Different methods of solution →

Static Electric Field	Static megnatic Field
(1.) Formula method (Coulombs force law).	(1.) Formula Method (Biot savart law)
(2.) Gauss law method.	(2.) Ampere's circuital law method.
(3.) Solving Laplace & Poission eq^n of electric field.	(3.) Solving Laplace & poission of eq^n of magnetic field.

Relation between network theory & EMT →

Network Theory		EMT
* G [Conductance (℧)]	→	σ [Conductivity ($\frac{℧}{m}$)]
* C [capacitance (Farad)]	→	ϵ [permitivity ($\frac{Farad}{meter}$)]
* L [Inductance (Henry)]	→	μ [Permeability ($\frac{Henry}{meter}$)]
* V [Voltage (volts)]	→	$\bar{E}$ [Electric Field ($\frac{Volt}{meter}$)]
* I [Current (Ampere)]	→	$\bar{H}$ [megnatic field ($\frac{Amp}{meter}$)] (OR) $\bar{J}$ ($\frac{amp}{m^2}$)
* Ψ [Electric flux (coulambs)]	→	$\bar{D}$ [Electric flux density ($\frac{coulambs}{m^2}$)]
* ϕ [Megnatic flux (weber)]	→	$\bar{B}$ [megnatic flux density ($\frac{weber}{m^2}$)]

Source		Result	
(1.) Static charge (Q) (charge is not moving)	→	Static electric field $\bar{E}(x,y,z)$	maxwell's eq^n $\bar{\nabla}\times\bar{E}=0$ $\nabla\times\bar{H}=\bar{J}$
(2) Dc current [I (dc)] (Charge is moving with Constant Velocity)	→	Static magnetic field $\bar{H}(x,y,z)$	
(3) Ac current (Charge is moving with acceleration)	→	Time varying electric & magnetic fields $\bar{E}(x,y,z,t)$ $\bar{H}(x,y,z,t)$	$\bar{\nabla}\times\bar{E}=\frac{-\partial B}{\partial t}$ $\bar{\nabla}\times\bar{H}=J+\frac{\partial D}{\partial t}$

✱ Static ~~Magnetic~~ Electric field →

(1.) Coulomb's Force law →

$$Q_1 \xleftrightarrow[\quad d \quad]{} Q_2$$

$F \propto Q_1$ $\qquad F \propto \dfrac{1}{d^2}$

$F \propto Q_2$

$$F \propto \frac{Q_1 Q_2}{d^2}$$

$$F = \text{constant} \ \frac{Q_1 Q_2}{d^2}$$

→ (1.) Balance units $\left(\frac{1}{4\pi\epsilon}\right)$.

→ (2.) Introduce physical phenomena (ϵ)

$$F = \frac{Q_1 Q_2}{4\pi\epsilon d^2} \quad \text{———— (i)}$$

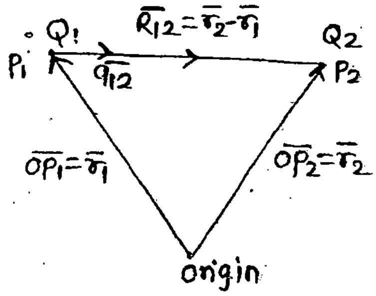
Coulomb's Force law in vector form →

$\overline{F_{12}}$ = Force on Q_2 due to Q_1 charge

$$= \frac{Q_1 Q_2}{4\pi\epsilon |\overline{R_{12}}|^2} \, \overline{a_{12}}$$

$$= \frac{Q_1 Q_2}{4\pi\epsilon |\overline{R_{12}}|^2} \cdot \frac{\overline{R_{12}}}{|\overline{R_{12}}|}$$

$$= \frac{Q_1 Q_2}{4\pi\epsilon |\overline{R_{12}}|^3} \cdot \overline{R_{12}}$$



$$\boxed{\overline{F_{12}} = \frac{1 \, Q_1 Q_2}{4\pi\epsilon |R_{12}|^3} \cdot \overline{R_{12}}}$$

Unknown $\overline{R_{12}}$

where; Q_1 = Point charge at point P_1

Q_2 = Point charge at point P_2

$|\overline{R_{12}}|$ = Distance between Q_1, Q_2

$\epsilon = \epsilon_0 \epsilon_r$

ϵ = Permitivity of the medium $\left(\frac{F}{m}\right)$

ϵ_0 = Permitivity of free space (or) air

$$\boxed{\begin{aligned} \epsilon_0 &= 8.854\times10^{-12}\ \text{Farad/m} = \frac{1}{36\pi}\times10^{-9}\ (F/m) \\ \frac{1}{4\pi\epsilon_0} &= \frac{1}{4\pi\times\frac{1}{36\pi}\times10^{-9}} = 9\times10^{9} \end{aligned}}$$

$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ → Relative permittivity (No units)

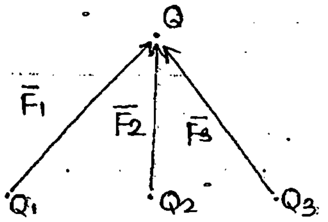
$$\boxed{\bar{F}_{21} = \text{Force on } Q_1 \text{ due to } Q_2 = -\bar{F}_{12}}$$

Note →

$F \propto Q_1$, $y \propto x \Rightarrow y = mx$ (linear eqn)

$F \propto Q_2$

Coulombs Force law is linear wrt charge; but wrt distance it is non-linear.



$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

Que → $Q_1 (3\times10^{-4} c)$ located at $P_1(1,2,3)$ & $Q_2(-10^{-4} C)$ located at $P_2(2,0,5)$ in air then find force on Q_2 due to Q_1.

Ans → $\vec{F}_{12} = \frac{Q_1 Q_2 \vec{R}_{12}}{4\pi\epsilon |\bar{R}_{12}|^3}$

$\bar{R}_{12} = \overline{OP}_2 - \overline{OP}_1$

$= (2-1)a_x + (0-2)a_y + (5-3)a_z$

$\bar{R}_{12} = a_x - 2a_y + 2a_z$

$|R_{12}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$

$$\bar{F}_{12} = (3\times10^{-4})(-10^{-4}) \times 9\times10^{9} \times \frac{1}{3^3}(a_x - 2a_y + 2a_z)$$

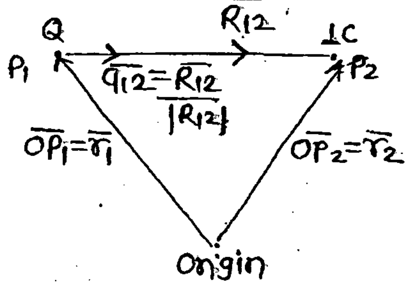
$= -10(a_x - 2a_y + 2a_z)$

$= -10a_x + 20a_y - 20a_z$

$$\boxed{\bar{F}_{12} = -10a_x + 20a_y - 20a_z}$$

Electric Field Intensity → It is defined as force on unit +ve test charge (+1C).

$$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\epsilon|\vec{R_{12}}|^2}\vec{a_{12}}$$



$$\frac{\vec{F_{12}}}{Q_2} = \frac{Q_1}{4\pi\epsilon|\vec{R_{12}}|^2}\cdot\vec{a_{12}}$$

$$\vec{E} = \frac{Q\,(\text{Col})}{4\pi\epsilon|\vec{R_{12}}|^2}\vec{a_{12}} \left(\frac{\text{Volt}}{m}\right) = \frac{Q\,\vec{R_{12}}}{4\pi\epsilon|\vec{R_{12}}|^3}$$

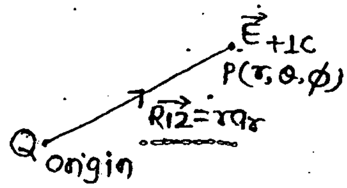
$$\frac{\text{Farad}}{m}\cdot m^2$$

Unknown ⇒ $\vec{R_{12}} = \vec{OP_2} - \vec{OP_1}$ $= P_2 - P_1$ $=$ Field ~~source~~ Point − Source point

Note:- Electric Field is linear wrt charge.

Que.→ Find electric field intensity in all the regions due to a point charge Q located at the origin.

Ans.→ Given that charge Q → solve in spherical
[In spherical sys. $P(r,\theta,\phi)$ represents all the regions]



$\vec{R_{12}}$ = field point − source point

$= r a_r$

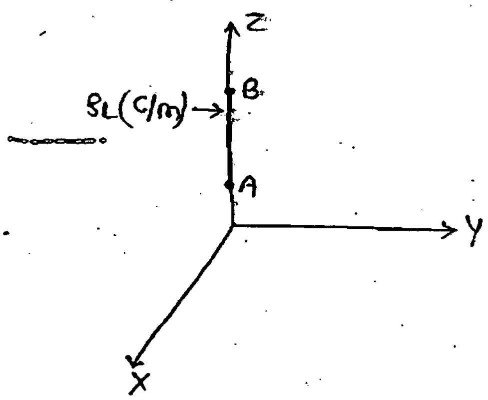
$|\vec{R_{12}}| = \sqrt{r^2+0^2+0^2} = r$

$$\vec{E_{12}} = \frac{Q}{4\pi\epsilon|R_{12}|^3}\cdot\vec{R_{12}}$$

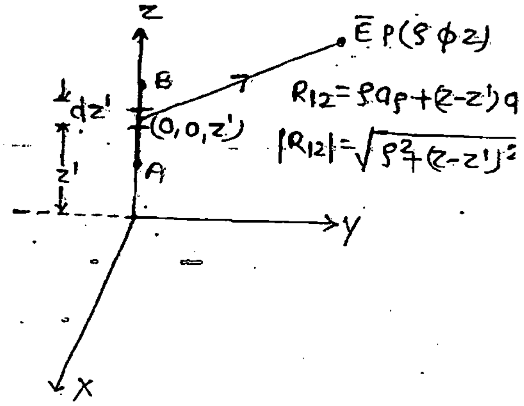
$$\vec{E_{12}} = \frac{Q}{4\pi\epsilon r^3}\cdot(r a_r)$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon r^2}\cdot a_r} \qquad \boxed{E \propto \frac{1}{r^2}}$$

Que.→ Find $\vec{E}$ in all the regions due to finite long line [having (ρ_L) C/m] lying on z-axis from point A to point B as shown below.



Sol^n→ Given line charge → cylindrical systems
In cylindrical sys. $P(\rho\ \phi\ z)$ represents all the regions.



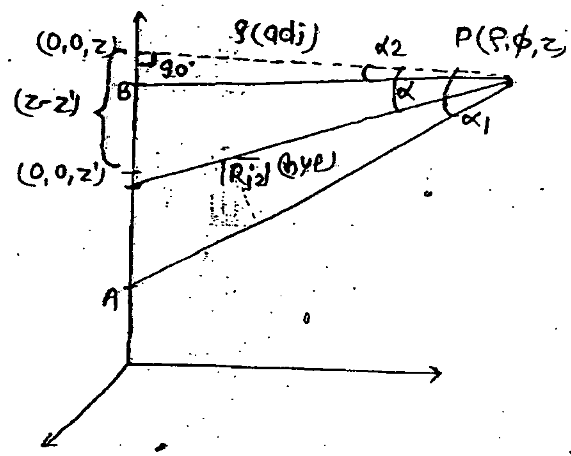
$$\vec{E} = \frac{Q}{4\pi\epsilon|\vec{R_{12}}|^3}\vec{R_{12}}$$

$$\vec{dE} = \frac{dQ}{4\pi\epsilon\,|\vec{R_{12}}|^3}\vec{R_{12}}$$

$$\vec{dE} = \frac{\rho_L dz'\,[\rho a_\rho + (z-z')\,a_z]}{4\pi\epsilon\left[\sqrt{\rho^2+(z-z')^2}\right]^3}$$

$$\vec{E} = \int \vec{dE} = \int_{z'=A}^{B} \frac{\rho_L dz'\,[\rho a_\rho + (z-z')a_z]}{4\pi\epsilon\left(\sqrt{\rho^2+(z-z')^2}\right)^3}$$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon}\int_{z'=A}^{B} \frac{\frac{\rho}{|\vec{R_{12}}|}a_\rho + \frac{(z-z')}{|\vec{R_{12}}|}a_z}{|\vec{R_{12}}|^2}\,dz' \quad \text{------ (i)}$$



$$\cos\alpha = \frac{\rho}{|\vec{R_{12}}|}, \quad |\vec{R_{12}}| = \frac{\rho}{\cos\alpha} = \rho\sec\alpha$$

$$\sin\alpha = \frac{z-z'}{|\vec{R_{12}}|}$$

$$\tan\alpha = \frac{z-z'}{\rho}\ ;\ z-z' = \rho\tan\alpha$$

differentiate wrt z'

$$0-1 = \rho\sec^2\alpha\cdot\frac{d\alpha}{dz'}$$

$$dz' = -\rho\sec^2\alpha\, d\alpha$$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon}\int_{\alpha=\alpha_1}^{\alpha_2} \frac{\cos\alpha\, a_\rho + \sin\alpha\, a_z}{\rho^2\sec^2\alpha}(-\rho\sec^2\alpha\, d\alpha)$$

$$\vec{E} = \frac{-\rho_L}{4\pi\epsilon\rho}\int_{\alpha=\alpha_1}^{\alpha_2}(\cos\alpha\, d\alpha)\, a_\rho + \int_{\alpha=\alpha_1}^{\alpha_2}(\sin\alpha\, d\alpha)\, a_z$$

$$\vec{E} = \frac{-\rho_L}{4\pi\epsilon\rho}\left[(\sin\alpha)_{\alpha_1}^{\alpha_2} a_\rho + (-\cos\alpha)_{\alpha_1}^{\alpha_2} a_z\right]$$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon\rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\, a_\rho + (\cos\alpha_2 - \cos\alpha_1)\, a_z\right] \text{ ----(ii)}$$

<u>Case→</u> Find $\vec{E}$ in all the regions due to infinite long line [having ρ_L (C/m)] lying along z-axis.

$\tan\alpha = \frac{z-z'}{\rho}$

α_1 :- angle of point A.

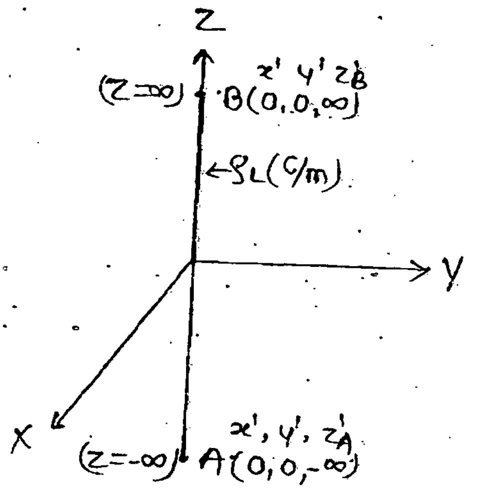
$\tan\alpha_1 = \frac{z-z_A'}{\rho} = \frac{z-(-\infty)}{\rho} = \infty$

$\alpha_1 = \tan^{-1}(\infty) = \frac{\pi}{2}$

α_2 :- angle of point B

$\tan\alpha_2 = \frac{z-z_B'}{\rho} = \frac{z-\infty}{\rho} = -\infty$

$\alpha_2 = \tan^{-1}(-\infty) = -\pi/2$



from eqⁿ (2)

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon\rho}\left[-(-1-1)\, a_\rho + 0\right]$$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon\rho}(2a_\rho)$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon\rho}\, a_\rho}$$

Note → (1) The electric field lines (electric flux lines (ψ)) or electric flux density ($\vec{D}$)] start $\perp$ to +ve charge surface in outward dirn & ends on a -ve charge surface.

(2) for ∞ long line along z-axis, the $\bar{E}$ value do not depand on z.

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon\rho}(a_\rho) = \frac{\rho_L}{2\pi\epsilon\sqrt{x^2+y^2}}(a_\rho)$$

* ρ is $\perp$ distance from line charge to field point.
* a_ρ is $\perp$ unit vector from line charge to field point.

(3.) *Due to Q the $\vec{E}$ is $\vec{E} = \frac{Q}{4\pi\epsilon r^2}a_r \;\Big|\; \left(E \propto \frac{1}{r^2}\right)$

*Due to ∞ long line with ρ_L (C/m²) the $\bar{E} = \frac{\rho_L}{2\pi\epsilon\rho}a_\rho \;\Big|\; E \propto \frac{1}{\rho}$

* $\vec{E}$ in all the regions due to a ∞ area surface having $\rho_s\left(\frac{C}{m^2}\right)$ surface charge density is

$$\boxed{E = \frac{\rho_s}{2\epsilon}\hat{a}_N}$$

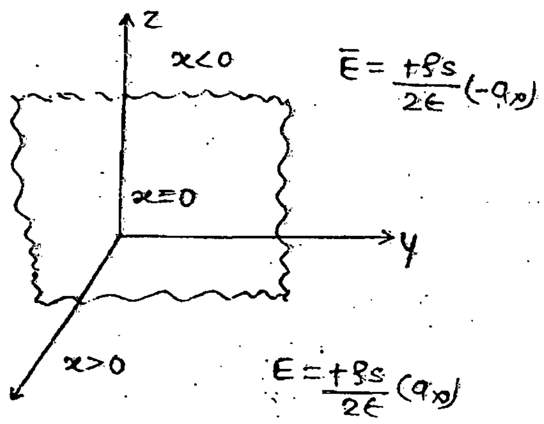
E is independant of distance.
Unknown is $\hat{a}_N$

Que → Find $\bar{E}$ in all the regions due to surface charge having surface charge density ρ_s (C/m²) on $x=0$ plane (yz plane located at $x=0$ distanc

Soln →

$$\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_N$$

$$\bar{E} = \begin{cases} \frac{\rho_s}{2\epsilon}a_x & x>0 \\ \frac{\rho_s}{2\epsilon}(-a_x) & x<0 \end{cases}$$



$\frac{1}{54}$ Electric Flux $(\psi) = \iint \bar{D}\cdot\bar{ds}$

Electric flux crossing $y=2$ plane for $0<x<4,\ 0<y<2$

y component of $\bar{D}$ crosses $y=$ constant surface.

$$\psi\Big|_{\substack{\text{crossing}\\ y=2\text{ surface}}} = \iint (yz\,a_y)\cdot(dy\,dz\,a_x + dx\,dz\,a_y + dy\,dx\,a_z)$$

$$= \int_{x=0}^{4}\int_{y=0}^{2} (2z)\,dx\,dz$$

$$= 2\left(\frac{z^2}{2}\right)_{z=0}^{2}(x)_{x=0}^{4}$$

$$= 16 \text{ Coulombs}.$$

$\frac{2}{54}$ $\bar{D} = 5(\rho-3)^2 a_\rho$

ψ crossing $\rho=4$ surface for $0<\phi<\pi,\ -5<z<5$

$$\psi\Big|_{\substack{\text{crossing}\\ \rho=4\text{ surface}}} = \iint \bar{D}\cdot\bar{ds}$$

$$= \iint D_\rho a_\rho\,\bar{ds}.\ \rho\,d\phi\,dz$$

$$= \iint 5(\rho-3)^2 a_\rho\cdot(\rho\,d\phi\,dz\,a_\rho)$$

$$= \int_{z=-5}^{5}\int_{\phi=0}^{\pi} 5(4-3)^2 4\,d\phi\,dz$$

$$= 5(1)^2(4)\int_{\phi=0}^{\pi} d\phi\int_{z=-5}^{5} dz$$

$$= 5(1)(4)(\pi)(10)$$

$$= 200\pi \text{ coulombs}.$$

$\frac{3}{54}$ $\bar{D} = z\rho(\cos^2\phi)\,\bar{a}_z\ \ c/m^2$

Find volume charge density at $(1, \pi/4, 3)$
$\quad\rho\quad\phi\quad z$

$$\rho_v = \vec{\nabla}\cdot\vec{D}$$

$$\frac{c}{m^2} \doteq \frac{1}{m}\cdot\frac{c}{m^2}$$

$$\rho_v = \cancel{\partial D_z}\ \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

$$\rho_v = \frac{\partial D_z}{\partial z} = \frac{\partial}{\partial z}(z\rho\cos^2\phi) = \rho\cos^2\phi\,\frac{\partial}{\partial z}(z)$$

$$= \rho\cos^2\phi\,(1)$$

$$\underset{\rho\ \ \phi\ \ z}{\rho_v(1,\ \pi/4,\ 3)} = 1\left[\cos(\pi/4)\right]^2$$

$$= 0.5\ C$$

$\frac{4}{54}$

$$D = z\rho(\cos^2\phi)\ \bar{a}_\rho\ C/m^2$$

$$\rho_v = \bar{\nabla}\cdot\bar{D}$$

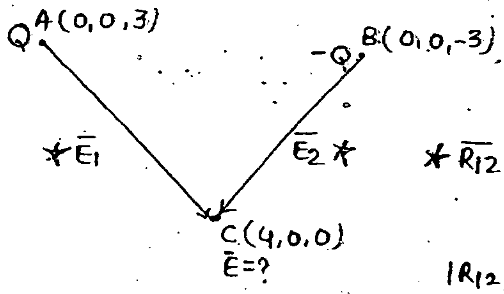
$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\times z\rho\cos^2\phi)$$

$$= \frac{z\cos^2\phi\cdot 2\rho}{\rho}$$

$$= 2z\cos^2\phi$$

$$\rho_v\Big|_{(1,\frac{\pi}{4},3)} = 3\ C/m^2$$

$\frac{5}{54}$



$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

* $\bar{R}_{12}$ = field point − source point

$$= 4a_x - 3a_z$$

$$|R_{12}| = \sqrt{4^2+3^2} = 5$$

* $\bar{R}_{12} = 4a_x + 3a_z$

$$|R_{12}| = 5$$

$$E_1 = \frac{Q}{4\pi\epsilon(5)^3}(4a_x - 3a_z) \qquad E_2 = \frac{Q}{4\pi\epsilon(5)^3}(4a_x + 3a_z)$$

$$E = E_1 + E_2 = \frac{Q}{4\pi\epsilon(5)^3}\left[\overset{4a_x - 4a_z}{\cancel{8a_x} - 6a_z}\right]$$

$$= \frac{Q}{4\pi\epsilon(5)^3}(-6a_z)$$

→ −ve z-dirn

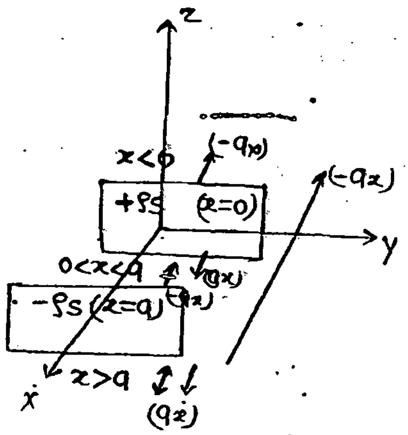
Que.→ Find $\bar{E}$ in all the region due to a surface charge having $+\rho_s\left(\frac{C}{m^2}\right)$ on $x=0$ surface; $-\rho_s(C/m^2)$ on $x=a$ surface.

Soln→

$x=0$ surface (yz surface)

$x=a$ surface (yz surface)

Because of ρ_s the $\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_N$



$x<0$; $\bar{E} = \bar{E}_{(+\rho_s)} + \bar{E}_{(-\rho_s)}$

$= \frac{+\rho_s}{2\epsilon}(-a_x) + \left(\frac{-\rho_s}{2\epsilon}\right)(-a_x)$

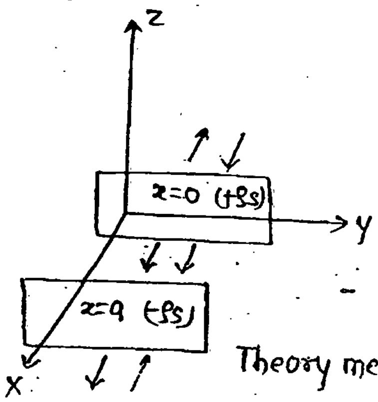
$= 0$

$0<x<a$; $\bar{E} = \frac{(+\rho_s)}{2\epsilon}(a_x) + \left(\frac{-\rho_s}{2\epsilon}\right)(-a_x)$

$= \frac{\rho_s}{\epsilon}a_x$

$x>a$; $\bar{E} = \frac{(+\rho_s)}{2\epsilon}(a_x) + \left(\frac{-\rho_s}{2\epsilon}\right)(+a_x) = 0$

$$\boxed{\bar{E} = \begin{cases} 0 & ; x<0 \\ \frac{\rho_s}{\epsilon}a_x & ; 0<x<a \\ 0 & ; x>a \end{cases}}$$



Theory method

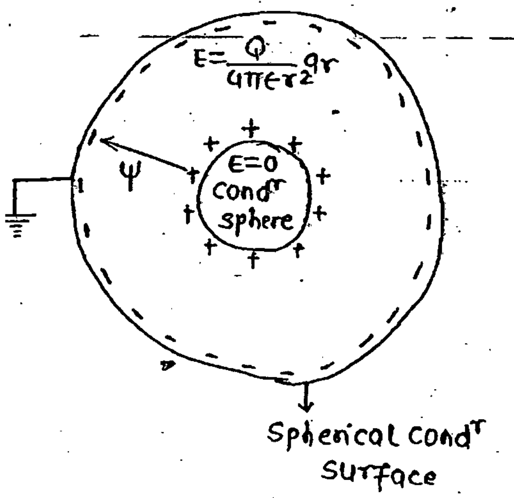
★ Electric flux→

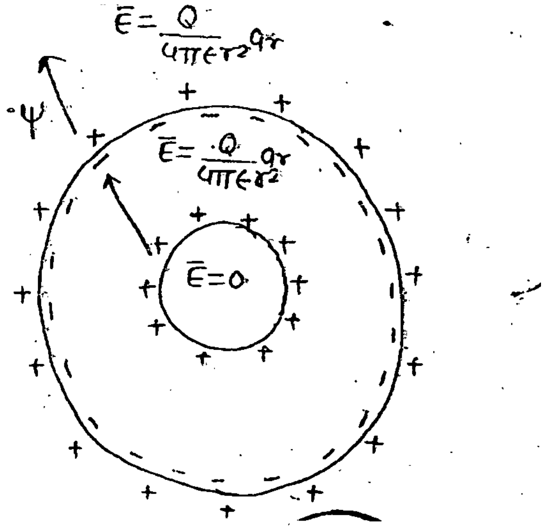
flux ⟶ Flow

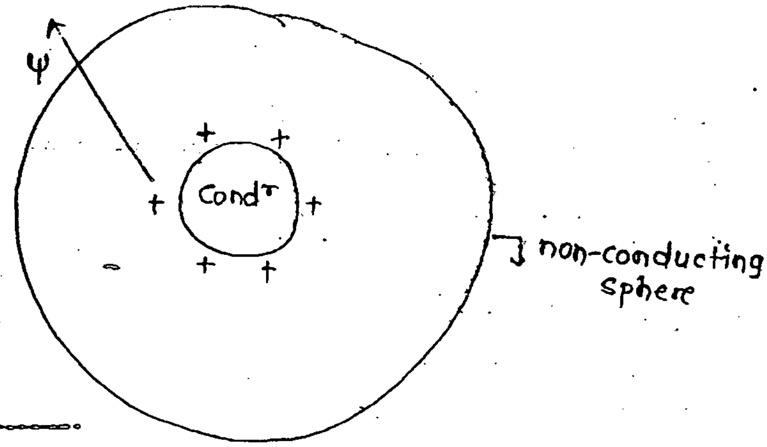
electric flux → Flow of electric effect.

Ψ = Electric flux

$\boxed{\Psi = Q}$







★ Electric flux density ($\vec{D}$) → It is defined as ratio of electric flux to area of the surface (through which electric flux is crossing).

$$\vec{D} = \frac{\text{Electric flux}}{\text{area of the surface (through which electric flux is crossing)}} \times \hat{a}_N$$

$$\vec{D} = \frac{\psi}{\text{Area}}\,\hat{a}_N \left(\frac{\text{coulombs}}{m^2}\right)$$

★★★

Given $\vec{D}$ then electric ~~field~~ flux (ψ) is given by

$$\psi = \iint \vec{D}\cdot\vec{ds}$$

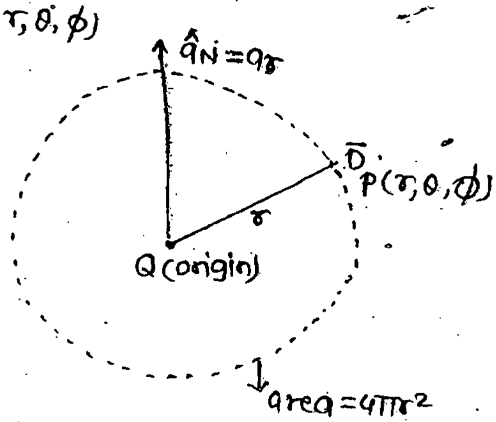
$$\left(\frac{\psi}{\text{area}}\right)(\text{area})$$

Not Required

Que→ Find $\vec{D}$ in all the region due to point charge Q located at the origin

Soln→

point charge ⟶ spherical co-ordinate (r, θ, ϕ)



$$\bar{D} = \frac{\psi}{area} \times \hat{a}_N = \frac{Q}{area}\hat{a}_N$$

$$\bar{D} = \frac{Q}{4\pi r^2} a_r \text{ ------(i)}$$

We know that for point charge Q located at origin

$$\bar{E} = \frac{Q}{4\pi\epsilon r^2} a_r$$

$$\epsilon\bar{E} = \frac{Q}{4\pi r^2} a_r \text{ ------(ii)}$$

Eqn (i) & (ii)

$$\boxed{D = \epsilon E}$$

Gauss law→ This says that a total electric flux crossing a closed surface is equal to the charge enclosed (inside) by that closed surface (or) Gaussian surface.

mathematicaly $\psi = Q$ ------(i)

By defination $\psi = \iint \bar{D}\cdot\bar{ds}$ ------(ii)

From above eqn

$$\boxed{Q_{enclosed} = \oint\!\!\oint \bar{D}\cdot\bar{ds}} \text{ ------(iii)}$$

Unknown quantity.

Que→ Find $\vec{E}$ in all the regions due to ∞ long line charge having $+\rho_l$ (C/m) lying along the z-axis.

Soln→

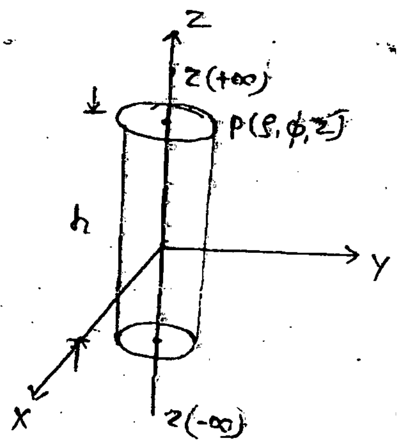
$$\oint\!\!\oint \bar{D}\cdot\bar{ds} = Q_{enclosed} \text{ ----(i)}$$

We know that in cylindrical sys.

$$\oint\!\!\oint \bar{D}\cdot\bar{ds} = D_\rho\, 2\pi\rho h \text{ -----(2)}$$

$Q_{enclosed}$ for "h" height = $Q_{enclosed} = \rho_L h$ ------ (3.)

Put (2), (3) in (1.) $D_\rho(2\pi\rho h) = \rho_L h$



$$D_\rho = \frac{\rho_L}{2\pi\rho}$$

$$\bar{D} = D_\rho a_\rho$$

$$\bar{D} = \frac{\rho_L}{2\pi\rho} a_\rho \text{ ------ (4.)}$$

$$\bar{E} = \frac{D}{\epsilon} \text{ ------ (5.)}$$

From (4) & (5.)

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho} \text{ ------ (6.)}$$

$$* \oint\!\!\oint \vec{D}\cdot\vec{ds} = \oint\!\!\oint (D_\rho a_\rho + D_\phi a_\phi + D_z a_z)\cdot \vec{ds}$$

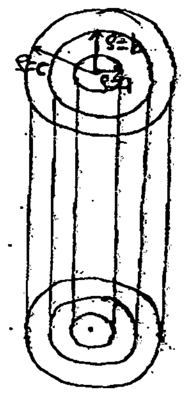
$$= \oint\!\!\oint (D_\rho a_\rho)\cdot(\rho\, d\phi\, dz\, a_\rho)$$

$$= \oint\!\!\oint D_\rho \rho\, d\phi\, dz$$

$$= D_\rho \rho \int_{\phi=0}^{2\pi} d\phi \cdot \int_{z=-h/2}^{h/2} dz = D_\rho(2\pi\rho h)$$

(IES conventional).

Que.→ Find E in all the regions of a ∞ long ~~line~~ coaxial cable along z-axis having $+\rho_{si}$ (C/m²) on $\rho = a$ cylindrical surface; $-\rho_{so}\left(\frac{C}{m^2}\right)$ on $\rho = b$ cylindrical surface as shown below.



soln → Coaxial cable ⟶ cylindrical sys.

*(1.) $\bar{E}$ in the region $0<\rho<a$

$\oiint \bar{D}\cdot\bar{ds} = Q_{enclosed}$

$D_\rho(2\pi\rho h) = 0$

$D_\rho = 0$

$\bar{D} = D_\rho a_\rho = 0$ ------(i)

$\rho_s = \frac{Q}{Area}$

$\rho_{si} = \frac{Q}{2\pi a h}$ | $\rho_{so} = \frac{-Q}{2\pi b h}$

$Q = \rho_{si}\cdot 2\pi a h$ $\quad Q = -\rho_{so}\, 2\pi b h$

$\rho_{si}(2\pi a h) = -\rho_{so}(2\pi b h)$

$$\boxed{\rho_{so} = -\rho_{si}\left(\frac{a}{b}\right)}$$

*(2.) $\vec{E}$ in the region $a<\rho<b$

$\oiint \bar{D}\cdot\bar{ds} = Q_{enclosed}$

$$D_\rho \underbrace{(2\pi\rho h)}_{\text{Gaussian surface area}} = \rho_{si}\underbrace{(2\pi a h)}_{\text{Charge surface area}}$$

$D_\rho = \rho_{si}\left(\frac{a}{\rho}\right)$

$\bar{D} = D_\rho a_\rho = \rho_{si}\left(\frac{a}{\rho}\right)a_\rho$ ------(ii)

*(3.) $\bar{E}$ in the region $\rho>b$

$\oiint \vec{D}\cdot\vec{ds} = Q_{enclosed}$

$D_\rho(2\pi\rho h) = Q_{inner}$ at a $+ Q_{outer}$ at b

$= \rho_{si}(2\pi a h) + \rho_{so}(2\pi b h)$

$= \rho_{si}(2\pi a h) - \rho_{si}\left(\frac{a}{\cancel{b}}\right)(2\pi \cancel{b} h)$

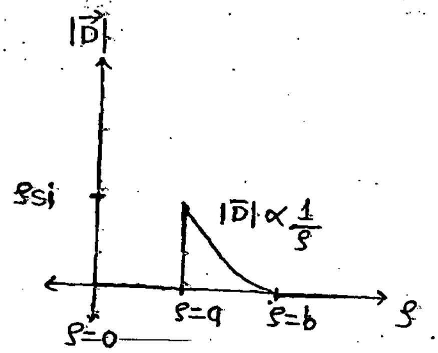
$D_\rho(2\pi\rho h) = 0$

$D_\rho = 0$

$\bar{D} = 0$ ------(iii)

$$\bar{D} = \begin{cases} 0 & ;\ 0<\rho<a \\ \rho_{si}\left(\frac{a}{\rho}\right)a_\rho & ;\ a<\rho<b \\ 0 & ;\ \rho>b \end{cases}$$

$$\bar{E} = \begin{cases} 0 & ;\ 0<\rho<a \\ \frac{\rho_{si}}{\epsilon}\left(\frac{a}{\rho}\right)a_\rho & ;\ a<\rho<b \\ 0 & ;\ \rho>b \end{cases}$$



Divergence →

$$\mathrm{div}\,\vec{A} = \vec{\nabla}\cdot\vec{A} = \lim_{\Delta v \to 0} \frac{\oiint \vec{A}\cdot\vec{ds}}{v} \text{ ------ (i)}$$

Maxwell equation →

Let $\bar{A}=\bar{D}$ in eqn (i)

$$\overline{\nabla\cdot D} = \lim_{\Delta v \to 0} \frac{\oiint \bar{D}\,\bar{ds}}{\Delta v} \text{ ------ (ii)}$$

~~From eqn (i) & (ii)~~

& Gauss law $\oiint \bar{D}\,\bar{ds} = Q$ ------ (iii)

From eqn (ii) & (iii)

$$\bar{\nabla}\cdot\bar{D} = \lim_{\Delta v = 0} \frac{Q}{\Delta v} \text{ ------ (iv)}$$

by defination:

$$\rho_v = \lim_{\Delta v = 0} \frac{Q}{\Delta v} \text{ ------ (v)}$$

from (v) in (iv)

$$\boxed{\bar{\nabla}\cdot\bar{D} = \rho_v}$$

Divergence Theorem →

Gauss law $\oiint \bar{D}\cdot\bar{ds} = Q$ ----- (i)

By defination $Q = \iiint \rho_v \cdot dv$ ----- (ii) $\quad \because \oiint$ surface = volume

from (ii) & (i)

$$\oiint \bar{D}\cdot\bar{ds} = \iiint \rho_v \cdot dv \text{ ----- (iii)}$$

from maxwell eqn $\bar{\nabla}\cdot\bar{D} = \rho$ ----- (iv)

From eq^n (iii) & (iv)

$$\boxed{\oint\!\!\oint \bar{D}\cdot\bar{ds} = \iiint (\bar{\nabla}\cdot\bar{D})\, dv}$$

Let $\bar{D} = \bar{A}$

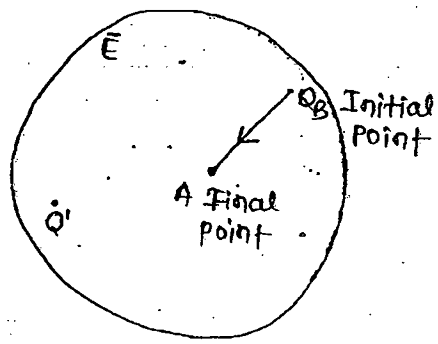
$$\boxed{\oint\!\!\oint \bar{A}\cdot\bar{ds} = \iiint (\bar{\nabla}\cdot\bar{A})\, dv}$$

* **Work done →** Work done in moving a Q (coulomb) charge in presence of a electric field ($\bar{E}$) is given by

$$W \propto Q \quad V$$

$$W = Q\left(-\int_B^A \bar{E}\cdot\bar{dl}\right)$$

$$\boxed{W = -Q\int_{\text{initial}(B)}^{\text{final }A} \bar{E}\cdot\bar{dl}} \quad \text{---(i)}$$



$$\boxed{W_{capacitor} = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}}$$

($\because Q = CV$)

$W \propto QV$

$W \propto QV$

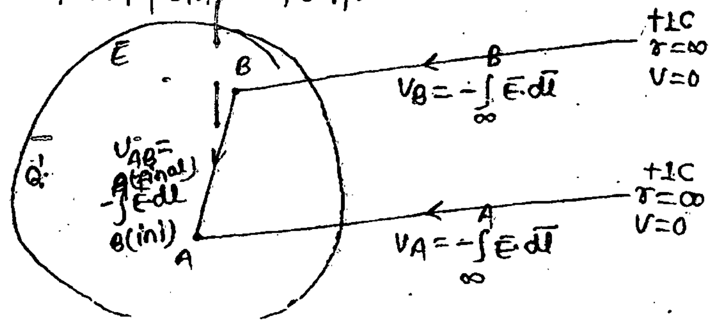
* If $\bar{E}$ is uniform (i.e. $\bar{E}$ is not a function of x, y, z) then

$$W = -Q\bar{E}\cdot\int_B^A \bar{dl}$$

$$\boxed{W = -Q\vec{E}\cdot\vec{L}_{BA}}$$

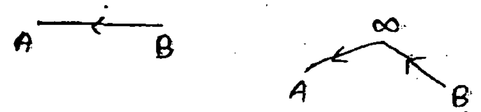
Where $\vec{L}_{BA}$ is length vector from point B to A.

* **Potential difference →** It is defined as work done in moving a +1C charge from point B to A.



$$\boxed{\begin{aligned} V_{AB} &= \text{Potential at point A wrt B} \\ &= \text{Potential diff b/n point A \& point B} \\ V_{AB} &= -\int_{\text{ini}(B)}^{\text{Fin}(A)} \bar{E}\cdot\bar{dl} \end{aligned}}$$

$$V_{AB} = -\int_B^A \bar{E}\cdot\bar{dl} = -\left[\int_B^{\infty} \bar{E}\cdot\bar{dl} + \int_{\infty}^{A} \bar{E}\cdot\bar{dl}\right]$$

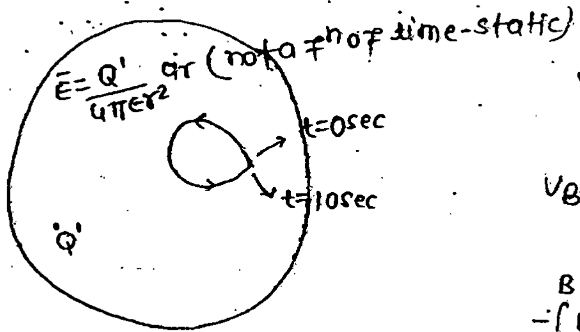


$$V_{AB} = -\left[-\int_{\infty}^{B} \bar{E}\cdot\bar{dl} + \int_{\infty}^{A} \bar{E}\cdot\bar{dl}\right]$$

$$= -\left[V_B - V_A\right]$$

$$\boxed{V_{AB} = V_A - V_B}$$

Observation →



$$V_{AB} = -\int_B^B E\cdot dl$$

$$V_{BB} = \underset{(10s)}{V_B} - \underset{(0s)}{V_B} = 0$$

$$-\int_B^B E\,dl = 0$$

$$-\oint E\,dl = 0$$

$$\boxed{\oint \bar{E}\cdot\bar{dl} = 0}$$

Note:- (1.) $\oint \bar{E}\cdot\bar{dl} = 0$ ----(i)

Stokes ~~theorem~~

$$\oint E\cdot dl = \iint (\bar{\nabla}\times\bar{E})\,\bar{ds} \text{ -----(ii)}$$

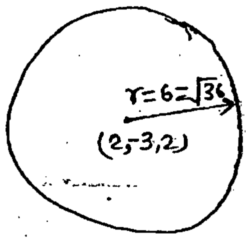
from (1) & (2)

$$0 = \iint (\bar{\nabla}\times\bar{E})\cdot \bar{ds}$$

$$\iint 0\, ds = \iint (\bar{\nabla}\times\bar{E})\cdot \bar{ds}$$

$$\boxed{\bar{\nabla}\times\bar{E} = 0 \text{ i.e. static E field is irrotational.}}$$

7/54

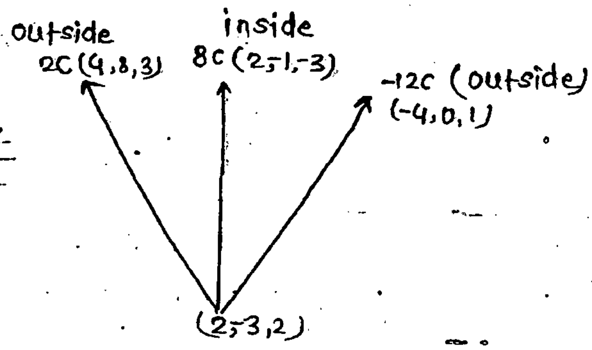


$R_{12} = 2a_x + 11a_y + a_z$

$|R_{12}| = \sqrt{2^2+11^2+1^2}$

$> \sqrt{36}$

Ans (c)



8/54 Line charge with $\rho_L = 15\,nC/m$ located at $x=2, y=4$ parabola to z-axis. Find flux crossing the cube $-5<x<5$
$-5<y<5$
$-5<z<5$

Ans. (a)

* line is parallel to z-axis, line length inside the cube $= 5-(-5) = 10m$

ψ charge inside the cube $= \rho_L l = 15\times10^{-9}\times10$

$= \underline{150\,nC.}$

9/54 Flux leaving the side of cube $-5<x<5$
(only one side) $-5<y<5$
$-5<z<5$

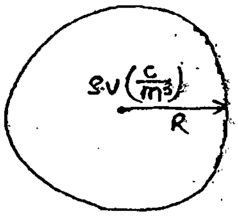
6 finite area surfaces form a closed surface $= \psi = Q = 6nC$

1 finite area .... $\dfrac{\text{closed surface}}{6} = \dfrac{\psi}{6} = \dfrac{\phi}{6} = \dfrac{6nC}{6} = \underline{1nC.}$

10/54 2-∞ area parallel planes form a closed surface $= \psi = \phi = 6nC$

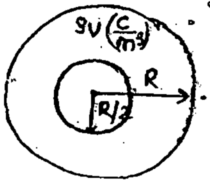
One ∞ area .... $\dfrac{\text{closed surface}}{2} = \dfrac{\psi}{2} = \dfrac{\phi}{2} = \underline{3nC}$

(11/54)



$$\rho_v = \frac{\text{charge}}{\text{Volume}} = \frac{Q}{\frac{4}{3}\pi R^3}$$

(i)



Electric field at $r = R/2$

$$\oint \bar{D}\cdot d\bar{s} = Q_{enclosed}$$

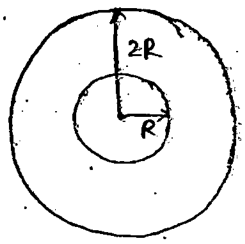
$$D_r \cdot 4\pi r^2 = \rho_v \cdot \frac{4}{3}\pi (R/2)^3$$

$$D_r \cdot 4\pi \left(\frac{R}{2}\right)^2 = \rho_v \frac{4\pi}{3}\left(\frac{R}{2}\right)^3$$

$$D_r = \frac{\rho_v R}{6}$$

$$E_r = \frac{D_r}{\epsilon} = \frac{\rho_v R}{6\epsilon}$$

(ii) Electric field at $r = 2R$;



$$\oint \bar{D}\cdot d\bar{s} = Q_{enclosed}$$

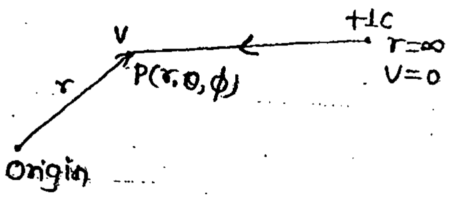
$$D_r \cdot 4\pi r^2 = \rho_v \cdot \frac{4}{3}\pi (R)^3$$

$$D_r \cdot 4\pi (2R)^2 = \rho_v \cdot \frac{4}{3}\pi (R)^3$$

$$D_r = \frac{\rho_v R}{12}, \quad E = \frac{D}{\epsilon} = \frac{\rho_v R}{12\epsilon}$$

<u>Que.→</u> Find the potential field in all the regions of a point charge Q Coulomb located at the origin.

Solⁿ→.



$$V = -\int_{r=\infty}^{r} \bar{E}\cdot d\bar{l} \text{ ------ (i)}$$

for point charge Q located at origin

$$\bar{E} = \frac{Q}{4\pi\epsilon r^2} a_r \text{ ------- (ii)}$$

From (ii) & (i)

$$V = -\int_{\infty}^{r} \left(\frac{Q}{4\pi\epsilon r^2} a_r\right) dr\, a_r$$

$$V = -\int_{\infty}^{r} \frac{Q}{4\pi\epsilon r^2} a_r$$

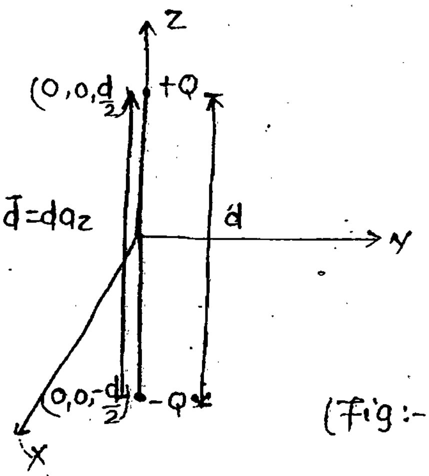
$$= \frac{-Q}{4\pi\epsilon} \int_{\infty}^{r} \frac{1}{r^2} a_r$$

$$= \frac{-Q}{4\pi\epsilon} \left(\frac{-1}{r} \right)_{\infty}^{r}$$

$$V = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r} - \frac{1}{\infty} \right)$$

$$\boxed{V = \frac{Q}{4\pi\epsilon r}} \qquad V \propto \frac{1}{r}$$

* **Electric dipole →** It has +ve, -ve charges of equal magnitude, they are seprated by a distance 'd' which is very small compare to the distance r where we measure electric, potential fields.



(Fig:-1)

* Electric dipole moment $\vec{P}$ is defined as

$$\boxed{\begin{aligned} \bar{P} &= Q\bar{d} \quad \text{(coulomb·meter)} \\ &= Q.d a_z \end{aligned}}$$

* Polarisation $(\bar{P})$ is defined as total electric dipole moment per unit volume

$$\bar{P} = \frac{\bar{P}_{total}}{Volume} = \frac{\sum_i \bar{P}_i}{Volume} \left(\frac{cm}{m^3} \right)$$

$$\boxed{\bar{P}\left(\frac{c}{m^2}\right)}$$

IES

Que.→ Find $\vec{E}$, V in all the regions due to electric dipole shown in fig.1.

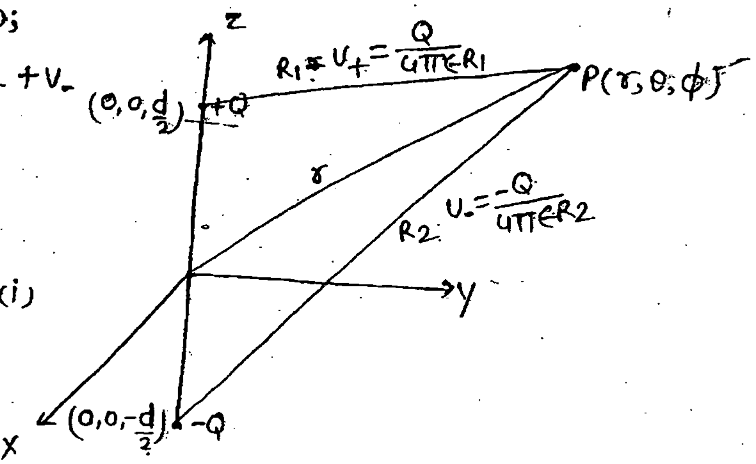
Sol^n→ **V:-** Given point charges – spherical sys. In spherical $P(r, \theta, \phi)$ represents all the regions.

Potential is linear, so;

$$V_{P(r,\theta,\phi)} = V_+ + V_-$$

$$= \frac{Q}{4\pi\epsilon R_1} + \frac{-Q}{4\pi\epsilon R_2}$$

$$= \frac{Q}{4\pi\epsilon}\left(\frac{R_2 - R_1}{R_1 R_2}\right) \text{ ------- (i)}$$

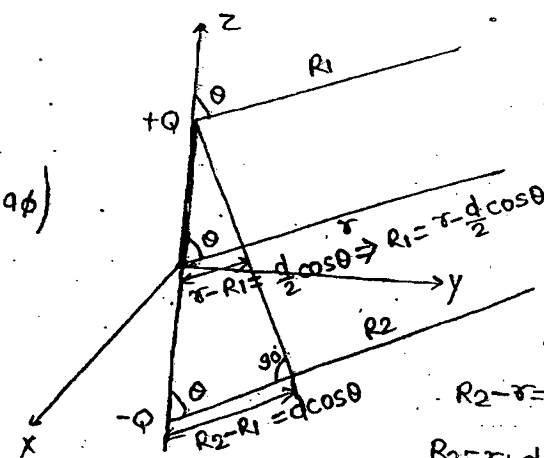


★★★ At all points on xy plane (z=0 plane)

$R_1 = R_2$; so $V = 0$

This plane is called as equipotential surface

$$V = \frac{Q \cdot d\cos\theta}{4\pi\epsilon r^2} \text{ ------- (2.)}$$



For P: $P(r,\theta,$

$R_2 - r = \frac{d}{2}\cos\theta$

$R_2 = r + \frac{d}{2}\cos\theta$

$$\vec{E} = -\vec{\nabla}V$$

$$= -\left(\frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} a_\phi\right)$$

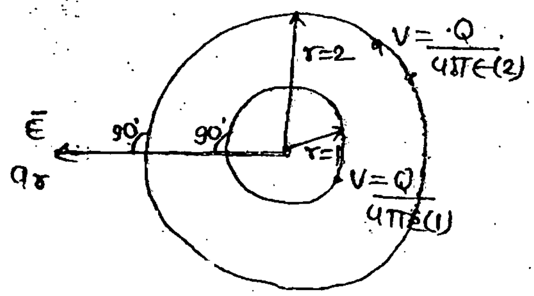
$$\vec{E} = -\left[\frac{\partial}{\partial r}\left(\frac{Q \cdot d\cos\theta}{4\pi\epsilon r^2}\right) a_r + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{Q d\cos\theta}{4\pi\epsilon r^2}\right) a_\theta + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\phi}\left(\frac{Q \cdot d\cos\theta}{4\pi\epsilon r^2}\right) a_\phi\right]$$

$$\vec{E} = -\left[\frac{Q d\cos\theta}{4\pi\epsilon}\left(\frac{-2}{r^3}\right) a_r + \frac{1}{r}\left(\frac{Qd}{4\pi\epsilon r^2}\right)(-\sin\theta) a_\theta\right]$$

$$\vec{E} = \frac{Qd}{4\pi\epsilon r^3}\left(2\cos\theta\, a_r + \sin\theta\, a_\theta\right) \text{ ------- (3.)}$$

Dueto
Note→ (1.) Point charge ~~due to~~

$$V=\frac{Q}{4\pi\epsilon r} \quad \boxed{V\propto\frac{1}{r}} \quad \Big| \quad \vec{E}=\frac{Q}{4\pi\epsilon r^2}a_r \quad \boxed{E\propto\frac{1}{r^2}}$$



* For a point charge equipotential surfaces are concentric spheres whose center is located at the point charge.
* The equipotential surfaces, electric field lines meet at 90° angle (or) ⊥ to each other.

(2.) Due to electric dipole

$\because a\cdot b=|a||b|\cos\theta$

$$V=\frac{Qd\cos\theta}{4\pi\epsilon r^2}=\frac{p\cdot a_r}{4\pi\epsilon r^2}=\frac{Q d\, a_z\cdot a_r}{4\pi\epsilon r^2}=\frac{Qd\,|a_z||a_r|\cos\theta}{4\pi\epsilon r^2}$$

$$\boxed{V\propto\frac{1}{r^2}}$$

due to dipole $\boxed{E\propto\frac{1}{r^3}}$

Note:- $Q=CV$; $W_{capacitor}=\frac{1}{2}CV^2=\frac{1}{2}QV=\frac{1}{2}\frac{Q^2}{C}$

(1.) Energy store in the sys. of point charges $(Q_1, Q_2, \ldots Q_N)$ is

$$W=\frac{1}{2}\sum_{i=1}^{N}Q_iV_i \quad (\text{Joule})$$

where; V_1 = potential at the locn ~~of Q~~ $_1$ charge due to other charges.

V_N = Potential at the locn of Q_N charge due to other charges.

Energy stored in electric field is farad · volt^2 = Joule

(2.) NT → $Q = CV$ ∴ $\omega_{capacitor} = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}Q^2/C$

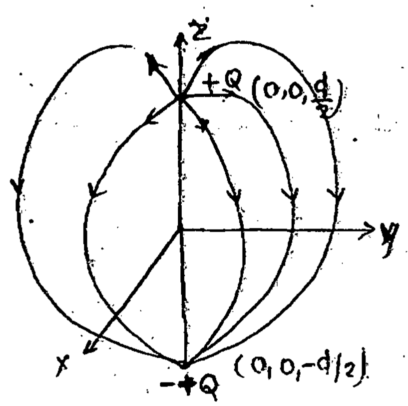
EMT → $\bar{D} = \epsilon\bar{E}$ ∴ $\omega_{electric\ field} = \frac{1}{2}\epsilon|\bar{E}|^2 = \frac{1}{2}\bar{D}\cdot\bar{E} = \frac{1}{2}|D|^2/\epsilon$

$\frac{Farad}{m} \cdot \frac{V^2}{m^2} = \frac{Joule}{m^3}$

$= \iiint \frac{1}{2}\epsilon|E|^2 dv = \iiint \frac{1}{2}\bar{D}\cdot\bar{E}\, dv = \iiint \frac{1}{2}|D|^2/\epsilon$

(3.) Electric field energy density $= \frac{1}{2}\epsilon|E|^2 = \frac{1}{2}\bar{D}\cdot\bar{E} = \frac{1}{2}\frac{|\bar{D}|^2}{\epsilon}$ $\left(\frac{Joule}{m^3}\right)$

* Electric dipole & image theory →



xy plane (z=0) plane is equipotential plane (or) surface

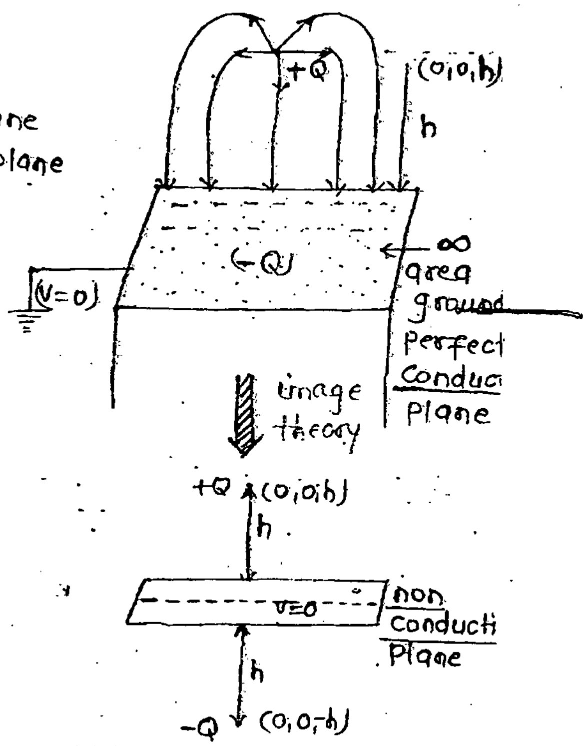
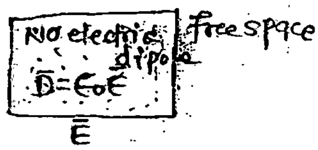


*Image theory → This says the charge Q in the presence of a ∞ area perfect conducting grounded plane is replaced by a charge itself, its image, & zero potential cond^r plane is replaced by zero potential non-cond^r plane.

* Dielectric material & polarization ($\vec{P}$) →

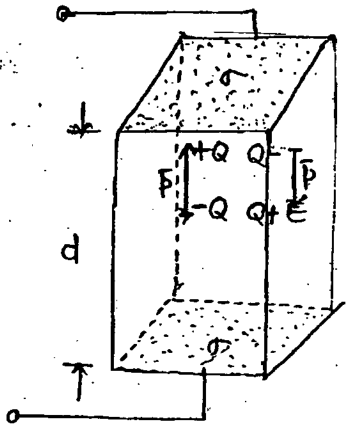
In free space {no free $\bar{e}$, no electric dipole, no megnatic dipole}

$\bar{D} = \epsilon_0\bar{E}$ -----(i)



Dielectric	Insulator
$I_c=0$	$I_c=0$
Electric dipole	Not ×

In dielectric material, electric dipoles are present, when external voltage (electric field) is not applied then the situation is shown below.

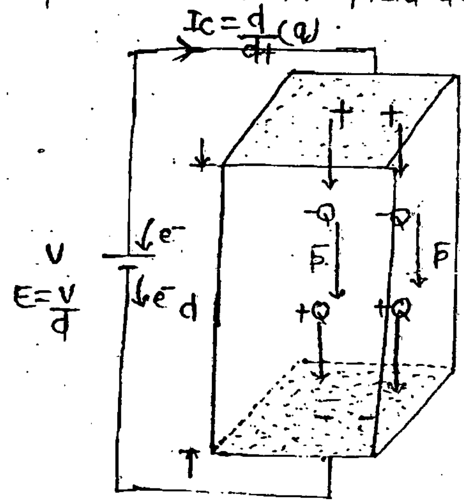


$V=0$

$E=\frac{V}{d}=\frac{0}{d}=0$

$\bar{P}=\frac{\Sigma p_i}{\text{volume}}=\frac{0}{\text{volume}}=0$ (Because of dipole)

when external electric field is applied then the electric dipole align in the dir^n of external electric field as shown below.



I_c = cond^n current

$I_d=\frac{d\psi}{dt}$ (amp) (displacement current)

$I_c=I_d$

$\bar{P}=\frac{p_{total}}{\text{volume}}\neq 0$

$\bar{D}=\epsilon_0\bar{E}$ -------(i) {free space eq^n}

$\bar{D}=\epsilon_0\bar{E}+\bar{P}$ ----(ii)

In this case $\bar{p}$ (C/m^2) is extra compare to free space. so $\bar{p}$ is added to eq^n (i) [free space eq^n]

$\boxed{\bar{D}=\epsilon_0\bar{E}+\bar{P}}$ Applicable for any materials

Case → ~~Linear dielectric material~~ →

- linear ⇔ o/p ∝ i/p

$$\bar{P} \propto \bar{E}$$

$$P = \text{Constant}\ \bar{E}$$

→ Balance units (ϵ_0)

→ Introduce physical phenomenon (χ_e) chi

$$\bar{P} = \chi_e \epsilon_0 \bar{E} \quad \text{------} (3.)$$

where χ_e → Electrical suceptibility (Receive)
(unit less)

From eqn (3) in (2.)

$$\bar{D} = \epsilon_0 \bar{E} + \chi_e \epsilon_0 \bar{E}$$

$$\bar{D} = \epsilon_0 (1 + \chi_e) \bar{E} \quad \text{------} (4.)$$

Let $\epsilon_r = (1 + \chi_e)$ ------- (5.)

$\therefore \epsilon_r = 1 + \chi_e$ · (For free space $\chi_e = 0$)

ϵ_r ↓ Relative Permitivity (Unit less)
1 ↓ Free space

From eqn (5.) in (4.)

$$\bar{D} = \epsilon_0 \epsilon_r \bar{E} \quad \text{------} (6.)$$

Let $\epsilon_0 \epsilon_r = \epsilon$ ------ (7.)

From (7.) in (6.) $\boxed{\bar{D} = \epsilon \bar{E}}$ ----- (8)

Applicable for only linear material.

*****Note** → (1.) $\bar{D} = \epsilon_0 \bar{E} + \bar{P}$ (For any material) ---- (i)

(2.) For linear material

$$\bar{D} = \epsilon E \quad \text{-----} (2)$$

$\epsilon = \epsilon_0 \epsilon_r$ | $\epsilon_r = 1 + \chi_e$
$\chi_e = \epsilon_r - 1$

$$\bar{P} = \chi_e \epsilon_0 \bar{E}$$

$$\bar{P} = (\epsilon_r - 1) \epsilon_0 \bar{E} \quad \text{-----} (3)$$

from eqⁿ (3.) $\bar{E} = \dfrac{\bar{P}}{(\epsilon_r - 1)\epsilon_0}$ ------(4)

from eqⁿ (4) in (i)

$$\bar{D} = \cancel{\epsilon_0}\frac{\bar{P}}{(\epsilon_r-1)\cancel{\epsilon_0}} + \bar{P}$$

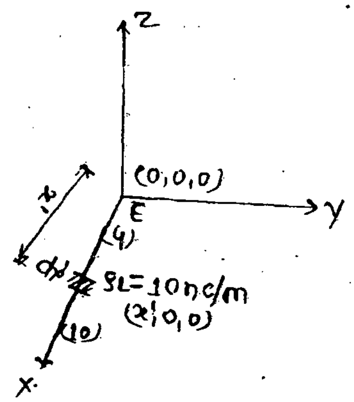
$$\bar{D} = \bar{P}\left(\frac{1}{\epsilon_r - 1} + 1\right)$$

$$\bar{D} = \bar{P}\left(\frac{\epsilon_r}{\epsilon_r - 1}\right)$$

$$\bar{P} = \left(\frac{\epsilon_r - 1}{\epsilon_r}\right)\bar{D}$$

$$\boxed{\bar{P} = \left(1 - \frac{1}{\epsilon_r}\right)\bar{D}}$$

12/55 $dE = \dfrac{dQ}{4\pi\epsilon|R_{12}|^3}\bar{R}_{12}$



R_{12} = Field point − Source point

$= -x'a_x$ [(0,0,0) − (x',0,0)]

$|R_{12}| = \sqrt{(-x')^2} = x'$

$$dE = \frac{\rho_L dx'}{4\pi\epsilon_0 (x')^3}(-x'a_x) = \frac{-\rho_L dx'}{4\pi\epsilon_0 (x')^2}a_x$$

$$\bar{E} = \int_4^{10} \frac{-\rho_L dx'}{4\pi\epsilon_0 (x')^2}a_x = \frac{-10\times10^{-9}}{4\pi\epsilon_0}\int_{x'=4}^{10}\frac{dx'}{(x')^2}a_x$$

$$\bar{E} = -10\times10^{-9}\times 9\times10^{9}\left(\frac{-1}{x'}\right)_{x=4}^{10} a_x$$

$$\bar{E} = 90\left(\frac{1}{10} - \frac{1}{4}\right)a_x$$

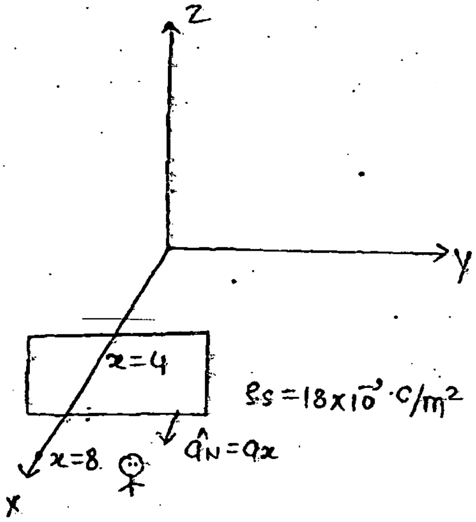
$$\boxed{\bar{E} = -13.5\,a_x}$$

20/55 $\vec{E}$ due to ρ_s is $\vec{E} = \dfrac{\rho_s}{2\epsilon}\hat{a}_N$

given $\rho_s = 10\,nc/m^2$ at $x = 4$ then $\bar{E}$ at $x = 8$? (8,0,6)

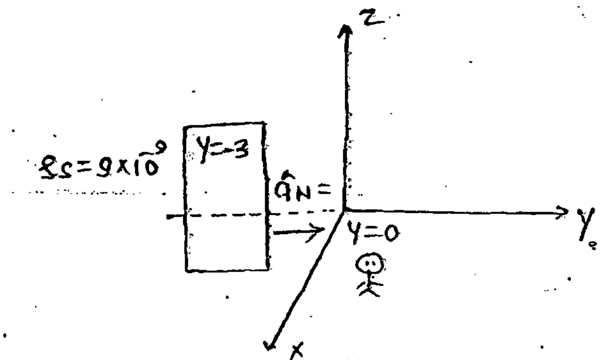
$\rho_s = 9\,nc/m^2$ at $y = -3$ then $\bar{E}$ at $y = 0$? ~~(8,0,6)~~

$\rho_s = -24\,c/m^2$ at $z = 0$ then $\bar{E}$ at $z = 6$? (8,0,6)



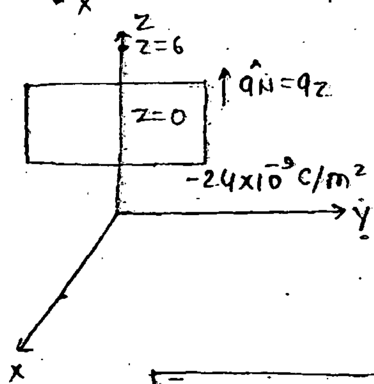
$$\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_N = \frac{18\times10^{-9}}{2\cdot\frac{1}{36\pi}\times10^{-9}}\,a_x$$

$$\boxed{\bar{E} = 324\pi a_x}$$



$$\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_N = \frac{9\times10^{-9}}{2\times\frac{1}{36\pi}\times10^{-9}}(a_y)$$

$$\boxed{E = 162\pi a_y}$$



$$\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_N = \frac{-24\times10^{-9}}{2\times\frac{1}{36\pi}\times10^{-9}}(a_z)$$

$$\boxed{\bar{E} = -432\pi a_z}$$

$$\boxed{\bar{E}_{total} = 324\pi a_x + 162\pi a_y - 432\pi a_z}$$

21/55

- for $+\rho_s$ at $x = 0$

  $-\rho_s$ at $x = a$

$$\bar{E} = \begin{cases} 0 & ; x<0 \\ \frac{\rho_s}{\epsilon}a_x & ; 0<x<a \\ 0 & , x>a \end{cases}$$

given $\rho_s = 10\times10^{-9}\ \frac{C}{m^2}$ | $a = 10$

$$\bar{E} = \begin{cases} 0 & x<0 \\ \frac{10\times10^{-9}}{\frac{1}{36\pi}\times10^{-9}}a_x = 360\pi a_x & 0<x<a \\ 0 & x>a \end{cases}$$

(4, 1, 0) w.r.t.

23/56 Potential at the origin

$$\bar{E} = y a_x + x a_y$$

$$V = -\int_{(0,0,0)}^{(4,1,0)} \bar{E}\, d\bar{l}$$

$$V = -\int_{(0,0,0)}^{(4,1,0)} (y a_x + x a_y)(dx a_x + dy a_y + dz a_z)$$

$$V = -\left[\int_0^4 y\,dx + \int_0^1 x\,dy\right]$$

We need relation b/w (x, y)

(x, y)

(4, 1) (0, 0)

Slopes are equal

$$\frac{y-0}{x-0} = \frac{1-0}{4-0} \quad ; \quad y = \frac{x}{4}$$

$$V = -\left[\int_{x=0}^{4} \frac{x}{4}\,dx + \int_{y=0}^{1} 4y\,dx\right]$$

$$\boxed{V = -4V}$$

24/56

$$\bar{E} = 4a_x - 3a_y + 5a_z \quad , \quad Q = 5C$$

Q = 5C

(9, 4, 1) (dirⁿ) (2m) (6, 2, -3)

If $\bar{E}$ is uniform the $WD = -Q\bar{E}\cdot\bar{l}_{BA}$

$$\bar{l}_{BA} = \hat{a} \times (2m)$$

$$\hat{a} = \frac{(9-6)a_x + (4-2)a_y + (1+3)a_z}{\sqrt{3^2+2^2+4^2}}$$

$$\hat{a} = \frac{3a_x + 2a_y + 4a_z}{\sqrt{29}}$$

$$\vec{L}_{BA} = 2\hat{a} = 2\left(\frac{3a_x + 2a_y + 4a_z}{\sqrt{29}}\right)$$

$$WD = -5(4a_x - 3a_y + 5a_z) \cdot \frac{2}{\sqrt{29}}(3a_x + 2a_y + 4a_z)$$

$$\boxed{WD = -48.28 \text{ Joule}}$$

25/56

$$V = 3x^2y - yz$$

$$\bar{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z\right)$$

$$= -\left[6xy\,a_x + (3x^2 - y)a_y + (-y)a_z\right]$$

$$= -\left[6xy\,a_x + (3x^2 - y)a_y - y\,a_z\right]$$

(a) $V|_{(1,0,-1)} = 0 - 0 = \text{vanish}$

$E|_{(1,0,-1)} = -\left[3(1)^2 - (-1)\right]a_y = -4a_y$ (not vanishing)

(b)

~~26~~

Capacitance →

$$Q = CV, \quad C = \frac{Q}{V}$$

$z=d$

d

$z=0$

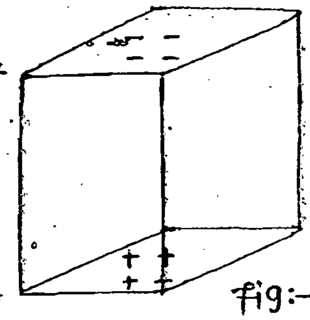


fig:-1

$$C_{+-} = \frac{\text{charge on +ve plate}}{V_{+-}}$$

$$= \frac{Q}{V_{+-}}$$

$$= \frac{\iint \bar{D} \cdot d\bar{s}}{-\int \bar{E} \cdot d\bar{l}} \quad \text{------(i)}$$

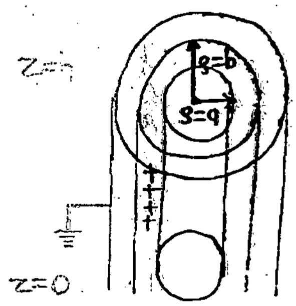
* If $\bar{E}$ is uniform (i.e. E is not ~~uniform~~ a f^n of x, y, z) $C = \frac{\epsilon A}{d}$
* If $\bar{E}$ is non-uniform then to find capacitance use eq^n (i)

Case(i)→ Capacitance are prallel plate capacitor shown in fig(1.)

$$\bar{E} = \frac{\rho_s}{\epsilon}a_z \quad ; \quad 0 < z < d \quad \text{------(ii)}$$

If eqⁿ (ii) is uniform then $\boxed{C = \frac{\epsilon A}{d}}$

Case (2) → Capacitance of a coaxial capacitor between $\rho = a$ & $\rho = b$ for h height



$$\bar{E} = \frac{\bar{D}}{\epsilon} = \frac{\rho_{si}}{\epsilon}\frac{a}{\rho}a_\rho \; ; \; a<\rho<b$$

$\bar{E}$ is non uniform. so we use eqⁿ (i) to find capacitance

$$C_{ab} = \frac{Q}{V_{ab}} = \frac{\iint \epsilon \bar{E} d\bar{s}}{-\int_{\rho=b}^{a} \bar{E} d\bar{l}} = \frac{\iint \epsilon \left(\frac{\rho_{si}}{\epsilon}\cdot\frac{a}{\rho}\right)a_\rho(\rho\, d\phi dz \cdot a_\rho)}{-\int_b^a \left(\frac{\rho_{si}}{\epsilon}\cdot\frac{a}{\rho}\right)a_\rho(d\rho \cdot a_\rho)}$$

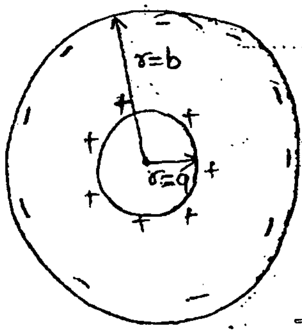
$$C_{ab} = \frac{\epsilon \cdot \frac{\rho_{si}}{\epsilon} \cdot a \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{h} dz}{-\frac{\rho_{si}}{\epsilon} a \left[\int_{\rho=b}^{a} \frac{d\rho}{\rho}\right]}$$

$$C_{ab} = \frac{\epsilon(2\pi)h}{-(\ln\rho)_{\rho=b}^{a}} = \frac{2\pi\epsilon h}{-(\ln a - \ln b)} = \frac{2\pi\epsilon h}{\ln\frac{b}{a}}$$

$$\boxed{C_{ab} = \frac{2\pi\epsilon h}{\ln\left(\frac{b}{a}\right)}}$$

Case (3) → Capacitor of a spherical capacitor between $r = a$ & $r = b$

$$\bar{E} = \frac{Q}{4\pi\epsilon r^2} a_r \text{ ------ (iii)}$$



$\bar{E}$ is non-uniform because the fⁿ of r

$$C_{ab} = \frac{Q}{V_{ab}} = \frac{\iint \epsilon \bar{E} \cdot d\bar{s}}{-\int_{r=b}^{a} \bar{E} \cdot d\bar{l}} = \frac{\iint \epsilon \cdot \frac{Q}{4\pi\epsilon r^2} a_r \cdot (r^2 \sin\theta \, d\theta d\phi a_r)}{-\left[\int_{r=b}^{a} \left(\frac{Q}{4\pi\epsilon r^2} a_r \cdot dr a_r\right)\right]}$$

$$C_{ab} = \frac{\epsilon \int_{\theta=0}^{\pi} \sin\theta \cdot \int_{\phi=0}^{2\pi} d\phi}{-\left[\int_{r=b}^{a} \frac{dr}{r^2}\right]}$$

$$= \frac{\epsilon(2)(2\pi)}{-\left(\frac{-1}{r}\right)_{r=b}^{a}}$$

$$= \frac{4\pi\epsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

$$\boxed{C_{ab} = \frac{4\pi\epsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)}}$$

* If $(b=\infty)$ then only $r=a$ sphere is present, it is called Isolated spherical cond^r of radius a, whose capacitance

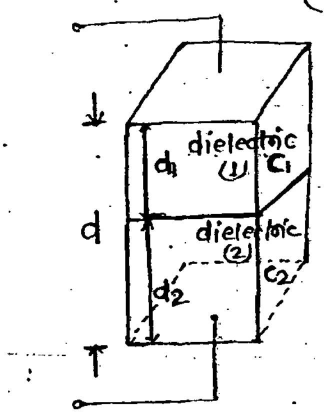
$$\boxed{C = 4\pi\epsilon a}$$

Case(4) → Two capacitors in series (C_1 & C_2)

$$C = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{C_1C_2}{C_1+C_2}$$

Series → current is same → Energy is same in both dielectrics of C_1 & C_2

(distance between capacitor plates are divided)



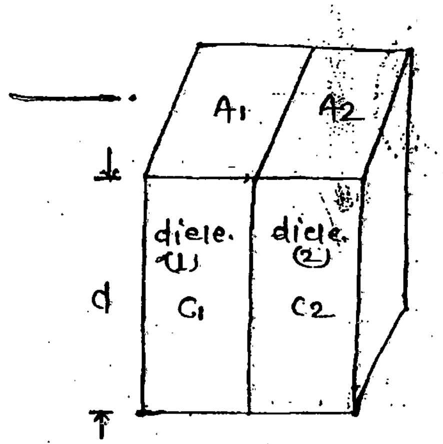
$$\boxed{C_1 = \frac{\epsilon_1 A}{d_1}} \; ; \; \boxed{C_2 = \frac{\epsilon_2 A}{d_2}}$$

Case(5) → Two capacitors are in parallel.

$$C = C_1 + C_2$$

Parallel → Current is devided → energy is devided

(Area of the plate is devided).

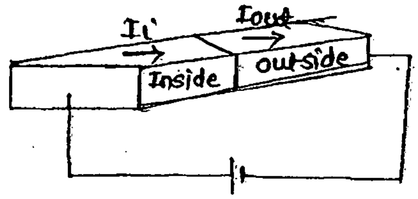


$$\boxed{C_1 = \frac{\epsilon_1 A_1}{d}} \; ; \; \boxed{C_2 = \frac{\epsilon_2 A_2}{d}}$$

| IES conventional)

Continuity Equation →

$$I_i = \frac{dq_i}{dt}$$



$$I = \frac{dq}{dt}$$

$$I = \iint \bar{J} \cdot ds$$

$$I = -\frac{dq}{dt} \text{ -----(i)}$$

$$I_{out} = -\frac{dq_i}{dt}$$

To have a contineous current flow

$$I_{out} = \frac{-dq_i}{dt}$$

By defination; $I = \oint\!\!\oint \bar{J} \cdot ds$ ------(ii)

$$q = \iiint \rho_v dv \text{ ------(iii)}$$

from ②,③ in (i)

$$\oint\!\!\oint \bar{J} \cdot ds = \frac{-d}{dt} \iiint \rho_v dv \text{ ----- (iv)}$$

from divergance theorem;

$$\oint\!\!\oint \bar{J} \cdot \bar{ds} = \iiint (\bar{\nabla} \cdot \bar{J})\, dv \text{ ----(v)}$$

from (v) in (iv)

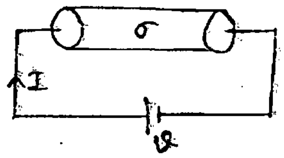
$$\iiint (\bar{\nabla} \cdot \bar{J})\, dv = \iiint \left(\frac{-\partial \rho_v}{dt} \right) dv \text{ ---(vi)}$$

Compare $\boxed{\vec{\nabla}\cdot\vec{J} = -\dfrac{\partial \rho_v}{\partial t}}$ ------ (vii)

Solution $\rightarrow$

$$\bar{\nabla}\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t} \quad \text{----- (vii)}$$

$$\bar{J} = \sigma\bar{E} \text{ for a cond}^r \quad \text{---- (viii)}$$



From (viii) in (vii)

$$\vec{\nabla}\cdot\sigma\vec{E} = -\frac{\partial \rho_v}{\partial t}$$

$$\sigma\,\bar{\nabla}\cdot\bar{E} = -\frac{\partial \rho_v}{\partial t} \quad \text{------ (ix)}$$

$$\because \bar{D} = \epsilon\bar{E}\,, \quad \bar{E} = \frac{\bar{D}}{\epsilon} \quad \text{------ (x)}$$

from (x) in (ix); $\sigma\nabla\cdot\dfrac{\bar{D}}{\epsilon} = -\dfrac{\partial \rho_v}{\partial t}$ ----- (xi)

$$\frac{\sigma}{\epsilon}(\nabla\cdot\bar{D}) = -\frac{\partial \rho_v}{\partial t} \quad \text{----- (xii)}$$

$$(\bar{\nabla}\cdot\bar{D}) = \rho_v \quad \text{------ (xiii)}$$

from (xiii) in (xii)

$$\frac{\sigma}{\epsilon}(\rho_v) = -\frac{\partial \rho_v}{\partial t}$$

$$\boxed{\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon}\rho_v = 0} \quad \text{------ (xiv)}$$

$$\left(D + \frac{\sigma}{\epsilon}\right)\rho_v = 0 \quad \left\{D = \frac{\partial}{\partial t}\right\}$$

roots $\Rightarrow$ $D + \dfrac{\sigma}{\epsilon} = 0\,, \quad D = -\dfrac{\sigma}{\epsilon}$

Solution $\Rightarrow$ top side variable = (constant). $e^{(\text{root})\,(\text{down side variabl}}$

$$\rho_v(t) = \rho_v = (\text{constant})e^{-\frac{\sigma}{\epsilon}(t)} \quad \text{------ (xv)}$$

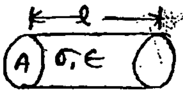
Let ρ_o is the initial charge density at $t=0$

then $\rho_v(t) = \rho_o e^{-t/\left(\frac{\epsilon}{\sigma}\right)}$ ------- (xvi)

$$\rho_v(t) = \rho_0\, e^{\frac{-t}{\tau_r}}$$

where τ_r = relaxation time constant of material $\left(\frac{\epsilon}{\sigma}\right)$

Observation →



$C = \frac{\epsilon A}{l}$ & $R = \frac{l}{\sigma A}$

$R \cdot C = \frac{\epsilon \cancel{A}}{\cancel{l}} \times \frac{\cancel{l}}{\sigma \cancel{A}}$

$\boxed{RC = \frac{\epsilon}{\sigma}}$ For any material

IES

★ Poisson Equation →

Homogeneous [Homo → same, geneous → Type] medium :- The medium parameters μ, ϵ, σ do not change with distance.

[(μ, ϵ, σ) are not fu^n of x, y, z]

Non-Homogeneous → μ, ϵ, σ are f^n of x, y, z

maxwell eq^n $\bar{\nabla} \cdot \bar{D} = \rho_v$ —— (i)

$D = \epsilon \bar{E}$ —— (2)

From (2.) in (i) $\nabla \cdot (\epsilon E) = \rho_v$ —— (3.)

$E = -\nabla V$ —— (4.)

From (4.) in (3.)

$$\nabla \cdot [\epsilon(-\nabla V)] = \rho_v$$

$$\boxed{\nabla \cdot (\epsilon \nabla V) = -\rho_v} \quad \text{—— (5.)}$$

Eq^n(5.) is the poission eq^n of non-Homogeneous medium.

★ For Homogeneous medium ϵ, μ, σ are not f^n of x, y, z

so eq^n (5.) becomes

$$\epsilon(\bar{\nabla} \cdot \bar{\nabla} V) = -\rho_v$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}} \quad \text{—— (6.)}$$

Eqn (6.) is called poisson eqn for homogeneous medium.

Laplace eqn →

If $\rho_v = 0$ (But Q, ρ_l, ρ_s can be present) in eqn (5.) then

$$\boxed{\nabla \cdot (\epsilon \nabla V) = 0} \quad \text{———} (7.)$$

Eqn (7.) is called Laplace eqn of non-homogeneous medium.

* If $\rho_v = 0$ in eqn (6.) then

$$\boxed{\nabla^2 V = 0} \quad \text{———} (8.)$$

Eqn (8.) is called Laplace eqn of Homogeneous eqn.

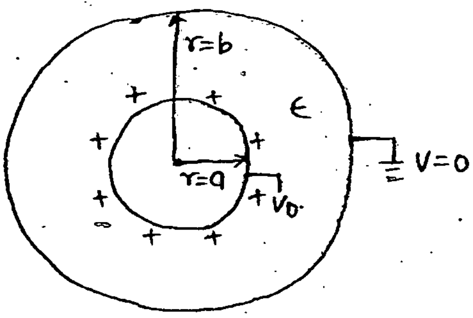
Note →

(1.) Solving diode problem $\xrightarrow{\text{Use}}$ Poisson eqn
(space charge region ρ_v is present)

(2.) Solving Q, ρ_s, ρ_l problems $\xrightarrow{\text{Use}}$ Laplace eqn

IES

Que → Find charge on the +ve plate (Q), V (between $r=a, r=b$), $\bar{E}, \bar{D}$, capacitance of a spherical configuration shown below.



Ans → Given $V = V_0$, at $r=a$

$V = 0$, at $r=b$

the medium b/n $r=a, r=b$ is homogeneous $\rho_v = 0$

$$\nabla^2 V = 0 \quad \text{———} (1)$$

given spherical surface, so

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0 \quad \text{———} (2)$$

given that $V=V_0$ at $r=a$ and $V=0$ at $r=b$, so;

V is a fn of r only. so only 1st term is present.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)=0 \quad \text{---------- (3)}$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)=0\,(r^2)=0$$

Take integral both sides

$$\int\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)dr=\int 0\,dr$$

$$r^2\frac{\partial V}{\partial r}+C_2=0+C_1$$

$$r^2\frac{\partial V}{\partial r}=C_1-C_2$$

$$r^2\frac{\partial V}{\partial r}=A \qquad (C_1, C_2, A \text{ are constant})$$

$$\frac{\partial V}{\partial r}=\frac{A}{r^2}$$

$$\partial V=\frac{A}{r^2}\partial r$$

Take integral both sides

$$\int\partial V=\int\frac{A}{r^2}\partial r$$

$$V=\frac{-A}{r}+B \text{ ---- (4.) } (B \text{ is constant})$$

given $V=V_0$ at $r=b$ ------ (5.)

From (5.) in (4.)

$$\cancel{V_0=-A}$$

$$0=\frac{-A}{b}+B$$

$$B=\frac{A}{b} \text{ ---------- (6.)}$$

From (6.) in (4.)

$$V=\frac{-A}{r}+\frac{A}{b}=A\left(\frac{1}{b}-\frac{1}{r}\right)$$

$$V=A\left(\frac{1}{b}-\frac{1}{r}\right) \text{ ------ (7)}$$

given $V = V_0$ at $r = a$ ——— (8)

From (8.) in (7)

$$V_0 = A\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$A = \frac{V_0}{\left(\frac{1}{b} - \frac{1}{a}\right)} \quad \text{——— (9)}$$

From (9) in (7)

$$V = \frac{V_0}{\left(\frac{1}{b} - \frac{1}{a}\right)}\left(\frac{1}{b} - \frac{1}{r}\right) \quad \text{——— (10)}$$

$$V = \frac{V_0\left(\frac{1}{r} - \frac{1}{b}\right)}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

E →

$$\bar{E} = -\nabla V = -\left(\frac{\partial V}{\partial r} a_r\right)$$

$$= -\frac{\partial}{\partial r}\left[\frac{V_0\left(\frac{1}{r} - \frac{1}{b}\right)}{\left(\frac{1}{a} - \frac{1}{b}\right)}\right] a_r$$

$$= \frac{-V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{b}\right) a_r$$

$$\bar{E} = \frac{-V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}\left(-\frac{1}{r^2}\right) a_r$$

$$\bar{E} = \frac{V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \frac{1}{r^2} a_r \quad \text{——— (11.)}$$

D →

$$\bar{D} = \epsilon \bar{E}$$

$$\bar{D} = \frac{\epsilon V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \frac{1}{r^2} a_r \quad \text{——— (12.)}$$

Q (charge on the +ve plate at $r=a$) →

$$Q = \iint \bar{D} \cdot \bar{ds}$$

$$Q = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\epsilon V_0}{\cancel{r^2}\left(\frac{1}{a}-\frac{1}{b}\right)} a_r \cdot (\cancel{r^2}\sin\theta \, d\theta \, d\phi \, a_r)$$

$$Q = \frac{\epsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$Q = \frac{\epsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)} \times 2 \times 2\pi$$

$$Q = \frac{4\epsilon V_0 \pi}{\left(\frac{1}{a}-\frac{1}{b}\right)} \quad \text{——— (13)}$$

Capacitance →

$$C_{ab} = \frac{Q}{V_{ab}} = \frac{Q}{V_a - V_b} = \frac{Q}{V_0 - 0}$$

$$C_{ab} = \frac{Q}{V_0}$$

$$C_{ab} = \frac{4\pi\epsilon \cancel{V_0}}{\cancel{V_0}\left(\frac{1}{a}-\frac{1}{b}\right)}$$

$$C_{ab} = \frac{4\pi\epsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)} \quad \text{——— (14)}$$