# Some Applications of Trigonometry

# **Case Study Based Questions**

## Case Study 1

There is fire incident in the house. The house door is locked so, the fireman is trying to enter the house from the window. He places the ladder against the wall such that its top reaches the window as shown in the figure.



Based on the above information, solve the following questions:

Q1. If window is 6 m above the ground and angle made by the foot of ladder to the ground is 30°, then length of the ladder is:

- a. 8 m
- b. 10 m
- c. 12 m
- d. 14 m

Q2. If fireman place the ladder 5 m away from the wall and angle of elevation is observed to be 30°, then length of the ladder is:

a. 5 m b.  $\frac{10}{\sqrt{3}}$  m c.  $\frac{15}{\sqrt{2}}$  m d. 20 m

Q3. If fireman places the ladder 2.5 m away from the wall and angle of elevation is observed to be 60°, then find the height of the window: (Take  $\sqrt{3}$  = 1.73)

- a. 4.325 m
- b. 5.5 m
- c. 6.3 m
- d. 2.5 m

Q4. If the height of the window is 8 m above the ground and angle of elevation is observed to be 45°, then horizontal distance between the foot of ladder and wall is:

a. 2 m

b. 4 m

c. 6 m

d. 8 m

Q5. If the fireman gets a 9 m long ladder and window is at 6 m height, then how far should the ladder be placed?

- a. 5 m
- b. 3√5m
- c. 3m
- d. 4 m

## **Solutions**





A group of students of class-X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name is Delhi Memorial, originally called All-India War Memorial, Monumental Sandstone Arch in New Delhi is dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.



Based on the given information, solve the following questions:

Q1. What is the angle of elevation, if they are standing at a distance of  $42\sqrt{3}$  m away from the monument?

a. 0°

b. 30°

- c. 45°
- d. 60°

Q2. They want to see the tower (monument) at an angle of 60°. So, they want to know the distance where they should stand and hence find the distance. [Use $\sqrt{3}$  = 1.732]

- a. 24.24 m
- b. 20.12 m
- c. 42 m
- d. 25.64 m

Q3. If the altitude of the Sun is at 30°, then the height of the vertical tower that will cast a shadow of Length 30 m is:

a.  $10\sqrt{3}$  m b.  $\frac{10}{\sqrt{3}}$  m c.  $\frac{20}{\sqrt{3}}$  m d.  $20\sqrt{3}$  m

Q4. The ratio of the length of a rod and its shadow is 24:8 $\sqrt{3}$ . The angle of elevation of the Sun is:

a. 30°

b. 60°

c. 45°

d. 90°

Q5. The angle formed by the line of sight with the horizontal when the object viewed is above the horizontal level, is:

- a. angle of elevation
- b. angle of depression
- c. corresponding angle
- d. complete angle

# **Solutions**

1. Let the angle of elevation be 0. Given that, Height of the monument (BC) = 42 m



2. Let the required distance be x m.

Given, angle of elevation (0) =  $60^{\circ}$ 



Now, in right-angled ∆ABC,



3. Let the height of the - Sun vertical tower be C hm. Given of angle hm elevation  $(\theta) = 30^{\circ}$ and length of the 30 Ŵ shadow (AB) = 30 m A В 30 m Now, In right-angled AABC, BC  $tan\theta =$ AB

$$\Rightarrow \qquad \tan 30^\circ = \frac{h}{30}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow \qquad h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ m}$$

So, option (a) is correct.

# 4. Given, the ratio of the length of a rod and its shadow is $24:8\sqrt{3}$ .

Sun

Let AC = 24kand BC =  $8\sqrt{3}k$ where k is a positive 24 k integer. Now, let angle of θ elevation of the Sun B С 8 √3 k is θ. In right-angled ∆ACB;  $\tan \theta = \frac{AC}{BC}$  $=\frac{24k}{8\sqrt{3}k}=\frac{3}{\sqrt{3}}$  $\tan\theta = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$  $\Rightarrow$  $tan\theta = tan 60^{\circ}$  $\Rightarrow$  $\theta = 60^{\circ}$ *.*.. So, option (b) is correct.

5. The angle formed by the line of sight with the horizontal, when the object viewed is above the horizontal level, is angle of elevation. So, option (a) is correct.

## Case Study 3

Suppose a straight vertical tree is broken at some point due to storm and the broken part is inclined at a certain distance from the foot of the tree.



Based on the above information, solve the following questions:

Q1. If the top of upper part of broken tree touches ground at a distance of 30 m (from the foot of the tree) and makes an angle of inclination 30°, then find the height of remaining part of the tree.

Q2. Find the height of the straight vertical tree.

Q3. If the height of a tree is 6 m, which is broken by wind in such a way that its top touches the ground and makes an angle 30° with the ground. Find the length of broken part of the tree.

#### OR

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If AB = 10\sqrt{3} m and AD = 2\sqrt{3} m, then find CD.
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# **Solutions**

1. Let AB be the tree of height h m and let it broken at height of x m, as shown in figure. Clearly CD = AC = (h-x) m Now, in right-angled  $\Delta$ CBD, we have

$$\tan 30^{\circ} = \frac{BC}{BD} = \frac{x}{30}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{x}{30}$$

$$\Rightarrow \qquad x = \frac{30}{\sqrt{3}}$$

$$D \xrightarrow{30^{\circ}}{} 30 \text{ m} \xrightarrow{B} + 1$$

$$=\frac{30}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{30\sqrt{3}}{3}=10\sqrt{3}$$
 m

Thus, the height of remaining part of the tree is  $10\sqrt{3}$ m.

2. In right-angled ACBD,

$$\cos 30^{\circ} = \frac{DB}{DC} = \frac{30}{DC}$$

$$\Rightarrow \qquad \frac{\sqrt{3}}{2} = \frac{30}{DC}$$

$$\Rightarrow \qquad DC = \frac{60}{\sqrt{3}} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3}$$

$$\Rightarrow \qquad DC = 20\sqrt{3} \text{ m}$$

from part (1), BC = x =  $10\sqrt{3}$ m Thus, the height of the straight vertical tree AB = DC + BC = $20\sqrt{3}$ + $10\sqrt{3}$  =  $30\sqrt{3}$ m

3. Here, h=6 m and 0-30°

Now, in right-angled ABCD, we have



So, broken part of tree, AC = 6 - x = 6 - 2 = 4 m. Or Clearly, BD = AB - AD =  $(10\sqrt{3} - 2\sqrt{3})$  m =  $8\sqrt{3}$  m Now, in right-angled  $\Delta$ BCD, we have  $\sin 60^\circ = \frac{BD}{DC} \implies \frac{\sqrt{3}}{2} = \frac{8\sqrt{3}}{DC} \implies DC = 16$  m

#### Case Study 4

Sandeep and his sister Dolly visited at their uncle's place-Birth, Himachal Pradesh.

During day time Sandeep, who is standing on the ground spots a paraglider at a distance of 24 m from him at an elevation of 30°. His sister Dolly is also standing on the roof of a 6 m high building, observes the elevation of the same paraglider as 45°. Sandeep and Dolly are on the opposite sides of the paraglider.



Based on the given information, solve the following questions:

Q1. Find the distance of paraglider from the ground.

Q2. Find the value of PD.

Q3. Find the distance between the paraglider and the Dolly.

#### Or

Find the distance between Sandeep and base of the building.

# **Solutions**

1. In the right–angled  $\triangle AQD$ , we have

$$\sin 30^{\circ} = \frac{DQ}{AD} \Rightarrow \frac{1}{2} = \frac{DQ}{24}$$
  
$$\Rightarrow \qquad DQ = 12 \text{ m}$$
  
Thus, distance of paraglider from the ground is  
12 m.

- We have PQ = BC = 6 m Now, as DQ = 12 m
   ∴ DP = DQ - PQ = 12 - 6 = 6 m
- **3.** In right–angled  $\triangle$ BPD, we have

$$\sin 45^\circ = \frac{DP}{BD} \Rightarrow \frac{1}{\sqrt{2}} = \frac{6}{BD} \Rightarrow BD = 6\sqrt{2} m$$

Thus, the distance of paraglider from the girl is  $6\sqrt{2}$  m.

 $\cos 30^\circ = \frac{AQ}{AD}$ 

$$\Rightarrow \qquad \frac{\sqrt{3}}{2} = \frac{AQ}{24} \Rightarrow AQ = 12\sqrt{3} \text{ m}$$

In right-angled ∆BPD, we have

$$\cos 45^{\circ} = \frac{BP}{BD}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{2}} = \frac{BP}{6\sqrt{2}} \Rightarrow BP = 6m$$

Thus, the distance between Sandeep and base of the building = AQ + BP

$$= 12\sqrt{3} + 6 = 6(2\sqrt{3} + 1)$$
 m.

#### Case Study 5

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O. Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30° and the angle of elevation of the top of Section A is 45°.



Based on the given information, solve the following questions: [CBSE 2023]

Q1. Find the length of the wire from the point O to the top of Section B.

Q2. Find the height of the Section A from the base of the tower.

Q3. Find the distance AB.

#### Or

#### Find the area of AOPB.

# **Solutions**

1. Let the length of the wire from the point O to the top of section B, i.e., OB = Im.



 $\therefore$  The height of the Section A from the base of the tower = AP = 36 cm.

3. Now, in right-angled ∆BPO,

$$\tan 30^{\circ} = \frac{BP}{OP} \implies \frac{1}{\sqrt{3}} = \frac{BP}{36}$$
$$\implies BP = \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{36\sqrt{3}}{3} = 12\sqrt{3} \text{ cm}$$
$$\therefore AB = AP - BP \qquad [\because AP = 36 \text{ cm}]$$
$$= 36 - 12\sqrt{3} = 12(3 - \sqrt{3}) \text{ cm}$$
So, required distance AB is 12 (3 -  $\sqrt{3}$ ) cm.

Since,  $\Delta BPO$  is a right-angled triangle.

∴ Area of 
$$\triangle OPB = \frac{1}{2} \times base \times height$$
  
=  $\frac{1}{2} \times OP \times BP = \frac{1}{2} \times 36 \times 12\sqrt{3}$   
=  $216\sqrt{3}$  cm<sup>2</sup>.

#### **Case Study 6**

A boy is standing on the top of light house. He observed that boat P and boat Q are approaching the light house from opposite directions. He finds that angle of depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of the light house is 100 m.



Based on the above information, answer the following questions. Q1. What is the measure of APD?

Q2. If ZYAQ = 30°, then ZAQD is also 30°, why?

Q3. Find length of PD.

Find length of QD.

# **Solutions**

1. Let a boy is standing on the top (A) of light house (AD). Here XYII PQ and AP is traversal.



Because, XY||PQ and AQ is a traversal. So, alternate interior angles are equal.

- $\therefore$   $\angle YAO = \angle AOD.$
- 3. In right–angled ∆ADP,

 $\Rightarrow$ 

$$\tan 45^\circ = \frac{AD}{PD}$$
$$1 = \frac{100}{PD}$$

⇒ PD = 100 m.
∴ Boat P is 100 m from the light house.

Or In right–angled ∆ADQ,

$$\tan 30^{\circ} = \frac{AD}{DQ}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{100}{DQ}$$

$$\Rightarrow \qquad QD = 100\sqrt{3} \text{ m}$$

 $\therefore$  Boat Q is 100  $\sqrt{3}$  m from the light house.

# Solutions for Questions 7 to 21 are Given Below

#### **Case Study 7**

#### Visit to Temple

There are two temples on each bank of a river. One temple is 50 m high. A man, who is standing on the top of 50 m high temple, observed from the top that angle of depression of the top and foot of other temple are 30° and 60° respectively. (Take  $\sqrt{3} = 1.73$ )



Based on the above information, answer the following questions.

(i)	Measure of $\angle ADF$ is equal to							
	(a)	45°	(b)	60°	(c)	30°	(d)	90°
(ii)	Mea	asure of $\angle ACB$	is eq	ual to				
	(a)	45°	(b)	60°	(c)	30°	(d)	90°
(iii) Width of the river is								
	(a)	28.90 m			(b)	26.75 m		
	(c)	25 m			(d)	27 m		
(iv)	Hei	ght of the other	tem	ple is				
	(a)	32.5 m			(b)	35 m		
	(c)	33.33 m			(d)	40 m		
(v)	Angle of depression is always							
	(a)	reflex angle			(b)	straight		
	(c)	an obtuse angle	e		(d)	an acute angle		

### Observation of a Balloon

There are two windows in a house. First window is at the height of 2 m above the ground and other window is 4 m vertically above the lower window. Ankit and Radha are sitting inside the two windows at points G and F respectively. At an instant, the angles of elevation of a balloon from these windows are observed to be 60° and 30° as shown belc<sup>…</sup>



#### **Case Study 9**

#### Stunt by Circus Artist

A circus artist is climbing through a 15 m long rope which is highly stretched and tied from the top of a vertical pole to the ground as shown below. Based on the above information, answer the following questions.

- (i) Find the height of the pole, if angle made by rope to the ground level is 45°.
  - (a) 15 m (b)  $15\sqrt{2} \text{ m}$ (c)  $\frac{15}{\sqrt{3}} \text{ m}$  (d)  $\frac{15}{\sqrt{2}} \text{ m}$



- (ii) If the angle made by the rope to the ground level is 45°, then find the distance between artist and pole at ground level.
  - (a)  $\frac{15}{\sqrt{2}}$  m (b)  $15\sqrt{2}$  m (c) 15 m (d)  $15\sqrt{3}$  m
- (iii) Find the height of the pole if the angle made by the rope to the ground level is 30°.

(b) 4 m

(a) 2.5 m (b) 5 m (c) 7.5 m (d) 10 m

(iv) If the angle made by the rope to the ground level is 30° and 3 m rope is broken, then find the height of the pole.

(a) 2 m

(c) 5 m

(d) 6 m

- (v) Which mathematical concept is used here?
   (a) Similar Triangles
   (c) Application of Trigonometry
- (b) Pythagoras Theorem
- (d) None of these

# Case Study 10

#### **Fire Incident**

There is fire incident in the house. The house door is locked so, the fireman is trying to enter the house from the window. He places the ladder against the wall such that its top reaches the window as shown in the figure.



Based on the above information, answer the following questions.

- (i) If window is 6 m above the ground and angle made by the foot of ladder to the ground is 30°, then length of the ladder is
  - (a) 8 m (b) 10 m (c) 12 m (d) 14 m
- (ii) If fireman place the ladder 5 m away from the wall and angle of elevation is observed to be 30°, then length of the ladder is

(a) 5 m (b) 
$$\frac{10}{\sqrt{3}}$$
 m (c)  $\frac{15}{\sqrt{2}}$  m (d) 20 m

(iii) If fireman places the ladder 2.5 m away from the wall and angle of elevation is observed to be 60°, then find the height of the window. (Take  $\sqrt{3} = 1.73$ )

(a) 4.325 m (b) 5.5 m (c) 6.3 m (d) 2.5 m

(iv) If the height of the window is 8 m above the ground and angle of elevation is observed to be 45°, then horizontal distance between the foot of ladder and wall is

- (a) 2 m (b) 4 m (c) 6 m (d) 8 m
- (v) If the fireman gets a 9 m long ladder and window is at 6 m height, then how far should the ladder be placed?
  - (a) 5 m (b)  $3\sqrt{5} \text{ m}$  (c) 3 m (d) 4 m

### **Repairing of Electric Fault**

An electrician has to repair an electric fault on the pole of height of 8 m. He needs to reach a point 2 m below the top of the pole to undertake the repair work.



## **Case Study 12**

#### Application of Trigonometry for Moving Car

Rohit is standing at the top of the building observes a car at an angle of 30°, which is approaching the foot of the building with a uniform speed. 6 seconds later, angle of depression of car formed to be 60°, whose distance at that instant from the building is 25 m.



Based on the above information, answer the following questions.

(i) Height of the building is

(a) 2572 m $(b) 50 m$ $(c) 2575 m$ $(d) 25$	(a)	25√2 m	(b) 50 m	(c)	$25\sqrt{3}$ m	(d) 25 m
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(ii)	i) Distance between two positions of the car is							
	(a) 40 m	(b) 50 m	(c)	60 m	(d)	75 m		
(iii)	(iii) Total time taken by the car to reach the foot of the building from starting point is							
	(a) 4 sec.	(b) 3 sec.	(c)	6 sec.	(d)	9 sec.		
(iv)	iv) The distance of the observer from the car when it makes an angle of 60° is							
	(a) 25 m	(b) 45 m	(c)	50 m	(d)	75 m		
(v)	(v) The angle of elevation increases							
	(a) when point of observation moves towards the object							
	(b) when point of observa	tion moves away from the o	bjec	t				
	(c) when object moves aw	ay from the observer						
	(d) None of these							

#### **Light House**

A boy is standing on the top of light house. He observed that boat *P* and boat *Q* are approaching to light house from opposite directions. He finds that angle of depression of boat *P* is 45° and angle of depression of boat *Q* is 30°. He also knows that height of the light house is 100 m.



- (a)  $100\sqrt{2}$  m
- (c) 50 m

(b)  $100\sqrt{3}$  m

(d) 100 m

#### Visit to Exhibition

In an exhibition, a statue stands on the top of a pedestal. From the point on ground where a girl is clicking the photograph of the statue the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the top of the statue is 60° and from the same point, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the statue is 60° and from the same point.



#### **Case Study 15**

#### Diwali Celebration

Teewan, Arun and Pankaj were celebrating the festival of Diwali in open ground with firecrackers. There is a pedestal in the ground. All of sudden Teewan stands on pedestal and release sky lantern from the top of pedestal.



Based on the above information answer the following questions. (Take  $\sqrt{3} = 1.73$ )

- (i) Which one is a pair of angle of depression?
  - (a)  $(\angle x, \angle y)$  (b)  $(\angle y, \angle z)$  (c)  $(\angle z, \angle t)$  (d)  $(\angle r, \angle q)$

(ii)	If the position of Pankaj is 25 m away from the base of pedestal and $\angle r = 30^\circ$ , then find the height of pedestal.								
	(a) 14.45 m	(b) 15.5 m	(c) 16.36 m	(d) 17.36 m					
(iii)	If the height of pedestal is Pankaj is	30 m, $\angle t = 45^{\circ}$ and $\angle z = 3^{\circ}$	30°, then the horizontal d	istance between Arun and					
	(a) 24.5 m	(b) 19.5 m	(c) 20 m	(d) 21.9 m					
(iv)	If the vertical height of sky Teewan and sky lantern is	lantern from the top of pe	destal is 12 m and $\angle y = 3$	0°, then distance between					
	(a) 20 m	(b) <sup>o</sup> 16.97 m	(c) 24 m	(d) 19.86 m					
(v)	If $\angle q = 60^\circ$ and position of	Arun is 15 m away from th	e base of pedestal, then fin	nd the height of pedestal.					
	(a) 16.25 m	(b) 25 m	(c) 25.95 m	(d) 26 m					

#### Hoardings on the Road

Two hoardings are put on two poles of equal heights standing on either side of the road. From a point between them on the road the angle of elevation of the top of poles are 60° and 30° respectively. Height of the each pole is 20 m



Based on the above information, answer the following questions. (Take  $\sqrt{3} = 1.73$ ).

(i) Find the length of PO.

	(a) 20 m	(b)	$20\sqrt{3}$ m	(c)	$\frac{20}{\sqrt{3}}$ m	(d)	None of these
(ii)	Find the length of RO.				<b>V</b> 5		
	(a) 20 m	(b)	$20\sqrt{3}$ m	(c)	$\frac{20}{\sqrt{3}}$ m	(d)	None of these
(iii)	The width of the road is				<b>V</b> 0		
	(a) 31.23 m	(b)	35.68 m	(c)	39.73 m	(d)	46.24 m
(iv)	If the angle of elevation ma	de by	y pole PQ is 45°, then t	he le	ngth of PO =		
	(a) 20 m	(b)	$20\sqrt{3}$ m	(c)	$\frac{20}{\sqrt{3}}$ m	(d)	None of these

- (v) Angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level is known as
  - (a) angle of depression (b) angle of elevation (c) right Angle (d) reflex angle

# Applications of Trigonometry - Broken Tree

Suppose a straight vertical tree is broken at some point due to storm and the broken part is inclined at a certain distant from the foot of the tree. Based on the above information, answer the following questions

Based on the above information, answer the following questions.

- (i) If the top of upper part of broken tree touches ground at a distance of 30 m (from the foot of the tree) and makes an angle of inclination 30°, then the height of remaining part of the tree is
  - (a)  $\sqrt{3}$  m (b)  $30\sqrt{3}$  m (c)  $\frac{30}{\sqrt{3}}$  m (d) 30 m
- (ii) If the top of broken part of a tree touches the ground at a point whose distance from foot of the tree is equal to height of remaining part, then its angle of inclination is
  - (a)  $30^{\circ}$  (b)  $60^{\circ}$  (c)  $45^{\circ}$  (d) None of these
- (iii) The angle of elevation are always
  - (a) obtuse angle (b) acute angle (c) right angle (d) reflex angle

(iv) If 
$$AB = 10\sqrt{3}$$
 m,  $AD = 2\sqrt{3}$  m, then  $CD =$ 

(a) 9 m (b) 11 m (c) 14 m (d) 16 m

(v) If the height of a tree is 6 m, which is broken by wind in such a way that its top touches the ground and makes an angles 30° with the ground. At what height from the bottom of the tree is broken by the wind?

(a) 2 m (b) 4 m (c) 8 m (d) 10 m

## **Case Study 18**

## Trigonometry in Bridge Design

One day while sitting on the bridge across a river Arun observes the angles of depression of the banks on opposite sides of the river are 30° and 60° respectively as shown in the figure. (Take  $\sqrt{3} = 1.73$ )









(i)	If the bridge is at a height o	f 6 n	, then $AD =$				
	(a) 6 m	(b)	$\frac{\sqrt{3}}{6}$ m	(c)	$6\sqrt{3}$ m	(d)	$rac{6}{\sqrt{3}}$ m
(ii)	BD =						
	(a) 6 m	(b)	$6\sqrt{3}$ m	(c)	$\sqrt{3}$ m	(d)	$10\sqrt{3}$ m
(iii)	Width of the river is						
	(a) 10.85 m	(b)	13.87 m	(c)	15.85 m	(d)	19.85 m
(iv)	The angles of elevation and	dep	ression are always				
	(a) acute angles	(b)	obtuse angles	(c)	right angles	(d)	straight angles
(y)	If $BD = 21$ m, then height of	of the	bridge is				
	(a) 7 m	(b)	21 m	(c)	$7\sqrt{3}$ m	(d)	$\frac{7}{\sqrt{3}}$ m

#### Hot Air Balloon

Karan and his sister Riddhima visited at their uncle's place-Bir, Himachal Pradesh. During day time Karan, who is standing on the ground spots a paraglider at a distance of 24 m from him at an elevation of 30°. His sister Riddhima is also standing on the roof of a 6 m high building, observes the elevation of the same paraglider as 45°. Karan and Riddhima are on the opposite sides of the paraglider.

Based on the above information, answer the following questions.

(i)	The	distance	of	paraglider	from	the ground is	
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(a) 10 m (b) 12 m (c) 18 m

(b) 7 m

(b) 6 m

(ii) The value of PD is

(a) 6 m

(iii) The distance between the paraglider and the Riddhima is

(a)  $\sqrt{2}$  m

(iv) In the given figure,  $\angle AOP$  is



- (a) Reflex angle
- (c) Straight angle

(v) If *A* and *B* are two objects and the eye of an observer is at point *O*, then the line of sight will be

- (a) OA
- (c) Both (a) and (b)

(b) *OB* 

(c) 8 m

(c)  $6\sqrt{2}$  m

(d) None of these

(b) Angle of elevation

(d) Angle of depression





(d) 22 m

#### Flying Pigeon

A boy 4 m tall spots a pigeon sitting on the top of a pole of height 54 m from the ground. The angle of elevation of the pigeon from the eyes of boy at any instant is 60°. The pigeon flies away horizontally in such a way that it remained at a constant height from the ground. After 8 seconds, the angle of elevation of the pigeon from the same point is 45°

Based on the above information, answer the following questions. (Take  $\sqrt{3}$  =1.73)

- (i) Find the distance of first position of the pigeon from the eyes of the boy.
   (a) 54 m
   (b) 100 m
  - (c)  $\frac{100}{\sqrt{3}}$  m (d)  $100\sqrt{3}$



(iii) Find the distance between the boy and the pole.

(a) 50 m (b) 
$$\frac{50}{\sqrt{3}}$$
 m (c)  $50\sqrt{3}$  m (d)  $60\sqrt{3}$ 

- (iv) How much distance the pigeon covers in 8 seconds?
  (a) 12.13 m
  (b) 19.60 m
  (c) 21.09 m
  (v) Find the speed of the pigeon.
  - (a) 2.63 m/sec (b) 3.88 m/sec (c) 6.7 m/sec (d) 9.3 m/sec

#### **Case Study 21**

#### Wooden Stool

Aditi purchase a wooden bar stool for her living room with square top of side 2 m and having height of 6 m above the ground. Also each leg is inclined at an angle of 60° to the ground as shown in the figure (not drawn to scale).



Based on the above information, answer the following questions. (Take  $\sqrt{3} = 1.73$ )

(i)	Find the length of the each	leg.					
	(a) 5.9 m	(b)	6.93 m	(c)	7.3 m	(d)	8.2 m
(ii)	Find the length of <i>GH</i> . (a) 0.53 m	(b)	1 m	(c)	1.15 m	(d)	2.73 m
(iii)	The length of second step is	5					
	(a) 4.3 m	(b)	4.99 m	(c)	5.68 m	(d)	6.78 m
(iv)	The length of $PQ =$						
	(a) 1.56 m	(b)	2.31 m	(c)	3.34 m	(d)	5.68 m
(v)	The length of first step is						
	(a) 4.78 m	(b)	5.34 m	(c)	6.62 m	(d)	7.82 m



m

(d) 26.32 m

# **HINTS & EXPLANATIONS**

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- (i) (c) : Since, AE || FD
- $\angle EAD = \angle ADF = 30^{\circ}$

[Alternate interior angles]

130°1 ····· > E

(ii) (b): Since, AE || BC  $\therefore \angle EAC = \angle ACB = 60^{\circ}$ [Alternate interior angles]

(iii) (a): In  $\triangle ABC$ ,

30°C 50 m 60°  $\tan 60^\circ = \frac{AB}{\Rightarrow} \Rightarrow \sqrt{3} = \frac{50}{3}$ R

$$\Rightarrow BC = \frac{50}{\sqrt{3}} = 28.90 \text{ m}$$
(iv) (c) : In  $\triangle ADF$ , tan 30° =  $\frac{AF}{FD}$   

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB - BF}{FD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{50 - CD}{\frac{50}{\sqrt{3}}}$$
[ $\because FD = BC = \frac{50}{\sqrt{3}}$ ]  

$$\Rightarrow \frac{50}{3} = 50 - CD \Rightarrow CD = 50 - \frac{50}{3} = \frac{100}{3} = 33.33 \text{ m}$$

(v) (d)

(i) (b): The person who makes small angle of 8. elevation is more closer to the balloon.

∴ Radha is more closer to the balloon.

(ii) (b): In 
$$\triangle EFD$$
,  $\tan 30^\circ = \frac{ED}{DF}$   
 $\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{DF}$   
 $\Rightarrow DF = h\sqrt{3}$  m  
(iii) (a): In  $\triangle GCE$ ,  
 $\tan 60^\circ = \frac{EC}{GC} = \frac{h+4}{DF}$   
 $\Rightarrow \sqrt{3} = \frac{h+4}{\sqrt{3}h} \Rightarrow 3h = h+4 \Rightarrow h = 2$ 

(iv) (c): Height of the balloon from the ground = BE = BC + CD + DE = 2 + 4 + 2 = 8 m(v) (b)

(i) (d): Let h be the height of the pole. 9. In  $\Delta ABC$ ,

$$\frac{h}{15} = \sin 45^{\circ} \Rightarrow \frac{h}{15} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow h = \frac{15}{\sqrt{2}} \text{ m}$$

$$C = \frac{15}{\sqrt{2}} \text{ m}$$

(ii) (a): Let x be the required distance. In  $\triangle ABC$ ,

$$\frac{x}{15} = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$x = \frac{15}{\sqrt{2}} \text{ m}$$

(iii) (c): Let h be the height of the pole. In right triangle ABC,

$$\frac{h}{15} = \sin 30^\circ = \frac{1}{2}$$

$$\implies h = \frac{15}{2} = 7.5 \text{ m}$$

(iv) (d): If 3 m rope is broken, then the length of the rope is 12 m.

In 
$$\triangle ABC$$
,  $\frac{h}{12} = \sin 30^\circ = \frac{1}{2}$   
 $\Rightarrow h = \frac{12}{2} = 6 \text{ m}$   
(v) (c)

In 
$$\triangle ABC$$
,  $\frac{BC}{AC} = \sin 30^{\circ}$   
 $\Rightarrow \frac{6}{AC} = \frac{1}{2} \Rightarrow AC = 12 \text{ m}$ 

(ii) (b): In  $\triangle ABC$ ,  $\frac{AB}{AC} = \cos 30^{\circ}$  $\Rightarrow \frac{5}{AC} = \frac{\sqrt{3}}{2} \Rightarrow AC = \frac{10}{\sqrt{3}} \text{ m}$ 



 $\Lambda C$ 

 $1^A$ 

(iii) (a): Let BC be the height of window from ground. In  $\triangle ABC$ ,  $\frac{BC}{AB} = \tan 60^\circ$  $\Rightarrow \frac{BC}{2.5} = \sqrt{3}$  $\Rightarrow BC = 2.5 \times 1.73 = 4.325 \text{ m}$ 

(iv) (d): Let AB be the horizontal distance between the foot of ladder and wall.

In  $\triangle ABC$ ,  $\frac{BC}{AB} = \tan 45^\circ$ 8 m B <45°  $\Rightarrow \frac{8}{AB} = 1 \Rightarrow AB = 8 \text{ m}$ (v) (b): Let the required distance be x. In  $\triangle ABC$ ,  $(9)^2 = x^2 + (6)^2$ [By Pythagoras theorem] 9 m/ 6 m  $\Rightarrow$  81 - 36 =  $x^2 \Rightarrow 45 = x^2$  $\Rightarrow x=3\sqrt{5}$  m 11. (i) (b): Total height of pole = 8 m :. BD = AD - AB = (8 - 2)m = 6 m(ii) (a): In  $\triangle BDC$ ,  $\frac{BD}{BC} = \sin 60^{\circ}$  $\Rightarrow \frac{6}{BC} = \frac{\sqrt{3}}{2}$  $\Rightarrow BC = \frac{12}{\sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{2}} = 4\sqrt{3} \text{ m}$ (iii) (d): In  $\triangle BDC$ ,  $\frac{BD}{CD} = \tan 60^\circ \Longrightarrow \frac{6}{CD} = \sqrt{3} \Longrightarrow CD = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{2}} = 2\sqrt{3}m$ (iv) (b): If  $\triangle BCD$ ,  $\frac{BD}{CD} = \tan \theta \Longrightarrow 1 = \tan \theta$ [:: BD = CD] $\Rightarrow \theta = 45^{\circ}$ (v) (c): In  $\triangle BDC$ ,  $\angle B + \angle D + \angle C = 180^{\circ}$  $\therefore \ \angle B = 180^{\circ} - 60^{\circ} - 90^{\circ} = 30^{\circ}$ 12. (i) (c): In  $\triangle ABC$ ,  $\frac{AB}{BC} = \tan 60^{\circ}$  $\Rightarrow AB = 25 \times \sqrt{3}$ ∴ Height of building is  $25\sqrt{3}$  m. (ii) (b): In  $\triangle ABD$ ,  $\frac{AB}{BD} = \tan 30^{\circ}$  $\Rightarrow \frac{25\sqrt{3}}{BD} = \frac{1}{\sqrt{2}} \Rightarrow BD = 75 \text{ m}$ ∴ Distance between two positions of car = (75 – 25) m = 50 m.

(iii) (d): Time taken to cover 50 m distance = 6 sec.

... Time taken to cover 25 m distance = 3 sec.

Total time taken by  $car = 6 \sec + 3 \sec = 9 \sec$ 

(iv) (c): In  $\triangle ABC$ ,  $\frac{BC}{AC} = \cos 60^{\circ}$  $\Rightarrow \frac{25}{10} = \frac{1}{2}$  $\Rightarrow AC = 50 \text{ m}$ (v) (a) **13.** (i) (b):  $\angle XAC = 45^{\circ}$  (Given)  $\therefore \angle ACD = 45^{\circ}$ [Alternate interior angles] (ii) (b) (iii) (c): In  $\triangle ACD$ ,  $\frac{AD}{DC} = \tan 45^\circ$  $\Rightarrow \frac{100}{DC} = 1 \Rightarrow DC = 100 \text{ m}$ (iv) (d): In  $\triangle ABD$ ,  $\frac{AD}{BD} = \tan 30^{\circ}$  $\Rightarrow \frac{100}{BD} = \frac{1}{\sqrt{2}}$  $\Rightarrow BD = 100\sqrt{3} \text{ m}$ (v) (a): In  $\triangle ADC$ ,  $\frac{AD}{AC} = \sin 45^\circ \Rightarrow \frac{100}{AC} = \frac{1}{\sqrt{2}} \Rightarrow AC = 100\sqrt{2} \text{ m}$ 14. (i) (a): In  $\triangle ACD$ ,  $\tan 45^\circ = \frac{CD}{AC} = 1$  $\therefore AC = CD = 20 \text{ m} \qquad \dots(i)$ (ii) (b): Let, BD = h m be the height of the statue. In  $\triangle ABC$ ,  $\tan 60^\circ = \frac{BC}{AC} \Longrightarrow \frac{BD + CD}{AC} = \sqrt{3}$  $\Rightarrow \frac{20+h}{20} = \sqrt{3} [\text{using}(i)] \Rightarrow h = 20 (\sqrt{3}-1) \text{ m}.$ (iii) (d): Since, in  $\triangle ACD$ ,  $\angle DAC = 45^{\circ}$  $\therefore AC = CD (say x)$ In  $\triangle BAC$ , tan60° =  $\frac{BC}{AC}$  $\Rightarrow \frac{1.6+x}{x} = \sqrt{3}$  $\Rightarrow 1.6 = x (\sqrt{3} - 1)$ 

$$\Rightarrow x = \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = 0.8 (\sqrt{3} + 1) \text{ m}$$
  
(iv) (c): In  $\triangle ABC$ ,  
$$\tan 60^{\circ} = \frac{BC}{AC} \Rightarrow \frac{39}{AC} = \sqrt{3}$$
  
$$\Rightarrow AC = \frac{39}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 13\sqrt{3} \text{ m}.$$
  
(v): (a): In  $\triangle ACD$ ,  $\sin 45^{\circ} = \frac{CD}{AD}$   
$$\Rightarrow \frac{35}{AD} = \frac{1}{\sqrt{2}}$$
  
$$\Rightarrow AD = 35\sqrt{2} \text{ m}$$

39 m

A B

D 35\_m

15. (i) (c)

(ii) (a): Let *AB* be the height of pedestal.

In  $\Delta ABC$ ,

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow AB = \frac{25}{\sqrt{3}} = \frac{25}{1.73} = 14.45 \text{ m}$$

(iii) (d): Let x be the distance between Arun and Pankaj. AB 30 m

45%

30 m

<u>∕30</u>°

A

30

S

R

12 m

In 
$$\triangle ABD$$
,  $\tan 45^\circ = \frac{AB}{BD}$   
 $\Rightarrow BD = 30 \text{ m}$ 

Now, in  $\triangle ABC$ ,

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{30}{30+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 30 (\sqrt{3} - 1) = 30 \times 0.73 = 21.9 \text{ m}$$

(iv) (c): In  $\triangle ARS$ ,

$$\sin 30^\circ = \frac{RS}{AS}$$

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$$\Rightarrow \frac{12}{AS} = \frac{1}{2} \Rightarrow AS = 12 \times 2 = 24 \text{ m}$$

(v) (c) : In 
$$\triangle ABD$$
,  $\frac{AB}{BD} = \tan 60^{\circ}$   
 $\Rightarrow \frac{AB}{15} = \sqrt{3}$   
 $\Rightarrow AB = 15 \times 1.73 = 25.95 \text{ m}$   
16. (i) (c) : In  $\triangle OPQ$ , we have  
 $\tan 60^{\circ} = \frac{PQ}{PO}$   
 $\Rightarrow \sqrt{3} = \frac{20}{PO}$   
 $\Rightarrow PO = \frac{20}{\sqrt{3}} \text{ m}$   
(ii) (b): In  $\triangle ORS$ , we have  
 $\tan 30^{\circ} = \frac{RS}{OR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{OR} \Rightarrow OR = 20\sqrt{3} \text{ m}$   
(iii) (d): Clearly, width of the road = PR  
 $= PO + OR = \left(\frac{20}{\sqrt{3}} + 20\sqrt{3}\right) \text{m}$   
 $= 20\left(\frac{4}{\sqrt{3}}\right) \text{m} = \frac{80}{\sqrt{3}} \text{m} = 46.24 \text{ m}$   
(iv) (a): In  $\triangle OPQ$ , if  $\angle POQ = 45^{\circ}$ , then  
 $\tan 45^{\circ} = \frac{PQ}{PO} \Rightarrow 1 = \frac{20}{PO} \Rightarrow PO = 20 \text{ m}$   
(v) (b)  
17. (i) (c): Let AB be the tree of height h m and let it  
broken at height of x m, as shown in figure.  
Clearly  $CD = AC = (h - x) \text{ m}$   
Now, in  $\triangle CBD$ , we have  
 $\tan 30^{\circ} = \frac{x}{30}$   
 $\Rightarrow x = \frac{30}{\sqrt{3}} \text{ m}$ .  
Thus, the height of remaining  $D = \frac{30}{\sqrt{3}} \text{ m}$ .

(ii) (c): In this case, BD = BC = x m

$$\therefore \quad \text{If } \theta \text{ be the angle of inclination, then} \\ \tan \theta = \frac{BC}{BD} = 1$$

$$\Rightarrow \tan \theta = \tan 45^{\circ}$$

$$\implies \ \theta = 45^{\circ}$$

(iii) (b): The angle of elevation and depression are always acute angles.

(iv) (d): Clearly, BD = AB - AD $=(10\sqrt{3}-2\sqrt{3})m=8\sqrt{3}m$ Now, in  $\triangle BCD$ , we have  $\sin 60^\circ = \frac{BD}{DC}$  $\Rightarrow \frac{\sqrt{3}}{2} = \frac{8\sqrt{3}}{DC} \Rightarrow DC = 16 \text{ m}$ (v) (a): Here,  $h = 6 \text{ m}, \theta = 30^{\circ}$ DC = AC = (6 - x) mNow, in  $\triangle BCD$ , we have  $\sin 30^\circ = \frac{BC}{CD}$ 30°  $\Rightarrow \frac{1}{2} = \frac{x}{6-x}$  $\Rightarrow 6 - x = 2x$  $\Rightarrow 3x = 6 \Rightarrow x = 2$ **18.** (i) (d): Clearly,  $\angle DAC = 60^{\circ}$ So, in  $\triangle ADC$ , we have  $\tan 60^\circ = \frac{CD}{AD} \Rightarrow \sqrt{3} = \frac{6}{AD}$  $\Rightarrow AD = \frac{6}{\sqrt{3}}m$ (ii) (b): Clearly, ∠DBC = 30° So, in  $\triangle BDC$ , we have CD

$$\tan 30^\circ = \frac{CD}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{6}{BD}$$

$$\Rightarrow BD = 6\sqrt{3} \text{ m}$$
(iii) (b): Width of the riv

(iii) (b): Width of the river = AB = AD + BD=  $\frac{6}{\sqrt{3}} + 6\sqrt{3}$ =  $6\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) = 6\left(\frac{4}{\sqrt{3}}\right) = \frac{24}{\sqrt{3}}$  m = 13.87 m

(iv) (a): The angle of elevation and angle of depression are always acute angles.

(v) (c): In  $\triangle BCD$ , if BD = 21 m, then

$$\tan 30^\circ = \frac{CD}{BD}$$
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{CD}{21} \Rightarrow CD = \frac{21\sqrt{3}}{3} = 7\sqrt{3} \text{ m}$$

CD

**19.** (i) (b): In the right  $\triangle ADQ$ , we have

$$\sin 30^\circ = \frac{DQ}{AD} \implies \frac{1}{2} = \frac{DQ}{24}$$
  
 $\implies DQ = 12 \text{ m}$ 

Thus, distance of paraglider from the ground is 12 m.

(ii) (a): We have 
$$PQ = BC = 6 \text{ m}$$
  
Now, as  $DQ = 12 \text{ m}$   
 $\therefore DP = DQ - PQ = 12 - 6 = 6 \text{ m}$ 

(iii) (c): In right  $\triangle BDP$ , we have

$$\sin 45^\circ = \frac{DP}{BD} \implies \frac{1}{\sqrt{2}} = \frac{6}{BD}$$

$$\Rightarrow BD = 6\sqrt{2} \text{ m}$$

6 m

x m

Thus, the distance of paraglider from the girl is  $6\sqrt{2}$  m.

(iv) (d):  $\angle AOP$  given in figure, is the angle of depression.

(v) (c): If *A* and *B* are two objects and the eye of an observer is at point *O*, then line of sight will be both *OA* and *OB*.

**20.** (i) (c): Distance of first position of pigeon from the eyes of boy = AC



In  $\triangle ABC$ ,

$$\sin 60^\circ = \frac{BC}{AC} \Rightarrow AC = \frac{CH - BH}{\sin 60^\circ} = \frac{54 - 4}{\sqrt{3}/2} = \frac{100}{\sqrt{3}} \text{ m}$$

(ii) (b): If the distance increases, then the angle of elevation decreases.

(iii) (b): Distance between boy and pole = AB

Now, in  $\triangle ABC$ ,

$$\tan 60^\circ = \frac{BC}{AB} \implies \sqrt{3} AB = 50 \implies AB = \frac{50}{\sqrt{3}} m$$

(iv) (c): In 
$$\triangle AED$$
,  $\tan 45^\circ = \frac{ED}{AD}$   
 $\Rightarrow AD = BC = 50 \text{ m}$  (::  $ED = BC$ )  
Now, distance between two positions of pigeon  $= EC$   
 $= BD = AD - AB$ 

$$= \left(50 - \frac{50}{\sqrt{3}}\right) \mathbf{m} = \frac{50(1.73 - 1)}{1.73} = 21.09 \,\mathrm{m}$$

(v) (a): Speed of pigeon =  $\frac{\text{Distance covered}}{\text{Time taken}}$ =  $\left(\frac{21.09}{8}\right)$  m/sec = 2.63 m/sec

- **21.** Given, side of square top = 2 m
- $\therefore AB = HT = QR = CD = 2 \text{ m}$

Also, *AC* and *BD* are perpendicular to the ground. So, AH = HQ = QC. (By B.P.T. Theorem) (i) (b): In  $\triangle AEC$ ,  $\sin 60^\circ = \frac{AC}{AE} \Rightarrow \frac{\sqrt{3}}{2} = \frac{6}{AE} \Rightarrow AE = 6.93 \text{ m}$  $\therefore$  Length of each leg *i.e.*, AE = BF = 6.93 m. (ii) (c): In  $\triangle AGH$ ,  $\tan 60^\circ = \frac{AH}{GH} \Rightarrow \sqrt{3} = \frac{2}{GH}$  $\Rightarrow GH = 1.15 \text{ m}$ (iii) (a): Length of second step = GH + HT + TU= 1.15 + 2 + 1.15 = 4.3 m (iv) (b): In  $\triangle APQ$ ,

$$\tan 60^\circ = \frac{AQ}{PQ} \Rightarrow \sqrt{3} = \frac{4}{PQ} \Rightarrow PQ = \frac{4}{\sqrt{3}} \text{ m} = 2.31 \text{ m}$$

(v) (c) : Length of first step = *PQ* + *QR* + *RS* =2.31 + 2 + 2.31 = 6.62 m