MIND MAP : LEARNING MADE SIMPLE

CHAPTER - 3

Matrices

Transpose of a Matrix

If $A=[a_{ij}]_{m\times n}$, then its transpose A' $(A^T)=[a_{ji}]_{n\times m}$ *i.e.* if $A=\begin{pmatrix}2 & 1\end{pmatrix}$ then $A^T=\begin{pmatrix}1\\2\end{pmatrix}$.

Also, $(A')'=A$, $(kA)'=kA'$, $(A+B)'=A'+B'$, $(AB)'=B'A'$.

- A is symmetric matrix if $A=A'$ *i.e.* $A'=-A$.
- A is skew – symmetric if $A=-A'$ *i.e.* $A'=-A$.
- A is any matrix, then–

$$A=\frac{1}{2}\{(A+A')+(A-A')\}=\text{sum of a symmetric and a skew-symmetric matrix.}$$

$(A+A')$ → S.M; $(A-A')$ → Skew, S.M.

For *eg* if $A=\begin{pmatrix}2 & 8\\6 & 4\end{pmatrix}$, then $A=\frac{1}{2}\left\{\begin{pmatrix}2 & 7\\7 & 4\end{pmatrix}+\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\right\}$.

Definition and its types

A matrix of order m×n is an ordered rectangular array of numbers or functions having 'm' rows and 'n' columns. The matrix $A=[a_{ij}]_{m\times n}$ is given by

$$A=\begin{bmatrix}a_{11} & a_{12} & ...a_{1n}\\a_{21} & a_{22} & ...a_{2n}\\a_{m1} & a_{m2} & ...a_{mn}\end{bmatrix}_{m\times n}$$

- Column matrix : It is of the form $[a_{ij}]_{m\times 1}$
- Row matrix : It is of the form $[a_{ij}]_{1\times n}$
- Square matrix : Here, $m=n$ (no. of rows = no. of columns)
- Diagonal matrix : All non-diagonal entries are zero i.e $a_{ij}=0\,\forall i\neq j$
- Scalar matrix : $a_{ij}=0, i\neq j$ and $a_{ij}=k$ (Scalar), $i=j$
- Identity matrix : $a_{ij}=0, i\neq j$ and $a_{ij}=1, i=j$
- Zero matrix : All entries are zero.

Equality of two matrix

$A=[a_{ij}]=[b_{ij}]=B$ if, A and B are of same order and $a_{ij}=b_{ij}\ \forall$ i and j.

Operations on matrices

Addition

If A, B are two matrices of same order, then $A+B=[a_{ij}+b_{ij}]$ The addition of A and B follows: $A+B=B+A$, $(A+B)+C=A+(B+C)$, $-A=(-1)A$, $k(A+B)=kA+kB$, k is scalar and $(k+l)A=kA+lA$, k and l are constants.

- If $A=\begin{pmatrix}2 & 3\\2 & 4\end{pmatrix}$, $B=\begin{pmatrix}-3 & 2\\-4 & 5\end{pmatrix}$ then $A+B=\begin{pmatrix}-1 & 5\\-2 & 9\end{pmatrix}$
- If $A=\begin{pmatrix}2 & 3\end{pmatrix}_{1\times 2}$, $B=\begin{pmatrix}4\\5\end{pmatrix}_{2\times 1}$, then $AB=(2\times 4+3\times 5)=\begin{pmatrix}2 & 3\end{pmatrix}_{1\times 1}$

Multiplication

If $A=[a_{ij}]_{m\times n}$ and $B=[b_{jk}]_{n\times p}$, then $AB=C=[C_{ik}]_{m\times p}$, $[C_{jk}]=\sum_{j=1}^{n}a_{ij}b_{jk}$. Also, $A(BC)=(AB)C$, $A(B+C)=AB+AC$ and $(A+B)C=AC+BC$, but $AB\neq BA$ (always).

Elementary operations on a matrix

$R_i\leftrightarrow R_j$ or $C_i\leftrightarrow C_j$

$R_i\to kR_i$ or $C_i\to kC_i$

$R_i\to R_i+kR_j$ or $C_i\to C_i+kC_j$

Inverse of a matrix

If A, B are square matrices such that $AB=BA=I$ then $B=A^{-1}$ i.e., A is the inverse of B and vice-versa.

Inverse of a square matrix, if it exists, is unique

For eg: If $A=\begin{pmatrix}1 & 2\\3 & 4\\5 & 6\end{pmatrix}$, then after $R_1\longleftrightarrow R$, A becomes $\begin{pmatrix}3 & 4\\1 & 2\\5 & 6\end{pmatrix}$

If A and B are invertible matrices of the same order, then $(AB)^{-1}=B^{-1}A^{-1}$. By elementary transformations, we can convert $A=IA$ to $I=A^{-1}A$. This is one process of finding the inverse of a given square matrix A.