, 4, ...

Remember that whole numbers and natural numbers are included within integers. Now we will discuss the spread of integers.

9.2 Rational Number and its necessity :

Integers are obtained from whole numbers by putting '–' sign before the natural numbers, similarly various opposite facts need to be expressed by putting '–' sign before fractions. For example, if we indicate 2500 meter height from the sea level i.e. $\frac{5}{2}$ km height is indicated as the fraction $\frac{5}{2}$, then to indicate $\frac{5}{2}$ km depth $-\frac{5}{2}$ can be used. But a number like $-\frac{5}{2}$ does not occure whithin the numbers that we have found till now that neither this number belongs to whole number nor to fractions. Therefore, it is needed to expand the collection of numbers covering all those integers, fractions and the new type of numbers which we obtained by putting '–' sign before fractions.

When we discussed ratio we saw that ratio is the comparison between two terms i.e. one term is how many times of the other. Let the ratio of two number be 2:5, it can be expressed in the form of $\frac{2}{5}$.

Rational Number

When we express a ratio in the form of $\frac{\text{numerator}}{\text{denominator}}$, the numerator and denominator may be any integer. In this way by putting '–' sign before the fraction, the numbers may be included in a family. By expressing the ratio in the form of $\frac{\text{numerator}}{\text{denominator}}$ and using any integer for numerator and denominator, an extended form of fractions can be obtained which are termed as rational numbers. Therefore $\frac{p}{q}$ is a rational number if $q \neq 0$. Here p and q are numerator and denominator of the rational number respectively.

Therefore, $\frac{4}{6}, \frac{-3}{2}, \frac{7}{-3}, \frac{-4}{-10}, \frac{0}{3}, \frac{0}{-10}, \frac{5}{1}, \frac{-2}{1}$ are all rational numbers.

Note : (i) any integer can be expressed in the form of rational number. For example $0=\frac{0}{1}, 1=\frac{1}{1}, 2=\frac{2}{1}, -3=\frac{-3}{1}$ etc. Therefore all integers are rational numbers.

(ii) We have already seen that all fractions are rational numbers. Example : $\frac{1}{2}, \frac{3}{2}$, $2\frac{3}{4}=\frac{11}{4}, \frac{9}{11}$ etc. Therefore all fractions are rational numbers.

(iii) Take a decimal number. Let it be 0.5. If it is expressed in the form of $\frac{5}{10}$, then it is also a rational number. Therefore all decimal numbers are rational numbers.

Try yourself :

(1) Are the natural numbers rational numbers?

(2) Are the rational numbers integers?

(3) Write five such rational numbers which are not integers.

(4) Is 0.33 a rational number?

(5) Write one number in $\frac{p}{q}$ form (p, q are integers and $q \neq 0$) for each of the following :

(i) Numerator positive and denominator negative.

(ii) Both numerator and denominator are negative.

(iii) Numerator negative and denominator positive.

(iv) Both of numerator and denominator are positive.

9.3 Equivalent rational numbers

During the discussion on fraction you have seen that when both the numerator and denominator of a fraction is multiplied or divided by any non zero integer, then the value remains same. Similarly if the numerator and denominator of a rational number is multiplied or divided by same non zero integer, then the value of the rational number remains same.

For example, few equivalent rational numbers of $\frac{-2}{3}$ are given below –

$$\frac{-2}{3}=\frac{-2\times 2}{3\times 2}=\frac{-4}{6},\qquad \frac{-2}{5}=\frac{(-2)\times(-1)}{5\times(-1)}=\frac{2}{-5},\ \frac{-2}{5}=\frac{(-2)\times(-5)}{5\times(-5)}=\frac{10}{-25}\ \text{etc.}$$

$$\frac{12}{15}=\frac{12\div 3}{15\div 3}=\frac{4}{5},\qquad \frac{-10}{12}=\frac{-10\div(-2)}{12\div(-2)}=\frac{5}{-6}\ \text{etc.}$$

Try yourself :

(1) Write 3 equivalent rational numbers for each of the following

(i) $\frac{3}{4}$ (ii) $\frac{5}{-8}$ (iii) $\frac{-7}{11}$ (iv) 0.75 (v) $\frac{-12}{-18}$

(2) Can you say $\frac{-3}{4}$ and $\frac{3}{-4}$ are equivalent rational numbers ?

9.4 Positive and negative rational numbers

Please recall that by putting '–' sign before the fraction we formed the rational numbers. For eg. $\frac{2}{3}$ is a fraction. If we put '–' sign before it then we will get $-\frac{2}{3}$. In rational number form, we can write $-\frac{2}{3}$, as $\frac{-2}{3}$ or $\frac{2}{-3}$ again with the concept of equivalent rational number –

$\frac{2}{-3}=\frac{2\times(-1)}{-3\times(-1)}=\frac{-2}{3}$ Therefore $-\frac{2}{3}=\frac{-2}{3}=\frac{2}{-3}$; So, rational numbers can be divided into two categories i.e. positive and negative. For example $\frac{2}{3}$ is a positive rational number and $-\frac{2}{3}=\frac{-2}{3}=\frac{2}{-3}$ is a negative rational number. But in which category do $\frac{-2}{-3}$ lie– whether it is positive or negative? With the concept of equivalent rational number we get $\frac{-2}{-3}=\frac{(-2)\times(-1)}{(-3)\times(-1)}=\frac{2}{3}$, which is already mentioned as positive rational number. i.e. $\frac{-2}{-3}$ is a positive rational number. In general, a rational number $\frac{p}{q}$ (where *p,*

q are integers and $q \neq 0$) will be positive if p and q both are either positive or negative. On the other hand $\frac{p}{q}$ is negative if p and q have the opposite sign.

From the description of rational number we can say –

$-3 = \frac{-3}{1} = \frac{3}{-1}$ is a negative rational number. $5 = \frac{5}{1} = \frac{-5}{-1}$ is a positive rational number.

Therefore positive integers are positive rational numbers and negative integers are negative rational numbers. What will you say about integer 0? We have already learnt that '0' is neither positive nor negative. Similarly as a rational number '0' is neither positive nor negative.

9.5 Rational numbers in standard form :

A rational number can be expressed in different ways by using the concept of equivalent rational number. For example

$$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{-6}{-10} = \frac{-9}{-15} = \frac{-12}{-20} \text{ etc.}$$

$$\frac{-2}{3} = \frac{-4}{6} = \frac{-6}{9} = \frac{-8}{12} = \frac{4}{-6} = \frac{6}{-9} = \frac{8}{-12} \text{ etc.}$$

Hence there is a need of standard form of rational number with the help of which the discussion of rational number can be done easily.

A rational number $\frac{p}{q}$ is said to be in standard form when

(i) p is either positive or negative integer but q is positive integer.

(ii) There is no common factor of p and q except 1.

For eg, Standard form of $\frac{4}{8}$ is $\frac{1}{2}$ because $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

Standard form of $\frac{-12}{20}$ is $\frac{-3}{5}$ because, $\frac{-12}{20} = \frac{-12 \div 4}{20 \div 4} = \frac{-3}{5}$

Standard form of $\frac{6}{-2}$ is $\frac{-3}{1} = -3$ because, $\frac{6}{-2} = \frac{6 \div (-2)}{-2 \div (-2)} = \frac{-3}{1} = -3$

Standard form of $\frac{-15}{-12}$ is $\frac{5}{4}$ because, $\frac{-15}{-12} = \frac{-15 \div (-3)}{-12 \div (-3)} = \frac{5}{4}$ etc.

Example 1 : Express in standard form (i) $\frac{-15}{21}$ (ii) $\frac{20}{-65}$

(i) $\frac{-15}{21} = \frac{-15 \div 3}{21 \div 3} = \frac{-5}{7}$ (Remember, here both numerator and denominator are divided by the HCF of 15 and 21 i.e. 3)

(ii) $\frac{20}{-65} = \frac{20 \div 5}{-65 \div 5} = \frac{4}{-13} = \frac{-4}{13}$ (HCF of 20 and 65 is 5)

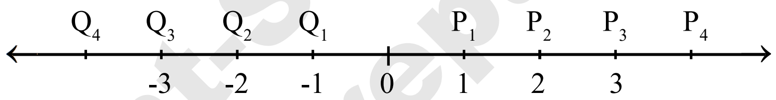
Example 2 : Convert to standard form (i) $\frac{6}{-11}$ (ii) $\frac{24}{-45}$

Solution : (i) $\frac{6}{-11} = \frac{6 \div (-1)}{(-11) \div (-1)} = \frac{-6}{11}$ Or (i) $\frac{6}{-11} = \frac{6 \times (-1)}{(-11) \times (-1)} = \frac{-6}{11}$

(ii) $\frac{24}{-45} = \frac{24 \div (-3)}{(-45) \div (-3)} = \frac{-8}{15}$

9.6 Representation of rational number in number line

You have already learnt how can the integers be placed in number line. It is shown below again



Here some points are taken at an equal interval on a straight line. One of these units is considered as '0' and the points on right side of zero are identified as –1, –2, –3,... In this way all integers are placed in a line as points.

Let us discuss how can the rational number be placed with the help of points in a number line.

As the rational numbers are extended form of fraction hence let us first discuss the placing of fractions in a number line.

In a number line, to place a number, for example 5, we first take '0' as a point on the numberline and 1 is marked at certain interval and then the 5th point reached after equal intervals on the right of '0' is marked as –5

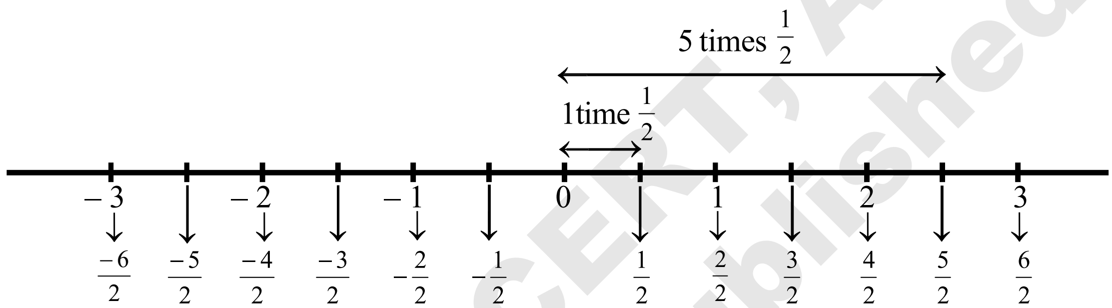
Let, us place a fraction $\frac{5}{2}$ in a number line.

We know $\frac{5}{2} = 5 \times \frac{1}{2}$, i.e. 5 times of $\frac{1}{2}$. We know that $\frac{1}{2}$ is greater than 0 and less than

1. The point representing $\frac{1}{2}$ will be the middle point of '0' and 1. That means the place of $\frac{1}{2}$ is the midpoint of 0 and 1 which divides the line into equal parts.

Now if we take the length of $\frac{1}{2}$ and proceed five equal lengths of $\frac{1}{2}$ towards right from 0, then we get a point which represents $\frac{5}{2}$.

Similarly we get each point on the right of 0 for the fractions of $\frac{2}{2}=1$, $\frac{3}{2}$, $\frac{4}{2}=2$.



Now, where will the rational number $-\frac{5}{2}$ be placed ?

It is understood from the above diagram that, 4 times of $\frac{1}{2}$ is $\frac{4}{2}=2$

6 times of $\frac{1}{2}$ is $\frac{6}{2}=3$

That means, 5 times of $\frac{1}{2}$ i.e. $\frac{5}{2}$, will be placed between $\frac{4}{2}=2$ and $\frac{6}{2}=3$.

Again, –1 and 1 are both integers having opposite sign.1 is positive and –1 is negative. Besides –1 and 1 there are some numbers in both side i.e. left side and right side of '0' respectively. Therefore $\frac{1}{2}$ is placed at which distance from the right of '0', $-\frac{1}{2}$ will be placed at the same distance from the left of '0'.

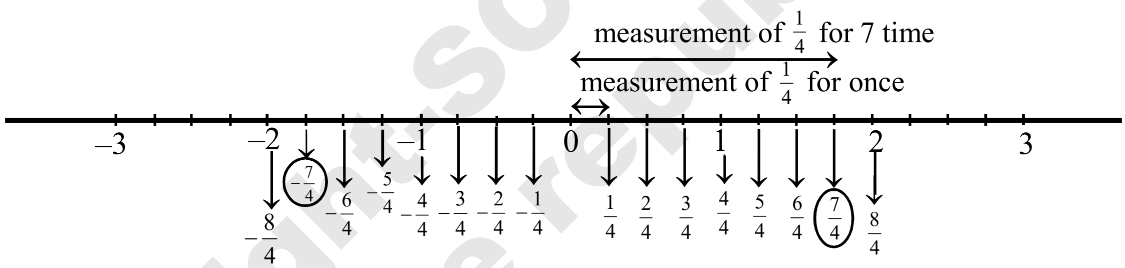
Now at which distance the positive integer 2 is placed to the right of '0' and at the same distance the negative integers –2 are placed to the left of 0. Likewise, at which distance the positive integer 3 is placed to the right of 0 at the same distance the negative integer –3 is placed to the left of '0'. But $\frac{5}{2}$ is placed in the middle of $\frac{4}{2}=2$ and $\frac{6}{2}=3$.

Therefore the rational number $-\frac{5}{2}=\frac{-5}{2}$ will exist in between $\frac{-4}{2}=-2$ and $\frac{-6}{2}=-3$.

Similarly $-\frac{7}{4}$ is obtained by using '–' sign before the fraction $\frac{7}{4}$. Therefore the distance of $\frac{7}{4}$ from '0' in its right side is equal to the distance of $\frac{-7}{4}$ from '0' on its left side.

Again $\frac{7}{4}=7\times\frac{1}{4}$ i.e. 7 times of $\frac{1}{4}$. If we equally divide the distance of 0 to 1 in four equal parts and proceed from '0' towards right side, the first point will be $\frac{1}{4}$ and subsequently we get the points $\frac{2}{4}, \frac{3}{4}, \frac{4}{4}=1$ etc. If we proceed upto 2, the points on the right side of the '0' are $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}=1, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}=2, \frac{4}{4}$ etc.

As the point $\frac{7}{4}$ lies in between $\frac{4}{4}=1$ and $\frac{8}{4}=2$ hence $\frac{-7}{4}$ lies in between –1 and –2.



In this way any rational number can be placed in number line. Do yourself – Try to put the rational numbers $\frac{3}{4}, \frac{2}{5}, -\frac{5}{3}$ and $-\frac{13}{5}$ in a number line.

9.7 Comparison of Rational number :

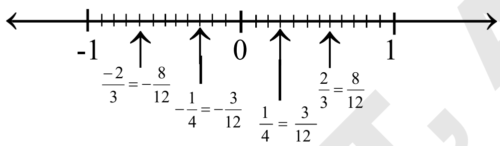
You have already become familiar with the rational numbers. You have already learnt about the comparison of integer and fraction earlier. Now we will discuss the comparison of rational numbers like $-\frac{3}{5}, -\frac{1}{2}, -\frac{1}{3}$ and others. By observing the number line you will understand the place of a number. In a number line the numbers to the right of Zero is greater than the numbers to its left side example :

(i) 2<3 because 3 is on the right side of 2 and similarly –2 is on the right side of –3. Hence –2>–3.

(ii) 15<32 because 15, is on the left side of 32. But –15>–32 because – 15 is on the right side of –32.

Now recall the comparison of $\frac{1}{4}$ and $\frac{2}{3}$. The LCM of the denominators of two fractions is $4\times3=12$

$\frac{1}{4}=\frac{1\times3}{4\times3}=\frac{3}{12}$, $\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}$ but 3<8, so $\frac{3}{12}<\frac{8}{12}$ i.e. $\frac{1}{4}<\frac{2}{3}$. On the other hand in case of $-\frac{1}{4}$ and $-\frac{2}{3}$, $-\frac{1}{4}=\frac{-1}{4}=\frac{-3}{12}$ and $-\frac{2}{3}=\frac{-2}{3}=\frac{-8}{12}$, –3>–8 therefore $\frac{-3}{12}>\frac{-8}{12}$ i.e. $\frac{-1}{4}>-\frac{2}{3}$



The ideas which have been achieved from the above mentioned points (i), (ii) and (iii) are discussed below –

(A) The positive integers can easily be compared because the values of the numbers are easily understood (175<225 because in a number line 175 is placed on the left side of 225)

(B) In case of two negative numbers decision can be taken considering the corresponding the positive forms of the number. Let us compare –15 and – 32. The corresponding positive numbers are 15 and 32 and it is known that 15 < 32. Hence the sign of inequality will have to be reverse. i.e. –15 > –32. By observing the place of these numbers in number line, they can be compared too. –15> –32 because –15, lies on the right side of –32.

(C) The method used in (B) can also be applied in case of the rational number. Let us compare $-\frac{1}{4}$ and $-\frac{2}{3}$. At first we should compare $\frac{1}{4}$ and $\frac{2}{3}$ for comparing these two positive rational numbers of same denominator.

Example : $\frac{1}{4}=\frac{1\times3}{4\times3}=\frac{3}{12}$, $\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}$

Here 8 > 3, i.e. $\frac{2}{3}>\frac{1}{4}$

The inequality sign will be reversed in corresponding negative rational numbers i.e. $\frac{-2}{3}<-\frac{1}{4}$, see the diagram of number line.

(D) Any negative rational number is smaller than any positive rational number, For eg, $-\frac{2}{3} < 1$, $-\frac{7}{3} < \frac{2}{5}$

(E) Any negatiove rational number is smaller than '0' and positive rational number is greater than '0'.

Example : Which one is greater $-\frac{2}{5}$ or $-\frac{3}{7}$?

Solution : At first convert $\frac{2}{5}$ and $\frac{3}{7}$ to the fraction of same denominator here the L.C.M of 5 and 7 is $5 \times 7 = 35$

$\therefore \frac{2}{5} = \frac{2\times 7}{5\times 7} = \frac{14}{35},\quad \frac{3}{7} = \frac{3\times 5}{7\times 5} = \frac{15}{35}$, Here $\frac{14}{35} < \frac{15}{35}$ Therefore $\frac{2}{5} < \frac{3}{7}$, So, $\frac{-2}{5} > \frac{-3}{7}$

Try to solve : (i) Which one is smaller between $-\frac{4}{5}$ and $-\frac{5}{7}$?

(ii) Which one is greater between $-3\frac{2}{7}$ and $-3\frac{3}{5}$ (Hint : $-3\frac{2}{7} = -\frac{23}{7}$, $-3\frac{3}{5} = -\frac{18}{5}$)

9.8 Identification of rational numbers between two definite rational numbers.

Please recall the natural numbers. 1, 2, 3, 4, 5, ... (infinity) are natural numbers. Think yourself and try to give the answers to the following.

(i) Write the natural numbers between 5 and 17.

(ii) Write five rational numbers between 30 and 41.

(iii) How many natural numbers are there in between 4 and 5.

Here we are going to discuss few questions about integer. We know that

– 4, –3, –2, –1, 0, 1, 2, 3, 4, ... are integer.

Try to give answers to the following questions–

(a) Write the integer between – 4 and 4.

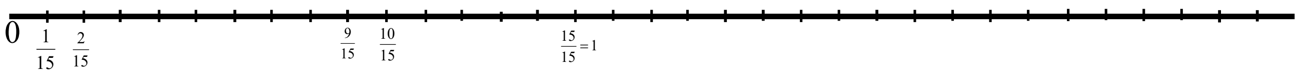
(b) Write the rational numbers between –1and 7 which are less than '0'.

(c) Is there any rational number between –5 and – 4.

(d) Let a and b are two continued integers. Is there any integer between a and b.

Now we will explain all the points of discussion on rational numbers.

Let us find out the rational numbers between 0 and 1. As we have discussed earlier,



the LCM of 3 and 5 is 15 and the two fractions are converted to equivalent rational number having same denominator as shown below–

$$\frac{2}{3}=\frac{2\times5}{3\times5}=\frac{10}{15}$$

$$\frac{3}{5}=\frac{3\times3}{5\times3}=\frac{9}{15}$$

Hence $\frac{2}{3}>\frac{3}{5}$ are placed in number line as shown in the diagram. From the diagram you have noticed that there is no rational number between $\frac{9}{15}=\frac{3}{5}$ and $\frac{10}{15}=\frac{2}{3}$.

But $\frac{3}{5}=\frac{9}{15}=\frac{9\times10}{15\times10}=\frac{90}{150}$ and $\frac{2}{3}=\frac{10}{15}=\frac{10\times10}{15\times10}=\frac{100}{150}$. Between $\frac{90}{150}$ and $\frac{100}{150}$ the rational numbers are $\frac{91}{150},\frac{92}{150},\frac{93}{150},\frac{94}{150},\frac{95}{150},\frac{96}{150},\frac{97}{150},\frac{98}{150}$ and $\frac{99}{150}$.

Similarly if the two fractions are converted to two equivalent fraction of same denominator of the equivalent fraction by multiplying with 100, then we get 99 nos of rational numbers. This method can be applied to identify the rational numbers between two definite rational numbers.

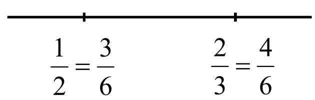
Example : (i) Find a rational number between $\frac{1}{2}$ and $\frac{2}{3}$.

(ii) Find out 5 rational number between $\frac{1}{2}$ and $\frac{2}{3}$.

(iii) Find out 10 rational number between $\frac{1}{2}$ and $\frac{2}{3}$.

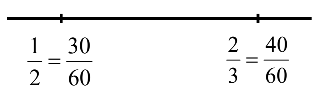
Solution : Let us convert $\frac{1}{2}$ and $\frac{2}{3}$ to the number of same denominator. Here L.C.M of 3 and 2 is $2\times3=6$

$$\frac{1}{2}=\frac{1\times3}{2\times3}=\frac{3}{6} \text{ and } \frac{2}{3}=\frac{2\times2}{3\times2}=\frac{4}{6}$$



Hence $\frac{1}{2}=\frac{3}{6}$ and $\frac{2}{3}=\frac{4}{6}$; $\frac{4}{6}>\frac{3}{6}$

Again, $\frac{1}{2}=\frac{3}{6}=\frac{3\times2}{6\times2}=\frac{6}{12}$ and $\frac{2}{3}=\frac{4}{6}=\frac{4\times2}{6\times2}=\frac{8}{12}$



If you observe, you will understand that you get *a* rational number like $\frac{7}{12}$ between $\frac{6}{12}$ and $\frac{8}{12}$.

(ii) It is seen in the above example (i) that $\frac{1}{2}=\frac{3}{6}$ and $\frac{2}{3}=\frac{4}{6}$. Now by multiplying both the numerator and denominator by 6 we get $\frac{1}{2}=\frac{3}{6}=\frac{3\times6}{6\times6}=\frac{18}{36}$ and $\frac{2}{3}=\frac{4}{6}=\frac{4\times6}{6\times6}=\frac{24}{36}$.

Therefore, the rational numbers between $\frac{18}{36}$ and $\frac{24}{36}$ are $\frac{19}{36},\frac{20}{36},\frac{21}{36},\frac{22}{36}$ and $\frac{23}{36}$.

(iii) Again $\frac{1}{2}=\frac{3}{6}=\frac{3\times11}{6\times11}=\frac{33}{66}$ and $\frac{2}{3}=\frac{4}{6}=\frac{4\times11}{6\times11}=\frac{44}{66}$ Now 10 rational numbers between $\frac{1}{2}=\frac{33}{66}$ and $\frac{2}{3}=\frac{44}{66}$ are $\frac{34}{66},\frac{35}{66},\frac{36}{66},\frac{37}{66},\frac{38}{66},\frac{39}{66},\frac{40}{66},\frac{41}{66},\frac{42}{66}$ and $\frac{43}{66}$.

Do yourself : (i) Find out 5 rational number betwen, $-\frac{5}{7}$ and $\frac{2}{3}$. Is there any integer in between them?

(ii) Which is greater between $-\frac{2}{3}$ and $-\frac{5}{7}$? Can you find 5 rational numbers between them?

Exercise -9.1

1. Select the correct options from the following sentences –
   (i) All natural numbers are integers.
   (ii) An integer may not be a natural number.
   (iii) If a number is rational then the number must be an integer.
   (iv) There are infinite numbers of rational numbers between two integers.
   (v) All fractions are integers.
   (vi) All fractions are rational numbers.
   (vii) 0 is rational number.
   (viii) Each integer is rational.
2. Write 3 equal rational numbers for the following fractions. (Keep in mind that there are infinite numbers of equivalent rational number for every number)

   (i) $\frac{-4}{5}$ (ii) $\frac{2}{-3}$ (iii) $\frac{-7}{21}$ (iv) $\frac{1}{-9}$ (v) $\frac{40}{64}$

3. Are the following pairs equal?

(i) $\frac{-3}{13}, \frac{6}{-26}$ (ii) $\frac{7}{-3}, \frac{1}{-3}$

4. Replace x and y in such a way so that the equity exists.

(i) $\frac{9}{-40} = \frac{-9}{x}$ (ii) $\frac{-5}{35} = \frac{y}{-70}$

5. Express in standard form

(i) $\frac{5}{-2}$ (ii) $\frac{7}{-14}$ (iii) $\frac{25}{-45}$ (iv) $2\frac{3}{7}$ (v) $\frac{-18}{10}$

6. Which is smaller in each of the pair of the following rational numbers.

(a) $\frac{7}{14}, \frac{-2}{4}$ (b) $\frac{-1}{3}, \frac{-2}{5}$ (c) $\frac{-8}{5}, \frac{-7}{4}$ (d) $\frac{-2}{-3}, \frac{16}{12}$

7. Write 5 rational number between the following pair of numbers. (Keep in mind that there are several rational numbers in each of the following pairs)

(i) –1 and 1 (ii) $-\frac{3}{4}, \frac{3}{4}$ (iii) –3, –2 (iv) $\frac{-2}{5}, \frac{-2}{3}$ (v) $\frac{5}{8}, \frac{3}{7}$

8. Put the following rational numbers in number line –

(i) $\frac{2}{3}$ (ii) $-\frac{4}{7}$ (iii) $\frac{3}{8}$ (iv) $-2\frac{3}{5}$ (v) $3\frac{4}{9}$

9. $\frac{31}{5}$ is a rational lies to the right of 0. What is the rational number which lies at the left of 0 in same distance ? What will be the rational number which lies to the centre of the two rational numbers?

10. (i) What will be the greatest integer among the integers which are smaller than $\frac{1}{2}$?

(ii) What will be the smallest integer among all the integers which are greater than $\frac{1}{2}$?

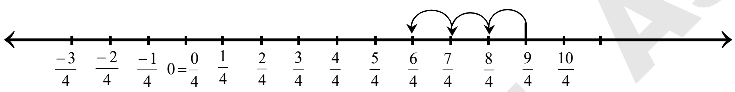
9.9 Operations of Rational numbers :

We have already discussed on addition, subtraction, multiplication and divisions of integers and fractions. Here the idea of addition, subtraction, multiplication and divisions of integers and fractions is extended to rational numbers.

9.9.1 Addition

Let us discuss addition with the help of an example. Let $\frac{9}{4}$ and $\frac{-3}{4}$ be added. Number line was taken by measuring the units of two consecutive points. The number line is arranged by taking $\frac{1}{4}$ as the distance between two consecutive points (keep in mind that the denominator of the two numbers to be added is 4)

Now the place of $\frac{9}{4}$ is assured in the number line.



Now we have towards forward to move left from $\frac{9}{4}$ making 3 jumps because the number to be added is $\frac{-3}{4}$. Here '–' sign (negative) indicates left movement. i.e. making 3 steps towards left we have reached $\frac{6}{4}$. Therefore $\frac{9}{4}+(\frac{-3}{4})=\frac{6}{4}$

This could be done in the following way $\frac{9}{4}+(\frac{-3}{4})=\frac{9+(-3)}{4}=\frac{9-3}{4}=\frac{6}{4}=\frac{3}{2}$ Likewise the addition of two rational numbers with same numerator can be accomplished.

Remember that –

(i) In case of rational numbers with same denominators, the addition can be done by adding the numerator while denominator remains same.

(ii) If the denominators are different then we should find out the LCM of denominators to convert the rational numbers to equivalent rational number so that denominators are same.

Example : Let us add $\frac{2}{9}$ and $\frac{-7}{15}$

Solution : (i) $\frac{2}{9}=\frac{2\times 5}{9\times 5}=\frac{10}{45}$

and $\frac{-7}{15}=\frac{-7\times 3}{15\times 3}=\frac{-21}{45}$

Now, $\frac{2}{9}+\frac{-7}{15}=\frac{10}{45}+\frac{-21}{45}=\frac{10+(-21)}{45}=\frac{-11}{45}$

This addition can be done in the following way in short method

$$\frac{2}{9}+\frac{-7}{15}=\frac{2\times 5+(-7)\times 3}{45}$$ (Recall what we have done in case of fractions)

$$=\frac{10-21}{45}=\frac{-11}{45}$$

9.9.2 Additive inverse of rational numbers

Now, take two rational numbers, say $\frac{2}{3}$ and $-\frac{2}{3}$

$$\frac{2}{3}+\left(-\frac{2}{3}\right)=\frac{2}{3}+\left(\frac{-2}{3}\right)=\frac{2+(-2)}{3}=0$$

Likewise, $\left(-\frac{2}{3}\right)+\frac{2}{3}=\left(\frac{-2}{3}\right)+\frac{2}{3}=\frac{(-2)+2}{3}=0$

i.e. $\frac{2}{3}+\left(-\frac{2}{3}\right)=\left(-\frac{2}{3}\right)+\frac{2}{3}=0$, so, like integer rational number $\frac{2}{3}$ is additive inverse of $-\frac{2}{3}$ and vice-versa.

If the sum of two rational number is 0 then they are additive inverse of each of other.

Try yourself : Determine the additive inverse of the following rational numbers.

(i) 0, $\quad -1$, $\quad \frac{3}{5}$, $\quad \frac{-9}{2}$, $\quad 2\frac{1}{3}$, $\quad -3\frac{4}{5}$

9.9.3 Subtraction

Like fractions, subtraction can be done in case of rational number too. Let us take an example. Let $\frac{2}{5}$ be subtracted from $\frac{8}{9}$.

Now, $\frac{8}{9}-\frac{2}{5}=\frac{8\times 5}{9\times 5}-\frac{2\times 9}{5\times 9}=\frac{40}{45}-\frac{18}{45}=\frac{40-18}{45}=\frac{22}{45}$. Again we know that in case of any integer *a* and *b*.

$$a-b=a+(-b)$$

Let us see what we get if the method is used in case of substraction of rational numberin the way it is used in integer.

$$\frac{8}{9}-\frac{2}{5}=\frac{8}{9}+\left(-\frac{2}{5}\right)=\frac{8}{9}+(\frac{-2}{5})=\frac{8\times 5}{9\times 5}+\left(\frac{-2\times 9}{5\times 9}\right)=\frac{40}{45}+\left(\frac{-18}{45}\right)=\frac{40+(-18)}{45}=\frac{40-18}{45}=\frac{22}{45}$$

Therefore it is seen that the value of $\frac{8}{9}-\frac{2}{5}$ and $\frac{8}{9}+\left(\frac{-2}{5}\right)$ is same. We can say from this that subtraction from *'a'* (rational number) a rational number *'b'* means addition of additive inverse of *'b'* with *'a'*.

Remember

$\frac{2}{3}+\frac{-2}{3}=\frac{2+(-2)}{3}$

$=\frac{0}{3}=0$

Example : Find out (i) $\frac{6}{7}-\frac{1}{9}$ (ii) $3\frac{2}{3}-(-2\frac{4}{5})$

Solution : (i) $\frac{6}{7}-\frac{1}{9}=\frac{6}{7}+(\frac{-1}{9})=\frac{6\times9+(-1)\times7}{63}$

$=\frac{54-7}{63}=\frac{47}{63}$

It can be done in this way $\frac{6}{7}-\frac{1}{9}=\frac{6\times9-1\times7}{63}=\frac{54-7}{63}=\frac{47}{63}$

(ii) $3\frac{2}{3}-(-2\frac{4}{5})=\frac{11}{3}-\left(-\frac{14}{5}\right)=\frac{11}{3}+\frac{14}{5}$ (additive inverse of $-\frac{14}{5}$ is $\frac{14}{5}$)

$=\frac{11}{3}+\frac{14}{5}$

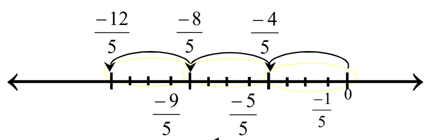
$=\frac{55+42}{15}=\frac{97}{15}=6\frac{7}{15}$

LCM of 3, 5
$=3\times5$
$=15$

9.9.4 Multiplication :

You have learnt about the multiplication of fraction. In this way multiplication of rational numbers can be done. The example '$\frac{-4}{5}\times3$' is taken to understand the activity of 'repeated addition'.

$\left(-\frac{4}{5}\right)\times3$ means addition of $\left(-\frac{4}{5}\right)$ for three times. It is shown in the number line.



The points are shown at a distance of $\frac{1}{5}$ from left side of 0. For negative sign we have to move thrice towards left of 0 for 4 divisions. We have reached $-\frac{12}{5}$ in this way after 3 jumps. Therefore $\left(\frac{-4}{5}\right)\times 3=\frac{-12}{5}$. We can do this in the following way –

$\frac{-4}{5}\times 3=\frac{-4\times 3}{5}=\frac{-12}{5}$ We get the same rational number in both the methods.

Let us multiply a rational number $\frac{-2}{5}$ with a negative integer.

$\frac{-2}{5}\times(-7)=\frac{(-2)\times(-7)}{5}=\frac{14}{5}$ but it can be written as $-7=\frac{-7}{1}$

Therefore, $\frac{-2}{5}\times(-7)=\frac{-2}{5}\times\frac{-7}{1}=\frac{(-2)\times(-7)}{5\times 1}=\frac{14}{5}$

Likewise, $\frac{-3}{4}\times\frac{7}{2}=\frac{(-3)\times 7}{4\times 2}=\frac{-21}{8}$

Therefore, multiplication of two rational number is,

$$=\frac{\text{multiplication of numerators of the two numbers}}{\text{multiplication of denominators of the two numbers}}$$

Example : Find out the value (i) $\frac{4}{7}\times\left(\frac{-5}{3}\right)$ (ii) $\frac{-7}{6}\times\frac{-5}{11}$

Solution : (i) $\frac{4}{7}\times\left(\frac{-5}{3}\right)=\frac{4\times(-5)}{7\times 3}=\frac{-20}{21}$ (ii) $\frac{-7}{6}\times\frac{-5}{11}=\frac{(-7)\times(-5)}{6\times 11}=\frac{35}{66}$

Do yourself : Determine : (i) $\frac{-3}{5}\times\frac{4}{7}$ (ii) $\frac{-8}{13}\times\frac{-11}{9}$ (iii) $\frac{21}{-16}\times\frac{-8}{7}$

9.9.5 Division :

We have learnt how to find reciprocals of a fraction earlier. We extend this idea of reciprocal to rational numbers. Therefore we get that

(i) Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$ (ii) Reciprocal of $\frac{-5}{7}$ is $\frac{7}{-5}$ i.e. $\frac{-7}{5}$

(iii) Reciprocal of –7 is $\frac{-1}{7}$ etc. (iv) Reciprocal of 0 cannot be obtained

To divide a rational number with another rational number, the first rational number can be multiplied by the reciprocal of the second rational number in the same way in which we had done division of fractions. An example given below–

(i) $\frac{-5}{6}\div\frac{7}{11}=\frac{-5}{6}\times\frac{11}{7}=\frac{(-5)\times 11}{6\times 7}=\frac{-55}{42}$

(ii) $\frac{-9}{5} \div \left(\frac{-6}{25}\right) = \frac{-9}{5} \times \frac{-25}{6} = \frac{(-9)\times(-25)}{5\times 6}$

$= \frac{9\times 25}{5\times 6} = \frac{15}{2}$

Exercise - 9.2

1. Find the sum :

(a) $\frac{3}{6}+\frac{5}{3}$ (b) $\frac{-5}{6}+\frac{4}{7}$ (c) $\frac{-8}{15}+\left(\frac{-3}{20}\right)$ (d) $1+\left(\frac{-8}{9}\right)$

(e) $\frac{8}{-21}+\left(\frac{-4}{35}\right)$ (f) $-3\frac{4}{5}+2\frac{1}{6}$ (g) $-4\frac{2}{3}+\left(-3\frac{2}{7}\right)$

2. Subtract the following :

(i) $\frac{51}{14}-\frac{3}{2}$ (ii) $\frac{2}{3}-\left(-\frac{1}{3}\right)$ (iii) $1-\left(-\frac{8}{9}\right)$ (iv) $\left(-4\frac{2}{3}\right)-\left(-3\frac{2}{7}\right)$

(v) $\frac{8}{-12}-\left(\frac{-4}{35}\right)$ (vi) $-2\frac{1}{9}-5$ (vii) $7-\left(-2\frac{2}{9}\right)$

3. Find the product :

(i) $-\frac{15}{14}\times\frac{2}{3}$ (ii) $\frac{3}{-11}\times\frac{-2}{5}$ (iii) $\left(\frac{-6}{21}\right)\times\left(\frac{7}{-8}\right)$ (iv) $\frac{6}{5}\times\left(\frac{-9}{11}\right)$

(v) $\left(\frac{-7}{12}\right)\times\left(\frac{-2}{13}\right)$ (vi) $(-1)\times\left(-\frac{4}{5}\right)$ (vii) $\left(-\frac{6}{7}\right)\times(-1)$ (viii) $\frac{3}{5}\times(-1)$

4. Find out the value of :

(i) $(-5)\div(-1)$ (ii) $-1\div\left(\frac{3}{5}\right)$ (iii) $-1\div\left(\frac{-3}{5}\right)$ (iv) $\left(-\frac{3}{7}\right)\div\frac{1}{21}$

(v) $\frac{7}{-3}\div(-21)$ (vi) $21\div\left(\frac{-7}{3}\right)$ (vii) $\frac{6}{13}\div\left(\frac{-4}{65}\right)$ (viii) $\frac{-1}{8}\div\left(-\frac{1}{8}\right)$

5. Answer as per the instruction –

(i) What is Additive inverse of $\frac{-8}{9}$ (ii) What is additive inverse of -1

(iii) $\frac{2}{3} \div \frac{2}{3} = 1$ (Say correct or incorrect) (iv) $1 \div \frac{4}{3} = \frac{3}{4}$ (Say correct or incorrect)

(v) If a and b are two rational numbers then, $a \times (-b) = -(a \times b)$ and $-a \times b = -(a \times b)$ । Now take any two rational numbers in place of a and b and verify the above equity.

(vi) What is the reciprocal of $\frac{-3}{7}$?

What we have to learnt

1. The numbers which can be expressed in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$, numbers are called rational numbers.
2. All the integers and fractional numbers are rational numbers.
3. Rational numbers can be grouped into positive and negative rational numbers.
4. ‘0’ (Zero) is a rational number which is not included in positive or negative rational number.
5. If denominator of a rational number is a positive integer and there is no common factor of the numerator and denominator of that rational number except 1, then the rational number is said to be in standard form.
6. There is innumerable rational number between two rational numbers.
7. The rational numbers obey the closure, commutative associative and distributive property.
8. In addition of two rational number whose denominators are the same, the numerators are simply added keeping the denominator same.
9. To subtract one rational number from another rational number the additive inverse of the rational number to be subtracted is added with the other rational number.
10. To divide one rational number by another non-zero rational number, the first rational number is multiplied by the reciprocal of the second rational number.
11. Addition subtraction multiplication and division of two rational numbers yields a rational number. The denominator must be a non-zero number in division only i.e. $\frac{a}{b}$, where a and b are integers $b \neq 0$ ।