MODEL PAPER - II

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 100***

General Instructions

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 carry one mark each.

1. Write matrix C for which A + B + C = 0 where

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

2. Let A be a square matrix of order 3 × 3. For what value of k, $|3A| = k\,|A|$.

3. If $f : R - \{-1\} \to R - \{+1\}$ be defined as $f(x) = \dfrac{x}{x+1}$, find $f^{-1}(x)$.

4. What is the value of $\tan\left(\cos^{-1}\dfrac{8}{17}\right)$?

5. Evaluate $\int \sqrt{4x - x^2}\,dx$.

6. Write a vector of magnitude 6 units in the direction of $\hat{i} - 2\hat{j} + 2\hat{k}$

7. Find the value of λ so that the vectors $\hat{i} - \hat{j} - \hat{k}$, $\hat{j} - \hat{k}$ and $\lambda\hat{i} - \hat{j} + \hat{k}$ are coplaner.

8. Evaluate : $\int_{-\pi/2}^{\pi/2} \left(\sin^5 x + x^3 - 2x\right) dx$

9. Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z-3}{6}$ is perpendicular to the plane $3x - y - 2z = 7$.

10. If A is a matrix of order 2×3 and B is of order 3×5, what is the order of $(AB)'$?

SECTION B

Question number 11 to 22 carry 4 marks each.

11. Using properties of determinants, show that

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = \left(1 + a^2 + b^2\right)^3$$

12. Prove that $\sin\left[\tan^{-1}\left(\frac{1 - x^2}{2x}\right) + \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)\right] = 1.$

OR

Prove that $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$.

13. A binary operation * on the set {0, 1, 2, 3, 4, 5} is defined as

$$a * b = \begin{cases} a + b & \text{if } a + b < 6 \\ a + b - 6 & \text{if } a + b \geq 6 \end{cases}$$

show that zero is the identity element for this operation and each non zero element 'a' of the set is invertible with 6 – a being the inverse of a

14. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

OR

If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

15. The function f (x) is defined as

$$f(x) = \begin{cases} \frac{e^{ax} - e^{bx}}{x} & x > 0 \\ 4 & x = 0 \\ \frac{a \log x}{1-x} & x < 0 \end{cases}$$

If f (x) is continuous at x = 0, find the value of 'a' and 'b'

16. A and B throw a die alternately till one of them gets a 5 and wins the game. Find their respective probabilities of winning if A starts the game. Why gambling is not a good way of earning money?

17. Find the intervals in which the function $f(x) = 2x^3 - 12x^2 + 18x - 7$ is increasing or decreasing.

OR

Show that the curves $2x = y^2$ and $2xy = k$ cut at right angle if $k^2 = 8$.

18. Evaluate : $\int \frac{x^2 - 1}{x^4 + 1} dx$

OR

Evaluate : $\int \frac{2 \sin 2\theta - \cos \theta}{6 - \cos^2 \theta - 4 \sin \theta} d\theta$

19. Solve $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$; $y(0) = 0$

20. Form the differential equation of the family of circles in the first quadrant which touches the coordinate axes.

21. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ intersect each other. Also find their point of intersection.

22. Given $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$. Express $\vec{b} = \vec{b_1} + \vec{b_2}$ where $\vec{b_1}$ is parallel to $\vec{a}$ and $\vec{b_2}$ is perpendicular to $\vec{a}$.

OR

The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

SECTION C

Question number 23 to 29 carry 6 marks each.

23. Two schools decided to award prizes to their teachers for two qualities– knowledge and guidance. School A decided to award a total of Rs. 3200 for the values to 4 and 3 teachers respectively while school B decided to award a total of Rs. 1600 for the values to 1 and 2 teachers respectively. Represent the above situation by a system of linear equations and solve using matrices. Which quality you prefer to be rewarded most and why?

24. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$

25. Evaluate : $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$

26. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

27. Find the vector equation of the plane through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which is at a unit distance from the origin.

28. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their output 5%, 4% and 2% are respectively

defective bolts. A bolt is drawn at random from the total production and is found to be defective. Find the probability that it is manufactured by machine B.

OR

An urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls in a random draw of three balls. Also find mean, variance and standard deviation of the distribution.

29. If a young man rides his motorcycle at 25 km/hour he has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 40 km/hour, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as LPP and solve it. 'Speed thrills but kills'. Comment.

ANSWERS

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SECTION A

1. $\begin{bmatrix} -3 & 4 \\ -3 & 0 \end{bmatrix}$

2. $k = 27$

3. $f^{-1}(x) = \dfrac{x}{1-x}$

4. $\dfrac{15}{8}$

5. $\dfrac{(x-2)\sqrt{4x-x^2}}{2} + 2\sin^{-1}\left(\dfrac{x-2}{2}\right) + c$

6. $2\hat{i} - 4\hat{j} + 4\hat{k}$

7. $\lambda = -1$

8. 0

9. $\lambda = -3$

10. 5×2

SECTION B

12. $x = \dfrac{4}{3}$

14. $\sqrt{2}a$

15. $a = -4,\ b = -8$

16. $P(A) = \dfrac{6}{11},\ P(B) = \dfrac{5}{11}$

17. Interval of increasing $(-\infty, 1) \cup (3, \infty)$

    Interval of decreasing $(1, 3)$

18. $\dfrac{1}{2\sqrt{2}} \log\left|\dfrac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right| + c$ **OR**

    $2\log\left|\sin^2\theta - 4\sin\theta + 5\right| + 7\tan^{-1}(\sin\theta - 2) + c$

19. $x = \tan^{-1} y - 1 + e^{-\tan^{-1} y}$

20. $(y - x)^2 (1 + y_1^2) = (x + yy_1)^2$

21. (1, 3, 2)

22. $\vec{b} = \left(\frac{3\hat{i} - \hat{j}}{2}\right) + \left(\frac{\hat{i} + 3\hat{j} - 6\hat{k}}{2}\right)$

OR

$\lambda = 1$

SECTION C

23. $4x + 3y = 3200,$

$x + 2y = 1600$

$x = 400, y = 600$

24. $\frac{1}{4}(\pi - 2)$ sq. units

25. $\frac{1}{\sqrt{2}} \log\left(\sqrt{2} + 1\right)$

26. Maximum Volume $= \frac{4\pi R^3}{3\sqrt{3}}$

27. $\vec{r} \cdot \left(2\hat{i} + \hat{j} + 2\hat{k}\right) + 3 = 0$

OR

$\vec{r} \cdot \left(\hat{i} - 2\hat{j} + 2\hat{k}\right) - 3 = 0$

28. $\frac{28}{69}$

OR

Mean $= \frac{9}{7}$, Variance $= \frac{24}{49}$, Standard deviation $= \frac{2\sqrt{6}}{7}$

29. Total distance = 30 km,

$\frac{50}{3}$ kms at 25 km/hour and

$\frac{40}{3}$ kms at 40 km/hour.