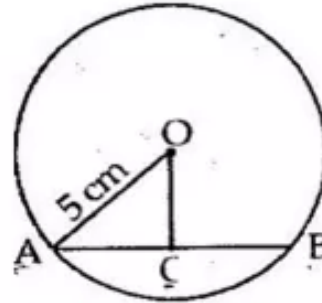# Chapter 17. Circle

Exercise 17(A)

Solution 1:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.

We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.
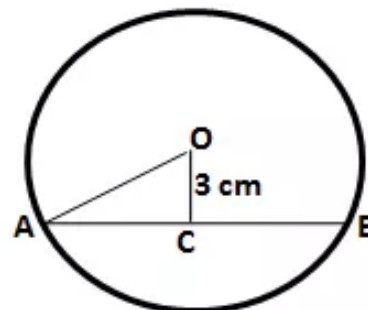$\therefore$ AC = CB = 3 cm
In $\triangle$OCA,

$$OA^2 = OC^2 + AC^2 \qquad \text{(By Pythagoras theorem)}$$
$$\Rightarrow \quad OC^2 = (5)^2 - (3)^2 = 16$$
$$\Rightarrow \quad OC = 4 \text{ cm}$$

Solution 2:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.

We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.
$\therefore$ AB=8 cm

$$\Rightarrow AC=CB=\frac{AB}{2}$$
$$\Rightarrow AC=CB=\frac{8}{2}$$
$$\Rightarrow AC=CB=4 \text{ cm}$$

In $\triangle$OCA,

$$OA^2 = OC^2 + AC^2 \qquad \text{(By Pythagoras theorem)}$$
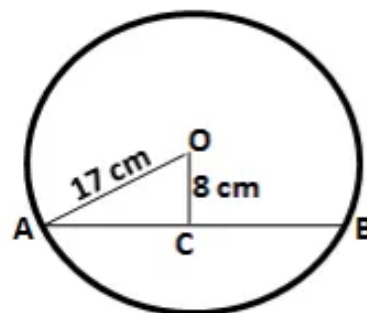$$\Rightarrow \quad OA^2 = (4)^2 + (3)^2 = 25$$
$$\Rightarrow \quad OA = 5 \text{ cm}$$

Hence, radius of the circle is 5 cm.

Solution 3:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.

We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.
$\therefore$ AC = CB

In $\triangle$OCA,

$$OA^2 = OC^2 + AC^2 \quad \text{(By Pythagoras theorem)}$$

$\Rightarrow \quad AC^2 = (17)^2 - (8)^2 = 225$

$\Rightarrow \quad AC = 15 \text{ cm}$

$\therefore \quad AB = 2\,AC = 2 \times 15 = 30 \text{ cm}.$

Solution 4:

Let AB be the chord of length 24 cm and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.
We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.
$\therefore$ AC = CB = 12 cm

In $\triangle$OCA,

$$OA^2 = OC^2 + AC^2 \quad \text{(By Pythagoras theorem)}$$

$$= (5)^2 + (12)^2 = 169$$

$\Rightarrow \quad OA = 13 \text{ cm}$

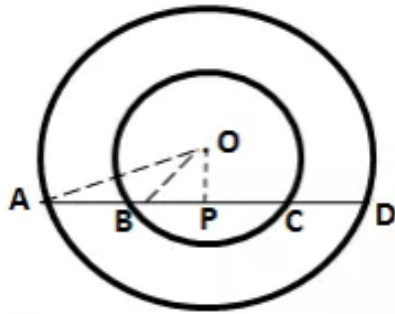$\therefore \quad$ radius of the circle = 13 cm.

Let A'B' be the new chord at a distance of 12 cm from the centre.

$\therefore \quad (OA')^2 = (OC')^2 + (A'C')^2$

$\Rightarrow \quad (A'C')^2 = (13)^2 - (12)^2 = 25$

$\therefore \quad A'C' = 5 \text{ cm}$

Hence, length of the new chord = $2 \times 5 = 10 \text{ cm}.$

Solution 5:

For the inner circle, BC is a chord and $OP \perp BC$.

We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ BP = PC

By Pythagoras Theorem,

$$OB^2 = OP^2 + BP^2$$

$$\Rightarrow \quad BP^2 = (20)^2 - (16)^2 = 144$$

$$\therefore \quad BP = 12 \text{ cm}$$

For the outer circle, AD is the chord and $OP \perp AD$.

We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AP = PD

By Pythagoras Theorem,

$OA^2 = OP^2 + AP^2$

=> $AP^2 = (34)^2 - (16)^2 = 900$

=> AP = 30 cm
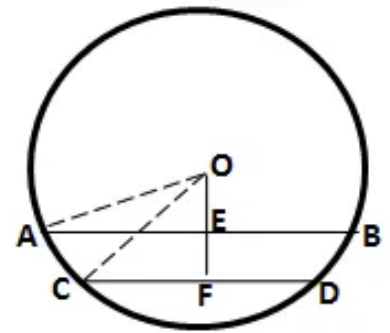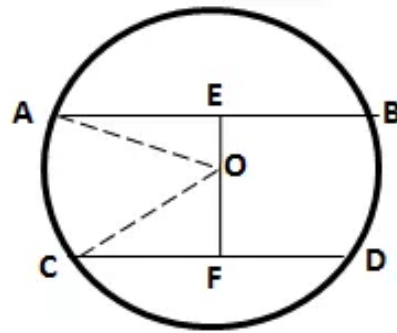
AB = AP - BP = 30 - 12 = 18 cm

Solution 6:

Let O be the centre of the circle and AB and CD be the two parallel chords of length 30 cm and 16 cm respectively.

Drop OE and OF perpendicular on AB and CD from the centre O.

OE ⊥ AB   and   OF ⊥ CD

∴ OE bisects AB and OF bisects CD

(Perpendicular drawn from the centre of a circle to a
chord bisects it)

$\Rightarrow \quad AE = \dfrac{30}{2} = 15\ cm\ ;\ \ CF = \dfrac{16}{2} = 8\ cm$

In right $\triangle OAE$,

$OA^2 = OE^2 + AE^2$

$\Rightarrow \quad OE^2 = OA^2 - AE^2 = (17)^2 - (15)^2 = 64$

∴ $\quad OE = 8\ cm$

In right $\triangle OCF$,

$OC^2 = OF^2 + CF^2$

$\Rightarrow \quad OF^2 = OC^2 - CF^2 = (17)^2 - (8)^2 = 225$

∴ $\quad OF = 15\ cm$

(i) The chords are on the opposite sides of the centre:

∴ $\quad EF = EO + OF = (8+15) = 23\ cm$
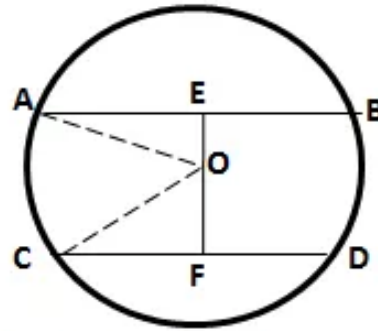
(ii) The chords are on the same side of the centre:

∴ $\quad EF = OF - OE = (15 - 8) = 7\ cm$

Solution 7:

Since the distance between the chords is greater than the radius of the circle (15 cm), so

the chords will be on the opposite sides of the centre.

Let O be the centre of the circle and AB and CD be the two parallel chords such that AB = 24 cm.
Let length of CD be 2x cm.
Drop OE and OF perpendicular on AB and CD from the centre O.

OE ⊥ AB   and   OF ⊥ CD

∴ OE bisects AB and OF bisects CD

$\left(\begin{array}{l}\text{Perpendicular drawn from the centre of a circle to a}\\\text{chord bisects it}\end{array}\right)$

$\Rightarrow \quad AE = \dfrac{24}{2} = 12 \text{ cm}; \quad CF = \dfrac{2x}{2} = x \text{ cm}$

In right △OAE,

$$OA^2 = OE^2 + AE^2$$

$\Rightarrow \quad OE^2 = OA^2 - AE^2 = (15)^2 - (12)^2 = 81$

∴   OE = 9 cm

∴   OF = EF − OE = (21 − 9) = 12 cm

In right △OCF,

$$OC^2 = OF^2 + CF^2$$

$\Rightarrow \quad x^2 = OC^2 - OF^2 = (15)^2 - (12)^2 = 81$

∴   x = 9 cm

Hence, length of chord CD = 2x = 2 × 9 = 18 cm

Solution 8:
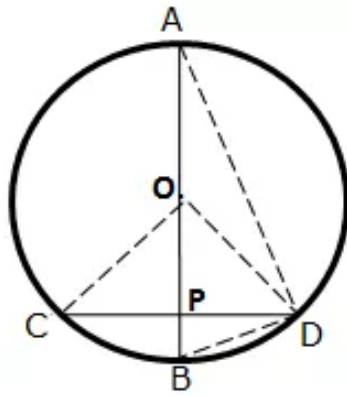
(i) OP ⊥ CD

∴ OP bisects CD

$\left(\begin{array}{l}\text{Perpendicular drawn from the centre of a circle to a}\\ \text{chord bisects it}\end{array}\right)$

$$\Rightarrow \quad CP = \frac{CD}{2}$$

In right △OPC,

$$OC^2 = OP^2 + CP^2$$

$$\Rightarrow \quad CP^2 = OC^2 - OP^2 = (15)^2 - (9)^2 = 144$$

$$\therefore \quad CP = 12\, cm$$

$$\therefore \quad CD = 12 \times 2 = 24 \ cm$$

(ii) Join BD.

$$\therefore \quad BP = OB - OP = 15 - 9 = 6\ cm$$

In right △BPD,

$$BD^2 = BP^2 + PD^2$$

$$= (6)^2 + (12)^2 = 180$$
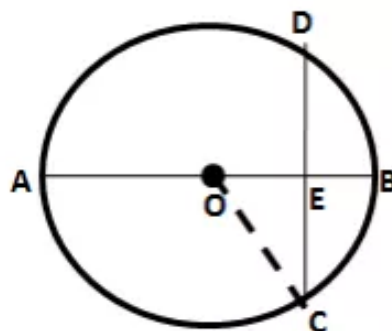
In △ADB, ∠ADB = 90°

(Angle in a semicircle is a right angle)

$$\therefore \quad AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad AD^2 = AB^2 - BD^2 = (30)^2 - 180 = 720$$

$$\therefore \quad AD = \sqrt{720} = 26.83\, cm$$

(iii) Also, BC = BD = $\sqrt{180}$ = 13.42 cm

Solution 9:

Let the radius of the circle be r cm.

∴ OE = OB − EB = r − 4

Join OC.

In right ΔOEC,

$$OC^2 = OE^2 + CE^2$$
$$\Rightarrow r^2 = (r-4)^2 + (8)^2$$
$$\Rightarrow r^2 = r^2 - 8r + 16 + 64$$
$$\Rightarrow 8r = 80$$
$$\therefore r = 10 \text{ cm}$$

Hence, radius of the circle is 10 cm.

Solution 10:

(i) AB is the chord of the circle and OM is perpendicular to AB.

So, AM = MB = 12 cm (Since ⊥ bisects the chord)

In right ΔOMA,

$$OA^2 = OM^2 + AM^2$$
$$\Rightarrow OA^2 = 5^2 + 12^2$$
$$\Rightarrow OA = 13 \text{ cm}$$

So, radius of the circle is 13 cm.

(ii) So, OA = OC = 13 cm (radii of the same circle)

In right ΔONC,

$$NC^2 = OC^2 - ON^2$$
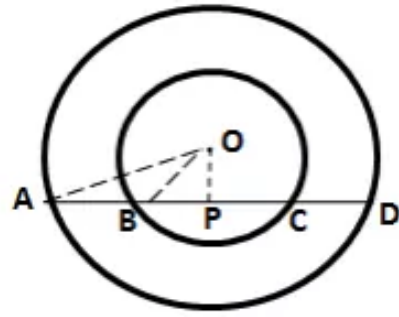$$\Rightarrow NC^2 = 13^2 - 12^2$$
$$\Rightarrow NC = 5 \text{ cm}$$

So, CD = 2NC = 10 cm

Exercise 17(B)

Solution 1:

Drop OP ⊥ AD

∴ OP bisects AD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

⇒   AP = PD   ----(i)

Now, BC is a chord for the inner circle and OP ⊥ BC

∴ OP bisects BC

(Perpendicular drawn from the centre of a circle to a chord bisects it)

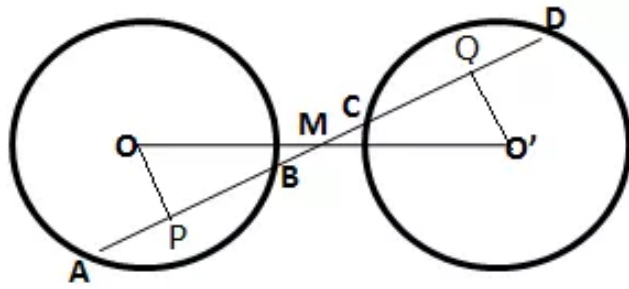⇒   BP = PC   ----(ii)

Subtracting (ii) from (i),

       AP – BP = PD – PC

⇒   AB = CD

Solution 2:

Given: A straight line Ad intersects two circles
of equal radii at A, B, C and D.
The line joining the centres OO' intersect AD at M
and M is the midpoint of OO'.
To prove: AB = CD.
Construction: From O, draw OP ⊥ AB and from O', draw O'Q ⊥ CD.
Proof:
In ΔOMP and ΔO'MQ,

$\angle$OMP = $\angle$O'MQ   (Vertically opposite angles)

$\angle$OPM = $\angle$O'QM $\left(\text{each} = 90^0\right)$

OM = O'M   (Given)

By Angle-Angle-Side criterion of congruence,

∴   ΔOMP ≅ ΔO'MQ,   (by AAS)
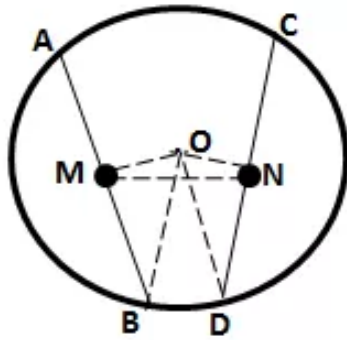
The corresponding parts of the congruent triangles are congruent

∴   OP = O'Q   (c.p.c.t)

We know that two chords of a circle or equal circles which are
equidistant from the centre are equal.

∴   AB = CD

Solution 3:

Drop OM ⊥ AB and ON ⊥ CD

∴ OM bisects AB and ON bisects CD.

$$\left(\begin{array}{l}\text{Perpendicular drawn from the centre of a circle to a}\\ \text{chord bisects it}\end{array}\right)$$

$$\Rightarrow \quad BM = \frac{1}{2}AB = \frac{1}{2}CD = DN \qquad ----(1)$$

Applying Pythagoras theorem,

$$\begin{aligned} OM^2 &= OB^2 - BM^2 \\ &= OD^2 - DN^2 \qquad \text{(by (1))} \\ &= ON^2 \end{aligned}$$

∴   OM = ON

$$\Rightarrow \quad \angle OMN = \angle ONM \qquad ----(2)$$

(Angles opp to equal sides are equal)

(i) $\angle OMB = \angle OND$ (both 90°)
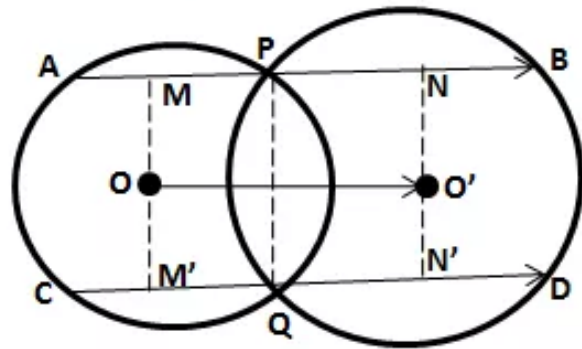
Subtracting (2) from above,

   $\angle BMN = \angle DNM$

(ii) $\angle OMA = \angle ONC$ (both 90°)

Adding (2) to above,

   $\angle AMN = \angle CNM$

Solution 4:

Drop OM and O'N perpendicular on AB and OM' and O'N' perpendicular on CD.

∴ OM, O'N, OM' and O'N' bisect AP, PB, CQ and QD respectively

$\left(\begin{array}{l}\text{Perpendicular drawn from the centre of a circle to a}\\ \text{chord bisects it}\end{array}\right)$

∴ $\quad MP = \dfrac{1}{2}AP,\ PN = \dfrac{1}{2}BP,\ M'Q = \dfrac{1}{2}CQ,\ QN' = \dfrac{1}{2}QD$

Now, $\quad OO' = MN = MP + PN = \dfrac{1}{2}(AP + BP) = \dfrac{1}{2}AB$ --- (i)

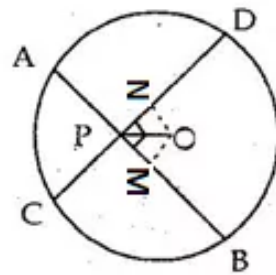and $\quad OO' = M'N' = M'Q + QN' = \dfrac{1}{2}(CQ + QD) = \dfrac{1}{2}CD$ --- (ii)

By (i) and (ii),

$\quad AB = CD$

Solution 5:

Drop OM and ON perpendicular on AB and CD.
Join OP, OB and OD.

∴ OM and ON bisect AB and CD respectively

$\begin{pmatrix} \text{Perpendicular drawn from the centre of a circle to a} \\ \text{chord bisects it} \end{pmatrix}$

∴ $\quad MB = \dfrac{1}{2} AB = \dfrac{1}{2} CD = ND \qquad ---(i)$

In rt ΔOMB, $\quad OM^2 = OB^2 - MB^2 \qquad ---(ii)$

In rt ΔOND, $\quad ON^2 = OD^2 - ND^2 \qquad ---(iii)$

From (i), (ii) and (iii),

$\qquad OM = ON$

In ΔOPM and ΔOPN,

$\qquad \angle OMP = \angle ONP \qquad (\text{both } 90^0)$

$\qquad OP = OP \qquad (\text{Common})$

$\qquad OM = ON \qquad (\text{Proved above})$

By Right Angle-Hypotenuse-Side criterion of congruence,

∴ $\qquad \Delta OPM \cong \Delta OPN (\text{by RHS})$

The corresponding parts of the congruent triangles are congruent.

∴ $\qquad PM = PN \qquad (\text{cp.c.t})$

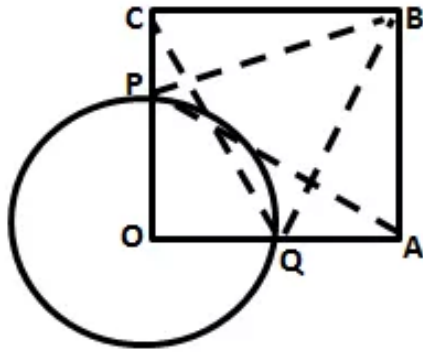Adding (i) to both sides,

$\qquad MB + PM = ND + PN$

⇒ $\qquad BP = DP$

Now, $\quad AB = CD$

∴ $\qquad AB - BP = CD - DP \qquad (\because BP = DP)$

⇒ $\qquad AP = CP$

Solution 6:

(i)

In ΔOPA and ΔOQC,

$\quad\quad$ OP = OQ $\quad\quad\quad\quad$ (radii of same circle)

$\quad\quad$ ∠AOP = ∠COQ $\quad\quad$ (both 90°)

$\quad\quad$ OA = OC $\quad\quad\quad\quad$ (sides of the square)

By Side – Angle – Side criterion of congruence,

∴ $\quad\quad$ ΔOPA ≅ ΔOQC (by SAS)

(ii)

Now, $\quad$ OP = OQ $\quad\quad\quad\quad\quad$ (radii)

and $\quad$ OC = OA $\quad\quad\quad\quad\quad$ (sides of the square)

∴ $\quad\quad$ OC – OP = OA – OQ

⇒ $\quad\quad$ CP = AQ $\quad\quad\quad\quad\quad$ – – – – (1)

In ΔBPC and ΔBQA,

$\quad\quad$ BC = BA $\quad\quad\quad\quad$ (sides of the square)

$\quad\quad$ ∠PCB = ∠QAB (both 90°)

$\quad\quad$ PC = QA $\quad\quad\quad\quad$ (by (1))

By Side – Angle – Side criterion of congruence,

∴ $\quad\quad$ ΔBPC ≅ ΔBQA (by SAS)

Solution 7:
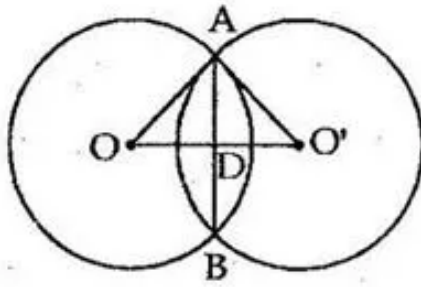
OA = 25 cm   and AB = 30 cm

$\therefore$ $AD = \frac{1}{2} \times AB = \left(\frac{1}{2} \times 30\right) cm = 15$ cm

Now in right angled $\triangle$ ADO,

$OA^2 = AD^2 + OD^2$

$\Rightarrow$ $OD^2 = OA^2 - OD^2 = 25^2 - 15^2$

$= 625 - 225 = 400$

$\therefore$ $OD = \sqrt{400} = 20$ cm

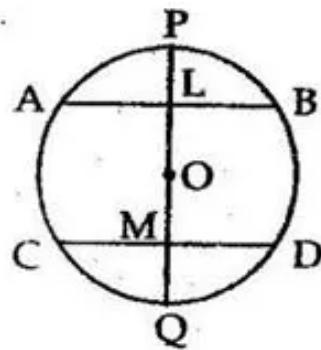Again, we have O'A = 17 cm.

In right angle $\triangle$ ADO'

$O'A^2 = AD^2 + O'D^2$

$\Rightarrow$ $O'D^2 = O'A^2 - AD^2 = 17^2 - 15^2$

$= 289 - 225 = 64$

$\therefore$ $O'D = 8$ cm

$\therefore$ $OO' = (OD + O'D)$

$= (20 + 8) = 28$ cm

$\therefore$ the distance between their centres is 28 cm.

Solution 8:

Given: AB and CD are the two chords of a circle with centre O.

L and M are the midpoints of AB and CD and O lies in the line joining ML

To Prove : AB ∥ CD.

Proof: AB and CD are two chords of a circle with centre O.

Line LOM bisects them at L and M.

Then,   OL $\perp$ AB

and,   OM $\perp$ CD

$\therefore$ $\angle$ALM = $\angle$LMD = 90^0

But they are alternate angles

$\therefore$ AB ∥ CD.

Solution 9:

In the circle with centre Q, QO ⊥ AD

∴   OA = OD   ---- (1)

$\begin{pmatrix} \text{Perpendicular drawn from the centre of a circle to a} \\ \text{chord bisects it} \end{pmatrix}$

In the circle with centre P, PO ⊥ BC

∴   OB = OC   ---- (2)

$\begin{pmatrix} \text{Perpendicular drawn from the centre of a circle to a} \\ \text{chord bisects it} \end{pmatrix}$

(i)

(1) – (2) gives,

          AB = CD   ----(3)

(ii)

Adding BC to both sides of equation (3)

          AB + BC = CD + BC

⇒   AC = BD

Solution 10:
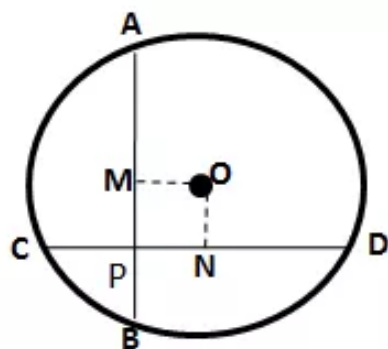
Clearly, all the angles of OMPN are 90⁰.

OM ⊥ AB and ON ⊥ CD

∴ $BM = \frac{1}{2}AB = \frac{1}{2}CD = CN$ ---- (i)

$\begin{pmatrix} \text{Perpendicular drawn from the centre of a circle to a} \\ \text{chord bisects it} \end{pmatrix}$

As the two equal chords AB and CD intersect at point P inside the circle,

∴   AP = DP   and   CP = BP   ---- (ii)

Now,   CN – CP = BM – BP   (by (i) and (ii))

⇒   PN = MP

∴   Quadrilateral OMPN is a square.

Exercise 17(C)

Solution 1:

In the given figure, ΔABC is an equilateral triangle.
Hence all the three angles of the triangle will be
equal to 60º.
i.e. ∠A=∠ B=∠C=60º
As the triangle is an equilateral triangle,
BO and CO will be the angle bisectors of ∠B and ∠C
respectively.

Hence ∠OBC =$\dfrac{\angle ABC}{2}$

$$=30º$$

and as given in the figure we can see that OB and OC
are the radii of the given circle
Hence they are of equal length.
The ΔOBC is an isosceles triangle with OB = OC
In ΔOBC,∠OBC = ∠OCB as they are angles opposite
to the two equal sides of an isosceles triangle.
Hence, ∠OBC = 30º and ∠OCB = 30º
Since the sum of all the angles of a triangle is 180º
Hence in triangle OBC, ∠OCB+∠OBC+∠BOC=180º
30º + 30º + ∠BOC = 180º
60º + ∠BOC = 180º
∠BOC = 180º – 60º
∠BOC = 120º
Hence ∠BOC = 120º and ∠OBC = 30º

Solution 2:

In the given figure we can extend the staight line OB to BD
and CO to CA
Then we get the diagonals of the square which intersect each
other at 90º by the property of Square.
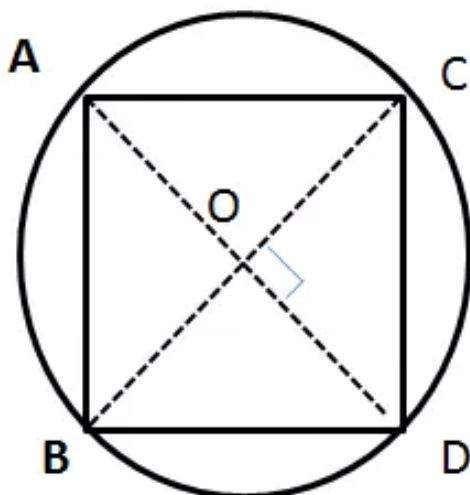
From the above statement we can see that

∠ COD = 90°.

The sum of the angle ∠ BOC and ∠OCD is 180° as BD is a straight line.

Hence ∠BOC + ∠OCD = ∠BOD = 180°

∠BOC + 90°= 180°

∠BOC = 180° – 90°

∠BOC = 90°

We can see that the ΔOCB is an isosceles triangle with sides OB and OC of equal length as they are the radii of the same circle.

In ΔOCB , ∠OBC = ∠OCB as they are oppsite angles to the two equal sides of an isosceles triangle.

Sum of all the angles of a triangle is 180°

so, ∠OBC+∠OCB+∠BOC=180°

∠OBC+∠OBC+90° = 180° as,∠OBC=∠OCB

2∠OBC = 180° – 90°

2∠OBC = 90°

∠OBC = 45°

as ∠OBC = ∠OCB So,

∠OBC = ∠OCB = 45°

Yes BD is the diameter of the circle.

Solution 3:
As given that AB is the side of a pentagon the angle subtended by each arm of the

pentagon at

the centre of the circle is $= \dfrac{360^o}{5} = 72^o$

Thus angle $\angle AOB = 72^o$
Similarly as BC is the side of a hexagon hence the angle subtended

by BC at the centre is $= \dfrac{360^o}{6}$

i.e. 60°
$\angle BOC = 60^o$
Now $\angle AOC = \angle AOB + \angle BOC = 72^o + 60^o = 132^o$
The triangle thus formed , $\triangle AOB$ is an isosceles triangle
with OA = OB as they are radii of the same circle.
Thus $\angle OBA = \angle BAO$ as they are opposite angles of equal sides
of an isosceles triangle.
The sum of all the angles of a triangle is 180°
so, $\angle AOB + \angle OBA + \angle BAO = 180^o$

$\quad 2\angle OBA + 72^o = 180^o$ as $,\angle OBA = \angle BAO$

$\quad 2\angle OBA = 180^o - 72^o$

$\quad 2\angle OBA = 108^o$

$\quad \angle OBA = 54^o$

as $\angle OBA = \angle BAO$ So,

$\quad \angle OBA = \angle BAO = 54^o$

The triangle thus formed , $\triangle BOC$ is an isosceles triangle
with OB = OC as they are radii of the same circle.
Thus $\angle OBC = \angle OCB$ as they are opposite angles of equal sides
of an isosceles triangle.
The sum of all the angles of a triangle is 180°
so, $\angle BOC + \angle OBC + \angle OCB = 180^o$

$\quad 2\angle OBC + 60^o = 180^o$ as $,\angle OBC = \angle OCB$

$\quad 2\angle OBC = 180^o - 60^o$

$\quad 2\angle OBC = 120^o$

$\quad \angle OBC = 60^o$
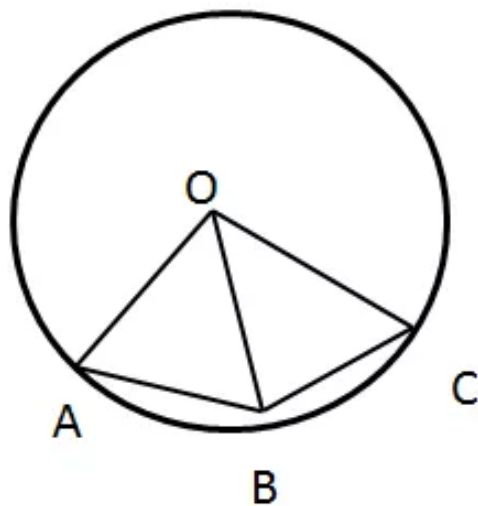
as $\angle OBC = \angle OCB$
So, $\angle OBC = \angle OCB = 60^o$
$\angle ABC = \angle OBA + \angle OBC = 54^o + 60^o = 114^o$

Solution 4:

We know that the arc of equal lengths subtend equal angles at the centre.

hence ∠AOB = ∠BOC = 48°

Then ∠AOC = ∠AOB + ∠BOC = 48° + 48° = 96°

The triangle thus formed , ΔBOC is an isosceles triangle with OB = OC as they are radii of the same circle.

Thus ∠OBC = ∠OCB as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, ∠BOC+∠OBC+∠OCB=180°

   2∠OBC + 48° = 180° as ∠OBC = ∠OCB

   2∠OBC = 180° – 48°

   2∠OBC= 132°

   ∠OBC = 66°

as ∠OBC = ∠OCB

So, ∠OBC = ∠OCB = 66°

The triangle thus formed , ΔAOC is an isosceles triangle with OA = OC as they are radii of the same circle.

Thus ∠OAC = ∠OCA as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, ∠COA+∠OAC+∠OCA =180°

   2∠OAC + 96° = 180° as ,∠OAC = ∠OCA

   2∠OAC= 180° – 96°

   2∠OAC= 84°

   ∠OAC = 42°

as ∠OCA = ∠OAC

So,∠OCA = ∠OAC = 42°

Solution 5:

We know that for two arcs are in ratio 3:2 then

∠AOB : ∠BOC = 3:2

As give ∠AOC = 96°

So, 3x=96

x = 32

Therefore ∠BOC = 2 x 32 = 64°

The triangle thus formed, ΔAOB is an isosceles triangle
with OA = OB as they are radii of the same circle.

Thus ∠OBA = ∠BAO as they are opposite angles of equal sides
of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, ∠AOB+∠OBA+∠BAO=180°

2∠OBA + 96° = 180° as ,∠OBA = ∠BAO

2∠OBA = 180° – 96°

2∠OBA = 84°

∠OBA = 42°

as ∠OBA = ∠BAO So,

∠OBA = ∠BAO = 42°

The triangle thus formed, ΔBOC is an isosceles triangle
with OB = OC as they are radii of the same circle.

Thus ∠OBC = ∠OCB as they are opposite angles of equal sides
of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, ∠BOC+∠OBC+∠OCB =180°

2∠OBC+64° = 180° as ,∠OBC = ∠OCB

2∠OBC = 180° – 64°

2∠OBC= 116°

∠OBC= 58°

as ∠OBC = ∠OCB So,

∠OBC = ∠OCB = 58°

∠ABC = ∠BOA + ∠OBC = 42° + 58° = 100°

Solution 6:

Since arc AB and BC are equal

so, ∠AOB = ∠BOC = 50°

Now

∠AOC = ∠AOB + ∠BOC = 50° + 50° = 100°

As arc AB, arc BC and arc CD so,

∠AOB = ∠BOC = ∠COD = 50°

∠AOD = ∠AOB + ∠BOC + ∠COD = 50° + 50° + 50° = 150°

Now, ∠BOD = ∠BOC + ∠BOD

     ∠BOD = 50° + 50°

     ∠BOD = 100°

The triangle thus formed , ΔAOC is an isosceles triangle

with OA = OC as they are radii of the same circle.

Thus ∠OAC = ∠OCA as they are opposite angles of equal sides

of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, ∠AOC+∠OAC+∠OCA =180°

    2∠OAC + 100° = 180° as ,∠OAC = ∠OCA

    2∠OAC= 180° – 100°

    2∠OAC= 80°

     ∠OAC = 40°

as ∠OCA = ∠OAC So,

   ∠OCA = ∠OAC = 40°

The triangle thus formed , ΔAOD is an isosceles triangle

with OA = OD as they are radii of the same circle.

Thus ∠OAD = ∠ODA as they are opposite angles of equal sides

of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, ∠AOD+∠OAD+∠ODA =180°

    2∠ODA + 150° = 180° as ,∠OAD = ∠ODA

    2∠ODA= 180° – 150°

    2∠ODA= 30°

     ∠ODA = 15°

as ∠OAD = ∠ODA So,

   ∠OAD = ∠ODA = 15°

Solution 7:

As AB is the side of a hexagon so the

$\angle AOB = \dfrac{360°}{6} = 60°$

AC is the side of an eight sided polygon so,

$\angle AOC = \dfrac{360°}{8} = 45°$

From the given figure we can see that:

$\angle BOC = \angle AOB + \angle AOC = 60° + 45° = 105°$

Again, from the figure we can see that $\triangle BOC$ is an isosceles triangle with sides BO = OC as they are the radii of the same circle.

Angles $\angle OBC = \angle OCB$ as they are opposite angles to the equal sides of an isoseles triangle.

Sum of all the angles of a triangle is 180°

$\angle OBC + \angle OCB + \angle BOC = 180°$

$2\angle OBC + 105° = 180°$ as, $\angle OBC = \angle BOC$

$2\angle OBC = 180° - 105°$

$2\angle OBC = 75°$

$\angle OBC = 37.5° = 37°30'$

As, $\angle OBC = \angle BOC$

$\angle OBC = \angle BOC = 37.5° = 37°30'$

Solution 8:

We know that when two arcs are in ratio 2:1 then the subtended by them is also in ratio 2:1

As given arc AB is twice the length of arc BC

Therefore, arc AB : arc BC = 2:1

Hence, ∠AOB : ∠BOC = 2:1

Now given that ∠AOB = 100°

so, ∠BOC = $\frac{1}{2}$∠AOB = $\frac{1}{2}$ × 100° = 50°

Now, ∠AOC = ∠AOB + ∠BOC = 100° + 50° = 150°

The triangle thus formed , ΔAOC is an isosceles triangle with OA = OC as they are radii of the same circle.

Thus ∠OAC = ∠OCA as they are opposite angles of equal sides of an isosceles triangle.

The sum of all the angles of a triangle is 180°

so, ∠COA+∠OAC+∠OCA =180°

2∠OAC + 150° = 180° as ,∠OAC = ∠OCA

2∠OAC= 180° – 150°

2∠OAC= 30°

∠OAC = 15⁰

as ∠OCA = ∠OAC So,

∠OCA = ∠OAC = 15°

Exercise 17(D)

Solution 1:
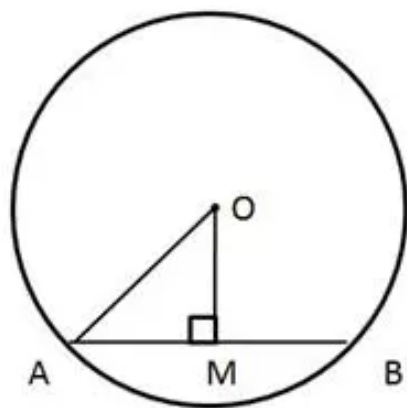
To find : OM
Given that AB = 24 cm
Since OM ⊥ AB
⇒ OM bisects AB
So, AM = 12 cm
In right ΔOMA,
$OA^2 = OM^2 + AM^2$
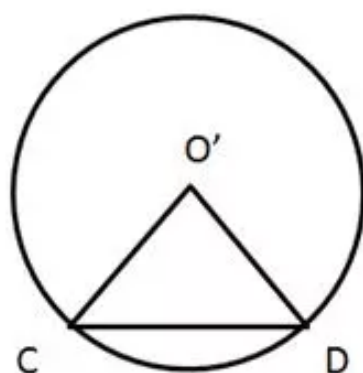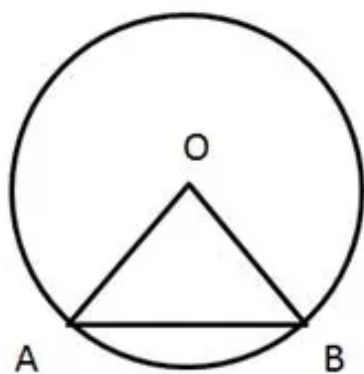$\Rightarrow OM^2 = OA^2 - AM^2$
$\Rightarrow OM^2 = 13^2 - 12^2$
$\Rightarrow OM^2 = 25$
$\Rightarrow OM = 5$ cm
Hence, the distance of the chord from the centre is 5 cm.

Solution 2:

Given : AB and CD are two equal chords of congruent circles with centres
O and O' respectively.
To prove : ∠AOB = ∠CO'D
Proof : In ΔOAB and ΔO'CD,
OA = O'C (∵ Radii of congruent circles)
OB = O'D (∵ Radii of congruent circles)
AB = CD (Given)
ΔOAB ≅ ΔO'CD (By SSS congruence criterion)
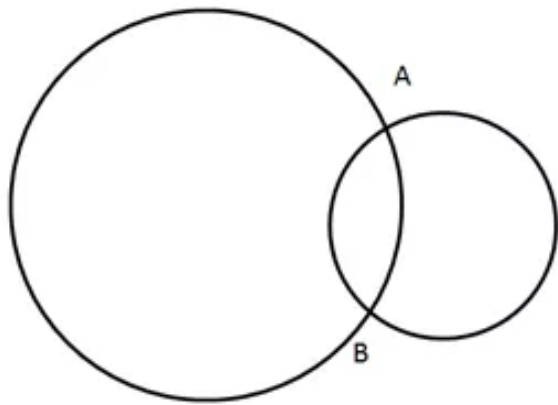∠AOB = ∠CO'D (cpct)

Solution 3:
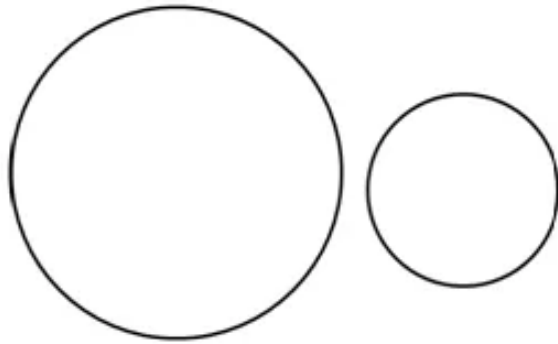
No point of intersection
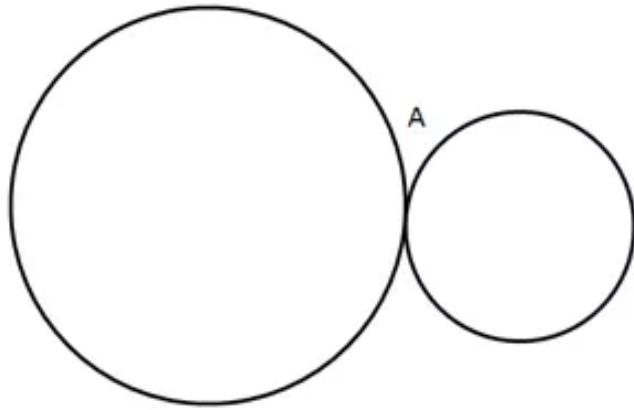
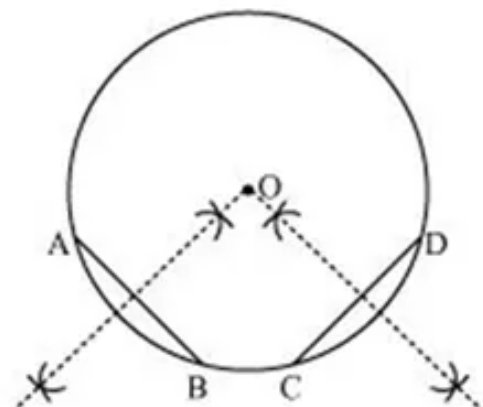One point of intersection

Two points of intersection

So, the circle can have 0, 1 or 2 points in common.

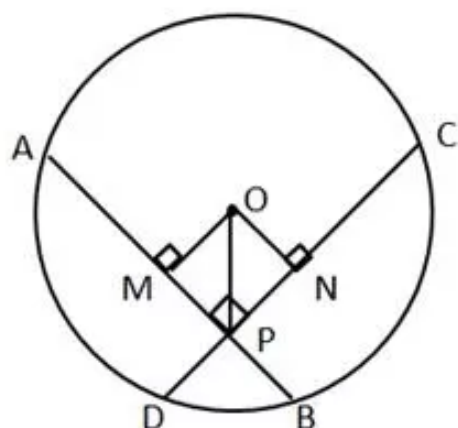The maximum number of common points is 2.

Solution 4:

To draw the centre of a given circle:
1. Draw the circle.
2. Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these chords.
3. Let these perpendicular bisectors meet at point O.
So, O will be the centre of the given circle.

Solution 5:

In ΔOMP and ΔONP,
OP = OP (common side)
∠OMP = ∠ONP (both are right angles)
OM = OM (side both the chords are equal, so the distance of the chords from the centre are also equal)
ΔOMP ≅ ΔONP (RHS congruence criterion)
⇒ MP = PN (cpct)
          ....(a)
(i) Since AB = CD (given)
⇒ AM = CN (⊥ drawn from the centre to the chord bisects the chord)
⇒ AM + MP = CN + NP (from (a))
⇒ AP = CP ...(b)
(ii) Since AB = CD
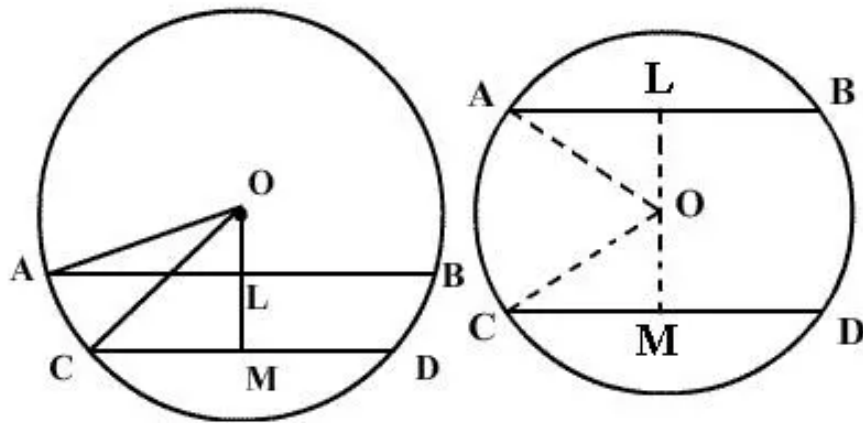⇒ AP + BP = CP + DP
⇒ BP = DP (from (b))
Hence proved.

Solution 6:

Given that AB = 16 cm and CD = 12 cm

So, AL = 8 cm and CM = 6 cm (⊥ from the centre to the chord bisects the chord)

In right triangles OLA and OMC,

By Pythagoras theorem,

$OA^2 = OL^2 + AL^2$ and $OC^2 = OM^2 + CM^2$

$\Rightarrow 10^2 = OL^2 + 8^2$ and $10^2 = OM^2 + 6^2$

$\Rightarrow OL^2 = 100 - 64$ and $OM^2 = 100 - 36$

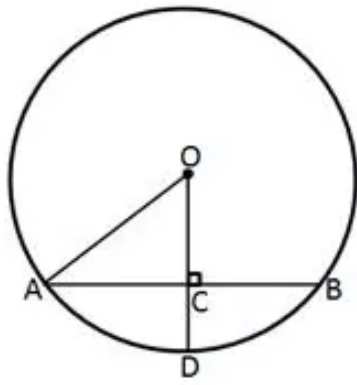$\Rightarrow OL^2 = 36$ and $OM^2 = 64$

$\Rightarrow OL = 6$ cm and $OM = 8$ cm

(i) In the first case, distance between AB and CD is

LM = OM − OL = 8 − 6 = 2 cm

(ii) In the second case, distance between AB and CD is

LM = OM + OL = 8 + 6 = 14 cm

Solution 7:

To find : CD

Given AB = 32 cm

$\Rightarrow$ AC = 16 cm (Since $\perp$ drawn from the centre to the chord, bisects the chord)

In right ΔOCA,

OA^2 = OC^2 + AC^2 (By Pythagoras theorem)

$\Rightarrow$ OC^2 = OA^2 − AC^2

$\Rightarrow$ OC^2 = 20^2 − 16^2

$\Rightarrow$ OC^2 = 144

$\Rightarrow$ OC = 12 cm

Since OD = 20 cm and OC = 12 cm

$\Rightarrow$ CD = OD − OC = 20 − 12 = 8 cm

Solution 8:

It is given in the question that point
P is the midpoint of the chord AB and and point Q is the
midpoint of the chord CD .

$\Rightarrow \angle APO = 90°$ $\left(\begin{array}{l}\text{as the straight line drawn from the centre of a circle}\\\text{to bisect a chord, which is not a diameter, is at the}\\\text{right angle to the chord}\end{array}\right)$

As chords AB and CD are equal therefore they are equidistant from the
centre i.e. PO = OQ   ($\because$ Equal chords of a circle are equidistant from the centre)
Now the ∆POQ is an isosceles triangle with OP = OQ as its two equal sides
Therefore ∠OPQ = ∠PQO , as they are opposite angles to the equal sides of
an isosceles triangle.
Sum of all the angles of a triangle is 180^0

$\Rightarrow \angle POQ + \angle OPQ + \angle PQO = 180°$

$\Rightarrow \angle OPQ + \angle POQ + 150° = 180°$ [Given: ∠POQ = 150°]

$\Rightarrow 2\angle OPQ = 180° - 150°$ [As, ∠OPQ = ∠PQO]

$\Rightarrow 2\angle OPQ = 30°$

$\Rightarrow \angle OPQ = 15°$

As ∠APO = 90°

$\Rightarrow \angle APQ + \angle OPQ = 90°$

$\Rightarrow \angle APQ = 90° - 15°$ [As, ∠OPQ = 15°]

$\Rightarrow \angle APQ = 75°$

Solution 9:

Given :
1. AOC is the diameter

2. Arc AXB = $\dfrac{1}{2}$ Arc BYC

From Arc AXB = $\dfrac{1}{2}$ Arc BYC we can see that

Arc AXB : Arc BYC = 1:2

$\Rightarrow \angle BOA : \angle BOC = 1:2$

Since AOC is the diameter of the circle hence,

∠AOC = 180°
Now,
Assume that ∠BOA = x° and   ∠BOC = 2x°

∠AOC = ∠BOA + ∠BOC = 180°

$\Rightarrow$ x + 2x = 180

$\Rightarrow$ 3x = 180

$\Rightarrow$ x = 60

Hence ∠BOA = 60° and ∠BOC = 120°

Solution 10:

From the given conditions given in the question
we can draw the circle with arc APB, arc BQC and arc CRA
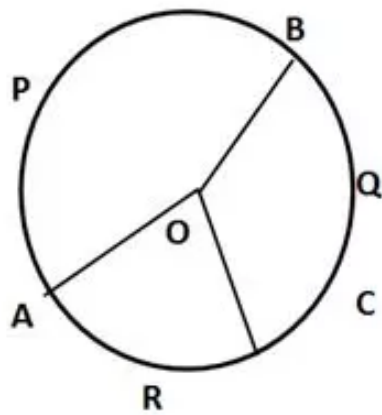
The given equation is

$$\frac{\text{Arc APB}}{2} = \frac{\text{Arc BQC}}{3} = \frac{\text{Arc CRA}}{4}$$

let

$$\frac{\text{Arc APB}}{2} = \frac{\text{Arc BQC}}{3} = \frac{\text{Arc CRA}}{4} = k \, (\text{Say})$$

then Arc APB = 2k, Arc BQC = 3k, Arc CRA = 4k

or

Arc APB : Arc BQC : Arc CRA = 2 : 3 : 4

$\Rightarrow \angle AOB : \angle BOC : \angle AOC = 2 : 3 : 4$

and therefore

and $\angle AOB = (2k)^\circ$, $\angle BOC = (3k)^\circ$ and $\angle AOC = (4k)^\circ$

Now,

Angle in a circle is 360°

So, 2k + 3k + 4k = 360

$\Rightarrow 9k = 360$

$\Rightarrow k = 40$

Hence

$\angle BOC = 3 \times 40 = 120^\circ$