HOTS (Higher Order Thinking Skills)

Que 1. In Fig. 8.53, ABCD is a parallelogram and E is the mid-point of AD. A line through D, drawn parallel to EB, meets AB produced at F and BC at L. Prove that

(i) AF = 2 DC (ii) DF = 2 DL

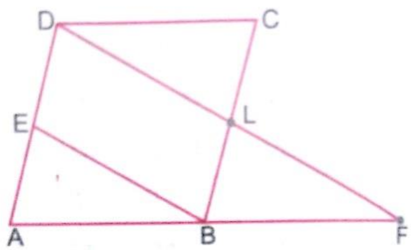

Fig. 8.53

Sol. (i) As EB\\ DL and ED\\ BL. Therefore, EBLD is a parallelogram.

∴ BL = ED = $\frac{1}{2}$ AD = $\frac{1}{2}$ BC = CL

Now in triangles DCL and FBL, we have

CL = BL (Proved above)

∠DLC = ∠FLB (Vertically opposite angles)

∠CDL = ∠BFL (Alternate angles)

∴ ΔDCL ≅ ΔFBL (By AAS congruence criterion)

∴ DC = BF and DL = FL

Now, BE = DC = AB

⇒ 2AB = 2DC ⇒ AF = 2DC

(ii) ∴ DL = FL ⇒ DF = 2DL

Que 2. PQ and RS are two equal and parallel line-segments. Any point M not lying on PQ or RS is joined to Q and S and lines through P parallel to QM and through R parallel to SM meet at N. prove that line segments MN and PQ are equal and parallel to each other.

Sol. Given: PQ = RS, PQ || RS, PN || QM, RN || MS

To prove: MN = PQ, MN || PQ

Proof: Since PQ = RS and PQ || RS

∴ PQSR is a parallelogram

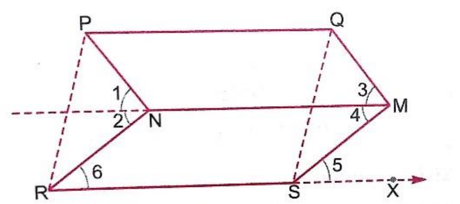

Fig. 8.54

$\Rightarrow$ PR = QS, PR || QS

Since PN || QM and MN is the transversal

$\therefore$ $\angle 1 = \angle 3$ (Corresponding angles) …(i)

Similarly, RN || MS

$\therefore$ $\angle 2 = \angle 4$ …(ii)

Adding (i) and (ii), we get

$\angle 1 + \angle 2 = \angle 3 + \angle 4$ i.e., $\angle PNR = \angle QMS$

Again, $\angle PRS = \angle QSX$ (Corresponding angles as PR || QS)

And $\angle 6 = \angle 5$ (Corresponding angles as RN || SM)

Subtracting the two equations, we get

$\angle PRS - \angle 6 = \angle QSX - \angle 5$ I.e., $\angle PRN = \angle QSM$

Now, in ΔPNR and ΔQMS,

PR = QS (Opp. Sides of $||^{gm}$)

$\angle PNR = \angle QMS$ (Proved above)

$\angle PRN = \angle QSM$

$\therefore$ $\Delta PNR \cong \Delta QMS$ (By AAS congruence criterion)

$\Rightarrow$ PN = QM (CPCT)

Also, PN = QM (Given)

$\therefore$ PNMQ is a parallelogram

$\Rightarrow$ PQ || MN and PQ = MN

Que 3. l, m and n are three parallel lines intersected by transversal p and q such that l, m and n cut-off equal intersepts AB and BC on p (Fig. 8.55). Show that l, m and n cut-off equal intercepts DE and EF on q also.

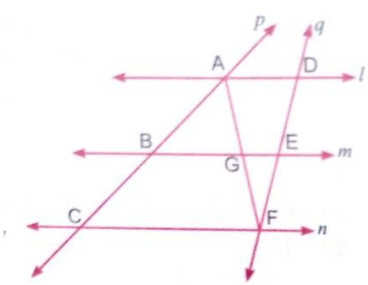

Fig. 8.55

Sol. We are given that AB = BC and have to prove that DE = EF.

Let us join A to F intersecting m at G.

The trapezium ACFD is divided into two triangles; namely ΔACF and ΔAFD.

In ΔACF, it is given that B is the mid-point of AC (AB = BC)

And BG || CF (Since m || n)

So, G is the mid-point of AF (By the converse of mid-point Theorem)

Now, in ΔAFD, we can apply the same argument as G is the mid-point of AF, GE || AD so E is the mid-point of DF,

i.e., DE = EF

In other words l, m and n cut-off equal intercepts on q also.