CBSE Class 09

Mathematics

Sample Paper 6 (2019-20)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. The product of two irrational numbers is

   a. always irrational

   b. always an integer

   c. always rational

   d. either irrational or rational

2. If $x + \frac{1}{x} = 7$, then the value of $x^3 + \frac{1}{x^3}$ is

   a. 231

   b. 320

c. 322

d. 233

3. The number of angles formed by a transversal with a pair of parallel lines are

a. 8

b. 4

c. 6

d. 3

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4. The construction of a $\triangle ABC$ with AB = 6 cm and $\angle A = 60^0$ is not possible when sum of BC and CA is equal to______.

a. 7 cm

b. 8 cm

c. 5.5 cm

d. 7.5 cm

5. If $p(x) = x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2}) =$

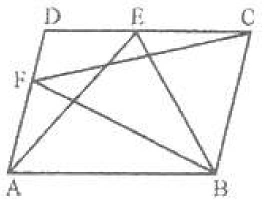
a. 1

b. $2\sqrt{2}$

c. 0

d. -1

6. In the figure, ABCD is a parallelogram, if area of $\triangle AEB$ is $16\ cm^2$, then area of $\triangle BFC$ is :



a. $8\,cm^2$.

b. $16\ cm^2$.

c. $24\ cm^2$.

d. $32\ cm^2$.

7. If $x^2 + kx - 3 = (x-3)(x+1)$, then the value of ‘k’ is

a. -3

b. 2

c. -2

d. 3

8. The area and length of one diagonal of a rhombus are given as 200 cm^2 and 10 cm respectively. The length of other diagonal is

a. 25 cm

b. 10 cm

c. 40 cm

d. 20 cm

9. The number of planks of dimensions $(5m \times 25cm \times 10cm)$ that can be placed in a pit which is 20 m long, 6 m wide and 80 cm deep is

a. 840.

b. 960.

c. 768.

d. 764.

10. Five cards – nine, ten, jack, queen and king of hearts are well-shuffled with their faces downwards. One card is picked at random. The probability that the drawn card is a king, is :

a. $\frac{1}{5}$

b. $\frac{2}{5}$

c. $\frac{4}{5}$

d. $\frac{3}{5}$

11. Fill in the blanks:

The product of two irrational numbers is ________.

12. Fill in the blanks:

2x = -5y in the form of ax + by + c = 0 is ________.

OR

Fill in the blanks:

The equation 2x + 5y = 7 has a unique solution, if x and y are ________.

13. Fill in the blanks:

A point both of whose coordinates are negative will lie in ________.

14. Fill in the blanks:

Angle inscribed in a semi-circle is a/an ________ angle.

15. Fill in the blanks:

If the radius of a sphere is 2r, then its volume will be ________.

16. If $\sqrt{2}$ = 1.414, find the value of $\sqrt{3} \div \sqrt{6}$ upto three places of decimals.

17. Write $(m + 2n - 5p)^2$ in the expanded form

18. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold ? ($1\ m^3 = 1000\ l$)

OR

The inner radius of pipe is 2.5 cm. How much water can 10 m of this pipe hold?

19. If two adjacent sides of a kite are 5 cm and 7 cm, find its perimeter.

20. If x = 2k - 1 and y = k is a solution of the equation 3x - 5y - 7 = 0, find the value of k.

21. Represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line.

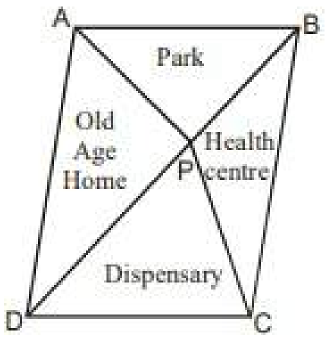
22. If x = 1 and y = 6 is solution of the equation $8x - ay + a^2 = 0$, find the values of a.

23. If $f(x) = 2x^3 - 13x^2 + 17x + 12$, find f (-3)

OR

Find the zeroes of the polynomial $(x-2)^2 - (x+2)^2$.

24. A plot is in the form of a parallelogram ABCD. Owner of this plot wants to build OLD AGE HOME, DISPENSARY, PARK and HEALTH CENTRE for elderly people as shown in the fig. P is a point on the diagonal BD.



Prove that area allotted to old age home and dispensary is same.

25. Following data gives the number of children in 40 families:
1, 2, 6, 5, 1, 5, 1, 3, 2, 6, 2, 3, 4, 2, 0, 0, 4, 4, 3, 2, 2, 0, 0, 1, 2, 2, 4, 3, 2, 1, 0, 5, 1, 2, 4, 3, 4, 1,

6, 2, 2.

Represent it in the form of a frequency distribution.

OR

The following is the distribution of weight (in kg) of 50 persons :

Weight (in kg)	**Number of persons**
50-55	12
55-60	8
60-65	5
65-70	4
70-75	5
75-80	7
80-85	6
85-90	3
Total	50

Draw a histogram for the above data.

26. Mukta had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with 3.5 cm radius?

27. Find three rational numbers between $2.\overline{2}$ and $2.\overline{3}$

OR

Locate $\sqrt{3}$ on the number line.

28. Plot the following points and check whether they are collinear or not: (1, 1), (2, -3), (-1, -2)

29. Find at least 3 solutions for the following linear equation in two variables: 2x – 3y + 7 = 0

OR

The cost of a toy horse is same as that of cost of 3 balls. Express this statement as a linear equation in two variables. Also draw its graph.

30. Construct a $\triangle$ABC in which BC = 5.6 cm, AC - AB = 1.6 cm and $\angle$B = 45°. Justify your construction.

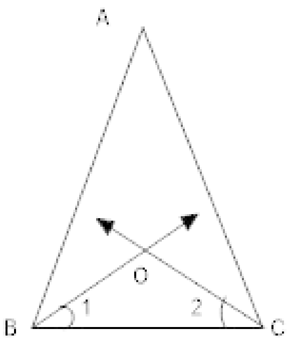
31. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Then prove that,

   i. D is the midpoint AC
   ii. MD is perpendicular to AC
   iii. CM = AM = $\frac{1}{2}$ AB

32. ABCD is a quadrilateral such that diagonal AC bisects the angles A and C. Prove that AB = AD and CB = CD.

OR

If $\triangle$ABC, the bisector of $\angle$ABC and $\angle$BCA intersect each other at the point O prove that BOC = 90° $\frac{1}{2}\angle A$.



33. Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs7 per m^2.

34. 1500 families with 2 children were selected randomly and the following data were

recorded

No. of girls in a family	No. of families
2	475
1	814
0	211

Compute the probability of a family, chosen at random, having.

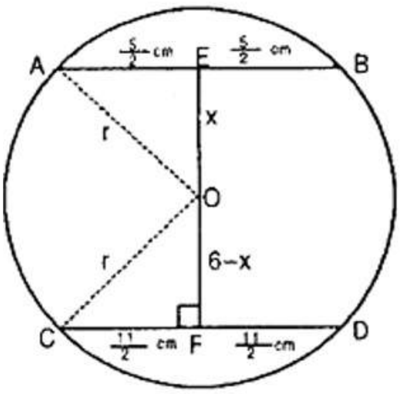
(i) 2 girls

(ii) 1 girl

(iii) No girl

Also check the sum of these probabilities

35. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If a distance between AB and CD is 6 cm, find the radius of the circle.



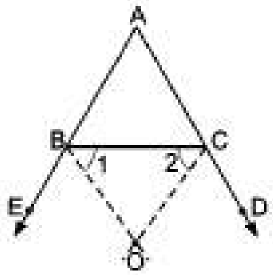
OR

ABCD is a cyclic quadrilateral whose diagonal AC and BD intersect at P. If AB = DC, Prove that:

i. $\triangle PAB \cong \triangle PDC$
ii. PA = PD and PC = PB
iii. AD | | BC.

36. In $\triangle$ABC in given figure, the sides AB and AC of $\triangle$ABC are produced to points E and

D respectively. If bisectors BO and CO of $\angle$CBE and $\angle$BCD respectively meet at point O, then prove that $\angle BOC = 90° - \frac{1}{2} \angle A$.



37. If the polynomials $(2x^3 + kx^2 + 3x - 5)$ and $(x^3 + x^2 - 2x + 2k)$ leave the same remainder, when divided by $(x - 3)$, then find the value of k. Also, find the remainder in first case.

OR

If $x + \frac{1}{x} = 3$, calculate $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$ and $x^4 + \frac{1}{x^4}$

38. A rectangular sheet of paper 30 cm $\times$ 18 cm can be transformed into the curved surface of a right circular cylinder in two ways i.e., either by rolling the paper along its length or by rolling it along with its breadth. Find the ratio of the volumes of the two cylinders thus formed.

OR

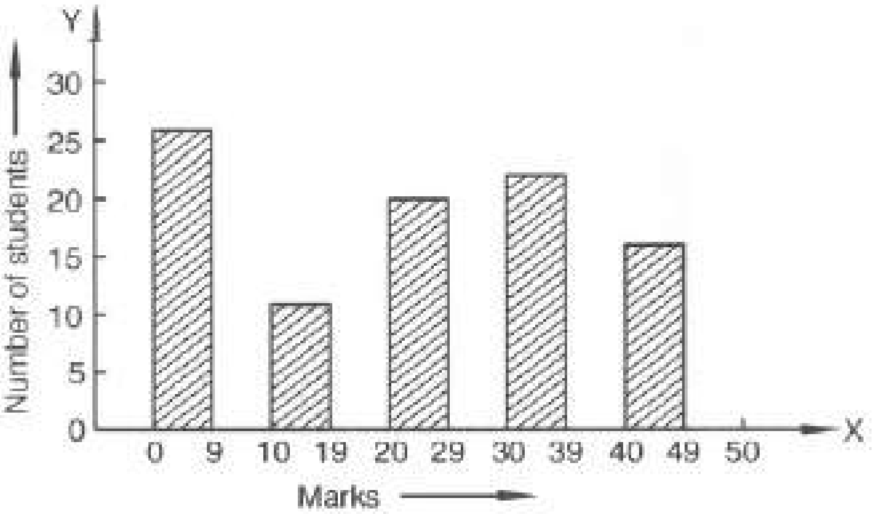
A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylinderical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of the conical part is 12 cm.

39. ABCD is a quadrilateral in which AB = AD, BC = DC and diagonals intersect at point E. Prove that

    i. AC bisects each of the angles A and C.
    ii. BE = ED
    iii. $\angle ABC = \angle ADC$. Is AE = EC ?

40. Given below figure is the bar graph indicating the marks obtained out of 50 in

mathematics paper by 100 students. Read the bar graph and answer the following questions:



i. It is decided to distribute workbooks on mathematics to the students obtaining less than 20 marks, giving one workbook to each of such students. If a workbook costs Rs.5, what sum is required to buy the workbooks?
ii. Every student belonging to the highest mark group is entitled to get a prize of Rs 10. How much amount of money is required for distributing the prize money?
iii. Every student belonging to the lowest mark-group has to solve 5 problems per day. How many problems, in all, will be solved by the students of this group per day?
iv. State whether true or false.
    a. 17% students have obtained marks ranging from 40 to 49.
    b. 59 students have obtained marks ranging from 10 to 29.
v. What is the number of students getting less than 20 marks?
vi. What is the number of students getting more than 29 marks?
vii. What is the number of students getting marks between 9 and 40?
viii. What is the number of students belonging to the highest mark group?
ix. What is the number of students obtaining more than 19 marks?

CBSE Class 09

Mathematics

Sample Paper 6 (20-20)

Solution

Section A

1. (d) either irrational or rational

   Explanation: $\sqrt{5} \times \sqrt{2} = \sqrt{10}$ is irrational number

   $\sqrt{5} \times \sqrt{5} = 5$ is rational number
2. (c) 322

   Explanation:

   $x + \frac{1}{x} = 7$

   $\Rightarrow \left(x + \frac{1}{x}\right)^3 = 7^3$

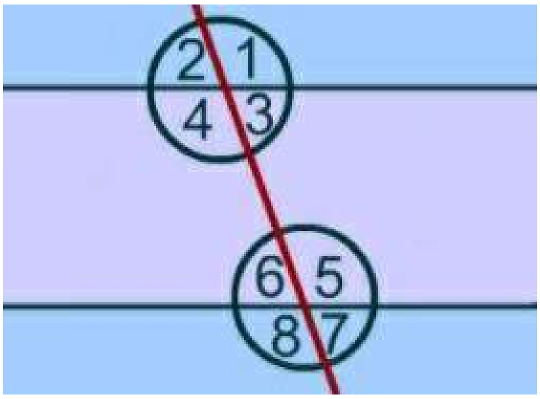
   $\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 343$

   $\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 7 = 343$

   $\Rightarrow x^3 + \frac{1}{x^3} = 343 - 21$

   $\Rightarrow x^3 + \frac{1}{x^3} = 322$
3. (a) 8

   Explanation:

   As we can see there are 4 angles formed at every point of intersection thus giving a total of 8 angles.

4. (c) 5.5 cm

   Explanation: To construct a triangle whose base, base angle and sum of other two sides are given, the sum of other two sides should be more than its base.

   But here, BC+CA<AB, so, we cannot construct it.

5. (a) 1

   Explanation:

   $p(x) = x^2 - 2\sqrt{2}x + 1$

   $\Rightarrow p\left(2\sqrt{2}\right) = \left(2\sqrt{2}\right)^2 - 2\sqrt{2}\left(2\sqrt{2}\right) + 1$

   $\Rightarrow\ p\left(2\sqrt{2}\right) = 8 - 8 + 1$

   $\Rightarrow p\left(2\sqrt{2}\right) = 1$

6. (b) $16\ cm^2$.

   Explanation:

   Given: Area of triangle ABE = 16 cm^2
   Since Parallelogram ABCD and $\triangle\text{ABE}$ are on the same base and between two parallels.
   Therefore,
   $\text{area}\left(\triangle\text{ABE}\right) = \frac{1}{2} \times \text{area}\left(\| gm\text{ABCD}\right)$ .... (i)
   Also,
   Since Parallelogram ABCD and ΔBFC are on the same base and between two parallels.
   Therefore,
   $\text{area}\left(\triangle\text{BFC}\right) = \frac{1}{2} \times \text{area}\left(\| gm\text{ABCD}\right)$ ....(ii)
   From eq.(i) and (ii), we have
   $\text{area}\left(\triangle\text{ABE}\right) = \text{area}\left(\triangle\text{BFC}\right)$
   $\Rightarrow \text{area}\left(\triangle\text{BFC}\right) = 16\ \text{cm}^2$

7. (c) -2

   Explanation:

$x^2 + kx - 3 = (x - 3)(x + 1)$

$\Rightarrow x^2 + kx - 3 = x^2 + (-3 + 1)\,x + (-3) \times 1$

$\Rightarrow \; x^2 + kx - 3 = x^2 - 2x - 3$

On comparing the term, we get $k = -2$

8. (c) 40 cm

   Explanation:

   Area of rhombus = $\frac{1}{2}$ x Product of diagonal

   $\Rightarrow$ 200 = $\frac{1}{2}$ x 10 x d_2

   $\Rightarrow$ d_2 = $\frac{200 \times 2}{10}$ = 40 cm

9. (c) 768.

   Explanation:

   Volume of planks = 25×500×10

   =125000 cm^3

   Volume of pit = 2000×600×80

   =96000000 cm^3

   Number of plank = $\frac{Volume\ of\ pit}{volume\ of\ plank}$

   = $\frac{96000000}{125000}$

   = 768

10. (a) $\frac{1}{5}$

    Explanation: Total number of possible outcomes = 5

    Number of Kings = 1

    The probability that the drawn card is a king = $\frac{1}{5}$

11. either irrational or rational

12. 2x + 5y = 0

OR

natural numbers

13. III quadrant

14. right

15. $\frac{32}{3}\pi r^3$

16. The given expression:

$\sqrt{3} \div \sqrt{6} = \frac{\sqrt{3}}{\sqrt{6}} = \sqrt{\frac{3}{6}}$

$= \sqrt{\frac{1}{2}}$

$= \frac{1}{\sqrt{2}}$

Now on multiplying numerator and denominator by $\sqrt{2}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$= \frac{1.414}{2}$

= 0.707

17. We have,

$(m + 2n - 5p)^2$

Using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (m)^2 + (2n)^2 + (-5p)^2 + 2 \times m \times 2n + 2 \times 2n \times (-5p) + 2 \times (-5p) \times m$

$= m^2 + 4n^2 + 25p^2 + 4mn - 20np - 10\ pm$

18. Capacity of the tank = $6 \times 5 \times 4.5\ m^3 = 135\ m^3$

∴ Volume of water it can hold = $135\ m^3$

= 135 × 1000 l = 135000 l

OR

Radius of the pipe (r) = 2.5 cm = 0.025 m

Length of the pipe (h) = 10 m

∴ Volume of the water which the pipe can hold = $\pi r^2 h$

= 3.14 × $(0.025)^2$ × 10 = 0.019625 m^3

19. Two pair of adjacent sides of a kite are equal.

So, the sides of the given kite are 5 cm, 5 cm, 7 cm, 7 cm

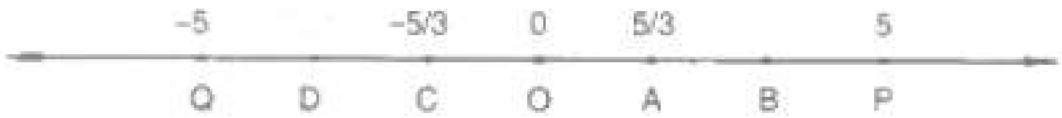
∴ Perimeter of the kite = 5 + 5 + 7 + 7 = 24 cm

20. It is given that x = 2k - 1 and y = k is a solution to the given equation.

∴ 3(2k - 1) - 5k - 7 = 0

⇒ 6k - 3 - 5k - 7 = 0 ⇒ k - 10 = 0 ⇒ k = 10.

21. In order to represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line, we first draw a number line and mark a point O on it to represent zero. Now, we find the points P and Q on the number line representing the positive integers 5 and -5 respectively.



Now, divide the segment OP into three equal parts. Let A and B be the points of division so that OA = AB = BP. By construction, OA is one-third of OP. Therefore, A represents the rational number $\frac{5}{3}$.

Point Q represents -5 on the number line. Now, divide OQ into three equal parts OC, CD and DQ. The point C is such that OC is one third of OQ. Since Q represents the number -5, therefore C represents the rational number $\frac{-5}{3}$.

22. We have,

$8x - ay + a^2 = 0$.....(i)

It is given that x = 1 and y = 6 is a solution of the equation $8x - ay + a^2 = 0$

On putting the corresponding value of *x* and *y* in (1), we get

∴ $8(1) - a(6) + a^2 = 0$

⇒ $8 - 6a + a^2 = 0$

⇒ $a^2 - 6a + 8 = 0$

$\Rightarrow$ a^2 - 4a - 2a + 8 = 0

$\Rightarrow$ a(a - 4) - 2(a - 4) = 0

$\Rightarrow$(a - 4)(a - 2) = 0

$\Rightarrow$ a - 4 = 0 or, a - 2 = 0

$\Rightarrow$ a = 4 or, a = 2

Hence, a = 4 or, a = 2.

23. We have,

f(x) = $2x^3 - 13x^2 + 17x + 12$

f (-3) = $2 \times (-3)^3 - 13 \times (-3)^2 + 17 \times (-3) + 12$

f(-3) = $2 \times (-27) - 13 \times 9 + 17 \times (-3) + 12$

= -54 - 117 - 51 + 12

= -222+12

= -210

OR

Suppose, $p(x) = (x-2)^2 - (x+2)^2$

$p(x) = 0$

$\Rightarrow (x-2)^2 - (x+2)^2 = 0$

$\Rightarrow (x-2+x+2)(x-2-x-2) = 0$

$\Rightarrow 2x(-4) = 0$

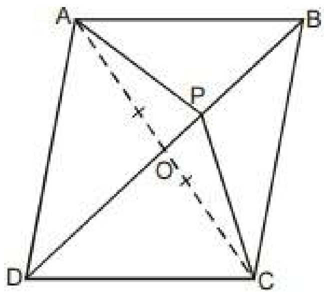
$\Rightarrow -8x = 0$

$\Rightarrow x = 0$

Hence, x = 0 is the only zero of p(x).

24. Join AC

Diagonal AC & BD of a ||gm ABCD bisect at O.



$\Rightarrow$ AO = OC and BO = OD

In $\triangle APC$, PO is median ($\because$ Median divides a triangle in two triangle equal in area)

$\therefore$ ar(APO) = ar(CPO) ....(i)

In $\triangle ADC$,

DO is median

$\therefore$ ar(ADO) = ar(DCO) ....(ii)

Adding (i) & (ii)

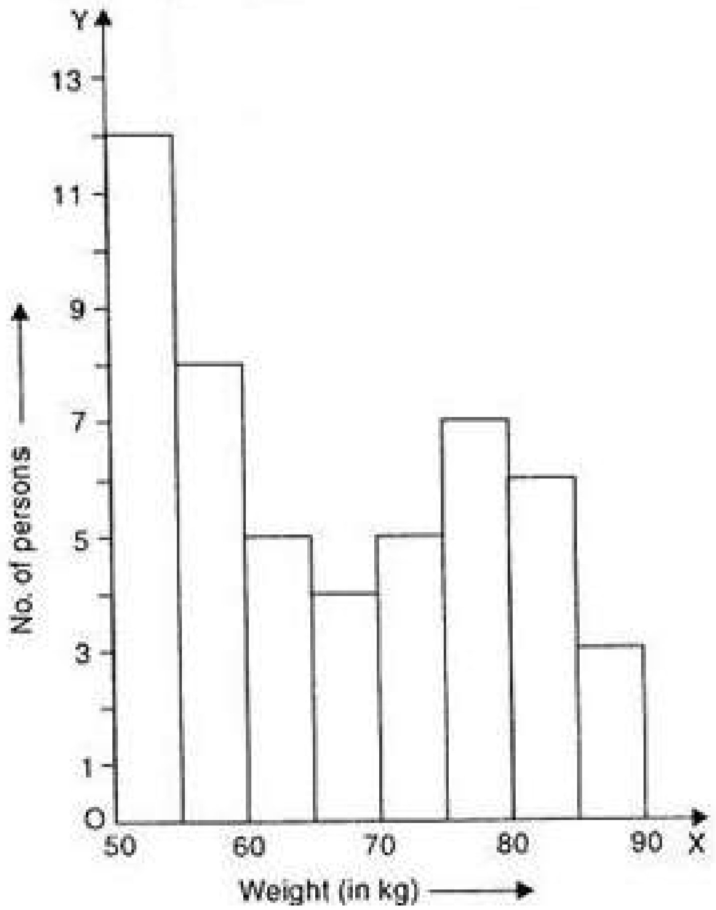
ar(APO) + ar(ADO) = ar(CPO) + ar(DCO)

$\Rightarrow$ ar(ADP) = ar(DPC)

25.

Number of children	Number of families
0	5
1	7
2	12
3	5
4	6
5	3
6	3

OR



26. Radius of the base of the cylindrical kaleidoscope = r = 3.5 cm

Height of kaleidoscope = h = 25 cm

Chart paper required = curved surface area of kaleidoscope

$= 2\pi rh$

$= 2\times \frac{22}{7} \times 3.5\times 25$

$= 2\times 22 \times 0.5 \times 25$

$= 550 \text{ cm}^2$

Hence the area of chart paper required is 550 cm^2.

27. Three rational numbers between $2.\overline{2}$ and $2.\overline{3}$

$2.\overline{2}$ means 2.2222222........ and $2.\overline{3}$ means 2.333333333....

so any number between them is the solution

fo example 3 numbers can be 2.23, 2.24. 2.25 etc

OR

Let point A represents 1 as shown in Figure.

Clearly, $OA = 1\ unit.$

Now, draw a right triangle OAB in which $AB = OA = 1\ unit.$

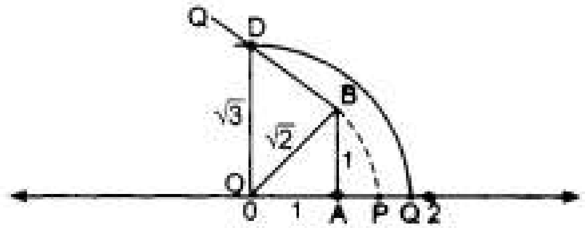
By Using Pythagoras theorem, we have

$OB^2 = OA^2 + AB^2$

$= 1^2 + 1^2$

$= 2$

$\Rightarrow OB = \sqrt{2}$



Taking O as centre and OB as a radius draw an arc intersecting the number line at point P.

Then p corresponds to $\sqrt{2}$ on the number line. Now draw DB of unit length perpendicular to OB.

By using Pythagoras theorem, we have

$OD^2 = OB^2 + DB^2$

OD^2 = $(\sqrt{2})^2$ + 12

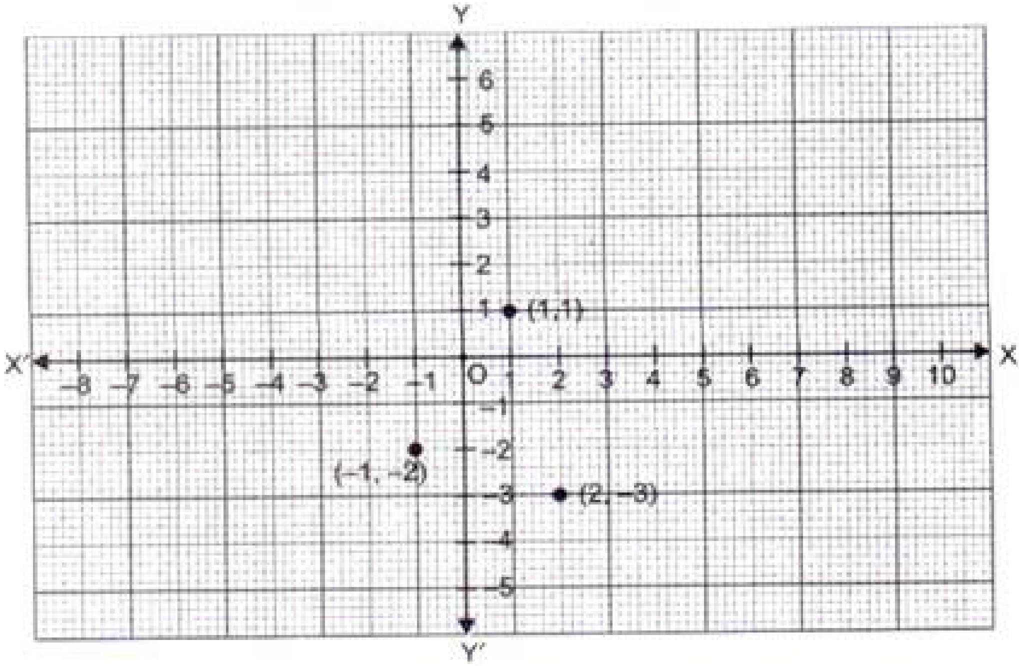
= 2 + 1 = 3

OD = $\sqrt{3}$

Taking O as centre and OD as a radius draw an arc which intersects the number line at the point Q.

Clearly, Q corresponds to $\sqrt{3}$.

28.



From the graph, we find that all the three points do not lie on the same straight line.

Hence, the given points are not collinear.

29. 2x – 3y + 7 = 0

⇒ 3y = 2x + 7

⇒ $y = \frac{2x+7}{3}$

Put x = 0, then $y = \frac{2(0)+7}{3} = \frac{7}{3}$

Put x = 1, then $y = \frac{2(1)+7}{3} = 3$

Put x = 2, then $y = \frac{2(2)+7}{3} = \frac{11}{3}$

Put x = 3, then $y = \frac{2(3)+7}{3} = \frac{13}{3}$

$\therefore (0, \frac{7}{3}), (1, 3), (2, \frac{11}{3}), (3, \frac{13}{3})$ are the solutions of the equation 2x – 3y + 7 = 0.

OR

Let the cost of toy horse be $Rs\ x$ and cost of one ball be $Rs\ y$.

$\therefore$ Cost of three balls $= 3y$

According to the given condition, we have

$x = 3y$...(i)

Taking $y = 1$, in equation (i), we get

$\therefore x = 3(1) = 3$

Taking $y = 2$, in equation (i), we get

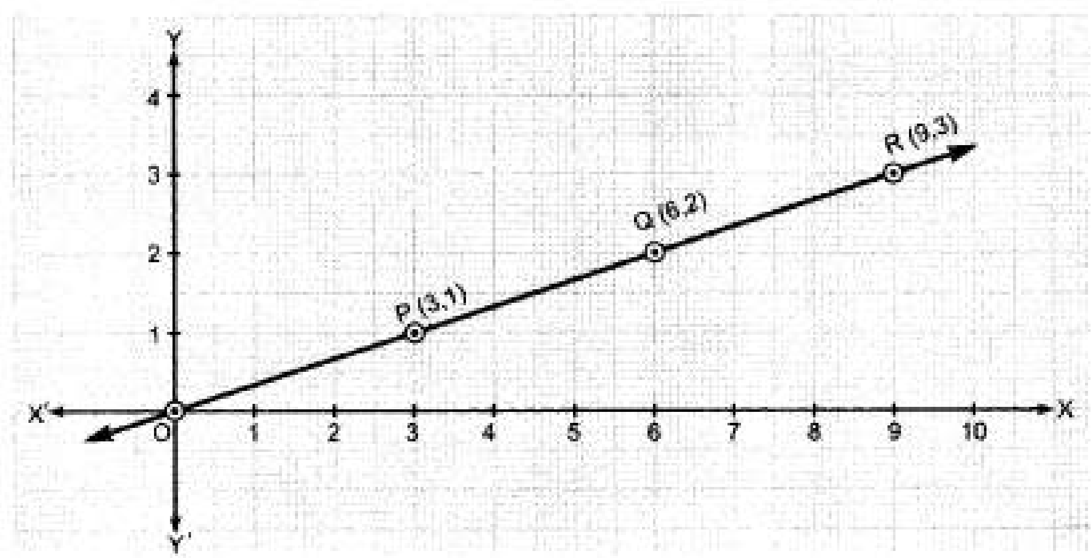
$\therefore x = 3(2) = 6$

Taking $y = 3$, in equation (i), we get

$\therefore x = 3(3) = 9$

x	3	6	9
y	1	2	3
	P	Q	R

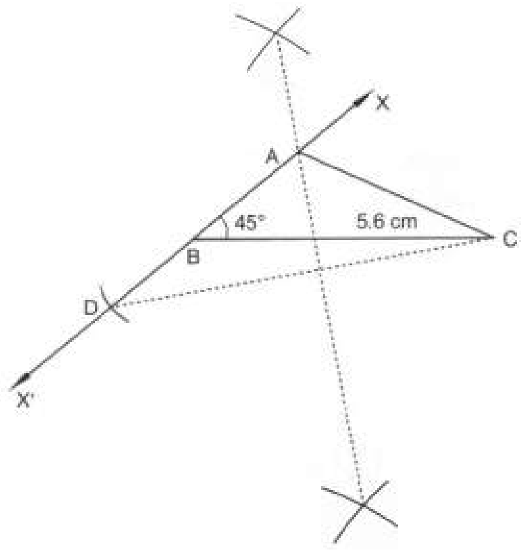
Now draw a graph taking $P(3,1), Q(6,2)\ and\ R(9,3)$ which is given below.



30. To draw the triangle ABC, we follow the following steps:

    Steps of Construction:

    i. Draw BC = 5.6 cm
    ii. At B, construct $\angle$CBX = 45^{0}
    iii. Produce XB to X' to form line XBX'.
    iv. From ray BX', cut-off line segment BD = 1.6 cm
    v. Join CD
    vi. Draw perpendicular bisector of CD which cuts BX at A.
    vii. Join CA to obtain the required triangle ABC.



Justification: Since A lies on the perpendicular bisector of CD.

$\therefore$ AC = AD = AB + DB = AB + 1.6

$\Rightarrow$ AC - AB = 1.6 cm

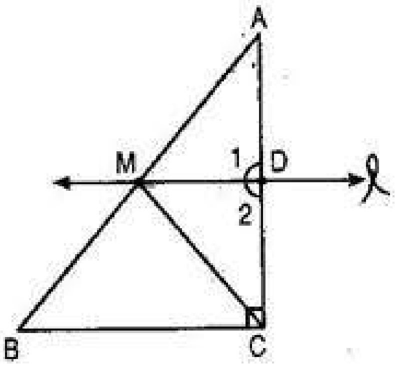
Hence, $\triangle$ABC is the required triangle.

31. i. In $\triangle$ABC, M is the mid-point of AB[Given]

MD $\parallel$ BC

$\therefore$ AD = DC[Converse of mid-point theorem]

Thus D is the mid-point of AC.



ii. $l \parallel$ BC (given) consider AC as a transversal.

$\therefore \angle 1 = \angle C$ [Corresponding angles]

$\Rightarrow \angle 1 = 90^\circ$ [$\angle C = 90^\circ$]

Thus MD $\perp$ AC.

iii. In $\triangle$AMD and $\triangle$CMD,

AD = DC[proved above]

$\angle 1 = \angle 2 = 90^\circ$ [proved above]

MD = MD[common]

$\therefore \triangle$AMD $\cong \triangle$CMD [By SAS congruency]

$\Rightarrow$ AM = CM[By C.P.C.T.]..........(i)

Given that M is the mid-point of AB.

$\therefore$AM = $\frac{1}{2}$ AB..........(ii)

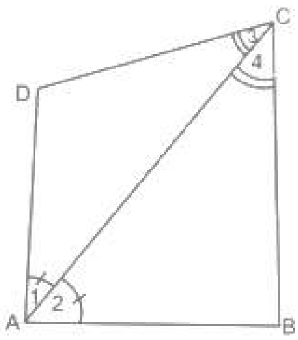
From eq. (i) and (ii),

CM = AM = $\frac{1}{2}$ AB

32. Given: A quadrilateral ABCD such that $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

To prove: AB = AD and CB = CD

Proof: In ΔABC and ΔADC, we have



$\angle 1 = \angle 2$ [Given]

AC = AC [Common side]

$\angle 3 = \angle 4$ [Given]

So, by SAS criterion of congruence, we have

$\Delta ABC \cong \Delta ADC$

$\therefore\ AB = AD$ [CPCT]

And CB = CD [CPCT]

Hence, proved.

OR

In $\triangle$BOC, we have

$\angle 1 + \angle 2 + \angle \mathrm{BOC} = 180^\circ \rightarrow (1)$

In $\triangle$ABC, we have

$\angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} = 180^\circ$

$\Rightarrow \angle A + 2(\angle 1) + 2(\angle 2) = 180^\circ$

$\Rightarrow \frac{\angle A}{2} + \angle 1 + \angle 2 = 90^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90 - \frac{\angle A}{2}$

Substituting this value of $\angle 1 + \angle 2$ in (1)

$90^\circ - \frac{\angle A}{2} + \angle \text{BOC} = 180^\circ$

$\angle \text{ BOC} = 90^\circ + \frac{\angle A}{2}$

So, $\angle \text{ BOC} = 90^\circ + \frac{\angle A}{2}$

33. We have, 2s = 50 m + 65 m + 65 m = 180 m

S = 180 ÷ 2 = 90 m

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{90(90-50)(90-65)(90-65)}$

$= \sqrt{90 \times 40 \times 25 \times 25} = 60 \times 25$

= $1500m^2$.

Cost of laying grass at the rate of Rs7 per m^2 = Rs(1500 × 7) = Rs10,500.

34. (i)Total no. of Families = 1500

No. of family having 2 girls = 475

$P(E) = \frac{475}{1500} = \frac{95}{300} = \frac{19}{60}$

(ii)No. of families having 1 girl = 814

$P(E) = \frac{814}{1500} = \frac{407}{750}$

(iii)No. of families having no girl = 211

$P(E) = \frac{211}{1500}$

Sum of these three probabilities = $\frac{475}{1500} + \frac{814}{1500} + \frac{211}{1500}$

= $\frac{475+814+211}{1500}$

= $\frac{1500}{1500}$

= 1

35. Let O be the centre of the circle.

Join OA and OC.

Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore AE = EB = \frac{1}{2}AB = \frac{1}{2} \times 5 = \frac{5}{2}cm$

And CF = FD = $\frac{1}{2}CD = \frac{1}{2} \times 11 = \frac{11}{2}cm$

Let OE = x

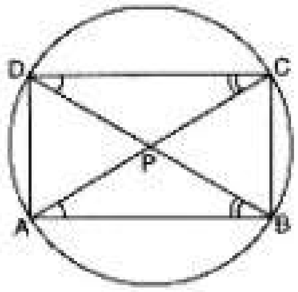
$\therefore OF = 6 - x$.$\because$OF = 6 - x

Let radius of the circle be r.
In right angled triangle AEO,
$AO^2 = AE^2 + OE^2$ $AO^2 = AE^2 + OE^2$
[Using Pythagoras theorem]
$\Rightarrow r^2 = \left(\frac{5}{2}\right)^2 + x^2$ ...(i)
Again In right angled triangle CFO,
$OC^2 = CF^2 + OF^2$
[Using Pythagoras theorem]
$\Rightarrow r^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$ ...(ii)
Equating eq. (i) and (ii),
$\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$
$\Rightarrow \frac{25}{4} + x^2 = \frac{121}{4} + 36 + x^2 - 12x$
$\Rightarrow 12x = \frac{121}{4} - \frac{25}{4} + 36$
$\Rightarrow 12x = \frac{96}{4} + 36$
$\Rightarrow 12x = 24 + 36$
$\Rightarrow 12x = 63$
$\Rightarrow x = 5$
Putting the value of x in eq. (i)
$r^2 = \left(\frac{5}{2}\right)^2 + 5^2$ $r^2 = \left(\frac{5}{2}\right)^2 + 5^2$
$\Rightarrow r^2 = 31.25$
$\Rightarrow r = 5.6$ cm (approx.)

OR

Given: ABCD is a cyclic quadrilateral whose diagonals AC and BD intersect at P. AB = DC.
To prove:

i. $\triangle PAB \cong \triangle PDC$
ii. PA = PD and PC = PB
iii. AD | | BC

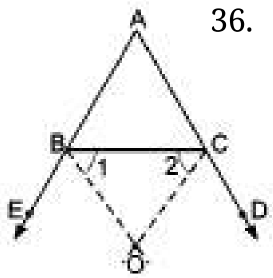


Proof :

i. In $\triangle PAB$ and $\triangle PDC$
$\angle PAB = \angle PDC$ |Angles in the same segment
$\angle PBA = \angle PCD$ |Angles in the same segment
AB = DC |Given
$\triangle PAB \cong \triangle PDC$
$\triangle PAB \cong \triangle PDC$ |ASA

ii. $\triangle PAB \cong \triangle PDC$ |Proved in (i)
$\therefore$ PA = PD |c.p.c.t
and PB = PC |c.p.c.t
$\Rightarrow$ PC = PB

iii. PA = PD
$\therefore \angle PDA = \angle PAD$ |Proved in (ii)
Also, $\angle PDA = \angle PCB$ |Angles opposite to equal sides
$\therefore \angle PAD = \angle PCB$ |Angles in the same segment
But these form a pair of equal alternate angles
AD || BC



36. As $\angle$ABC and $\angle$CBE form a linear pair
$\therefore \angle$ABC + $\angle$CBE = 180°..........(1)
Given, BO is the bisector of $\angle$CBE. Hence,
$\angle$CBE = 2$\angle$OBC.
$\Rightarrow \angle$CBE = 2$\angle 1$ ....(2)

Therefore, $\angle$ABC+ 2$\angle$1 =180° [from (1) & (2)]
$\Rightarrow$ 2$\angle$1 = 180° - $\angle$ABC
$\Rightarrow \angle$1 = 90° - $\frac{1}{2}\angle$ABC........(3)
Again, $\angle$ACB and $\angle$BCD form a linear pair

$\therefore \angle ACB + \angle BCD = 180°$.........(4)

Given, CO is the bisector of $\angle BCD$. Hence,

$\angle BCD = 2\angle 2$............(5)

So, $\angle ACB + 2\angle 2 = 180°$ [from (4) & (5)]

$\Rightarrow 2\angle 2 = 180° - \angle ACB$

$\Rightarrow \angle 2 = 90° - \frac{1}{2}\angle ACB$ ...(6)

Now in $\triangle OBC$, we have

$\angle 1 + \angle 2 + \angle BOC = 180°$ (Angle sum property of triangle) ...(7)

From (3), (6) and (7), we have

$90° - \frac{1}{2}\angle ABC + 90° - \frac{1}{2}\angle ACB + \angle BOC = 180°$.

$\Rightarrow \angle BOC = \frac{1}{2}(\angle ABC + \angle ACB)$...........(8)

Now, in $\triangle ABC$, we have

$\angle BAC + \angle ABC + \angle ACB = 180°$

or, $\angle ABC + \angle ACB = 180° - \angle BAC$............(9)

From (8) and (9), we have:-

$\Rightarrow \angle BOC = \frac{1}{2}(180° - \angle BAC)$

Hence, $\angle BOC = 90° - \frac{1}{2}\angle A$ Proved.

37. Let (x) = $2x^3 + kx^2 + 3x - 5$

and $g(x) = x^3 + x^2 - 2x + 2k$

When f (x) is divided by (x - 3), then the remainder is f(3).

When g (x) is divided by (x - 3), then the remainder is g(3).

Now, $f(3) = 2 \times (3)^3 + k \times (3)^2 + 3 \times 3 - 5$

$= 54 + 9k + 9 - 5 = 58 + 9k$

and $g(3) = (3)^3 + (3)^2 - 2 \times 3 + 2k$

$= 27 + 9 - 6 + 2k = 30 + 2k$

According to the question,

$f(3) = g(3)$

$\Rightarrow 58 + 9k = 30 + 2k$

$\Rightarrow 7k = -28$

$\Rightarrow k = -4$

Remainder of f(x) = 58 + 9k = 58 + 9 (-4) = 22

Remainder of g(x) = 30 + 2k = 30 + 2(-4) = 22

OR

We know that,

$(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow (3)^2 = x^2 + \frac{1}{x^2} + 2$ [$\because (x + \frac{1}{x}) = 3$]

$\Rightarrow 9 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 7$ ......(i)

Now,

$(x + \frac{1}{x})^3 = x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}(x + \frac{1}{x})$

$(3)^3 = x^3 + \frac{1}{x^3} + 3 \times 3$ [$\because (x + \frac{1}{x}) = 3$]

$\Rightarrow 27 = x^3 + \frac{1}{x^3} + 9$

$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 9$

$\Rightarrow x^3 + \frac{1}{x^3} = 18$ ....(ii)

Now,

$(x^2 + \frac{1}{x^2})^2 = x^4 + \frac{1}{x^4} + 2$

$\Rightarrow (7)^2 = x^4 + \frac{1}{x^4} + 2$ [using (i)]

$\Rightarrow 49 = x^4 + \frac{1}{x^4} + 2$

$\Rightarrow x^4 + \frac{1}{x^4} = 47$ .....(iii)

From (i), (ii), (iii) we have,

$x^2 + \frac{1}{x^2} = 7$, $x^3 + \frac{1}{x^3} = 18$ and $x^4 + \frac{1}{x^4} = 47$

38. Let V_1 and V_2 be the volume of two cylinders.

When the sheet is folded along its length, it forms a cylinder of height h_1=18cm and perimeter of base equal to 30cm.

Let r_1 be the radius of the base.

Then,

$2\pi r_1 = 30$

$\Rightarrow r_1 = \frac{15}{\pi}$

$\therefore V_1 = \pi_1^2 h_1 = \pi \times \frac{225}{\pi^2} \times 18 \text{ cm}^3$

$= \frac{225}{\pi} \times 18 \text{ cm}^3$

When the sheet is folded along its breadth, it forms a cylinder of height h_2 = 30 cm and perimeter of base equal to 30 cm.

Let r_2 be the radius of the base when h_2 = 30 cm.

$\Rightarrow 2\pi r_2 = 18$

$\Rightarrow r_2 = \frac{9}{\pi}$

$V_2 = \pi r_2^2 h_2 = \pi \times \left(\frac{9}{\pi}\right)^2 \times 30 \text{ cm}^3$

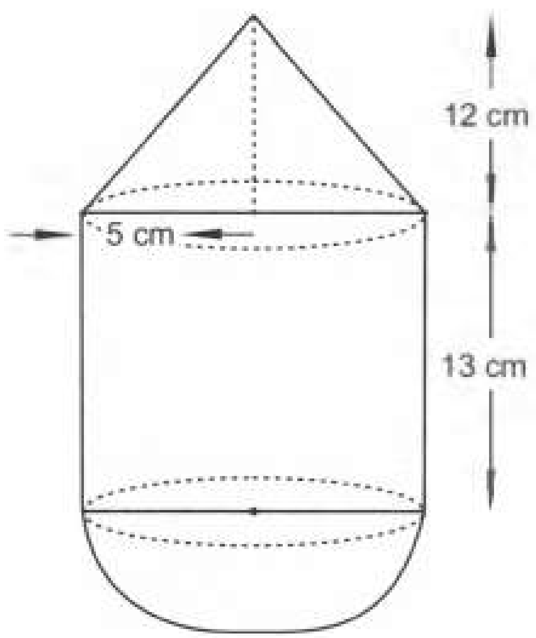
$= \frac{81 \times 30}{\pi} \text{ cm}^3$

$\therefore \frac{V_1}{V_2} = \frac{225 \times 18}{81 \times 30}$

$= \frac{5}{3}$

OR

Let r cm be the radius and h cm the height of the cylindrical part. Then, r = 5 cm and h = 13 cm.



Clearly, radii of the spherical part and base of the conical part are also r cm. Let h_1 cm be the height, l cm be the slant height of the conical part. Then,

$l^2 = r^2 + h_1^2$

$\Rightarrow \quad l = \sqrt{r^2 + h_1^2} \Rightarrow l = \sqrt{5^2 + 12^2}$ = 13 cm [$\because h_1$ = 12 cm, r = 5 cm]

Now, Surface area of the toy = Curved surface area of the cylindrical part + Curved surface area of hemisphrical part + Curved surface area of conical part

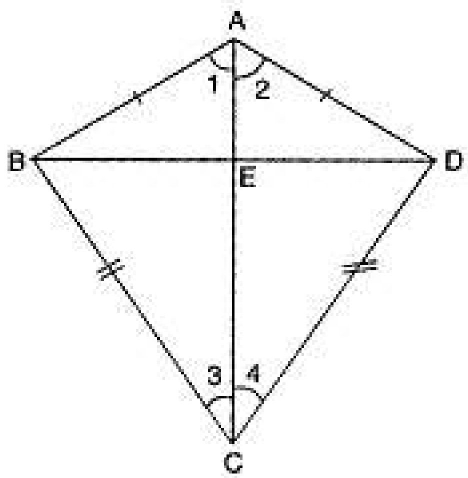
= $(2\pi rh + 2\pi r^2 + \pi rl)$ cm^2

= πr (2h + 2r +l) cm^2

= $\frac{22}{7} \times 5 \times (2 \times 13 + 2 \times 5 + 13)$ cm^2

= $\frac{22}{7} \times 5 \times 49$ cm^2 = 770 cm^2

39. $\triangle$ABC and $\triangle$ADC,



i. AB = AD, BC = DC . . . [Given]

   AC = AC . . . [Common]

   $\triangle ABC \cong \triangle ADC$ . . .[SSS axiom

   $\angle 1 = \angle 2$ . . . .[c.p.c.t.]

   AC bisects $\angle$A and $\angle 3 = \angle 4$ . . .[c.p.c.t.]

   AC bisects $\angle$C.

   Hence, AC bisects each of the angles A and C.

ii. In $\triangle$ABE and $\triangle$ADE,

   AB = AD . . . [Given]

   $\angle 1 = \angle 2$ . . .[As proved above]

   AE = AE . . . .[Common]

   $\therefore \triangle ABE \cong \triangle ADE$ . . . [SAS axiom]

   $\therefore$ BE = ED . . . [c.p.c.t.]

iii. $\triangle ABC \cong \triangle ADC$ . . . [As proved above]

$\therefore \angle ABC = \angle ADC$ . . . [c.p.c.t.]

40. i. Total number of students obtains less than 20 marks = 27 + 12 = 39

The cost of one work-book = Rs 5

$\therefore$ The cost of 39 work-books = 5 $\times$ 39 = Rs 195

ii. The number of students belonging to the highest marks group 40-49 = 17

The cost of a prize = Rs = 10

$\therefore$ The cost of 17 prizes = 10 $\times$ 17 = Rs 170

iii. The number of students belonging to the lowest mark group 0-9 = 27

The number of problems solved by 1 student = 5

$\therefore$ Total number of problems solved by 27 students = 5 $\times$ 27 = 135

iv.

a. Total number of students = 100

The number of students in range 40-49 = 17

Percentage of students obtaining marks ranging 40-49 = $\frac{17}{100} \times 100 = 17\ \%$

So, the given statement is true.

b. Total number of students in range 10-29 = 12 + 20 = 32

Percentage of students obtaining marks ranging 10-29 = $\frac{32}{100} \times 100 = 32\ \%$

So, the given statemnet is false.

v. Total number of students getting less than 20 marks = 27 + 12 = 39

vi. Total number of students getting more than 29 marks = 24 + 17 = 41

vii. Total number of students getting marks between 9 and 40 = 12 + 20 + 24 = 56

viii. The number of students belonging to the highest mark group 40-49 = 17

ix. The number of students obtaining more than 19 marks = 20 + 27 + 17 = 61.