CBSE Test Paper 03
Chapter 6 Work Energy & Power

1. The potential energy of a spring whose spring constant is k and is compressed by x is **1**
   a. $\frac{1}{4}kx^2$
   b. $\frac{1}{2}kx^2$
   c. kx^2
   d. $2kx^2$
2. Which of the following laws used to solve problems in collisions? **1**
   a. Kirchhoff's laws
   b. Newton's laws
   c. kepler's laws
   d. laws of conservation of linear momentum
3. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with an uniform speed of 7 m/s. It hits the floor of the elevator (length of the elevator = 3 m) and does not rebound. What is the heat produced by the impact? **1**
   a. 9.22 J
   b. 8.42 J
   c. 8.82 J
   d. 8.11 J
4. For a ball dropped from a tower of height h the total mechanical energy is **1**
   a. the difference of potential and kinetic energies
   b. the potential energy
   c. the sum of potential and kinetic energies
   d. the kinetic energy
5. Considering a completely inelastic collision of a particle of mass m_1 moving with initial velocity v_1 and a stationary particle of mass m_2 the final velocity is given by **1**

a. $\frac{m_2}{m_1+m_2}v_1$
b. $\frac{m_1}{m_1+2m_2}v_1$
c. $\frac{m_1}{m_1+m_2}v_1$
d. $\frac{m_1}{2m_1+m_2}v_1$

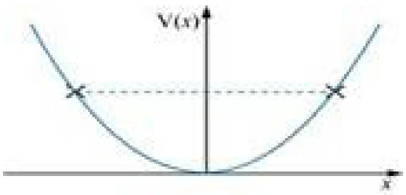
6. Is work done a scalar or a vector? **1**

7. If the momentum and total energy is conserved, which collision is occurred? **1**

8. In an elastic collision of two billiard balls, which of the following quantities remain conserved during the short time of collision of the balls (i.e., when they are in contact). **1**

   a. Kinetic energy.
   b. Total linear momentum? Give reason for your answer in each case

9. The potential energy function for a particle executing linear simple harmonic motion is given by V(x) = $\frac{kx^2}{2}$, where k is the force constant of the oscillator. For, k = 0.5 Nm^{-1}, the graph of V(x) versus x is shown in figure. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches x = ± 2 m. **2**



10. What are the conditions for work to be zero? **2**

11. Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2m. **2**

12. Consider a drop of small pebble of mass 1.00 g falling from a cliff of height 1.00 km. It hits the ground with a speed of 50.0 ms^{-1}. What is the work done by the unknown resistive force? **3**

13. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (electron mass = 9.11×10^{-31}kg, proton

mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J). **3**

14. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following figure is a possible result after collision? **3**

i. 

ii. 

iii. 

15. A balloon filled with helium rises against gravity increasing its potential energy. The speed of the balloon also increases as it rises. How do you reconcile this with the law of conservation of mechanical energy? You can neglect the viscous drag of air and assume that the density of air is constant. **5**

Answer

1. b. $\frac{1}{2}kx^2$

 Explanation: If a spring is stretched or compressed by a distance x from its natural length under the action of restoring force then

 F = -kx

 $dU = -F.dx$

 $U = \int_0^x -F.dx = \int_0^x kx.dx = k\left[\frac{x^2}{2}\right]_0^x$

 $U = \frac{1}{2}kx^2$

2. d. laws of conservation of linear momentum

 Explanation: If external force is absent then linear momentum of system remains conserved. In collision force of impact due to collision is internal and very large.

3. c. 8.82 J

 Explanation: Whole of the potential energy of bolt converted in to heat energy heat produced by the impact = mgh =$0.3 \times 9.8 \times 3 = 8.82J$

4. c. the sum of potential and kinetic energies

 Explanation: mechanical energy = sum of potential and kinetic energies

 a falling ball will have both these energies in between topmost and bottomost points of its motion so mechanical energy is the sum of potential and kinetic energies.

5. c. $\frac{m_1}{m_1+m_2}v_1$

 Explanation: For a completely inelastic collision momentum is remains conserved

 $p_i = p_f$

 $m_1v_1 + 0 = (m_1 + m_2)v$

 $v = \frac{m_1}{(m_1+m_2)}v_1$

6. Work done is a scalar physical quantity, it is a dot / scalar product of force and displacement.

7. Collision in which there is no net loss in kinetic energy in the system as a result of the collision, both momentum and kinetic energy are conserved. Such a collision is called elastic collision.

8. When two billiard balls collide each other then their linear momentum and kinetic energy remains conserved. Because here it is considered that there is not any non-conservative force (like air resistance/friction on surface etc.) and speed of ball is not so high so that they deformed on collision.

9. Total energy of the particle, E = 1J

   Force constant, k = 0.5 N m^{-1}
   Kinetic energy of the particle, $\mathrm{K} = \frac{1}{2}mv^2$
   According to the conservation law:
   E = V + K
   $1 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$
   At the moment of 'turn back', velocity (and hence K) becomes zero.
   $\therefore 1 = \frac{1}{2}kx^2$
   $\frac{1}{2} \times 0.5x^2 = 1$
   $x^2 = 4$
   $x = +2$
   Hence, the particle turns back when it reaches $x = \pm 2$ m.

10. There are the following three conditions for work to be zero:

    a. either no external force is acting on the body.
    b. or an external force is acting on the body but the body is at rest.
    c. or an external force is acting on the body and there is some displacement too but the displacement is in a direction perpendicular to that of force applied.

11. $\because WD = Fs\cos\theta$

    As the angle between horizontal distance 2 m and gravity vertically downward is 90^0. So WD

$WD = F_5 \cos 90^\circ = 0$

So work done by car against the gravity is zero.

12. We assume that the pebble is initially at rest on the cliff, i.e.

u = 0, m = 1.00 g = 10^{-3} kg

Given that v = 50 ms^{-1}, h =1.00 km = 10^3m

The change in KE of the pebble is given by the equation

$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2} \times 10^{-3} \times (50)^2 - 0 = 1.25\mathrm{J}$

Assuming that acceleration due to gravity g =10 ms^{-2} is constant, the work done by the gravitational force is

W_g = mgh = $10^{-3} \times 10 \times 10^3$ =10.0 J

If W_r is the work done by the resistive force on the pebble, then from the work-energy theorem we have, net change in kinetic energy = total work done

$\Delta K = W_g + W_r$

or $W_r = \Delta K - W_g$=1.25 - 10.0 = - 8.75 J

Hence, the work done by resistive force is 8.75 J, negative sign indicates that it is against the work done by gravitational force.

13. According to the question ,

Kinetic energy of the electron,

$K.E_e = 10000eV = 10000 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-15}$ J

Kinetic energy of the proton,

$K.E_P = 100eV = 100 \times 1000 \times 1.6 \times 10^{-19} = 1.6 \times -14\mathrm{J}$

For the velocity of an electron v_e, its kinetic energy is given by the relation:

$K.E_e = \frac{1}{2}mv_\epsilon^2$

$\therefore v_e = \sqrt{\frac{2 \times K.E_e}{m}}$

$= \sqrt{\frac{2 \times 1.60 \times 10^{-13}}{9.11 \times 10^{-31}}} = 5.93 \times 10^9\mathrm{m/s}$

For the velocity of a proton v_p, its kinetic energy is given by the relation:

$K.E_P = \frac{1}{2}mv_p^2$

$v_p = \sqrt{\frac{2 \times K.E_p}{m}}$

$\therefore v_p = \sqrt{\frac{2\times1.6\times10^{-14}}{1.67\times10^{-27}}} = 4.38 \times 10^6 \mathrm{m/s}$

Hence, the electron is moving faster than the proton.

The ratio of their speeds:

$\frac{v_e}{v_p} = \frac{5.93\times10^9}{4.38\times10^6} = 1354 : 1$

14. According to the question in each case, it can be observed that the total momentum before and after the collision in each case is constant as external forces are absent and internal forces are conservative in nature.

    so initial momentum = final momentum

    $p_i = p_f$

    For an elastic collision, the total kinetic energy of a system remains conserved before and after a collision while for an inelastic collision kinetic energy is not conserved as some energy is lost due to work done by the internal forces.

    Given the mass of ball bearings m, we can write:

    The total kinetic energy of the system before collision:

    $K.E. = \frac{1}{2}mV^2 + \frac{1}{2}(2m)0$

    $K.E. = \frac{1}{2}mV^2$

    Now,

    In Case (i)

    The total kinetic energy of the system after the collision:

    $K.E. = \frac{1}{2}m \times 0 + \frac{1}{2}(2m)\left(\frac{V}{2}\right)^2$

    $K.E. = \frac{1}{4}mV^2$

    Hence, in case (i) the kinetic energy of the system is not conserved.

    In Case (ii)

    The total kinetic energy of the system after the collision:

    $K.E. = \frac{1}{2}(2m) \times 0 + \frac{1}{2}mV^2$

    $K.E. = \frac{1}{2}mV^2$

    Hence, in case (ii) the kinetic energy of the system is conserved.

    In Case (iii)

    The Total kinetic energy of the system after the collision:

    $K.E. = \frac{1}{2}(3m)\left(\frac{V}{3}\right)^2$

    $K.E. = \frac{1}{6}mV^2$

Hence, the total kinetic energy of the system is not conserved in case(ii) & case(iii) in the process of collision.

15. As the dragging viscous force of air on balloon is neglected so there is Net Buoyant Force = Vpg

=Volume of air displaced $\times$ net density upward $\times$ g

$= V\left(p_{ar} - p_{Hz}\right) g(upward)$

Let a be the upward acceleration on balloon then

$ma = V\left(p_{ar} - p_{Hz}\right) g$ ...(i)

Where m = mass of balloon

V = Volume of air displacement by balloon = Volume of balloon

$p_{air} =$ density of air

$p_{He} =$ density of helium

$m\frac{dv}{dt} = V \cdot \left(p_{air} - P_{He}\right) g$

$mdv = V \cdot \left(p_{ar} - p_{Hz}\right) g \cdot dt$

Integrating both sides $mv = V \cdot \left(p_{ar} - p_{H\epsilon}\right) gt$

$v = \frac{V}{m}(p_{air} - p_{He})gt$

KE of balloon =$\frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = \frac{1}{2}m\frac{V^2}{m^2}(p_{air} - p_{He})^2 g^2 t^2 = \frac{V^2}{2m}(p_{air} - p_{He})^2 g^2 t^2$ ...(ii)

If the balloon rises to a height h, from (i)n

$a = \frac{V}{m}\left(p_{air} - p_{he}\right) g$

$h = ut + \frac{1}{2}at^2 = 0.t + \frac{1}{2}\left[\frac{V}{m}\left(p_{ar} - p_{He}\right) g\right] t^2$

$\therefore h = \frac{V}{2m}\left(p_{ar} - p_{He}\right) gt^2$ ...(iii)

From (ii) and (iii) rearranging the terms of (ii) according to h in (iii)

$\frac{1}{2}mv^2 = \left\{\frac{V}{2m}\left(p_{air} - p_{He}\right) gt^2\right\}.V\left(p_{air} - p_{He}\right) g$

$\frac{1}{2}mv^2 = \{h\}.V\left(p_{air} - p_{He}\right) g$

$\frac{1}{2}mv^2 = V \cdot \left(p_{air} - p_{He}\right) gh$

$\frac{1}{2}mv^2 = Vp_{air}gh - Vp_{He}gh$

$\frac{1}{2}mv^2 + p_{He}Vgh = P_{air}Vgh$

$KE_{balloon} + PE_{balloon} = Change\ in\ PE\ of\ air.$

So, as the balloon goes up, an equal volume of air comes down, increases in PE and KE of the balloon is at cost of PE of air (which comes down).