Sphere

Q.1. If A (– 1, 4 , – 3) is one end of the diameter AB of the sphere $x^2 + y^2 + z^2 - 2y + 2z - 15 = 0$ then find the coordinates of the other end point B.

Solution : 1

The given sphere is , $x^2 + y^2 + z^2 - 2y + 2z - 15 = 0$

The centre is (0, 1 , – 1) .

Point A is (– 1, 4 , – 3) .

Let point B is (α , β , γ) .Then

$(\alpha - 1) / 2 = 0 \Rightarrow \alpha = 1$.

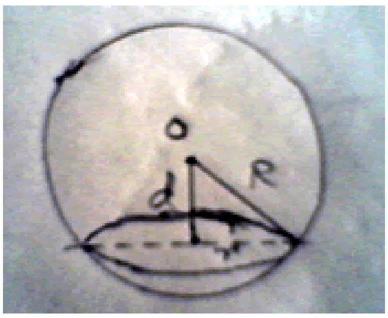
$(\beta + 4) / 2 = 1 \Rightarrow \beta = -2$

$(\gamma - 3) / 2 = -1 \Rightarrow \gamma = 1$.

Therefore required point is (1 , – 2 , 1) .

Q.2. Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle whose radius is unity.

Solution : 2



We have equation of sphere as :$x^2 + y^2 + z^2 - x + z - 2 = 0$

Then centre of sphere is $(1/2 , 0 , -1/2) = (-u, -v, -w)$

And radius = R = √(u2 + v2 + w2 – d) = √(1/4 + 1/4 + 2) = √(5/2)

Distance of centre from plane x + 2y – z = 4 is

d= [(1/2 + 1/2 – 4)/√(1 + 4 + 1)] = 3/√6.

Let radius of the circle be r then r = √(R2 – d2) = √(5/2 – 9/6) = √(2/2) = 1 . **[Proved.]**

Q.3. Show that the equation to a sphere passing through three points (2, 0, 0), (0, 2, 0)and (0, 0, 2) and having its centre on the plane 2x + 3y + 4z = 27 is

$x^2 + y^2 + z^2 - 6x - 6y - 6z + 8 = 0$.

Solution : 3

The general equation of sphere is : $x^{2} + y^{2} + z^{2} + 2ux + 2vy + 2wz + c = 0$

(2, 0, 0), (0, 2, 0) and (0, 0, 2) lie on it

Therefore, 22 + 0 + 0 + 4u + 0 + 0 + c = 0

Or, 4 + 4u + c = 0 ---------------- (1)

4 + 4v + c = 0 --------------- (2)

and 4 + 4w + c = 0 -------------- (3)

Centre (– u, – v, – w) lie on plane 2x + 3y + 4z = 27

Hence, – 3u – 3v – 4w = 27 ---------------------- (4)

Putting value of u, v, w from (1), (2) and (3) in (4), we get

– 2(– c – 4)/4 – 3(– c – 4)/4 – 4 (– c – 4)/4 = 27

Or, (2c + 8 = 3c + 12 + 4c + 16)/4 = 27

Or, 9c + 36 = 108 Or, 9c = 72 => c = 8.

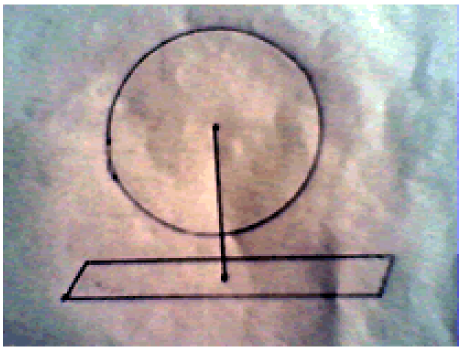
Therefore, 4 + 4u + 8 = 0 => u = – 3 [from (1)]

Similarly using (2) and (3) v = – 3 and w = – 3 ,

Putting u, v and w in general equation, we get x2 + y2 + z2 – 6x – 6y – 6z + 8 = 0 . **[Proved.]**

Q.4. Find the least distance of the plane $12x + 4y + 3z = 327$ from the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$.

Solution : 4



The given equation of sphere is , $x^{2} + y^{2} + z^{2} + 4x - 2y - 6z = 155$

Therefore, $u = 2$, $v = -1$, $w = -3$, $d = -155$,

The centre of the sphere is $(-u, -v, -w) = (-2, 3, 3)$.

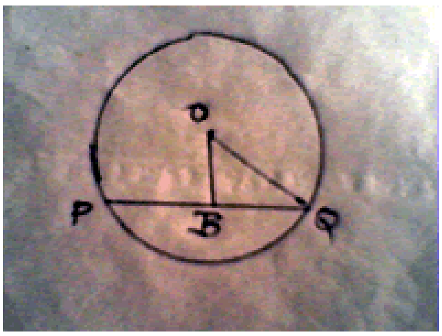
Distance of centre from the plane = $[12(-2) + 4 \times 1 + 3 \times 3 - 327]/\sqrt{(144 + 16 + 9)} = |(-24 + 4 + 9 - 327)/13| = 26$

Radius of the sphere = $\sqrt{(4 + 1 + 9 + 155)} = 13$,

Hence the least distance of plane from sphere = $26 - 13 = 13$.

Q.5. Find the radius of the circular section of the sphere $x^2 + y^2 + z^2 = 49$ cut by a plane $2x + 3y - z - 5\sqrt{(14)} = 0$.

Solution : 5



Equation of sphere is $x^2 + y^2 + z^2 = 49$ and plane is $2x + 3y - z - 5\sqrt{(14)} = 0$

In fig. above, O is the centre (0, 0, 0) of the sphere and PQ represents a plane.

Length of perpendicular (OB) = $|2\times0 + 3\times0 - 1\times0 - 5\sqrt{(14)}/\sqrt{(14)}| = |-5| = 5$ units.

BQ is the radius of the circle formed by cutting the sphere by plane.

Δ OBQ is a right angled triangle.

Therefore, $OB^2 + BQ^2 = OQ^2$

Or, $5^2 + BQ^2 = 49 \Rightarrow BQ^2 = 49 - 25 = 24$

Therefore, BQ = $\sqrt{(24)}$ units.

Hence, radius of the circle = $\sqrt{(24)}$ units.