Data Handling

Frequency Distribution Table and Terminology Related to It

Observe the information given in the following cases.

(1) The weights (in kg) of 15 students in the same class are as follows:

45, 50, 48, 47, 58, 52, 49, 54, 48, 51, 46, 57, 56, 50, 44

(2) Minimum temperature (in °C) of a city for each day of a week is given as follows:

1.5, 2, 0, 2.5, 3.5, 1, 1

(3) Runs scored by 8 players of a cricket team in a match are as follows:

Player	**Runs**
Harry	21
Venkat	16
Robin	74
Dinesh	09
Vikram	81
Laxmipati	42
Jairaj	36
Ysuf	27

It can be seen that in each case, we have some numeric information.

Numeric information collected for a particular purpose is known as raw data and each number involved in this raw data is known as score.

For example, in case (1), weight of each student is a score.

Similarly, in case (2), temperature of each day and in case (3), runs scored by each player are scores.

Raw data is found in unorganized form and in many real life situations, we have to deal with it.

Data can be of two types such as **primary data** and **secondary data**.

Primary data: When the data is collected by an investigator according to a plan for a particular objective then the collected data is called primary data.

For example, if a person collects information about the people using a particular mobile phone network in a particular locality, then the data collected by the person will be called primary data.

Secondary data: When the required data is taken from the data already collected by other private agency, government agency, an organization or any other party then the data is called secondary data.

For example, if an organization extracts the data from the records of census published by the government, then the data is called secondary data.

To draw meaningful inference, we organize the data into systematic pattern in the form of frequency distribution table.

Let us understand this with the help of an example.

Heights (in cm) of 30 students of a class are given as follows:

152, 160, 154, 151, 158, 165, 152, 160, 160, 152, 152, 161, 158, 160, 152, 165, 165, 155, 158, 158, 154, 158, 160, 161, 158, 161, 158, 155, 161, 160

Now, it can be seen that the lowest score is 151 and the highest score is 165.

The difference between the highest observation and lowest observation in a given data set is called the **range**. Range of the above data is 165 – 151 = 14.

It can be observed that few scores occur more than once in the data.

Number of times by which a score occurs in the data is called the frequency of that score.

Score 151 occurs just once, so its frequency is 1.

Similarly,

Score 152 occurs five times, so its frequency is 5.

Score 154 occurs two times, so its frequency is 2.

Score 155 occurs two times, so its frequency is 2.

Score 158 occurs seven times, so its frequency is 7.

Score 160 occurs six times, so its frequency is 6.

Score 161 occurs four times, so its frequency is 4.

Score 165 occurs three times, so its frequency is 3.

The sum of all frequencies or total frequency is 30 which gives us the total number of scores in the data. Total frequency is denoted by N.

Now, we can arrange these scores in a table according to their respective frequencies and such a table is known as frequency distribution table.

Frequency distribution table for the given data is as follows:

Height	**Tally Mark**	**Frequency**
151	ǀ	1
152	𝍸	5
154	ǀǀ	2
155	ǀǀ	2
158	𝍸ǀǀ	7
160	𝍸ǀ	6
161	ǀǀǀǀ	4
165	ǀǀǀ	3
	Total (N)	**30**

The bars in the second column are known as tally marks which are used to represent the numbers.

In tally marks representation, 1 is represented by one bar i.e., ǀ, 2 is represented by the group of two bars i.e., ǀǀ and 5 is represented by 𝍸 (four vertical bars are intersected by one bar diagonally). Similarly, each number is represented by putting that many of bars in a group.

We can make frequency distribution table by arranging the data in small groups or intervals also.

The table obtained in the video can be represented using tally marks as follows:

Group	Tally Mark	Frequency
0 – 10	||	2
10 – 20	~~||||~~ ~~||||~~ ||||	14
20 – 30	~~||||~~ ~~||||~~ ||||	14
30 – 40	~~||||~~ ~~||||~~	10
40 – 50	~~||||~~ |||	8

In the video, we have discussed about class limits, class size and class frequency. There is one more term that is used while talking about frequency table. The term is **class mark.**

Class mark is the arithmetic mean of the upper and lower limits of a class. It is also known as the mid value of the class interval.

Therefore,

$$\text{Class mark} = \frac{\text{Lower class limit} + \text{Upper class limit}}{2}$$

Let us now go through the given examples to understand this concept better.

Example 1:

The marks obtained by 10 students out of 100 are given below:
55, 79, 68, 85, 96, 48, 39, 67, 80, 72

Find the range of marks.

Solution:

From the given marks, we observe that the highest mark is 96 and the lowest mark is 39.

∴ Range of the marks = Highest mark – Lowest mark = 96 – 39 = 57

Example 2:

The number of runs scored by a cricket player in 25 innings are given below:

64, 94, 26, 35, 46, 49, 107, 56, 3, 36, 41, 73, 8, 63, 128, 17, 33, 68, 5, 11, 23, 77, 28, 85, 117

Prepare a frequency distribution table, taking the size of the class interval as 20, and answer the following questions:

(i)What are the class intervals of highest and lowest frequency.

(ii)What does the frequency 2 corresponding to the class interval (100 – 120) indicates?

(iii)What is the class mark of the class interval (100 – 120).

(iv)What is the range of the runs scored by the player?

Solution:

The frequency distribution for the given data is as follows:

Class interval (Runs scored)	Tally marks	Frequency
0 – 20	卌	5
20 – 40	卌 |	6
40 – 60	||||	4
60 – 80	||||	4
80 – 100	|||	3
100 – 120	||	2
120 – 140	|	1

(i) Class interval with the highest frequency is 20 – 40 whereas the class interval with the lowest frequency is 120 – 140.

(ii) The frequency 2 in the class interval 100 – 120 indicates that the player has scored runs in the range 100 to 120 twice in 25 innings.

(iii)Class mark of the interval 100 – 120 $= \frac{100+120}{2} = 110$

(iv) Range of the runs scored = Highest run – Lowest run = 128 – 3 = 125

Example 3:

Observe the given frequency distribution table and answer the following questions:

Salary per month(Rupees in thousands)	Number of employees(Frequency)
15	20
20	35
25	30
30	25
35	20
40	20
45	18
50	12

I. How many employees are involved in the survey?

II. How many employees earn Rs 25,000 per month?

III. What is the difference between the number of employees getting the highest salary and the number of employees getting the lowest salary?

IV. How many employees earn more than Rs 35,000 per month?

V. What is the monthly salary that is being paid to the maximum number of employees?

Solution:

I. Number of employees involved in the survey = Sum of all frequencies

∴ Number of employees involved in the survey = 20 + 35 + 30 + 25 + 20 + 20 + 18 + 12

⇒ Number of employees involved in the survey = 180

II. 30 employees earn Rs 25,000 per month.

III. Number of employees getting the highest salary = 12

Number of employees getting the lowest salary = 20

∴ Required difference = 20 – 12

⇒ Required difference = 8

IV. Number of employees earning more than Rs 35,000 per month = Sum of number of employees earning Rs 40,000, Rs 45,000 and Rs 50,000 per month

∴ Number of employees earning more than Rs 35,000 per month = 20 + 18 + 12

⇒ Number of employees earning more than Rs 35,000 per month = 50

V. Highest frequency in the table is 35 which represents the maximum number of employees in any salary group. Also, each employee in this group earns Rs 20,000 per month.

Thus, Rs 20,000 is the monthly salary that is being paid to the maximum number of employees.

Example 4:

Observe the given frequency distribution table and then answer the following questions.

Class interval (height in cm)	**Frequency (number of students)**
0 – 12	2
12 – 24	3
24 – 36	5
36 – 48	10
48 – 60	3

1. What is the size of the class intervals?
2. Which class interval has the highest frequency?
3. Which two classes have the same frequency?
4. How many students have height less than 36 cm?
5. What is the lower limit of the class interval 24 – 36?
6. What is the class mark of the class interval 48 – 60?

Solution:

1. The difference between the upper and lower class limits for each class interval is 12. Therefore, the class size is 12.
2. The class 36 – 48 has the highest frequency. 10 students height belong to this category.
3. The classes 12 – 24 and 48 – 60 have the same frequency.
4. The number of students having height less than 36 cm is 2 + 3 + 5 = 10.

5. The lower limit of the class interval 24 – 36 is 24.
6. $\text{Class mark} = \dfrac{\text{Lower class limit} + \text{Upper class limit}}{2}$

$$\Rightarrow \text{Class mark} = \frac{48+60}{2}$$
$$\Rightarrow \text{Class mark} = \frac{108}{2}$$
$$\Rightarrow \text{Class mark} = 54$$

Organise Data in the Form of Grouped Frequency Distribution Table

The ages of some residents of a particular locality are given as follows.

7, 28, 30, 32, 18, 19, 37, 36, 14, 27, 12, 8, 17, 24, 22, 2, 21, 5, 21, 36, 38, 25, 10, 25, 9.

How will you represent this raw data in a systematic form?

We represent such type of data with the help of ***grouped frequency distribution table.***

Let us now see how to draw it.

There are two ways to group the data to make frequency distribution table. These are as follows:

Inclusive method (Discontinuous form):

In this method, we group the data into small classes of convenient size. Let us take class size as 10 to group the data in different classes. In the above data, minimum value is 2 and maximum value is 38. The classes can be defined in inclusive method as 1 – 10, 11 – 20, 21 – 30 and 31 – 40. Here, both limits are inclusive in each class. Now, a number of residents falling in each group is obtained. All the given observations get covered in these four classes.

Now, frequency distribution table can be drawn as follows:

Class intervals	**Tally marks**	**Frequency**
1 – 10	𝍸 𝍷	6
11 – 20	𝍸	5
21 – 30	𝍸 𝍷𝍷𝍷𝍷	9
31 – 40	𝍸	5

Exclusive method (Continuous form):

First of all, we will choose the class intervals. In exclusive method, we take the class intervals as 0 – 10, 10 – 20, 20 – 30, 30 – 40 and obtain the number of residents falling in each group.

Now, the observations which are more than 0 but less than 10 will come under the group 0 – 10; the numbers which are more than 10 but less than 20 will come under the group 10 – 20 and so on.

We must note one thing, 10 occurs in two classes, which are 0 – 10 and 10 – 20. But it is not possible that an observation can be included in both classes. To avoid this, we can make any of lower limit or upper limit inclusive. Here, we adopt the convention that the common observation will belong to the higher class, i.e. 10 will be included in the class interval 10 – 20 and similarly we follow this for the other observations also.

The grouped frequency distribution table will be as follows:

Class intervals	**Tally marks**	**Frequency**
1 – 10	卌	5
10 – 20	卌 |	6
20 – 30	卌 |||	8
30 – 40	卌 |	6

The above frequency distribution tables help to draw many inferences.

We can also tell the frequency, class limits, class size, etc. from the above frequency distribution tables.

The most commonly used method to make frequency distribution table among the above discussed methods is exclusive method.

Class boundaries:

From the table given for inclusive or discontinuous method, it can be observed that there is a gap between the upper limit of a class and the lower limit of its next consecutive class. We can convert this table into a table having continuous classes without altering class size, class-marks and frequency column. For doing this, we just need to take the average of the upper limit of a class and the lower limit of its next consecutive class. This average is used as the **true upper limit** of that class and **true lower limit** of its next consecutive class.

Therefore,

True upper limit of the class

$= \frac{\text{Upper limit of the class} + \text{Lower limit of the next consecutive class}}{2}$ = True lower limit of the next consecutive class

For example, let us take two consecutive classes such as 1 – 10 and 11 – 20 from the table given for inclusive or discontinuous method.

Now,

True upper limit of the class 1 – 10 = $\frac{10+11}{2}$ = 10.5 = True lower limit of the class 11 – 20

In this manner, we obtain the continuous classes as 0.5 – 10.5, 10.5 – 20.5, 20.5 – 30.5 and 30.5 – 40.5.

Note: In this method, true lower limit of first class is obtained by subtracting the value added to its upper limit. Also, true upper limit of last class is obtained by adding the value subtracted from its lower limit.

There is one more method of finding the true upper and lower limits which is explained as follows:

Step 1: Find the difference by subtracting the upper limit of a class from the lower limit of the next consecutive class.

Step 2: Divide the difference by 2.

Step 3: Subtract the difference from the lower limit of each class to find the true lower limit of each class.

Step 4: Add the difference to the upper limit of each class to find the true upper limit of each class.

It can be observed that in the table given for inclusive or discontinuous method, difference between the upper limit of each class and the lower limit of its consecutive class is 1. On dividing this difference by 2, we get 0.5. Now, the continuous classes will be 0.5 – 10.5, 10.5 – 20.5, 20.5 – 30.5 and 30.5 – 40.5.

These methods are very helpful at times.

Now, we know that the range of a data set is the span from lowest value to highest value in the data. We should choose class intervals for a particular range of data very carefully.

Few points to be remembered while choosing class intervals are as follows:

1. Classes should not be overlapping and all values or observations should be covered in these classes.
2. The class size for all classes should be equal.
3. The number of class intervals is normally between five and ten.
4. Class marks and class limits should be taken as integers or simple fractions.

Let us now look at some more examples to understand the concept better.

Example 1:

Construct a frequency distribution table for the given data of weekly income of workers by using class intervals as 500 – 525, 525 – 550 and so on. The incomes for the 26 workers for a week are as follows.

540, 530, 545, 510, 520, 580, 570, 575, 555, 516, 527, 560, 550, 525, 535, 535, 565, 575, 585, 580, 560, 510, 515, 510, 520, 525

Solution:

The class intervals to be used are 500 – 525, 525 – 550 and so on. Therefore, the class size is 25. The frequency distribution table will be as follows.

Class intervals	**Tally marks**	**Frequency**
500 – 525	𝍸 ||	7
525 – 550	𝍸 |||	8
550 – 575	𝍸 |	6
575 – 600	𝍸	5

Example 2:

Observe the following distribution table.

Class intervals	**Frequency**
0 – 5	2

5 – 10	5
10 – 15	10
15 – 20	2
20 – 25	20
25 – 30	10
30 – 35	50
35 – 40	30

Form a frequency distribution table by taking the class intervals as 0 – 10, 10 – 20 and so on.

Solution:

Here, in the first class interval 0 – 10, we have to include both the classes (0 – 5 and 5 – 10) of the given table and to find the frequency of class interval (10 – 20), we include the classes 10 – 15 and 15 – 20. In the similar way, we can form the whole table. Thus, the new frequency distribution table will be as follows.

Class intervals	**Frequency**
0 – 10	7
10 – 20	12
20 – 30	30
30 – 40	80

Construction of Histograms when Class Size is Same

The frequency distribution table of the marks of 26 students in a particular subject is as follows.

Class interval (marks of students)	**Frequency (number of students)**
0 – 10	4
10 – 20	2
20 – 30	10
30 – 40	8
40 – 50	2

Can we represent this data graphically?

This data can be represented in the form of a histogram.

Let us now look at an example to understand this concept better.

Example 1:

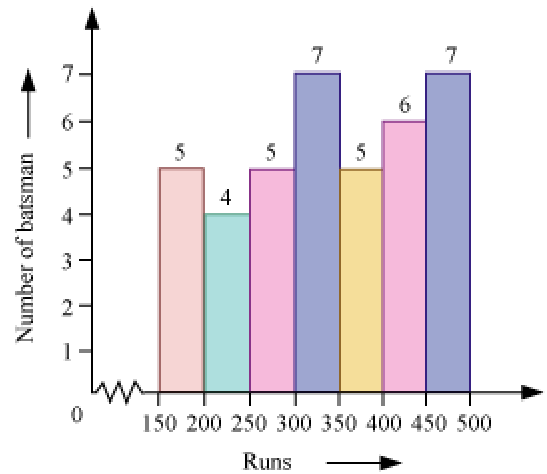
The given tally table represents the total runs scored by 39 batsmen in 10 different test matches.

Runs	**Tally marks**	**Frequency (Number of batsmen)**
150 – 200	𝍸	5
200 – 250	𝍷𝍷𝍷𝍷	4
250 – 300	𝍸	5
300 – 350	𝍸 𝍷𝍷	7
350 – 400	𝍸	5
400 – 450	𝍸 𝍷	6
450 – 500	𝍸 𝍷𝍷	7

Draw a histogram for the above given distribution table.

Solution:

In order to draw the histogram of the given frequency distribution table, we represent the runs on the horizontal axis and the number of batsmen on vertical axis. The height of each bar represents the frequency. The width of all the bars is same.



Here, we will use a broken line ( ) to indicate that the values between 0 – 150 are not represented.

Example 2:

The given table represents the data related to intelligent quotient (IQ) of the students of a class.

IQ	**Number of students**
61 – 70	**4**
71 – 80	**3**
81 – 90	**5**
91 – 100	**8**
101 – 110	**15**
111 – 120	**12**
121 – 130	**13**
Total	**60**

Draw a histogram for the above given distribution table.

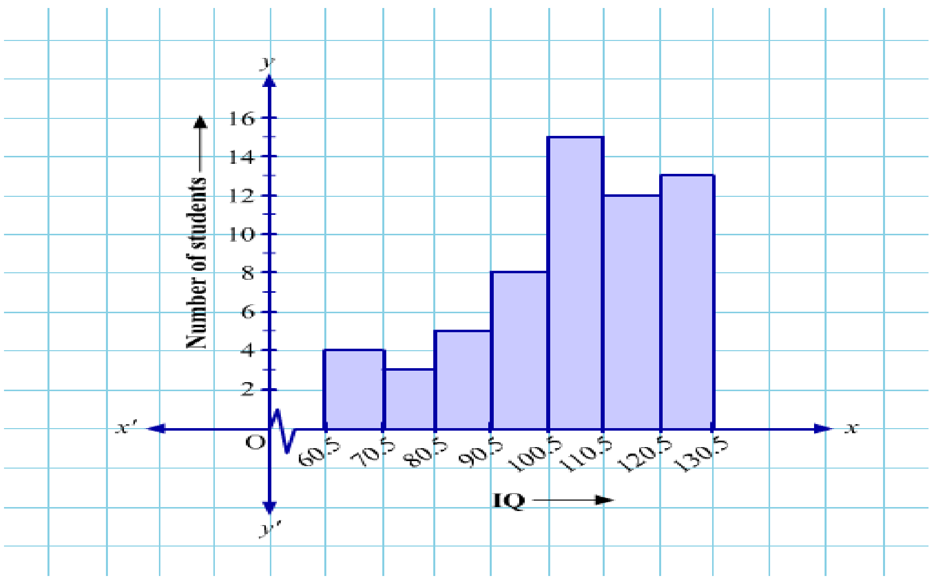
Solution:

In the given table the class intervals are of inclusive type, so we need to make them of exclusive type. Here, the difference between the upper limit of a class and lower limit of next class is 1. So, we need to subtract half of this i.e., 0.5 from lower limit and add 0.5 to upper limit of each class. Thus, we will get extended classes according to which, we can draw the histogram.

The modified table consisting extended classes is as follows:

Original Class	Extended Class	Number of students (Frequency)
61 – 70	60.5 – 70.5	4
71 – 80	70.5 – 80.5	3
81 – 90	80.5 – 90.5	5
91 – 100	90.5 – 100.5	8
101 – 110	100.5 – 110.5	15
111 – 120	110.5 – 120.5	12
121 – 130	120.5 – 130.5	13
Total		60

In order to draw the histogram of this frequency distribution table, we represent the IQ on the horizontal axis and the number of students on vertical axis. The height of each bar represents the frequency. The width of all the bars is same.



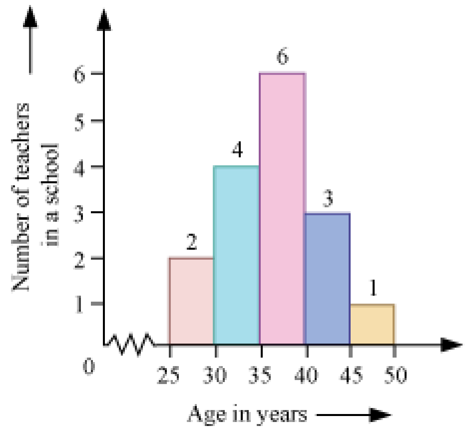
Here, we will use a broken line ( ) to indicate that the values between 0 – 60.5 are not represented.

Interpretation Of Histograms

An histogram is one of the most important tool used for representation of data. One can interpret a lot of information from a given histogram. So let us understand how to interpret information from a given histogram.

Example 1:

The following histogram shows the age of the teachers in a school.



Observe the histogram and answer the following questions.

1. **How many teachers age is 40 years or more but less than 45 years?**

2. **How many teachers are of age less than 40 years?**

3. **Which group contains the least number of teachers?**

4. **The age of maximum number of teachers lies in which group?**

Solution:

1. In order to find the number of teachers of the age 40 years or more but less than 45, we have to find the number of teachers in the age group 40 − 45. The number of teachers in the age group (40 − 45) is 3.

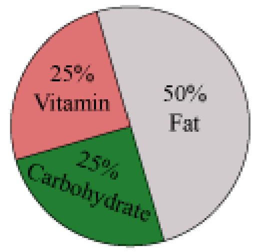
2. In order to find the number of teachers of age less than 40 years, we take into account the number of teachers of the age groups 25 − 30, 30 − 35, and 35 − 40. Thus, the number of teachers of age less than 40 years is 2 + 4 +6 = 12.

3. The age group 45 − 50 contains the least number of teachers.

4. The maximum numbers of teachers are in the age group 35 − 40.

Construction of Circle Graphs

Sometimes, the data is represented using circles. For example, the circle given below shows various nutrients present in a chocolate.



The representation of data in this form is called **pie-chart** or **sector graph** or **circle graph.**

A circle graph shows the relationship between a whole circle and its parts. The whole circle is divided into sectors and the size of each sector is proportional to the information it represents.

Now consider the following data that represents the numbers of people who watch channels 1, 2, and 3.

Channels	Percentage of people preferring the channel
1	30%
2	25%
3	45%

Can we draw a pie-chart of the given data?

Example 1:

The choice of food for a group of people is given below.

Favourite Food	Number of people
North Indian	50
South Indian	40
Others	30
Total	120

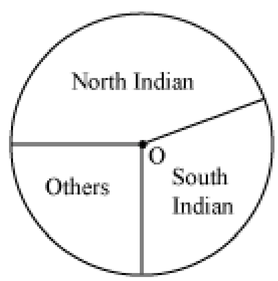
Draw a pie-chart for the given data.

Solution:

Firstly, we will find the central angle of each sector. Here, total number of people = 120. The central angle has been calculated in the following table.

Favourite Food	Number of people	In Fraction	Central Angle
North Indian	50	$\frac{50}{120}=\frac{5}{12}$	$\frac{5}{12}\times 360° = 150°$
South Indian	40	$\frac{40}{120}=\frac{1}{3}$	$\frac{1}{3}\times 360° = 120°$
Others	30	$\frac{30}{120}=\frac{1}{4}$	$\frac{1}{4}\times 360° = 90°$

Draw a circle of any radius. Then, draw the angle of sector for the north Indian food, which is 150°. Use the protractor to draw the angle of 150°. Then continue making the remaining angles (120° and 90°). The pie chart has been shown as follows.



Example 2:

Data regarding the maximum leaves taken in a month by the employees of a company is given in the following table.

Number of leaves	Number of employees

0 – 2	120
2 – 4	45
4 – 6	25
6 – 8	10
Total	200

Draw a pie-chart for the given data.

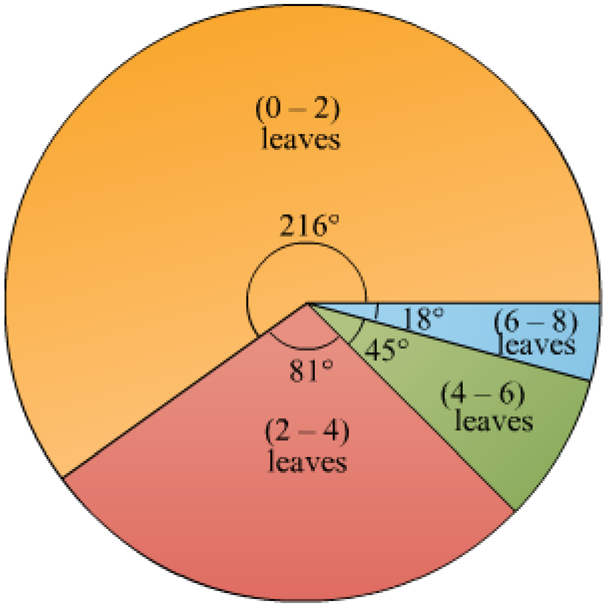
Solution:

Firstly, we will find the central angle of each sector. Here, total number of employees = 200. The central angle has been calculated in the following table.

Number of leaves	Number of employees	In Fraction	Central Angle
0 – 2	120	$\frac{120}{200} = \frac{3}{5}$	$\frac{3}{5} \times 360° = 216^{c}$
2 – 4	45	$\frac{45}{200} = \frac{9}{40}$	$\frac{9}{40} \times 360° = 81^{c}$
4 – 6	25	$\frac{25}{200} = \frac{1}{8}$	$\frac{1}{8} \times 360° = 45^{c}$
6 – 8	10	$\frac{10}{200} = \frac{1}{20}$	$\frac{1}{20} \times 360° = 18^{c}$

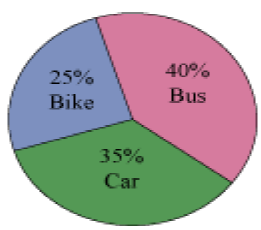
Draw a circle of any radius. Then, draw the angle for each sector.

The obtained pie-chart is as follows:



Interpretation of Circle Graphs

The following pie-chart represents the percentage of number of students who come to school by bus, car, or bike. The number of students studying in the school is 5000.



Can we find out how many students come by bike?

Let us see.

In the graph, the sector representing the number of students who come by bike is given to be 25%.

The total numbers of students are 5000.

Thus, number of students who come by bike = 25% of 5000 = $\frac{25}{100} \times 5000$ = 25 × 50 = 1250

Also find out how many students come by car and bus.

In the graph, the sector representing the number of students who come by car is given to be 35%.

Number of students who come by car = 35% of 5000 = $\frac{35}{100} \times 5000$ = 35 × 50 = 1750

In the graph, the sector representing the number of students who come by bus is given to be 40%.

Number of students who come by bus = 40% of 5000 = $\frac{40}{100} \times 5000$ = 40 × 50 = 2000

Which is the most common mode of transport?

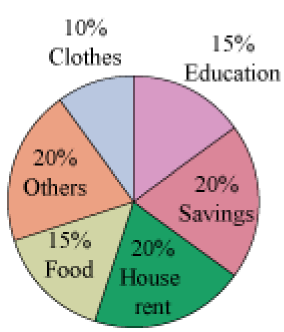
Since the number of students coming by the bus is highest, bus is the most common mode of transport.

In this way, we can interpret the information given in a pie-chart or a circle graph.

Let us now look at one more example.

Example:

Mr. Nair's expenditure on various items and his savings for a particular month has been represented in the following pie-chart.



Observe the given pie-chart and answer the following questions.

1. **On which of the represented items, the expenditure is least?**

2. **If the monthly savings of Mr Nair is Rs 4000, then what is the monthly expenditure on food?**

Solution:

1. The expenditure is least on clothes. It is 10%.

2. It is shown in the circle graph that his savings is 20%. 20% represents Rs 4000.

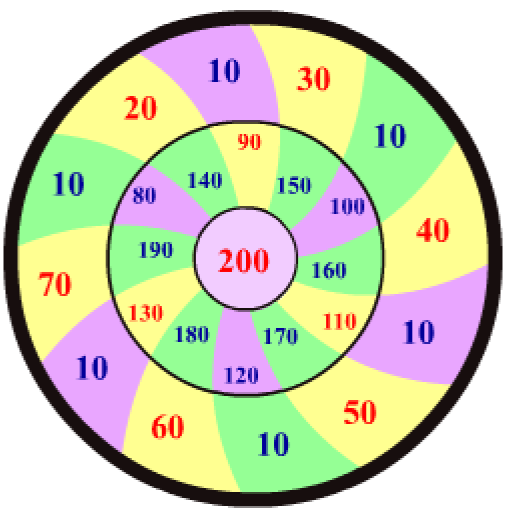
∴ 15% represents Rs $\frac{4000}{20} \times 15 =$ Rs 3000

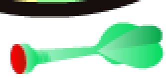
Thus, the monthly expenditure on food is Rs 3000.

Terminology Related to Probability

Observing an Experiment

It is not always possible to tell the exact outcome of a particular action. Take, for example, a dart board.





A dart is repeatedly thrown toward the dartboard, targeting a random number in each throw. We do not know which number is targeted in a particular throw. What we do know is that there is a fixed group of numbers and each time the targeted number is one of them.

We know that the likelihood of occurrence of an unpredictable event is studied under the theory of probability. So, we can say that there is a certain probability for each number to be targeted in the above experiment.

Let us learn more about probability and the meanings of terms associated with it, for example, ‘experiment’ and ‘outcome’.

Did You Know?

The word ‘probability’ has evolved from the Latin word ‘probabilitas’, which can be considered to have the same meaning as the word ‘probity’. In olden days in Europe, ‘probity’ was a measure of authority of a witness in a legal case, and it often correlated with the nobility of the witness.

The modern meaning of probability, however, focuses on the statistical observation of the likelihood of occurrence of an event.

Know More

Probability is widely applicable in daily life and in researches pertaining to different fields. It is an important factor in the diverse worlds of share market, philosophy, artificial intelligence or machine learning, statistics, etc. All gambling is based on probability. In gambling, one considers all possibilities and then tries to predict a result that is most likely to happen. The concept of probability is perhaps the most interesting topic to discuss in mathematics.

Terms Related to Probability

Experiment: When an operation is planned and done under controlled conditions, it is known as an experiment. For example, tossing a coin, throwing a die, drawing a card from a pack of playing cards without seeing, etc., are all experiments. A chance experiment is one in which the result is unknown or not predetermined.

Outcomes: Different results obtained in an experiment are known as outcomes. For example, on tossing a coin, if the result is a head, then the outcome is a head; if the result is a tail, then the outcome is a tail.

Random: An experiment is random if it is done without any conscious decision. For example, drawing a card from a well-shuffled pack of playing cards is a random experiment if it is done without seeing the card or figuring it out by touching.

Trial: A trial is an action or an experiment that results in one or several outcomes. For example, if a coin is tossed five times, then each toss of the coin is called a trial.

Sample space: The set of all possible outcomes of an experiment is called the sample space. It is denoted by the English letter ‘S’ or Greek letter ‘Ω’ (omega). In the experiment of tossing a coin, there are only two possible outcomes—a head (H) and a tail (T).

$\therefore$ Sample space (S) = {H, T}

Event: The event of an experiment is one or more outcomes of the experiment. For example, tossing a coin and getting a head or a tail is an event. Throwing a die and getting a face marked with an odd number (i.e., 1, 3 or 5) or an even number (2, 4 or 6) is also an event.

Know More

Initially, the word 'probable' meant the same as the word 'approvable' and was used in the same sense to support or approve of opinions and actions. Any action described as 'probable' was considered the most likely and sensible action to be taken by a rational and sensible person.

Whiz Kid

Equally Likely: If each outcome of an experiment has the same probability of occurring, then the outcomes are said to be equally likely outcomes.

Know Your Scientist

Girolamo Cardano (1501–1576) was a great Italian mathematician, physicist, astrologer and gambler. His interest in gambling led him to do more research on the concept of probability and formulate its rules. He was often short of money and kept himself solvent through his gambling skills. He was also a very good chess player. He wrote a book named *Liber de Ludo Aleae*. In this book about games of chance, he propounded the basic concepts of probability.

Solved Examples

Easy

Example 1:

A fair die is thrown. What is the sample space of this experiment?

Solution:

When a die is thrown, we can have six outcomes, namely, 1, 2, 3, 4, 5 and 6.

We know that sample space is the collection of all possible outcomes of an experiment.

∴ Sample space (S) = {1, 2, 3, 4, 5, 6}

Example 2:

Which of the following are experiments?

i)Tossing a coin

ii)Rolling a six-sided die

iii)Getting a head on a tossed coin

Solution:

Tossing a coin and rolling a six-sided die are experiments, while getting a head on a tossed coin is the outcome of an experiment.

Medium

Example 1:

What is the sample space when two coins are tossed together?

Solution:

When two coins are tossed together, we can get four possible outcomes. These are as follows:

i)A head (H) on one coin and a tail (T) on the other

ii)A head (H) on one coin and a head (H) on the other

iii)A tail (T) on one coin and a head (H) on the other

iv)A tail (T) on one coin and a tail (T) on the other

∴ Sample space (S) = {HT, HH, TH, TT}

Equally Likely Outcomes

There are many situations where on a particular day, you take a chance and the things do not go the way you want. However on the other days, they do.

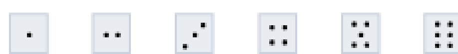
For example, suppose Prachi takes her umbrella everyday to her office. However, on one day she forgot to take the umbrella and it rained that day.

Sometimes it happens that you leave home just 10 minutes before the school timings and still manage to reach at time. Whereas the other day, when you left home 30 minutes earlier, still you could not reach at time because of a heavy traffic jam.

In these kinds of examples, chances of a certain thing occurring and not occurring are not equal. But there are also some cases where there are equal chances of an event to occur or not to occur.

For example, suppose you play a game with your friend where you toss a coin to decide who will play first. When you toss a coin, you can either get a tail or a head. There is no other possibility. Also, when you toss a coin, you cannot always get what you want out of Head or Tail. **There are equal chances of getting a head or a tail.** Such an experiment is called a **random experiment.** The results of an experiment are called **outcomes** of the experiment. Here, when you toss a coin, head or tail are the only two outcomes of this experiment.

Consider another example. Suppose you throw a dice while playing a game. There are 6 possible outcomes (1, 2, 3, 4, 5, or 6). There is no other possibility. Moreover, the chance of getting any of these outcomes is the same.



Let us note the results we obtain, when we throw a dice, once, twice, thrice, and so on. We will observe that as the number of throws increases, the chances of getting each of 1, 2 … 6 come closer and closer to one another. That is the numbers of each of the six outcomes become almost equal to each other. In this case, we may say that the different outcomes of the experiment are **equally likely,** i.e. each of the outcomes has the same chance of occurring.

Let us now look at an example.

Example 1:

Which of the following experiments results in equally likely outcomes?

1. **The school bus of Archit comes daily on time but the day he reaches early, the bus comes late.**

2. **Tossing of a coin 10 times**

Solution:

1. The chances of the bus to come on time or not on time are not equal. Thus, this experiment does not result in equally likely outcomes.

2. The experiment of tossing a coin 10 times will result in equally likely outcomes, since there are equal chances of getting a head or a tail.

Concept of Chance and Probability

In our daily life, various incidents happen and sometimes we know in advance that these incidents will happen. For example, the day after Saturday will be Sunday or the Sun will rise from the east. These are the events which are certain to happen. Similarly, there are events which are impossible such as March comes before February in a year, the apple goes up when dropped from the tree etc.

However, most of the events in our daily life have chances to happen in a particular way and there can be one or more ways in which an event can happen.

For example, India is going to play a cricket match against Bangladesh. Here, the result of this event can occur in various ways whether India will win, lose or draw the game.

Now, can we say India will win the match? Though match between India and Bangladesh is in favour of India, still we cannot say that India will win the match nor can we say that it will lose. This is again a matter of **chance**. We can only say that there is a chance for India to win the match.

Consider one more example now.

Suppose there are five balls of five different colours in a bag - blue, red, yellow, green, and black. Sonu is asked to draw a ball from the bag without looking into it. Can he be certain that he would draw a blue ball? No, it might be any one of the five balls.

Thereafter, Sonu draws one ball at a time without looking into the bag and records the colour of the ball. He then puts that ball inside the bag and again draws a ball. He performs this experiment 20 times and prepares the following table:

Times of drawing a ball	**Colour of the ball**
1	Green
2	Red
3	Green
4	Black
5	Blue

6	Red
7	Red
8	Red
9	Green
10	Green
11	Red
12	Black
13	Blue
14	Red
15	Yellow
16	Red
17	Yellow
18	Yellow
19	Green
20	Black

Can you say that after drawing a green ball, the next ball is always red in colour?

No, the table does not follow any pattern. It is a matter of chance that which colour will come after a particular colour.

In mathematics, we use probability to find the chance that a particular event can happen by considering all the cases which are possible.

The word probability is often used in day to day conversation also. People often use this word when they talk about the chances of an event to happen. We can often hear people saying that probably he is going to be our next Prime Minister or probably it will rain today. In these sentences, we are talking about the chances of an event to happen.

We can define probability as follows:

Probability is the measure or estimation of likelihood of happening of an event in a particular way.

Probability for an event to happen in a particular way depends upon all possible ways in which that event can happen.

For example, when a dice is rolled, the possible ways (positions) in which we get its top face are six such as 1, 2, 3, 4, 5 and 6. The probability to get any of these numbers on top face depends on all these six ways.

Similarly, probability is applicable in various situations and it can be very helpful to predict the future results.

Let us have a look at the following example.

Example:

Which of the following are certain events, impossible events, or matters of chance?

(i) Water always falls down.

(ii) When a coin is tossed, the outcome is Head.

(iii) Harry is older than his father.

(iv) In the musical chair game, Isha will get the chair.

(v) The size of the Sun is smaller than the size of the Earth.

(vi) When a dice is thrown, any one of the numbers among 1, 2, 3, 4, 5, and 6 shows up on the top face.

Solution:

The events **(i)** and **(vi)** are certain.

Water always falls down. It does not go up. A dice contains the number 1, 2, 3, 4, 5, and 6. Thus, when a dice is thrown, any one of the above numbers must show up on the top face.

The events **(iii)** and **(v)** are impossible to happen.

A son cannot be older than his father and the size of the Sun is greater than that of Earth.

The events **(ii)** and **(iv)** are matters of chance.

When a coin is tossed, the outcome may either be Head or Tail. In the musical chair game, Isha may or may not get the chair.

Outcomes Of An Experiment

Suppose Rahul is playing a game of ludo and he throws a dice.

Can we tell what the possible outcomes are?

There are six possible outcomes of the above event. Rahul can get the numbers 1, 2, 3, 4, 5, or 6 on the top face of the dice.

Now consider another example.Suppose you toss a coin. Then, what are the possible outcomes of this experiment?

There are two possible outcomes. We can either get a head or a tail.

Both the outcomes are equally likely.

Each outcome of an experiment or a collection of outcomes makes an event.

Let us now look at some more examples and find the possible outcomes of an experiment.

Example 1:

Two coins are tossed together. What are the possible outcomes?

Solution:

When a coin is tossed, we can get a head or a tail. Therefore, if two coins are tossed together, the possible outcomes are

1. Two heads (i.e. heads on both coins)
2. Two tails (i.e. tails on both coins)
3. One head and one tail

Example 2:

Suppose there are three pens of different colours in a box, out of which one is black, one is green and the third one is blue in colour. If a pen is drawn from the box, then what are the possible outcomes?

Solution:

There are three possible outcomes which are as follows.

1. Black pen
2. Green pen
3. Blue pen

Example 3:

In a bag, there are a total of four balls which are red, pink, yellow and blue in colour. If two balls are drawn out, then what are the possible outcomes?

Solution:

The possible outcomes are as follows.

1. One red and one pink ball
2. One red and one yellow ball
3. One red and one blue ball
4. One pink and one yellow ball
5. One pink and one blue ball
6. One yellow and one blue ball

Probability Of Events

Suppose Shashank throws a dice. There are six different outcomes. The outcomes of an experiment or a collection of outcomes makes an **event.**

In this example, getting the number 1 on the top face of the dice is an event. Similarly, getting the other numbers (2, 3, 4, 5, or 6) are also known as events.

Can we tell what will be the probability of getting 2 on the top face of the dice?

There are six possible outcomes and all are equally likely to occur. **The probability is the ratio of getting an outcome to the total number of outcomes.**

Probability of an event = $\dfrac{\textit{Number of outcomes that make an event}}{\textit{Total number of outcomes of the experiment}}$

Mathematically,

$$P(A) = \frac{n(A)}{n(S)}$$

Here,

$P(A)$ represents the probability of an event A.

$n(S)$ represents the total number of outcomes or number of elements in sample space.

$n(A)$ represents the number of outcomes that make event A or the number of elements in set A.

It should be noted that sample space S is the universal set here, so all elements of set A belong to set S i.e, $A \subseteq S$.

Now, let us find the solution of above discussed case.

In the above example, the total number of outcomes are 6.

∴ Probability of getting a number 2 on the top face $= \frac{1}{6}$

What is the probability of getting 6 on the top face of the dice?

Since all the outcomes are equally likely to occur,

∴ Probability of getting a number 6 on the top face $= \frac{1}{6}$

Similarly, for other numbers (1, 3, 4, and 5) as well, the probability of showing up on the top face is $\frac{1}{6}$.

This is how we can find out the probability of the occurrence of an outcome in an experiment.

Can we calculate the probability of the occurrence of a multiple of 3 on the top face of the dice?

Yes, we can.

Consider the multiples of 3 out of six possible outcomes. The multiples of 3 are 3 and 6 out of six possible outcomes.

Thus, probability of getting a multiple of 3

$$= \frac{\text{Number of multiples of 3}}{\text{Total number of possible outcomes}} = \frac{2}{6} = \frac{1}{3} \quad \frac{\text{Multiples of 3}}{\text{Total number of possible outcomes}} = \frac{2}{6} = \frac{1}{3}$$

Properties of probability:

Property 1: Probability of a certain event is 1 and probability of an impossible event is 0.

Probability of an impossible event is denoted by $P(\phi)$ and probability of a certain event is denoted by P(S).

Therefore, $P(\phi) = 0$ and $P(S) = 1$.

Proof:

We have

$$P(A) = \frac{n(A)}{n(S)}$$

Impossible event means that there is no way in which the event can occur. So, number of outcomes making event A will be 0 or set A will be empty set i.e., ϕ.

$$\therefore P(\phi) = \frac{n(\phi)}{n(S)} = \frac{0}{n(S)} = 0$$

Certain event means that there is only one possibility. So, the number of outcomes in sample space S as well as in set A will be equal i.e., 1.

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{n(S)}{n(S)} = 1$$

Hence proved.

Let us consider few examples based on this property.

Consider the events **"son being older than his father", "Saturday comes before Friday in a week"** and **"taking water from an empty mug".** All these are impossible events and thus, the probability of occurrence of each of these events is 0.

Now, consider the events **"Sun is larger than earth**" and **"Sunday comes before Monday in a week".** Both of these are certain events and thus, the probability of occurrence each of these events is 1.

Property 2: If the sample space S is a finite set and A is an event of S then probability of event A lies between 0 and 1 both inclusive.

Therefore, $0 \leq P(A) \leq 1$.

Proof:

Since $A \subseteq S$, we have

$\phi \subseteq A \subseteq S$

$\Rightarrow n(\phi) \leq n(A) \leq n(S)$

$\Rightarrow 0 \leq n(A) \leq n(S) \quad [n(\phi) = 0]$

On dividing the inequality by $n(S)$, we get

$\frac{0}{n(S)} \leq \frac{n(A)}{n(S)} \leq \frac{n(S)}{n(S)}$

$\Rightarrow 0 \leq P(A) \leq 1$

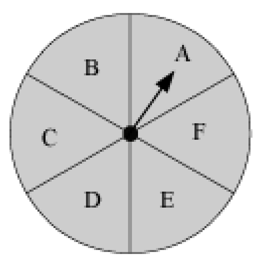
Hence proved.

These are very important properties related to probability which prove to be very helpful at times.

Now, let us have a look at some examples.

Example 1:

The given figure shows a wheel in which six English alphabets are written in six equal sectors of the wheel. Suppose we spin the wheel. What is the possibility of the pointer stopping in the sector containing alphabet A?



Solution:

The total number of possible outcomes is 6. The pointer can stop at six different sectors (A, B, C, D, E, F).

Thus, probability of pointer stopping in the sector containing A $= \frac{1}{6}$

Example 2:

A bag has 6 blue and 4 red balls. A ball is drawn from the bag without looking into the bag.

1. **What is the probability of getting a blue ball?**
2. **What is the probability of getting a red ball?**

Solution:

In a bag, there are 6 blue and 4 red balls.

∴ Total number of outcomes = 6 + 4 = 10

1. Getting a blue ball consists of 6 outcomes, since there are 6 blue balls.

   Probability of getting a blue ball $= \frac{6}{10} = \frac{3}{5}$

2. Getting a red ball consists of 4 outcomes, since there are 4 red balls.

   Probability of getting a red ball $= \frac{4}{10} = \frac{2}{5}$

Example 3:

When a dice is thrown, what is the probability of getting

(a) A prime number

(b) An even number

(c) An odd number

(d) A number less than or equal to 2

(e) A number more than or equal to 4

Solution:

When a dice is thrown, the total number of outcomes is 6.

(a) The prime numbers out of six possible outcomes are 2, 3, and 5. Thus, getting a prime number consists of 3 outcomes.

∴ Probability of getting a prime number $= \frac{3}{6} = \frac{1}{2}$

(b) Out of the possible outcomes, the even numbers are 2, 4, and 6. Thus, the number of outcomes of getting an even numbers is 3.

∴ Probability of getting an even number $= \frac{3}{6} = \frac{1}{2}$

(c) The odd numbers are 1, 3, and 5. Thus, the number of outcomes of getting an odd number is 3.

∴ Probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$

(d) The numbers less than or equal to 2 are 1 and 2. Thus, there are 2 possible

outcomes.

∴ Required probability $= \frac{2}{6} = \frac{1}{3}$

(e) The numbers more than or equal to 4 are 4, 5, and 6. Thus, there are 3 possible

outcomes.

∴ Required probability $= \frac{3}{6} = \frac{1}{2}$