Laws Of Motion

Newton's 1st Law of Motion

Inertia

A body at rest continues to be at rest, and a body in uniform motion continues to move with a uniform velocity. The property of a body to resist any change is called inertia.

Newton’s First Law

Every body continues to be in its state of rest or uniform motion in a straight line, unless compelled by some external force acting on it.

Example: Any object continues lying where it is, unless it is moved.

Newton’s first law defines force.

Newton’s first law defines Inertia.

Examples of Inertia

- When a horse starts suddenly, the rider falls backwards due to the inertia of rest of the upper part of his body.
- The dust particles in a carpet fall off when beaten with a stick. The beating sets the carpet in motion, whereas the dust particles tend to remain at rest.

Animations and videos:

Newton's 2nd Law of Motion

Momentum

- Momentum of a body is the product of its mass, *m* and velocity, *v*, and is denoted by *P*.
- $\vec{P} = m\vec{v}$
- It is a vector quantity.
- SI unit → kg ms^{-1}

Examples to Demonstrate the Importance of Momentum

- Much greater force is needed to push a truck than that needed to push a car for bringing them to the same speed in the same time.

- Greater opposing force is needed to stop a heavy body than that needed to stop a light body in the same time, if they are moving with the same speed.

- It is always easier to catch a lighter stone than a heavier one. Thus, the mass of a body is an important parameter that determines the effect of force on its motion.

- A bullet thrown with a moderate speed can be easily stopped, whereas the same bullet fired from a gun with a high speed can pierce human tissue. Thus, speed is another important parameter to be considered.

Newton's Second Law of Motion

- **Statement**

The rate of change of linear momentum of a body is directly proportional to the external force applied on the body, and this change takes place always in the direction of force applied.

Let,

m = Mass of the body

$\overrightarrow{v}$ = Velocity of the body

Therefore, the linear momentum of the body is,

$$\overrightarrow{P} = m\overrightarrow{v}$$

Let,

$\overrightarrow{F}$ = External force

$\Delta \overrightarrow{P}$ = Change in linear momentum

According to Newton's second law,

$$\frac{\Delta \overrightarrow{P}}{\Delta t} \propto \overrightarrow{F}$$

$$\overrightarrow{F} = k\frac{\Delta \overrightarrow{P}}{\Delta t}$$

Where, *k* is the constant of proportionality

Taking the limit $\Delta t \to 0, \frac{\Delta \overrightarrow{P}}{\Delta t}$ becomes $\frac{\mathrm{d}P}{\mathrm{d}t}$

$$\therefore \overrightarrow{F} = k\frac{\mathrm{d}\overrightarrow{P}}{\mathrm{d}t} = k\frac{\mathrm{d}}{\mathrm{d}t}\left(m\overrightarrow{v}\right) = km\frac{\mathrm{d}\overrightarrow{v}}{\mathrm{d}t}$$

$$\overrightarrow{F} = km\overrightarrow{a} \quad \left[\because \frac{\mathrm{d}\overrightarrow{v}}{\mathrm{d}t} = \overrightarrow{a}\right]$$

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$$\overrightarrow{F} = km\overrightarrow{a} \quad \left[\because \frac{\mathrm{d}\overrightarrow{v}}{\mathrm{d}t} = \overrightarrow{a}\right]$$

In S.I, *k* = 1

$$\therefore \overrightarrow{F} = m\overrightarrow{a}$$

Thus, the second law of motion gives us the measure of force.

Some Important Points about the 2nd Law

The second law of motion is consistent with the first law of motion.

$$F_x = \frac{dP_x}{dt} = ma_x$$
$$F_y = \frac{dP_y}{dt} = ma_y$$
$$F_z = \frac{dP_z}{dt} = ma_z$$

- Here, $\overrightarrow{F}$ refers to the total external force on the system (any internal forces in the system are not included in *F*).

Impulse

- Impulse of a force is a measure of the total effect of the force

Impulse = Force × Time

- Forces which act on bodies for a short time are called impulsive forces.

  Example: firing a gun, hitting a ball with a bat

- It is a vector quantity.

Problems Related To the Concept of Impulse

- A fielder lowers his hands while catching a cricket ball. Explain.

Solution:

According to Newton's 2nd law of motion,

$$\overrightarrow{F} = \frac{d\overrightarrow{P}}{dt}$$

$$\therefore \overrightarrow{F}\,dt = d\overrightarrow{P}$$

Integrating both sides within the limits

$$\int_0^t \overrightarrow{F}\,dt = \int_{\overrightarrow{P}_1}^{\overrightarrow{P}_2} d\overrightarrow{P}$$

Where, $\overrightarrow{P}_1$ is the initial linear momentum at $t = 0$, and $\overrightarrow{P}_2$ is the final linear momentum at time t

If $\overrightarrow{F'}$ is the force during this time,

$$F'[t]_0^t = \left[\overrightarrow{P}\right]_{\overrightarrow{P_1}}^{\overrightarrow{P_2}}$$

$\overrightarrow{F'} \times t = \overrightarrow{P}_2 - \overrightarrow{P}_1$ = Change in momentum

∴ Force × Time = Change in linear momentum

Force = $\frac{\text{Change in linear momentum}}{\text{Time}}$

Thus, we can see that by increasing the time of a catch, the fielder would apply a smaller force against the ball to stop it. Consequently, the ball will exert a smaller reaction force on the hands of the fielder.

- A cricket ball of mass 0.2 kg, moving with a speed of 15 ms^{-1}, is hit by a bat so that the ball is turned back with a velocity of 20 ms^{-1}. Calculate the impulse received by the ball.

Solution:

Mass, m = 0.2 kg

u = 15 ms^{-1}

v = −20 ms^{-1} (Since the direction of velocity of ball changes after being hit)

Impulse = Final momentum − Initial momentum

Impulse = $mv - mu = m(v - u)$

= 0.2 (−20 − 15)

= 0.2 (−35) = −7 kg ms−1

Do You Know

According to Einstein, the mass of the body varies with velocity as

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

where, m_0 = rest mass of the body
m = effective mass of the particle moving with velocity *v*

c = velocity of light in vacuum

If $v << c, m \simeq m_o$

Newton's 3rd Law of Motion

Statement

To every action, there is always an equal and opposite reaction.

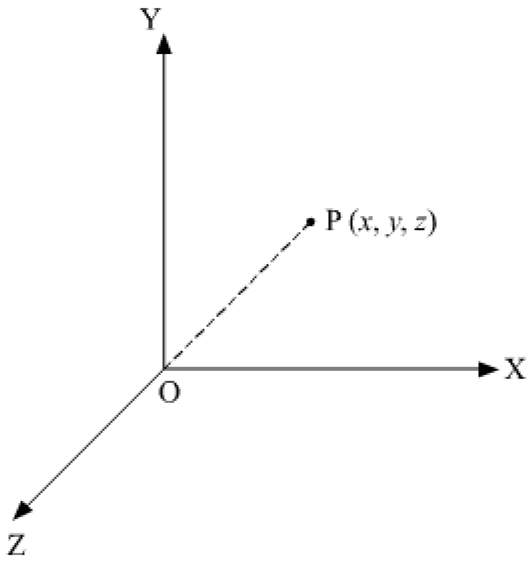
Some Important Points about the Third Law

- Forces always occur in pairs. Force exerted on a body A by a body B is equal and opposite to the force exerted on body B by body A.
- There is no cause-effect relationship implied in the third law. The force on body A by body B, and the force on body B by body A act at the same instant.
- Forces of action and reaction act always on different bodies. Hence, they never cancel each other.
- $\overrightarrow{F}_{\mathrm{AB}} = -\overrightarrow{F}_{\mathrm{BA}}$

Frames of References

Frame of Reference

In Physics, to locate physical quantities like position, velocity, acceleration, etc. of a particle, we need a frame of reference. The easiest way to choose the frame of reference is by using three mutually perpendicular axes and naming them X,Y and Z axes.



We add a clock to record the time along with the points in the frame of reference.

In the given frame of reference, position of a particle is given by

$$\overline{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

Velocity of the particle,

$$\overline{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$$

Railway stations, bus stands and electric poles are at rest with respect to the ground because they do not move with the passage of time. But the railway stations and electric poles are in motion with respect to the frame of the train.

Also, there is no specific rule to choose a frame of reference. We can choose a frame of reference according to our convenience to express the situation of a particle under study.

[[Q,1]]

A bus is moving with a constant velocity of 30 km/h. Find

(a) velocity of the bus with respect to a passenger sitting in it.

(b) velocity of the bus with respect to a bus stand.

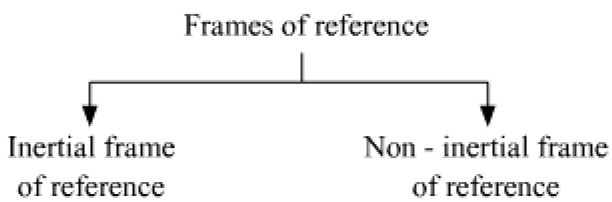
[[S]]

(a) Velocity of bus with respect to the ground = Velocity of a passenger sitting in the bus with respect to the ground

∴ Velocity of bus with respect to the passenger sitting in it, $\vec{V}_{BP} = 0$.

(b) $\vec{V}_{BG}$ = Velocity of bus = 30 m/s

Types of frame of reference



(I) Inertial frame of reference:

A frame of reference moving with a constant velocity or at rest with respect to the ground is called an inertial frame of reference. In this frame, Newton's laws of motion are valid. We take the ground (Earth) as an inertial frame of reference. Only real forces exist in this frame.

(II) Non-inertial frame of reference:

A frame of reference that is moving with an acceleration is called a non-inertial frame of reference.

A rotating frame is a non-inertial frame of reference because it also has acceleration.

Newton's laws of motion are not valid in a non-inertial frame of reference.

Types of Force:
(I) Real Force:

Real forces arise due to interaction between objects. These forces have specific source and origin outside the body experiencing the force and are explained on the basis of fundamental interactions.
Examples : (i) The revolution of earth around the sun in a circular path due to the gravitational force of attraction between the sun and the earth
(ii) Binding of protons and neutrons in the nucleus of an atom due to nuclear forces

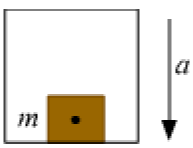
(II) Pseudo Force:

In Physics, pseudo forces arise when Newton's laws of motion are applied to a non-inertial (accelerating) frame of reference. These forces have no real existence but must be taken into account in an accelerating frame of reference to make Newton's laws of motion applicable to the system.

Example: Centrifugal force is a pseudo force that is considered when a body is moving in a circular path. The actual force that keeps the revolving body in the circular path is the centripetal force, which is directed towards the centre of the circle. The magnitude of centrifugal force is equal to the centripetal force but is directed away from the centre.

Special cases:

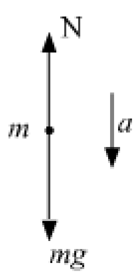
(i) Suppose a block of mass *m* is placed in an elevator moving down with a constant acceleration *a.*



Case (I)

For an inertial frame of reference (ground frame), free body diagram (FBD) is:

From Newton's second law of motion,



$mg - N = ma$

$\Rightarrow N = mg - ma$

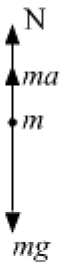
$N = m(g - a)$ …(I)

Case II

For a non-inertial frame of reference (i.e. from the lift),

ma = Pseudo force

From Newton's third law of motion,



$mg = N + ma$

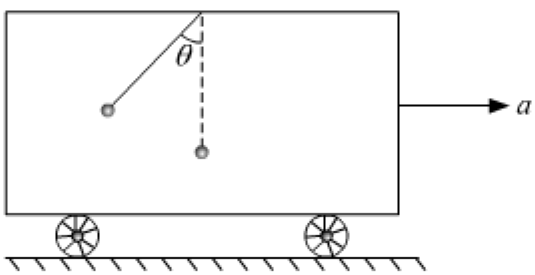
$\Rightarrow N = m(g - a)$ …(II)

If we are not going to consider pseudo force, then $N' = mg$ …(III)

From equations (I), (II) and (III), we conclude that pseudo force is necessary for a non-inertial frame of reference.

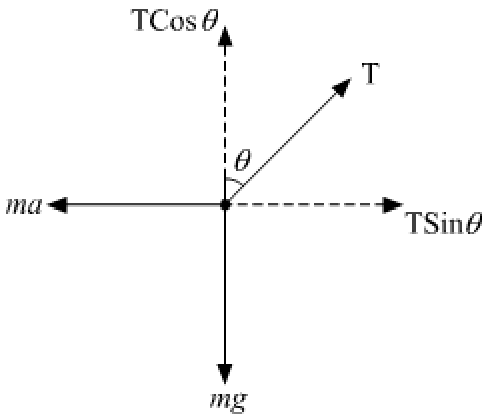
[[Q,2]]

In the figure, find the value of θ in equilibrium position.



[[S]]

From the accelerating frame, FBD for the given system,



T → Tension in the string

mg → Weight of the bob

ma → Pseudo force

Since the bob is in equilibrium,

$$\sum F_X = 0$$
$$\Rightarrow T \sin\theta = ma \qquad ...(i)$$
$$\sum F_Y = 0$$
$$\Rightarrow T \cos\theta = mg \qquad ...(ii)$$

(i) ÷ (ii) gives:

$$\tan\theta = \frac{a}{g}$$
$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Conservation Of Momentum

- Statement − In an isolated system, the vector sum of the linear momenta of all the bodies of the system is conserved and is not affected due to their mutual action and reaction.
- Collision of two bodies: Consider two bodies *A* and *B*. Let

$\vec{P}_A$ − Initial momentum of body *A*

$\vec{P}_B$ − Initial momentum of body *B*

$\vec{P}'_A$ − Momentum of body *A* after collision

$\vec{P}'_B$ − Momentum of body *B* after collision

Δt − Time for which the bodies collide

During collision,

$\vec{F}_{AB}$ − Force on *A* exerted by *B*

$\vec{F}_{BA}$ − Force on *B* exerted by *A*

According to Newton's Second Law,

$\vec{F}_{AB} \times \Delta t$ = Change in momentum of $A = \vec{P}'_A - \vec{P}_A$.

$\vec{F}_{BA} \times \Delta t$ = Change in momentum of $B = \vec{P}'_B - \vec{P}_B$.

According to Newton's Third Law,

$$\vec{F}_{AB} = -\vec{F}_{BA}$$
$$\therefore \vec{P}'_A - \vec{P}_A = -(\vec{P}'_B - \vec{P}_B)$$
$$\boxed{\vec{P}'_A + \vec{P}'_B = \vec{P}_A + \vec{P}_B}$$

∴Final momentum = Initial momentum

- An example: Recoil of a gun

Suppose,

m_b = Mass of bullet

m_g = Mass of gun

$\vec{v}_b$ = Velocity of the bullet

$\vec{v}_g$ = Velocity of the recoil of the gun

Before the bullet was fired from the gun, both the gun and the bullet were at rest. Therefore, total linear momentum of the system before firing is zero.

According to the principle of conservation of linear momentum, total linear momentum of the system after firing should be zero.

$$m_b \overrightarrow{v}_b + m_g \overrightarrow{v}_g = 0$$
$$m_g \overrightarrow{v}_g = -m_b \overrightarrow{v}_b$$
$$\overrightarrow{v}_g = \frac{-m_b \overrightarrow{v}_b}{m_g}$$

The negative sign indicates that the direction of recoil of the gun is opposite the direction of the speed of the bullet.

- **Person jumping from a boat**

  When a person jumps from a stationary boat, the boat is pushed away from the shore. The linear momentum of the boat is equal and opposite to the person, in accordance with the law of conservation of linear momentum.

- **Motion of rocket**

  Rockets use the law of conservation of momentum in their motion. In a rocket, oxygen is burnt in the combustion chamber. The hot and compressed gases are ejected through a narrow opening at a very high speed. As a result, the ejected gas acquires a large momentum in the backward direction. This imparts an equal forward momentum to the rocket, as per the principle of conservation of linear momentum. This makes the rocket move upward with high speed.

- **Explosion of a bomb**

  When a bomb is at rest, the linear momentum of the bomb is zero. When the bomb explodes into a number of pieces, the pieces are scattered horizontally in different directions, but the vector sum of linear momentum of these pieces is zero. This is in accordance with the law of conservation of linear momentum.

Equilibrium of Particle

- Equilibrium of a particle in mechanics refers to the situation when the net external force on the particles is zero. According to the first law, this means that the particle is either at rest or in uniform motion.

- If two forces $\vec{F_1}$ and $\vec{F_2}$ act on a particle, then equilibrium requires the condition:

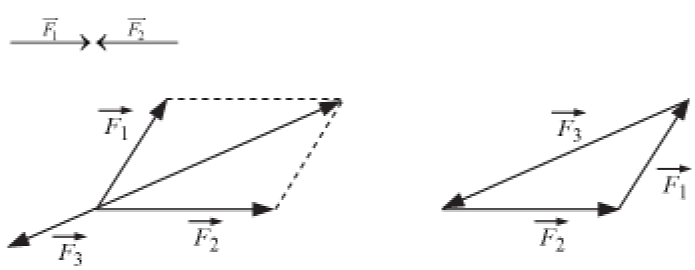
  $$\vec{F_1} = -\vec{F_2} \qquad \text{...(i)}$$

  i.e., the two forces on the particle must be equal and opposite

  Equilibrium under three concurrent forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ requires that the vector sum of the three forces is zero.

  $$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0 \qquad \text{...(ii)}$$

- **Equilibrium under Concurrent Forces**

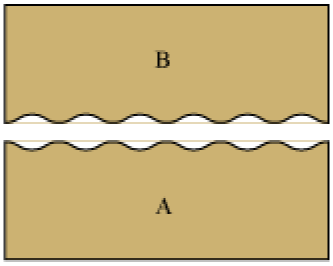


Contact Force: Friction

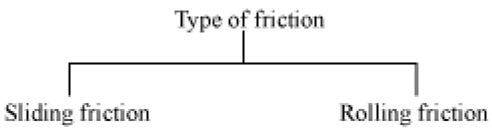
Contact force − A contact force on an object arises due to contact with another object (solid or fluid). Example - force of friction.

Friction − Friction is the property due to which a force is set up at the surface of contact of two bodies and which prevents any relative motion between them.

Cause of friction - No solid surface is perfectly smooth. Thus, when a body B is placed over another body A, the irregularities of the two surfaces get interlocked. When one of the bodies moves or tends to move over the other, there is always a force which opposes the motion. Such resistance to motion is called friction.



Types of friction:



- **Sliding friction −** Whenever a body slides or tends to slide over the surface of another body, the friction that comes into play is called sliding friction. It is of two types:

- **Static friction −** It is that opposing force which comes into play when a body tends to slide over the surface of another body. The maximum value of static friction which comes into play when the body is just on the point of sliding is called **limiting friction**.

- **Dynamic friction −** It is the opposing force that comes into play when a body is actually sliding over the surface of another body. Dynamic friction is also called kinetic friction.

- **Rolling friction −** Friction that arises when one body is rolling over another is called rolling friction.

Laws of static friction:

- **First law:** The magnitude of the limiting force of static friction (F_s) between any two bodies in contact is directly proportional to the normal reaction (N) between them.

  $F_s \propto\propto N$

  $F_s = \mu_s N$,

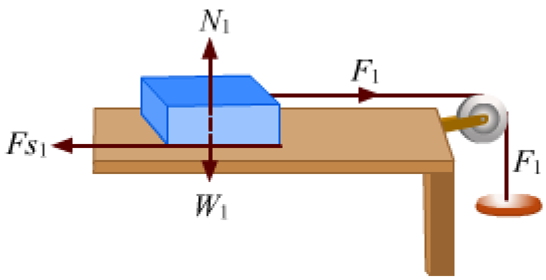
  where μ_s is proportionality constant and is called the coefficient of static friction.

- **Second law:** The limiting force of static friction is independent of the area of contact, as long as the normal reaction between two surfaces in contact remains the same.

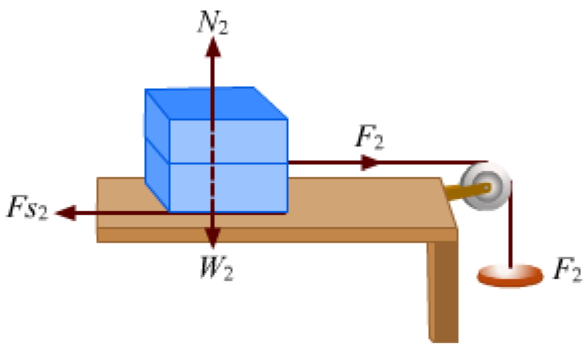
- **Third law:** The limiting force of static friction depends on the nature of material of the surfaces in contact.
- **Fourth law:** The direction of limiting friction force is always opposite to the direction in which one body is at verge of moving over the other body.

Experimental verification of laws of static friction:

First law:
A wooden block of weight (W_1) is placed on a horizontal table. The wooden block is tied to a string that passes over the pulley and that is attached to a light pan at the other end. We adjust the weight in the pan to increase the driving force (F_1) till the block begins to move. W_1 and F_1 are noted.



The experiment is repeated by placing another block on the lower block as shown below. W_2 and F_2 are noted.



When the block just moves, the applied force is equal to the limiting force of friction.

$F_1 = F_{s1}$ and $F_2 = F_{s2}$; and $W_1 = N_1$ and $W_2 = N_2$.

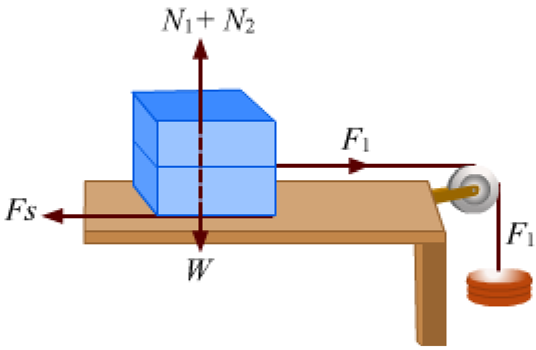
We find that

$$\frac{F_{s_1}}{N_1} = \frac{F_{s_2}}{N_2} = \text{constant}$$

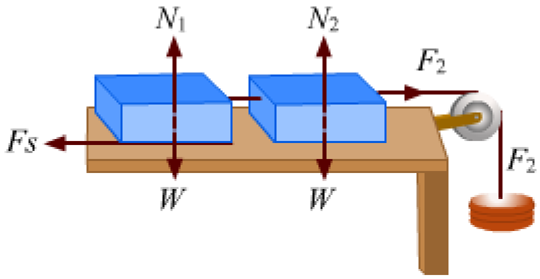
This shows that $F_s \propto N$.

Second law:

Two identical wooden blocks A and B are placed one above the other, as shown. Weights are added in the pan till the blocks just begin to move. Weights W and F_1 are noted.



Now, the wooden blocks A and B are placed one after another. In this case, the area of contact increases, but $W = N_1+N_2$ remains the same. Weights are added in the pan till the blocks begin to move. Weight F_2 is noted.



We find that

$F_1 = F_2$

This shows that the limiting force of static friction is independent of the area of the surfaces.

Third law:

- A wooden block is placed on a horizontal surface and the limiting force of static friction (as explained above), F_1, is noted. Now, another wooden block with a polished surface is placed on the horizontal table. Limiting force of static friction(as explained above is), $F_{2,}$ is noted. It is found that

  $F_1 \neq F_2$

  This show that limiting force of static friction depends on the nature of surface in contact.

- When instead of a wooden block, blocks of different materials are used to find the limiting force of static friction, different values of limiting force are noted. We get different values, which shows that the limiting force of static friction depends on the material of the surface in contact.

Fourth law:

- As in equilibrium $W_1 = N_1$ and $W_2 = N_2$, which implies F1 = F_{s1} and $F_2 = F_{s2}$ i.e, force of limiting friction is equal to the applied force. The direction of limiting friction force is always opposite to the given direction of motion of the block, which is on the right. So, the direction of limiting friction force is on the left.

Laws of kinetic friction

- **First law:** The magnitude of force of kinetic friction (F_k) between any two bodies in contact is directly proportional to the normal reaction (N) between them.

  $F_k \propto N$

  $F_k = \mu_k N$,

  where μ_k is the proportionality constant and is called the coefficient of kinetic friction.

- **Second law:** The force of kinetic friction is independent of the area of contact, as long as the normal reaction between two surfaces in contact remains the same.

- **Third law:** The force of kinetic friction depends on the nature of material of the surface in contact.

- **Fourth law**: The force of kinetic friction is approximately independent of relative velocity, provided relative velocity is neither too high nor too low.

Banking of Roads

Centripetal force: $F = \frac{Mv^2}{R}$

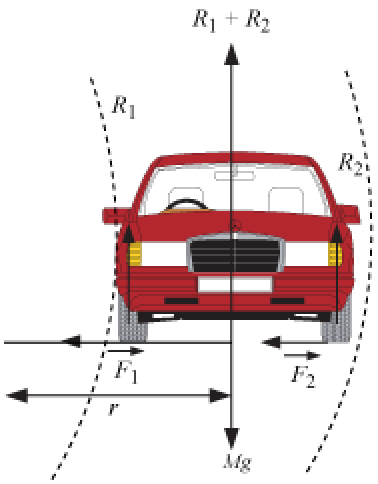
Where,

M = Mass of the body

v = Velocity of the body

R = Radius of the circular path

- **Motion of a car on a level road:**



If R_1 and R_2 are the normal reactions of the ground on the two tyres of a car of weight *M*g, going around on a circular turn of radius *r*, with velocity *v*, on a level road, then

$F_1 = \mu R_1$ and $F_2 = \mu R_2$

Where, *μ* is the coefficient of friction between the tyres and the road

The total force of friction provides the necessary centripetal force, i.e.,

$$F_1 + F_2 = \frac{Mv^2}{r}$$
$$\mu R_1 + \mu R_2 = \frac{Mv^2}{r}$$
$$\mu\left(R_1 + R_2\right) = \frac{Mv^2}{r} \qquad \ldots(\text{i})$$

The total normal reaction balances the weight of the car, i.e.,

$$R_1 + R_2 = Mg \qquad \ldots(\text{ii})$$

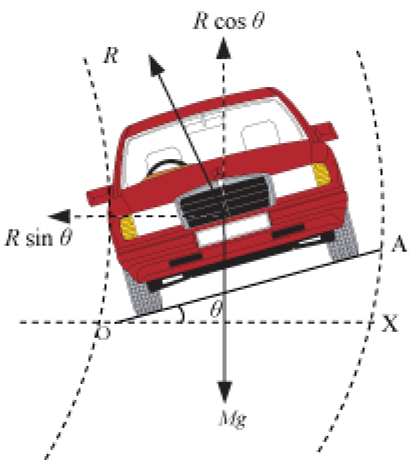
From equations (i) and (ii), we have

$$\mu Mg = \frac{Mv^2}{r}$$
$$\mu = \frac{v^2}{rg}$$

The above equation gives the maximum velocity with which the car can take a turn of radius *r*, when the coefficient of friction between the tyres and the road is equal to v^2/rg.

- **Motion of a car on a banked road:**

  For the vehicle to go round the curved track at a reasonable speed without skidding, the greater centripetal force is managed for it by raising the outer edge of the track a little above the inner edge. It is called banking of circular tracks.



  Consider a vehicle of weight *M*g, moving round a curved path of radius *r*, with a speed *v*, on a road banked through angle *θ*.

  The vehicle is under the action of the following forces:

- The weight *M*g acting vertically downwards

- The reaction *R* of the ground to the vehicle, acting along the normal to the banked road OA in the upward direction.

  The vertical component $R \cos \theta$ of the normal reaction *R* will balance the weight of the vehicle and the horizontal component $R \sin \theta$ will provide the necessary centripetal force to the vehicle. Thus,

  $R \cos \theta = Mg$ …(i)

  $R \sin \theta = \frac{Mv^2}{r}$ …(ii)

  On dividing equation (ii) by equation (i), we get

$$\frac{R\sin\theta}{R\cos\theta} = \frac{Mv^2/r}{Mg}$$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

As the vehicle moves along the circular banked road OA, the force of friction between the road and the tyres of the vehicle, $F = \mu R$, acts in the direction AO.

The frictional force can be resolved into two components:

- $\mu R \sin\theta$ in the downward direction
- $\mu R \cos\theta$ in the inward direction

Since there is no motion along the vertical,

$R \cos\theta = Mg + \mu R \sin\theta$ ……. (iii)

Let v_{max} be the maximum permissible speed of the vehicle. The centripetal force is now provided by the components $R \sin\theta$ and $\mu Mg \cos\theta$, i.e.,

$R \sin\theta + \mu R \cos\theta = \frac{Mv_{max}^2}{r}$ …….. (iv)

From equation (iii),we have

$Mg = R \cos\theta (1 - \mu \tan\theta)$…(v)

Again from equation (iv), we have

$\frac{Mv_{max}^2}{r} = R\cos\theta(\mu + \tan\theta)$ ...(vi)

On dividing equation (iv) by (v), we have

$$v_{\max} = \left[\frac{gr(\mu+\tan\theta)}{1-\mu\tan\theta}\right]^{1/2}$$