INTEGRALS / EXERCISE 7.2 (NCERT)

QNo.1. $\int \frac{2x}{1+x^2}\,dx = \int \frac{\frac{d}{dx}(1+x^2)}{1+x^2}\,dx = \log|1+x^2| + C$

$\left[\because \int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C\right]$

QNo.2 $\int \frac{(\log x)^2}{x}\,dx = \int (\log x)^2 \cdot \frac{1}{x}\,dx = \int (\log x)^2 \cdot \frac{d}{dx}(\log x)\,dx.$

$= \frac{(\log x)^{2+1}}{2+1} + C \quad \left[\because \int (f(x))^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C.\right.$

$= \frac{1}{3}(\log x)^3 + C$

QNo3. $\int \frac{1}{x + x\log x}\,dx = \int \frac{1}{x(1+\log x)}\,dx = \int \frac{1/x}{(1+\log x)}\,dx$

$= \log|1+\log x| + C \quad \left[\because \frac{f'(x)}{f(x)} \text{ type}\right]$

QNo4 $\int \sin x \sin(\cos x)\,dx.$

Sol. Put $\cos x = t \Rightarrow -\sin x.dx = dt$

$\therefore I = \int \sin t.(-dt) = -\int \sin t\,dt = -(-\cos t) + C =$

$= \cos t + C = \cos(\cos x) + C$

QNo 5. $\int \sin(ax+b)\cos(ax+b)\,dx. = \frac{1}{2}\int 2\sin(ax+b)\cos(ax+b)\,dx$

$= \frac{1}{2}\int \sin 2(ax+b)\,dx. = \frac{1}{2}\left(-\frac{\cos 2(ax+b)}{2a}\right) + C$

$= -\frac{1}{4a}\cos 2(ax+b) + C$

QNo 6 $\int \sqrt{ax+b}\,dx = \int (ax+b)^{1/2}\,dx = \frac{(ax+b)^{3/2}}{3/2 \cdot a} + C = \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$

QNo7. $\int x\sqrt{x+2}\,dx.$

Sol. Put $x+2 = t \Rightarrow dx = dt$ and $x = t-2$

$\therefore I = \int (t-2)\sqrt{t}\,dt = \int t^{3/2}\,dt - 2\int t^{1/2}\,dt$

$= \frac{t^{5/2}}{5/2} - 2\cdot\frac{t^{3/2}}{3/2} + C = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{3/2} + C$

Q No. 8: $\int x\sqrt{1+2x^2}\,dx = \int \frac{(1+2x^2)^{1/2}\cdot 4x}{4}\,dx = \frac{1}{4}\int (1+2x^2)^{1/2}\cdot 4x\cdot dx$

$= \frac{1}{4}\frac{(1+2x^2)^{3/2}}{3/2} + C \qquad [(f(x))^n\cdot f'(x)$ type

$= \frac{1}{6}(1+2x^2)^{\frac{3}{2}} + C$

Q No 9. $\int (4x+2)\sqrt{x^2+x+1}\,dx = \int (x^2+x+1)^{1/2}(2x+1)\cdot 2\,dx$

$= 2\cdot\frac{(x^2+x+1)^{3/2}}{3/2} + C \qquad [(f(x))^n f'(x)$ type$]$

$= \frac{4}{3}(x^2+x+1)^{\frac{3}{2}} + C$

Q No 10 $\int \left(\frac{1}{x-\sqrt{x}}\right) dx = \int \frac{1}{\sqrt{x}(\sqrt{x}-1)}\,dx = 2\int \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}-1}\,dx$

$= 2\log|\sqrt{x}-1| + C \qquad \left[\frac{f'(x)}{f(x)}\text{ type}\right]$

Q No 11. $\int \frac{x}{\sqrt{x+4}}\,dx = \int \frac{x+4-4}{\sqrt{x+4}}\,dx = \int \frac{x+4}{\sqrt{x+4}}\,dx - \int \frac{4}{\sqrt{x+4}}\,dx$

$= \int \sqrt{x+4}\,dx - 4\int (x+4)^{-1/2}\,dx = \frac{(x+4)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 4\frac{(x+4)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$

$= \frac{2}{3}(x+4)^{3/2} - 8\sqrt{x+4} + C = \frac{2}{3}\sqrt{x+4}\,(x+4-12) + C$

Q No 12. $\int (x^3-1)^{1/3}\cdot x^5\,dx = \int (x^3-1)^{1/3}\cdot x^3\cdot x^2\,dx$

Sol. Put $x^3-1 = t \Rightarrow 3x^2\,dx = dt$

$\therefore I = \int (t)^{1/3}(t+1)\frac{dt}{3} = \frac{1}{3}\left[\int t^{4/3}dt + \int t^{1/3}dt\right]$

$= \frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{7/3} + \frac{t^{4/3}}{4/3}\right] + C = \frac{1}{7}t^{7/3} + \frac{1}{4}t^{4/3} + C$

$= \frac{1}{7}(x^3-1)^{7/3} + \frac{1}{4}(x^3-1)^{4/3} + C$

Q No. 13. $\int \frac{x^2}{(2+3x^3)^3}dx = \frac{1}{9}\int (2+3x^3)^{-3} \cdot 9x^2\,dx$

$$= \frac{1}{9}\frac{(2+3x^3)^{-3+1}}{-3+1} + c \qquad \left[\left[f(x)\right]^n f'(x) \text{ type}\right]$$

$$= -\frac{1}{18}\frac{1}{(2+3x^3)^2} + c$$

Q No 14. $\int \frac{1}{x(\log x)^m}dx$; $x > 0$ and $m \neq 1$

Sol. $I = \int (\log x)^{-m} \cdot \frac{1}{x}dx = \frac{(\log x)^{-m+1}}{-m+1} + c$

$$= \frac{1}{1-m}(\log x)^{1-m} + c$$

Q No 15. $\int \frac{x}{9-4x^2}dx = \frac{1}{8}\int \frac{-8x}{9-4x^2}dx = \frac{1}{8}\log|9-4x^2| + c$

Q No 16 $\int e^{2x+3}dx = \frac{e^{2x+3}}{2} + c$

Q No 17 $\int \frac{x}{e^{x^2}}dx = \int e^{-x^2} \cdot x \cdot dx.$

Sol. Put $-x^2 = t \Rightarrow -2x\,dx = dt$

$$\therefore I = \int e^t \cdot \frac{dt}{-2} = -\frac{1}{2}\int e^t dt = -\frac{1}{2}e^t + c$$

$$= -\frac{1}{2}e^{-x^2} + c = -\frac{1}{2e^{x^2}} + c$$

Q No 18 $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx.$

Sol. Put $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}dx = dt$

$$\therefore I = \int e^t \cdot dt = e^t + c = e^{\tan^{-1}x} + c$$

Q.No 19 $\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$

Sol.. Dividing Numerator and denominator by e^x; We get

$$I = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx.$$

Now $\frac{d}{dx}(e^x+e^{-x}) = e^x + e^{-x}\cdot(-1) = e^x - e^{-x}$

$$\therefore I = \int \frac{\frac{d}{dx}(e^x+e^{-x})}{e^x+e^{-x}}\,dx = \log|e^x+e^{-x}| + c$$

Q No. 20 $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx = \frac{1}{2}\int \frac{2e^{2x}-2e^{-2x}}{e^{2x}+e^{-2x}}\,dx.$

$$= \frac{1}{2}\log|e^{2x}+e^{-2x}| + c \quad \left[\frac{f'(x)}{f(x)} \text{ type}\right]$$

Q No 21 $\int \tan^2(2x-3)\,dx = \int (\sec^2(2x-3)-1)\,dx$

$$= \int \sec^2(2x-3)\,dx - \int 1\,dx$$

$$= \frac{\tan(2x-3)}{2} - x. + c.$$

Q No 22 $\int \sec^2(7-4x)\,dx = \frac{\tan(7-4x)}{-4} + c$

Q No 23 $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int \sin^{-1}x \cdot \left(\frac{d}{dx}(\sin^{-1}x)\right)dx$

$$= \frac{(\sin^{-1}x)^2}{2} + c. = \frac{1}{2}(\sin^{-1}x)^2 + c \quad [f(x)\cdot f'(x) \text{ type}]$$

Q No 24 $\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\,dx = \frac{1}{2}\int \frac{4\cos x - 6\sin x}{6\cos x + 4\sin x}\,dx$

$$= \frac{1}{2}\int \frac{\frac{d}{dx}(6\cos x + 4\sin x)}{6\cos x + 4\sin x}\,dx = \frac{1}{2}\log|6\cos x + 4\sin x| + c$$

OR

$$I = \int \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}\,dx = \frac{1}{2}\int \frac{\frac{d}{dx}(3\cos x + 2\sin x)}{3\cos x + 2\sin x}\,dx$$

$$= \log|3\cos x + 2\sin x| + C$$

Q No. 25 $\int \frac{1}{\cos^2 x(1-\tan x)^2}\,dx = \int \frac{\sec^2 x\,dx}{(1-\tan x)^2}$

Sol.

$$= -\int (1-\tan x)^{-2}(-\sec^2 x)\,dx = \cdot$$

$$= -\frac{(1-\tan x)^{-2+1}}{-2+1} + C = \frac{1}{1-\tan x} + C \quad \{\cdot\ [f(x)]^n f'(x) \text{ type}$$

Q No 26 $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx.$

Sol. Put $\sqrt{x} = t \Rightarrow \frac{1}{2}(x)^{-1/2}dx = dt$ ie $\frac{1}{2\sqrt{x}}dx = dt$

$$\therefore I = \int \cos t \cdot 2dt = 2\int \cos t\,dt = 2\sin t + C = 2\sin\sqrt{x} + C$$

Q No 27 $\int \sqrt{\sin 2x}\cos 2x\,dx$

Sol. Put $\sin 2x = t \Rightarrow 2\cos 2x\,dx = dt$

$$\therefore I = \int \sqrt{t}\cdot\frac{dt}{2} = \frac{1}{2}\cdot\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{1}{3}t^{3/2} + C = \frac{1}{3}(\sin 2x)^{3/2} + C$$

Q No 28 $\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx = \int (1+\sin x)^{-1/2}\cdot\frac{d}{dx}(1+\sin x)\,dx$

$$= \frac{(1+\sin x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = 2\left(\sqrt{1+\sin x}\right) + C \quad \left[(f(x))^n f'(x) \text{ type}\right].$$

Q No 29. $\int \cot x \cdot \log\sin x\,dx.$

Sol. Since $\frac{d}{dx}(\log\sin x) = \frac{1}{\sin x}\cdot\cos x = \frac{\cos x}{\sin x} = \cot x.$

$$\therefore I = \int (\log\sin x)\cdot\frac{d}{dx}(\log\sin x)\,dx = \frac{(\log\sin x)^{1+1}}{1+1} + C$$

$$= \frac{1}{2}(\log\sin x)^2 + C.$$

Q No 30. $\int \frac{\sin x}{1+\cos x} dx = -\int \frac{-\sin x}{1+\cos x} dx = -\int \frac{\frac{d}{dx}(1+\cos x)}{(1+\cos x)} dx$

$= \log|1+\cos x| + c \quad \left[\frac{f'(x)}{f(x)} \text{ type}\right]$

Q No 31. $\int \frac{\sin x}{(1+\cos x)^2} dx = -\int (1+\cos x)^{-2}(-\sin x)\, dx$

$= -\frac{(1+\cos x)^{-2+1}}{-2+1} + C = \frac{1}{1+\cos x} + c \quad \left[(f(x))^n f'(x) \text{ type}\right]$

Q No 32. $\int \frac{1}{1+\cot x} dx = \int \frac{1}{1+\frac{\cos x}{\sin x}} dx = \int \frac{\sin x}{\sin x+\cos x} dx$

$= \frac{1}{2}\int \frac{2\sin x}{\sin x+\cos x} dx = \frac{1}{2}\int \frac{\sin x+\cos x+\sin x-\cos x}{\sin x+\cos x} dx$

$= \frac{1}{2}\int 1 \cdot dx + \frac{1}{2}\int \frac{-(\cos x-\sin x)}{\sin x+\cos x} dx$

$= \frac{1}{2}x - \frac{1}{2}\log|\sin x+\cos x| + c$

Q No. 33. $\int \frac{1}{1-\tan x} dx = \int \frac{1}{1-\frac{\sin x}{\cos x}} dx = \int \frac{\cos x}{\cos x-\sin x} dx$.

$= \frac{1}{2}\int \frac{2\cos x}{\cos x-\sin x} dx = \frac{1}{2}\int \frac{\cos x-\sin x+\cos x+\sin x}{\cos x-\sin x} dx$

$= \frac{1}{2}\int 1 \cdot dx - \frac{1}{2}\int \frac{\cos x+\sin x}{\sin x-\cos x} dx$

$= \frac{1}{2}x + \frac{1}{2}\log|\sin x-\cos x| + c$

Q No. 34 $\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} dx = \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}\cdot \cos^2 x} dx = \int \frac{\sqrt{\tan x}}{\tan x} \cdot \sec^2 x\, dx$

$= \int (\tan x)^{-1/2} \sec^2 x\, dx = \frac{(\tan x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$= 2\sqrt{\tan x} + c$

Q.No. 35 $\int \frac{(1+\log x)^2}{x}\,dx = \int (1+\log x)^2 \cdot \frac{1}{x}\,dx.$

$$= \frac{(1+\log x)^{2+1}}{2+1} + C = \frac{1}{3}(1+\log x)^3 + C \quad [(f(x))^n f'(x) \text{ type}]$$

Q.No 36. $\int \frac{(x+1)(x+\log x)^2}{x}\,dx = \int (x+\log x)^2 \left(\frac{x+1}{x}\right) dx$

$$= \int (x+\log x)^2 \left(1+\frac{1}{x}\right) dx \quad [(f(x))^n f'(x) \text{ type}$$

$$= \frac{(x+\log x)^{2+1}}{2+1} + C = \frac{1}{3}(x+\log x)^3 + C$$

Q.No. 37. $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}\,dx.$

Sol. Put $\tan^{-1} x^4 = t \Rightarrow \frac{1}{1+x^8} \cdot 4x^3\,dx = dt$

$$\therefore I = \int \sin t \cdot \frac{dt}{4} = \frac{1}{4}\int \sin t\,dt = -\frac{1}{4}\cot t + C$$

$$= -\frac{1}{4}\cos(\tan^{-1} x^4) + C$$

Q.No. 38 $\int \frac{10x^9 + 10^x \log_e 10}{x^{10}+10^x}\,dx$

Sol. $I = \int \frac{\frac{d}{dx}(x^{10}+10^x)}{x^{10}+10^x}\,dx = \log|x^{10}+10^x| + C$

$\therefore$ D is Correct option.

Q.No 39. $\int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{1}{\sin^2 x \cos^2 x}\,dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx$

$$= \int \sec^2 x\,dx + \int \operatorname{cosec}^2 x\,dx = \tan x - \cot x + C$$

$\therefore$ (B) is Correct option.

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