5. Fractions



Equivalent fractions



If one bhakari is divided equally between two people, each one will get **half** a bhakari. The fraction half is written as $\frac{1}{2}$. Here 1 is the numerator and 2 is the denominator.

• One bhakari was divided into four equal parts. Two of the parts were given away. This is shown as $\frac{2}{4}$. Here, 2 is the numerator and 4, the denominator. This, too, means that **half** a bhakari was given.





Six equal parts were made of one melon. They were shared equally by two people. It means that the part that each one got was $\frac{3}{6}$. Each one actually got **half** the melon. Thus, $\frac{3}{6}$ also shows the fraction 'one half'.

In the three examples above, the fraction 'half' has been shown by $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$ respectively. It means that the value of all three fractions is the same. This is written as $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$. Such fractions of equal value are called **equivalent fractions**.



Look at the coloured parts of the two equal circles shown alongside. One circle is divided into 3 equal parts and two of them are coloured. That is, the coloured part is $\frac{2}{3}$ of the circle.

The other circle of the same size is divided into six equal

parts and 4 of them are coloured. That is, $\frac{4}{6}$ of the whole circle is coloured. However, we see that the coloured parts of the two circles are equal. Therefore, $\frac{2}{3} = \frac{4}{6}$.

Thus, $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent fractions.

Obtaining equivalent fractions



Two of the 5 equal parts in the figure are coloured. The coloured part is $\frac{2}{5}$ of the whole figure.

When two lines are drawn across the same figure, it gets divided into 15 equal parts. So, now, the fraction that shows the coloured part is $\frac{6}{15}$.

However, the coloured part has not changed. Therefore, we see that $\frac{2}{5} = \frac{6}{15}$.

- **Teacher :** Do you see any special connection between the numerators and denominators of the fractions $\frac{2}{5}$ and $\frac{6}{15}$?
- **Sonu** : Three times 2 is 6 and three times 5 is fifteen.
- **Teacher :** We have also seen that $\frac{1}{2} = \frac{2}{4}$, $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{3} = \frac{4}{6}$. In two equivalent fractions, the numerator of one fraction is as many times the numerator of the other as the denominator of one is of the denominator of the other.

When the numerator and denominator of a fraction are multiplied by the same non-zero number, we get a fraction that is equivalent to the given fraction.

- **Nandu** : Can I get an equivalent fraction by dividing the numerator and denominator by the same number?
- **Teacher :** Of course! If the numerator and denominator have a common divisor, then the fraction obtained on actually dividing them by that divisor is equivalent to the given fraction. The numerator and denominator of the fraction $\frac{6}{15}$ can be divided by 3. On doing this division, we get the fraction $\frac{2}{5}$. It means that $\frac{6}{15} = \frac{2}{5}$.

If the numerator and denominator have a common divisor then the fraction we get on dividing them by that divisor is equivalent to the given fraction.

Teacher : Divide the numerator and denominator of $\frac{6}{12}$ by the same number to find an equivalent fraction.

Sonu obtained	6	$6 \div 2$	3	Minu obtained	6	6÷3	2
this fraction	12	$=\overline{12\div 2}$	$= \frac{1}{6}$	this fraction	$\overline{12}$	$=\overline{12\div3}$	$=\overline{4}$

Nandu : 6 and 12 can also be divided by 6. Will that do ?

Teacher : Sure. $\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$.

Remember that the fractions we get by dividing $\frac{6}{12}$ by 2 or 3 or 6 are all equivalent to $\frac{6}{12}$. That is $\frac{6}{12} = \frac{3}{6} = \frac{2}{4} = \frac{1}{2}$.

Example (1) Find a fraction with denominator 30 which is equivalent to $\frac{5}{6}$. $\frac{5}{6} = \frac{5}{30}$. We must find the right number for the box. Here, 5 times the denominator 6 is 30. What is five times the numerator 5? $\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$. Hence, the fraction $\frac{25}{30}$ with denominator 30 is equivalent to $\frac{5}{6}$.

Example (2) Find a fraction equivalent to $\frac{15}{40}$ but with denominator 8.

- $\frac{15}{40} = \frac{\square}{8}$. We must find the number for the box.
- 40 divided by 5 is 8.

So, we will get the number for the box by dividing 15 by 5. $15 \div 5 = 3$.

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- Therefore, $\frac{15}{40} = \frac{3}{8}$ Thus, the fraction $\frac{3}{8}$ is equivalent to the fraction $\frac{15}{40}$.
- **1.** Write the proper number in the box.

(1)
$$\frac{1}{2} = \frac{\Box}{20}$$
 (2) $\frac{3}{4} = \frac{15}{\Box}$
(5) $\frac{14}{26} = \frac{\Box}{13}$ (6) $\frac{\Box}{3} = \frac{4}{6}$

2. Find an equivalent fraction with denominator 18, for each of the following fractions.

- $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{6}$, $\frac{2}{9}$, $\frac{7}{9}$, $\frac{5}{3}$
- 4. From the fractions given below, pair off the equivalent fractions.

 $\frac{2}{3}$, $\frac{5}{7}$, $\frac{5}{11}$, $\frac{7}{9}$, $\frac{14}{18}$, $\frac{15}{33}$, $\frac{18}{27}$, $\frac{10}{14}$

Like fractions and unlike fractions

Fractions such as $\frac{1}{7}$, $\frac{4}{7}$, $\frac{6}{7}$ whose denominators are equal, are called 'like fractions'.

Fractions such as $\frac{1}{3}$, $\frac{4}{8}$, $\frac{9}{11}$ which have different denominators are called 'unlike fractions'.

Converting unlike fractions into like fractions

Example (1) Convert $\frac{5}{6}$ and $\frac{7}{9}$ into like fractions.

Here, we must find a common multiple for the numbers 6 and 9.

Multiples of 6 : 6, 12, 18, 24, 30, 36,

(3) $\frac{9}{11} = \frac{18}{\Box}$	(4) $\frac{10}{40} = \frac{1}{8}$
$(7) \frac{1}{\Box} = \frac{4}{20}$	$(8)\frac{\Box}{5} = \frac{10}{25}$

.....

3. Find an equivalent fraction with denominator 5, for each of the following fractions.

 $\frac{6}{15}$, $\frac{10}{25}$, $\frac{12}{30}$, $\frac{6}{10}$, $\frac{21}{35}$

5. Find two equivalent fractions for each of the following fractions.

 $\frac{7}{9}, \frac{4}{5}, \frac{3}{11}$

Multiples of 9 : 9, 18, 27, 36, 45

Here, the number 18 is a multiple of both 6 and 9. So, let us make 18 the denominator of both fractions.

 $\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18} \qquad \frac{7}{9} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18}$ Thus, $\frac{15}{18}$ and $\frac{14}{18}$ are like fractions, respectively equivalent to $\frac{5}{6}$ and $\frac{7}{9}$.

Here, 18 is a multiple of both 6 and 9. We could also choose numbers like 36 and 54 as the common denominators.

Example (2) Convert $\frac{4}{8}$ and $\frac{5}{16}$ into like fractions. As 16 is twice of 8, it is easy to make 16 the common denominator.

 $\frac{4}{8} = \frac{4 \times 2}{8 \times 2} = \frac{8}{16}$. Thus, $\frac{8}{16}$ and $\frac{5}{16}$ are the required like fractions.

Example (3) Find a common denominator for $\frac{4}{7}$ and $\frac{3}{4}$.

The number 28 is a multiple of both 7 and 4. So, make 28 the common denominator.

 $\frac{4}{7} = \frac{4 \times 4}{7 \times 4} = \frac{16}{28}, \ \frac{3}{4} = \frac{3 \times 7}{4 \times 7} = \frac{21}{28}.$ Therefore, $\frac{16}{28}$ and $\frac{21}{28}$ are the required like fractions.

Convert the given fractions into like fractions.

(1) $\frac{3}{4}$, $\frac{5}{8}$	(2) $\frac{3}{5}$, $\frac{3}{7}$	(3) $\frac{4}{5}$, $\frac{3}{10}$	(4) $\frac{2}{9}, \frac{1}{6}$
(5) $\frac{1}{4}$, $\frac{2}{3}$	(6) $\frac{5}{6}$, $\frac{4}{5}$	(7) $\frac{3}{8}$, $\frac{1}{6}$	(8) $\frac{1}{6}$, $\frac{4}{9}$

Comparing like fractions

Example (1) A strip was divided into 5 equal parts. It means that each part is $\frac{1}{5}$. The

coloured part is $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$. The white part is $\frac{2}{5} = \frac{1}{5} + \frac{1}{5}$. The coloured part is bigger than

the white part. This tells us that $\frac{3}{5}$ is greater than $\frac{2}{5}$. This is written as $\frac{3}{5} > \frac{2}{5}$.

Example (2) This strip is divided into 8 equal parts. 3 of the parts have one colour and 4 have another colour. Here, $\frac{3}{8} < \frac{4}{8}$.

.....

In like fractions, the fraction with the greater numerator is the greater fraction.

Comparing fractions with equal numerators

You have learnt that the value of fractions with numerator 1 decreases as the denominator increases.

Even if the numerator is not 1, the same rule applies so long as all the fractions have a common numerator. For example, look at the figures below. All the strips in the figure are alike.



Of two fractions with equal numerators, the fraction with the greater denominator is the smaller fraction.

Comparing unlike fractions



Teacher	:	Suppose we have to compare the unlike fractions $\frac{1}{5}$
		and $\frac{1}{7}$. Let us take an example to see how this is done.
		These two boys are standing on two blocks. How do we decide who is taller?
Sonu	:	But the height of the blocks is not the same. If both blocks are of the same height, it is easy to tell who is

3



Nandu	:	Now that they are on blocks of equal height, we see					
that the boy on the right is taller.							

Teacher : The height of the boys can be compared when they stand at the same height. Similarly, if fractions have the same denominators, their numerators decide which fraction is bigger.

Nandu : Got it! Let's obtain the same denominators for both fractions.



taller.

Sonu : 5×7 can be divided by both 5 and 7. So, 35 can be the common denominator.

$$\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35} \qquad \qquad \frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$$
$$\frac{21}{35} > \frac{20}{35} \qquad \text{Therefore,} \quad \frac{3}{5} > \frac{4}{7}$$

To compare unlike fractions, we convert them into their equivalent fractions so that their denominators are the same.

Write the proper symbol from \langle , \rangle , or = in the box.



Addition of like fractions



Example (1)
$$\frac{3}{7} + \frac{2}{7} = ?$$

Let us divide a strip into 7 equal parts. We shall colour 3 parts with one colour and 2 parts with another.

The part with one colour is $\frac{3}{7}$, and that with the other colour is $\frac{2}{7}$.

The total coloured part is shown by the fraction $\frac{5}{7}$.

It means that, $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$.

Example (2) Add : $\frac{3}{8} + \frac{2}{8} + \frac{1}{8}$.



The total coloured part is $\frac{3}{8} + \frac{2}{8} + \frac{1}{8} = \frac{3+2+1}{8} = \frac{6}{8}$.

When adding like fractions, we add the numerators of the two fractions and write the denominator as it is.

Example (3) Add : $\frac{2}{6} + \frac{4}{6}$ $\frac{2}{6} + \frac{4}{6} = \frac{2+4}{6} = \frac{6}{6}$. However, we know that $\frac{6}{6}$ means that all 6 of the 6 equal parts are taken. That is, 1 whole figure is taken. Therefore, $\frac{6}{6} = 1$. Note that :

If the numerator and denominator of a fraction are equal, the fraction is equal to one.

That is why, $\frac{7}{7} = 1$; $\frac{10}{10} = 1$; $\frac{2}{5} + \frac{3}{5} = \frac{2+3}{5} = \frac{5}{5} = 1$.

Remember that, if we do not divide a figure into parts, but keep it whole, it can also be written as 1.

This tells us that $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3}$ and so on.

You also know that if the numerator and denominator of a fraction have a common divisor, then the fraction obtained by dividing them by that divisor is equivalent to the given fraction.

$$\frac{5}{5} = \frac{5 \div 5}{5 \div 5} = \frac{1}{1} = 1$$
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1. Add :

 $(1) \frac{1}{5} + \frac{3}{5} \qquad (2) \frac{2}{7} + \frac{4}{7} \qquad (3) \frac{7}{12} + \frac{2}{12} \qquad (4) \frac{2}{9} + \frac{7}{9} \qquad (5) \frac{3}{15} + \frac{4}{15} \\ (6) \frac{2}{7} + \frac{1}{7} + \frac{3}{7} \qquad (7) \frac{2}{10} + \frac{4}{10} + \frac{3}{10} \qquad (8) \frac{4}{9} + \frac{1}{9} \qquad (9) \frac{5}{8} + \frac{3}{8} \\ \end{cases}$

2. Mother gave $\frac{3}{8}$ of one guava to Meena and $\frac{2}{8}$ of the guava to Geeta.

What part of the guava did she give them altogether?

3. The girls of Std V cleaned $\frac{3}{4}$ of a field while the boys of Std IV cleaned $\frac{1}{4}$. What part of the field was cleaned altogether?

Subtraction of like fractions

A figure is divided into 5 equal parts and 4 of them are coloured. That is, $\frac{4}{5}$ part of the figure is coloured.



Now, we remove the colour from one of the coloured parts. That is, we subtract $\frac{1}{5}$ from $\frac{4}{5}$. The remaining coloured part is $\frac{3}{5}$. Therefore, $\frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}$

When subtracting a fraction from another like fraction, we write the difference between the numerators in the numerator and the common denominator in the denominator.

Example (1) Subtract : $\frac{7}{13} - \frac{5}{13}$

These two fractions have a common denominator. So, we shall subtract the second numerator from the first and write the denominator as it is.

$$\frac{7}{13} - \frac{5}{13} = \frac{7-5}{13} = \frac{2}{13}$$

Example (2) If Raju got $\frac{5}{12}$ part of a sugarcane and Sanju got $\frac{3}{12}$ part, how much was the extra part that Raju got?

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To find out the difference, we must subtract.

$$\frac{5}{12} - \frac{3}{12} = \frac{5-3}{12} = \frac{2}{12}$$
. Thus, Raju got $\frac{2}{12}$ extra.

1. Subtract :

(1)	$\frac{5}{7} - \frac{1}{7}$	(2) $\frac{5}{8} - \frac{3}{8}$	(3) $\frac{7}{9} - \frac{2}{9}$	(4)	$\frac{8}{11}$ -	$-\frac{5}{11}$
(5)	$\frac{9}{13} - \frac{4}{13}$	(6) $\frac{7}{10} - \frac{3}{10}$	(7) $\frac{9}{12} - \frac{2}{12}$	(8)	$\frac{10}{15}$ -	$-\frac{3}{15}$

2. $\frac{7}{10}$ of a wall is to be painted. Ramu has painted $\frac{4}{10}$ of it. How much more needs to be painted?

☐ Addition and subtraction of unlike fractions

Example (1) Add: $\frac{2}{3} + \frac{1}{6}$ First let us show the fraction $\frac{2}{3}$ by colouring two of the three equal parts on a strip. You have learnt to add and to subtract fractions with common denominators. Here, we

have to add the fraction $\frac{1}{6}$ to the fraction $\frac{2}{3}$.

So let us divide each part on this strip into two equal parts. $\frac{4}{6}$ is a fraction equivalent to $\frac{2}{3}$. Now, as $\frac{1}{6}$ is to be added to $\frac{2}{3}$ i.e. to $\frac{4}{6}$, we shall colour one more of the six parts on the strip. Now, the total coloured part is $\frac{5}{6}$.

Therefore,
$$\frac{4}{6} + \frac{1}{6} = \frac{4+1}{6} = \frac{5}{6}$$
.
That is, $\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$.

Example (2) Add : $\frac{1}{2} + \frac{2}{5}$

Here, the smallest common multiple of the two denominators is 10. So, we shall change the denominator of both fractions to 10.

$$\frac{1}{2} + \frac{2}{5} = \frac{1 \times 5}{2 \times 5} + \frac{2 \times 2}{5 \times 2}$$
$$= \frac{5}{10} + \frac{4}{10}$$
$$= \frac{5+4}{10} = \frac{9}{10}$$

Example (4) Subtract : $\frac{3}{4} - \frac{5}{8}$

Let us make 8 the common denominator of the given fractions.

$\frac{3}{4}$ –	$\frac{5}{8}$	=	$\frac{3\times 2}{4\times 2} -$	$\frac{5}{8}$
		=	$\frac{6}{8} - \frac{5}{8}$	
		=	$\frac{6-5}{8}$	
		=	$\frac{1}{8}$	

Example (3) Add : $\frac{3}{8} + \frac{1}{16}$

Here, 16 is twice 8. So, we shall change the denominator of both fractions to 16.

$$\frac{3}{8} + \frac{1}{16} = \frac{3 \times 2}{8 \times 2} + \frac{1}{16}$$
$$= \frac{6}{16} + \frac{1}{16}$$
$$= \frac{6+1}{16} = \frac{7}{16}$$

Example (5) Subtract : $\frac{4}{5} - \frac{2}{3}$

The smallest common multiple of the denominators is 15. So, we shall change the denominator of both fractions to 15.

$$\frac{4}{5} - \frac{2}{3} = \frac{4 \times 3}{5 \times 3} - \frac{2 \times 5}{3 \times 5}$$
$$= \frac{12}{15} - \frac{10}{15}$$
$$= \frac{12 - 10}{15}$$
$$= \frac{2}{15}$$

1. Add :

(1) $\frac{1}{8} + \frac{3}{4}$	(2) $\frac{2}{21} + \frac{3}{7}$	(3) $\frac{2}{5} + \frac{1}{2}$	(4) $\frac{2}{7} + \frac{1}{2}$	(5) $\frac{3}{9} + \frac{3}{5}$
8 4	21 7	5 3	7 7 2	95

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2. Subtract :

(1)	3	1	$(2) \frac{3}{2}$	1	(2) $\begin{array}{c} 6 & 2 \\ \end{array}$ (4) $\begin{array}{c} 4 \\ - \end{array} = \begin{array}{c} 3 \\ - \end{array}$ (5)	2	1
(1)	10	20	$\binom{2}{4} - \frac{1}{4}$	2	$(3) \frac{14}{14} - \frac{7}{7}$ (4) 6 5 (3)	7	4

□ A fraction of a collection and a multiple of a fraction



- Meena has 5 rupees. Tina has twice as many rupees. That is, Tina has $5 \times 2 = 10$ rupees. Meena has half as many rupees as Tina, that is, $\frac{1}{2}$ of 10, or, 5 rupees. Ramu has to travel a distance of 20 km. If he has travelled $\frac{4}{5}$ of the distance by car, how many kilometres did he travel by car? $\frac{4}{5}$ of 20 km is $20 \times \frac{4}{5}$. So, we take $\frac{1}{5}$ of 20, 4 times. $\frac{1}{5}$ of 20 = 4. 4 times 4 is 4 × 4 = 16. It means that $20 \times \frac{4}{5} = 16$. Ramu travelled a distance of 16 kilometres by car. ······Q Problem Set 23 0··· 1. What is $\frac{1}{3}$ of each of the collections given below? (2) 21 balloons (3) 9 children (4) 18 books (1) 15 pencils 2. What is $\frac{1}{5}$ of each of the following ? (3) 15 litres (4) 25 cm (2) 30 km (1) 20 rupees 3. Find the part of each of the following numbers equal to the given fraction. (2) $\frac{7}{11}$ of 22 (3) $\frac{3}{8}$ of 64 (4) $\frac{5}{13}$ of 65 (1) $\frac{2}{3}$ of 30 ☐ Mixed fractions Half of each of the three circles is coloured. That is, 3 parts, each equal to $\frac{1}{2}$ of the circle, are coloured. $\frac{1}{2}$ The coloured part is $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, that is, $\frac{3}{2}$ or $1 + \frac{1}{2}$. $1 + \frac{1}{2}$ is written as $1 \frac{1}{2}$. $1\frac{1}{2}$ is read as 'one integer one upon two'. In the fraction $1\frac{1}{2}$, 1 is the integer part and $\frac{1}{2}$ is the fraction part. Hence, such fractions are called **mixed fractions** or **mixed numbers**. $2\frac{1}{4}$, $3\frac{2}{5}$, $7\frac{4}{9}$ are all mixed fractions. Fractions in which the numerator is greater than the denominator are called improper fractions.
 - $\frac{3}{2}$, $\frac{5}{3}$ are improper fractions. We can convert improper fractions into mixed fractions.

For example, $\frac{3}{2} = \frac{2+1}{2} = \frac{2}{2} + \frac{1}{2} = 1 + \frac{1}{2} = 1\frac{1}{2}$

Activities

1. Colour the Hats.



In the picture alongside : Colour $\frac{1}{3}$ of the hats red. Colour $\frac{3}{5}$ of the hats blue. How many hats have you coloured red? How many hats have you coloured blue? How many are still not coloured?

2. Make a Magic Spinner.



Take a white cardboard disc. As shown in the figure, divide it into six equal parts.

Colour the parts red, orange, yellow, green, blue and violet.

Make a small hole at the centre of the disc and fix a pointed stick in the hole.

Your magic spinner is ready.

What fraction of the disc is each of the coloured parts?

Give the disc a strong tug to make it turn fast. What colour does it appear to be now ?



The Clever Poet

There was a king who had a great love for literature. A certain poet knew that if the king read a good poem it made him very happy. Then the king would give the poet an award. Once, the poet composed a good poem. He thought if he showed it to the king, he would win a prize. So, he went to the king's palace. But, it was not easy to meet the king. You had to pass a number of gates and guards. The first guard asked the poet why he wanted to meet the king. So, the poet told him the reason. Seeing the chance of getting a share of the award, the guard demanded, 'You must

give me $\frac{1}{10}$ of your prize. Only then I will

let you go in.' The poet could do nothing but agree. The second guard stopped him and said, 'I will let you go in only if you promise me $\frac{2}{5}$ of your prize.' The third guard, too, was a greedy man. He said, 'I will not let you go, unless you promise me $\frac{1}{4}$ of your prize.' The king's palace was just a little distance away. Now, the poet told the guard, 'Why only $\frac{1}{4}$, I shall give you half the prize!' The guard was pleased and let him in.

The king liked the poem. He asked the poet, 'What is the prize you want?' 'I shall be happy if Your Majesty awards me 100 lashes of the whip.' The king was surprised. 'Are you out of your mind!' he exclaimed. 'I have never met anyone so crazy as to ask for a whipping!'



'Your Majesty, if you wish to know the reason, the three palace guards must be called here.' When the guards came, the poet explained, 'Your Majesty, all of them have a share in the 100 lashes that you have awarded to me. Each of them has fixed his own share of the prize I get. The first guard must get $\frac{1}{10}$ of the award, that is, lashes. The second must get $\frac{2}{5}$, which is , and the third must get half the award, that is, lashes!' The king could now see how greedy the guards were and how clever the poet was. He saw to it that each guard got the punishment he deserved. He gave the poet a prize for his poem. He also gave him an extra 100 gold coins for exposing the greed of the guards.

What was the clever idea of the poet which the king appreciated so much?

