**CBSE Test Paper 05**

**Chapter 1 Relations and Functions**

1. If A is a non empty set, an element e ∈ A is called the identity for the binary operation ∗, if
   a. a ∗ e = e = e ∗ a, ∀ a ∈ A
   b. a ∗ e = $a^2$ = e ∗ a, ∃ a ∈ A
   c. a ∗ e = a ≠ e ∗ a, ∀ a ∈ A
   d. a ∗ e = a = e ∗ a, ∀ a ∈ A
2. A relation R in a set A is called universal relation, if
   a. no element of A is related to any element of A
   b. each element of A is related to every element of A
   c. one element of A is related to all elements of A
   d. every element of A is related to one element of A
3. The period of the function $f(x) = sin^2 x + tan x$ is
   a. $\pi$
   b. $3\pi$
   c. $\frac{\pi}{2}$
   d. $2\pi$
4. + : $R \times R \rightarrow R$ and x : $R \times R \rightarrow R$ are
   a. commutative operations
   b. non-associative operations
   c. not commutative operations
   d. non transitive operations
5. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two invertible functions. Then gof is
   a. Non invertible
   b. invertible with (gof)-1= f-1og-1
   c. $I_y$
   d. $I_N$
6. The set of first elements of all ordered pairs in R, i.e., {x : (x, y) ∈R} is called the ________ of relation R.
7. A function f : X → Y is said to be a ________ function, if it is both one-one and onto.

8. A binary operation * on set X is said to be ________, if a * (b * c) = (a * b) * c, where a, b, c $\in$ X.

9. Let the function f : R $\rightarrow$ R be defined by f(x) = 4x - 1, $\forall$ x $\in$ R. Then, show that f is one-one.

10. What is a bijective function?

11. Find gof where f(x) = $8x^3$, g(x) = $x^{1/3}$

12. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.

13. If f : R $\rightarrow$ R is defined by f(x) = $x^2$ - 3x + 2, find f(f(x)).

14. Let C be the set of complex numbers. Prove that the mapping f : C $\rightarrow$ R given by f(z) = |z|, $\forall$z $\in$ C, is neither one-one nor onto.

15. Let A = N$\times$N and * be the binary operation on A define by (a, b) * (c, d) = (a + c, b + d) Show that * is commutative and associative.

16. If the function f : R $\rightarrow$ R is given by f(x)=$\frac{x+3}{3}$ and g: R$\rightarrow$ R is given by g(x) = 2x- 3, then find (i) fog (ii) gof. Is $f^{-1}$ = g?

17. Let f : R $\rightarrow$ R be the function defined by $f(x) = \frac{1}{2-\cos x}$, $\forall x \in R$. Then, find the range of f.

18. Consider f : $R_+$ $\rightarrow$ [-5, $\infty$] given by f(x) = $9x^2$ + 6x - 5. Show that f is invertible with $f^{-1}(y) = \left(\frac{(\sqrt{y+6})-1}{3}\right)$.

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## Chapter 1 Relations and Functions

### Solution

1. d. $a * e = e = e * a, \forall a \in A$

   **Explanation:** Let * be a binary operation on a set A. If there exist an element e $\in$ A such that $a * e = a = e * a, \forall a \in A$. Then e is called an identity element for the binary operation * on A.

2. b. each element of A is related to every element of A

   **Explanation:** The relation R = A x A is called Universal relation.

3. a. $\pi$

   **Explanation:** We have the function $f(x) = sin^2 x + tan x$ , therefore,
   $f(\pi+x) = \{\sin^2(\pi+x)+\tan(\pi+x)\} = \left\{\sin^2 x+\tan x\right\} = f(x)$.
   This implies that period of the function f(x) is $\pi$.

4. a. commutative operations

   **Explanation:** Since Addition and Multiplication are both commutative over the set of real numbers.

5. b. invertible with $(gof)^{-1} = f^{-1}og^{-1}$

   **Explanation:** If $f : X \to Y\ and\ g : Y \to Z$ be two bijective functions which are invertible then $(gof)^{-1} = f^{-1}og^{-1}$

6. domain
7. bijective
8. associative

9. For any two elements $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$, we have

   $4x_1 - 1 = 4x_2 - 1$

   $\Rightarrow 4x_1 = 4x_2$, i.e., $x_1 = x_2$

   **Hence f is one-one.**

10. A function f: X $\to$ Y is said to be bijective, if f is both one – one and onto.

11. gof (x) = g[f(x)]

$= g(8x^3)$

$= (8x^3)^{1/3}$

$= 2x$

12. Books x and x have same number of pages $\Rightarrow (x, x) \in R \therefore R$ is reflexive.
    If $(x, y) \in R$ ,then x and y have same no. of pages
    $\Rightarrow$ y and x have same no. of pages
    $\Rightarrow (y, x) \in R \therefore$ R is symmetric.
    Now if $(x, y) \in R, (y, z) \in R$. Then
    x and y have same no. of pages and y and z have same no. of pages. This implies x and z have same no. of pages.
    $\Rightarrow (x, z) \in R \therefore$ R is transitive.
    Since R is reflexive, symmetric and transistive, therefore, R is an equivalence relation.

13. Given: $f(x) = x^2 - 3x + 2$

    $= f[(x)] = f(x^2 - 3x + 2)$

    $= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

    $= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$

    $= x^4 - 6x^3 + 10x^2 - 3x$

14. The mapping $f : C \rightarrow R$
    Given, $f(z) = |z|, \forall z \in C$
    $f(1) = |1| = 1$
    $f(-1) = |-1| = 1$
    $f(1) = f(-1)$
    But $1 \neq -1$
    So, f(z) is not one-one. Also, f(z) is not onto as there is no pre-image for any negative element of R under the mapping f(z).

15. i. $(a, b) * (c, d) = (a + c, b + d)$
       $= (c + a, d + b)$
       $= (c, d) * (a, b)$
       Hence commutative
    ii. $\{(a, b) * (c, d)\} * (e, f)$

= (a + c, b + d) * (e, f)

= (a + c + e, b + d + f)

= (a, b) * (c + e, d + f)

= (a, b) * {(c, d) * (e, f)}

{(a, b) * (c, d)} * (e, f) = (a, b) * {(c, d) * (e, f)}

Hence * associative.

16. Given f : R $\rightarrow$ R such that f(x) = $\frac{x+3}{3}$ and
g : R $\rightarrow$ R such that $g(x) = 2x - 3$

   i. Clearly, fog : R $\rightarrow$ R and (fog)(x) = f[g(x)]
   = $f(2x-3)$ = $\frac{(2x-3)+3}{3}$
   = $\frac{2x}{3}$
   ii. Clearly, gof: R $\rightarrow$ R and $(gof)(x) = g[f(x)]$
   $= g\left(\frac{x+3}{3}\right) = \left[2\left(\frac{x+3}{3}\right)\right] - 3$
   $= \frac{2x+6}{3} - 3 = \frac{2x+6-9}{3}$
   $\Rightarrow (gof)(x) = \frac{2x-3}{3}$
   Note that, here $fog \neq I_R$ and $gof \neq I_R$

   Hence, $f^{-1} \neq g$ [$\because$ $f^{-1}$ = g iff gof = I and fog = I]

17. Given function, $f(x) = \frac{1}{2-\cos x}, \ \forall x \in R$
Let $y = \frac{1}{2-\cos x}$
$\Rightarrow$ 2y - y cosx = 1

$\Rightarrow$ y cosx = 2y - 1

$\Rightarrow \cos x = \frac{2y-1}{y} = 2 - \frac{1}{y} \Rightarrow \cos x = 2 - \frac{1}{y}$
$\Rightarrow -1 \leqslant \cos x \leqslant 1 \Rightarrow -1 \leqslant 2 - \frac{1}{y} \leqslant 1$
$\Rightarrow -3 \leqslant -\frac{1}{y} \leqslant -1$
$\Rightarrow 3 \geqslant \frac{1}{y} \geqslant 1$
$\Rightarrow \frac{1}{3} \leqslant y \leqslant 1$
So, Range of y, that is f(x) is $\left[\frac{1}{3}, 1\right]$

18. Consider f: $R_+ \rightarrow$ [-5, $\infty$] and f(x) = $9x^2 + 6x - 5$.

Let $x_1, x_2 \in$ R , then $f(x_1) = 9x_1^2 + 6x_1 - 5$ and $f(x_2) = 9x_2^2 + 6x_2 - 5$

Now, $f(x_1) = f(x_2)$ then $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9x_1^2 + 6x_1 = 9x_2^2 + 6x_2$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)[9(x_1 + x_2) + 6] = 0$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2 \therefore$ f is one-one.

Now, again $y = 9x^2 + 6x - 5$

$\Rightarrow 9x^2 + 6x - (5 + y) = 0$

$$\Rightarrow x = \frac{-6 \pm \sqrt{(6)^2 + 4 \times 9(5+y)}}{18} = \frac{-6 \pm 6\sqrt{1+5+y}}{18} = \frac{-6 \pm 6\sqrt{y+6}}{18} = \frac{\sqrt{y+6}-1}{3}$$

$$\therefore\ f(x) = f\left(\frac{\sqrt{y+6}-1}{3}\right) = 9\left(\frac{\sqrt{y+6}-1}{3}\right)^2 + 6\left(\frac{\sqrt{y+6}-1}{3}\right) - 5$$

$$= 9\left(\frac{y+6+1-2\sqrt{y+6}}{9}\right) + 2\left(\sqrt{y+6} - 1\right) - 5$$

$= y + 7 - 2\sqrt{y+6} + 2\sqrt{y+6} - 2 - 5 = y \therefore$ f is onto.

Therefore, f(x) is invertible and $f^{-1}(x) = \frac{\sqrt{y+6}-1}{3}$.