

1

Basic Concepts

1.1 History

Although this book will not follow a strictly historical development, to ‘set the scene’ this first chapter will start with a brief review of the most important discoveries that led to the separation of nuclear physics from atomic physics as a subject in its own right and later work that in its turn led to the emergence of particle physics from nuclear physics.¹

1.1.1 The origins of nuclear physics

Nuclear physics as a subject distinct from atomic physics could be said to date from 1896, the year that Henri Becquerel observed that photographic plates were being fogged by an unknown radiation emanating from uranium ores. He had accidentally discovered *radioactivity*: the fact that some nuclei are unstable and spontaneously decay. In the years that followed, the phenomenon was extensively investigated, notably by the husband and wife team of Pierre and Marie Curie and by Ernest Rutherford and his collaborators,² and it was established that there were three distinct types of radiation involved: these were named (by Rutherford) α -, β - and γ -rays. We know now that α -rays are bound states of two protons and two neutrons (we will see later that they are the nuclei of helium atoms), β -rays are electrons and γ -rays are photons, the quanta of electromagnetic radiation, but the historical names are still commonly used.

¹An interesting account of the early period, with descriptions of the personalities involved, is given in Se80. An overview of the later period is given in Chapter 1 of Gr87.

²The 1903 Nobel Prize in Physics was awarded jointly to Becquerel for his discovery and to Pierre and Marie Curie for their subsequent research into radioactivity. Rutherford had to wait until 1908, when he was awarded the Nobel Prize in Chemistry for his ‘investigations into the disintegration of the elements and the chemistry of radioactive substances’.

At about the same time as Becquerel's discovery, J. J. Thomson was extending the work of Perrin and others on the radiation that had been observed to occur when an electric field was established between electrodes in an evacuated glass tube, and in 1897 he was the first to definitively establish the nature of these 'cathode rays'. We now know the emanation consists of free *electrons*, (the name 'electron' had been coined in 1894 by Stoney) denoted e^- (the superscript denotes the electric charge) and Thomson measured their mass and charge.³ The view of the atom at that time was that it consisted of two components, with positive and negative electric charges, the latter now being the electrons. Thomson suggested a model where the electrons were embedded and free to move in a region of positive charge filling the entire volume of the atom – the so-called 'plum pudding model'.

This model could account for the stability of atoms, but could not account for the discrete wavelengths observed in the spectra of light emitted from excited atoms. Neither could it explain the results of a classic series of experiments performed in 1911 at the suggestion of Rutherford by his collaborators, Geiger and Marsden. These consisted of scattering α -particles by very thin gold foils. In the Thomson model, most of the α -particles would pass through the foil, with only a few suffering deflections through small angles. Rutherford suggested they should look for large-angle scattering and to their surprise they found that some particles were indeed scattered through very large angles, even greater than 90° . Rutherford showed that this behaviour was not due to multiple small-angle deflections, but could only be the result of the α -particles encountering a very small positively charged central *nucleus*. (The reason for these two different behaviours is discussed in Appendix C.)

To explain the results of these experiments Rutherford formulated a 'planetary' model, where the atom was likened to a planetary system, with the electrons (the 'planets') occupying discrete orbits about a central positively charged nucleus (the 'Sun'). Because photons of a definite energy would be emitted when electrons moved from one orbit to another, this model could explain the discrete nature of the observed electromagnetic spectra when excited atoms decayed. In the simplest case of hydrogen, the nucleus is a single *proton* (p) with electric charge $+e$, where e is the magnitude of the charge on the electron⁴, orbited by a single electron. Heavier atoms were considered to have nuclei consisting of several protons. This view persisted for a long time and was supported by the fact that the masses of many naturally occurring elements are integer multiples of a unit that is about 1 per cent smaller than the mass of the hydrogen atom. Examples are carbon and nitrogen, with masses of 12.0 and 14.0 in these units. However, it could not explain why not all atoms obeyed this rule. For example, chlorine has a mass of 35.5 in these

³J. J. Thomson received the 1906 Nobel Prize in Physics for his discovery. A year earlier, Philipp von Lenard had received the Physics Prize for his work on cathode rays.

⁴Why the charge on the proton should have exactly the same magnitude as that on the electron is a very long-standing puzzle, the solution to which is suggested by some as yet unproven, but widely believed, theories of particle physics that will be discussed briefly in Chapter 9.

units. At about the same time, the concept of *isotopism* (a name coined by Soddy) was conceived. *Isotopes* are atoms whose nuclei have different masses, but the same charge. Naturally occurring elements were postulated to consist of a mixture of different isotopes, giving rise to the observed masses.⁵

The explanation of isotopes had to wait 20 years until a classic discovery by Chadwick in 1932. His work followed earlier experiments by Irène Curie (the daughter of Pierre and Marie Curie) and her husband Frédéric Joliot.⁶ They had observed that neutral radiation was emitted when α -particles bombarded beryllium and later work had studied the energy of protons emitted when paraffin was exposed to this neutral radiation. Chadwick refined and extended these experiments and demonstrated that they implied the existence of an electrically neutral particle of approximately the same mass as the proton. He had discovered the *neutron* (*n*) and in so doing had produced almost the final ingredient for understanding nuclei.⁷

There remained the problem of reconciling the planetary model with the observation of stable atoms. In classical physics, the electrons in the planetary model would be constantly accelerating and would therefore lose energy by radiation, leading to the collapse of the atom. This problem was solved by Bohr in 1913. He applied the newly emerging quantum theory and the result was the now well-known Bohr model of the atom. Refined modern versions of this model, including relativistic effects described by the Dirac equation (the relativistic analogue of the Schrödinger equation that applies to electrons), are capable of explaining the phenomena of atomic physics. Later workers, including Heisenberg, another of the founders of quantum theory,⁸ applied quantum mechanics to the nucleus, now viewed as a collection of neutrons and protons, collectively called *nucleons*. In this case, however, the force binding the nucleus is not the electromagnetic force that holds electrons in their orbits, but is a short-range⁹ force whose magnitude is independent of the type of nucleon, proton or neutron (i.e. charge-independent). This binding interaction is called the *strong nuclear force*.

These ideas still form the essential framework of our understanding of the nucleus today, where nuclei are bound states of nucleons held together by a strong charge-independent short-range force. Nevertheless, there is still no single theory that is capable of explaining all the data of nuclear physics and we shall see that different models are used to interpret different classes of phenomena.

⁵Frederick Soddy was awarded the 1921 Nobel Prize in Chemistry for his work on isotopes.

⁶Irène Curie and Frédéric Joliot received the 1935 Nobel Prize in Chemistry for 'synthesizing new radioactive elements'.

⁷James Chadwick received the 1935 Nobel Prize in Physics for his discovery of the neutron.

⁸Werner Heisenberg received the 1932 Nobel Prize in Physics for his contributions to the creation of quantum mechanics and the idea of *isospin symmetry*, which we will discuss in Chapter 3.

⁹The concept of range will be discussed in more detail in Section 1.5.1, but for the present it may be taken as the effective distance beyond which the force is insignificant.

1.1.2 The emergence of particle physics: the standard model and hadrons

By the early 1930s, the 19th century view of atoms as indivisible *elementary particles* had been replaced and a larger group of physically smaller entities now enjoyed this status: electrons, protons and neutrons. To these we must add two electrically neutral particles: the *photon* (γ) and the *neutrino* (ν). The photon was postulated by Planck in 1900 to explain black-body radiation, where the classical description of electromagnetic radiation led to results incompatible with experiments.¹⁰ The neutrino was postulated by Fermi in 1930 to explain the apparent non-conservation of energy observed in the decay products of some unstable nuclei where β -rays are emitted, the so-called β -decays. Prior to Fermi's suggestion, β -decay had been viewed as a parent nucleus decaying to a daughter nucleus and an electron. As this would be a two-body decay, it would imply that the electron would have a unique momentum, whereas experiments showed that the electron actually had a momentum *spectrum*. Fermi's hypothesis of a third particle (the neutrino) in the final state solved this problem, as well as a problem with angular momentum conservation, which was apparently also violated if the decay was two-body. The β -decay data implied that the neutrino mass was very small and was compatible with the neutrino being massless.¹¹ It took more than 25 years before Fermi's hypothesis was confirmed by Reines and Cowan in a classic experiment in 1956 that detected free neutrinos from β -decay.¹²

The 1950s also saw technological developments that enabled high-energy beams of particles to be produced in laboratories. As a consequence, a wide range of controlled scattering experiments could be performed and the greater use of computers meant that sophisticated analysis techniques could be developed to handle the huge quantities of data that were being produced. By the 1960s this had resulted in the discovery of a very large number of unstable particles with very short lifetimes and there was an urgent need for a theory that could make sense of all these states. This emerged in the mid 1960s in the form of the so-called *quark model*, first suggested by Murray Gell-Mann and independently and simultaneously by George Zweig, who postulated that the new particles were bound states of three families of more fundamental physical particles.

¹⁰X-rays had already been observed by Röntgen in 1895 (for which he received the first Nobel Prize in Physics in 1901) and γ -rays were seen by Villard in 1900, but it was Planck who first made the startling suggestion that electromagnetic energy was quantized. For this he was awarded the 1918 Nobel Prize in Physics. Many years later, he said that his hypothesis was an 'act of desperation' as he had exhausted all other possibilities.

¹¹However, in Section 3.1.4 we will discuss recent evidence that neutrinos have very small, but non-zero, masses.

¹²A description of this experiment is given in Chapter 12 of Tr75. Frederick Reines shared the 1995 Nobel Prize in Physics for his work in neutrino physics and particularly for the detection of the electron neutrino.

Gell-Mann called these *quarks* (q).¹³ Because no free quarks were detected experimentally, there was initially considerable scepticism for this view. We now know that there is a fundamental reason why quarks cannot be observed as free particles (it will be discussed in Chapter 5), but at the time many physicists looked upon quarks as a convenient mathematical description, rather than physical particles.¹⁴ However, evidence for the existence of quarks as real particles came in the 1960s from a series of experiments analogous to those of Rutherford and his co-workers, where high-energy beams of electrons and neutrinos were scattered from nucleons. (These experiments will also be discussed in Chapter 5.) Analysis of the angular distributions of the scattered particles showed that the nucleons were themselves bound states of three point-like charged entities, with properties consistent with those hypothesized in the quark model. One of these properties was unexpected and unusual: quarks have fractional electric charges, in practice $-\frac{1}{3}e$ and $+\frac{2}{3}e$. This is essentially the picture today, where elementary particles are now considered to be a small number of physical entities, including quarks, the electron, neutrinos, the photon and a few others we shall meet, but no longer nucleons.

The best theory of elementary particles we have at present is called, rather prosaically, the *standard model*. This aims to explain all the phenomena of particle physics, except those due to gravity, in terms of the properties and interactions of a small number of *elementary* (or *fundamental*) *particles*, which are now defined as being point-like, without internal structure or excited states. Particle physics thus differs from nuclear physics in having a single theory to interpret its data.

An elementary particle is characterized by, amongst other things, its mass, its electric charge and its *spin*. The latter is a permanent angular momentum possessed by all particles in quantum theory, even when they are at rest. Spin has no classical analogue and is not to be confused with the use of the same word in classical physics, where it usually refers to the (orbital) angular momentum of extended objects. The maximum value of the spin angular momentum about any axis is $s\hbar$ ($\hbar \equiv h/2\pi$), where h is Planck's constant and s is the *spin quantum number*, or *spin* for short. It has a fixed value for particles of any given type (for example $s = \frac{1}{2}$ for electrons) and general quantum mechanical principles restrict the possible values of s to be $0, \frac{1}{2}, 1, \frac{3}{2}, \dots$. Particles with half-integer spin are called *fermions* and those with integer spin are called *bosons*. There are three families of elementary particles in the standard model: two spin- $\frac{1}{2}$ families of fermions called *leptons* and *quarks*; and one family of spin-1 *bosons*. In addition,

¹³Gell-Mann received the 1969 Nobel Prize in Physics for 'contributions and discoveries concerning the classification of elementary particles and their interactions'. For the origin of the word 'quark', he cited the now famous quotation 'Three quarks for Muster Mark' from James Joyce's book *Finnegans Wake*. Zweig had suggested the name 'aces', which with hindsight might have been more appropriate, as later experiments revealed that there were four and not three families of quarks.

¹⁴This was history repeating itself. In the early days of the atomic model many very distinguished scientists were reluctant to accept that atoms existed, because they could not be 'seen' in a conventional sense.

at least one other spin-0 particle, called the *Higgs boson*, is postulated to explain the origin of mass within the theory.¹⁵

The most familiar elementary particle is the electron, which we know is bound in atoms by the *electromagnetic interaction*, one of the four forces of nature.¹⁶ One test of the elementarity of the electron is the size of its magnetic moment. A charged particle with spin necessarily has an intrinsic magnetic moment $\boldsymbol{\mu}$. It can be shown from the Dirac equation that a point-like spin- $\frac{1}{2}$ particle of charge q and mass m has a magnetic moment $\boldsymbol{\mu} = (q/m)\mathbf{S}$, where $\mathbf{S}$ is its spin vector, and hence $\boldsymbol{\mu}$ has magnitude $\mu = q\hbar/2m$. The magnetic moment of the electron very accurately obeys this relation, confirming that electrons are elementary.

The electron is a member of the family of leptons. Another is the neutrino, which was mentioned earlier as a decay product in β -decays. Strictly speaking, this particle should be called the *electron neutrino*, written ν_e , because it is always produced in association with an electron (the reason for this is discussed in Section 3.1.1). The force responsible for β -decay is an example of a second fundamental force, the *weak interaction*. Finally, there is the third force, the (fundamental) *strong interaction*, which, for example, binds quarks in nucleons. The strong nuclear force mentioned in Section 1.1.1 is not the same as this fundamental strong interaction, but is a consequence of it. The relation between the two will be discussed in more detail later.

The standard model also specifies the origin of these three forces. In classical physics the electromagnetic interaction is propagated by electromagnetic waves, which are continuously emitted and absorbed. While this is an adequate description at long distances, at short distances the quantum nature of the interaction must be taken into account. In quantum theory, the interaction is transmitted discontinuously by the exchange of photons, which are members of the family of fundamental spin-1 bosons of the standard model. Photons are referred to as the *gauge bosons*, or ‘force carriers’, of the electromagnetic interaction. The use of the word ‘gauge’ refers to the fact that the electromagnetic interaction possesses a fundamental symmetry called *gauge invariance*. For example, Maxwell’s equations of classical electromagnetism are invariant under a specific phase transformation of the electromagnetic fields – the gauge transformation.¹⁷ This property is common to all the three interactions of nature we will be discussing and has profound consequences, but we will not need its details in this book. The weak and strong interactions are also mediated by the exchange of spin-1 gauge bosons. For the weak interaction these are the W^+ , W^- and Z^0 bosons (again the superscripts denote the electric charges) with masses about 80–90 times the mass of the proton.

¹⁵In the theory without the Higgs boson, all elementary particles are predicted to have zero mass, in obvious contradiction with experiment. A solution to this problem involving the Higgs boson will be discussed briefly in Chapter 9.

¹⁶Gravity is so weak that it can be neglected in nuclear and particle physics at presently accessible energies. Because of this, we will often refer in practice to the *three* forces of nature.

¹⁷See, for example, Appendix C.2 of Ma97.

For the strong interaction, the force carriers are called *gluons*. There are eight gluons, all of which have zero mass and are electrically neutral.¹⁸

In addition to the elementary particles of the standard model, there are other important particles we will be studying. These are the *hadrons*, the bound states of quarks. Nucleons are examples of hadrons,¹⁹ but there are several hundred more, not including nuclei, most of which are unstable and decay by one of the three interactions. It was the abundance of these states that drove the search for a simplifying theory that would give an explanation for their existence and led to the quark model in the 1960s. The most common unstable example of a hadron is the *pion*, which exists in three electrical charge states, written (π^+, π^0, π^-) . Hadrons are important because free quarks are unobservable in nature and so to deduce their properties we are forced to study hadrons. An analogy would be if we had to deduce the properties of nucleons by exclusively studying the properties of nuclei.

Since nucleons are bound states of quarks and nuclei are bound states of nucleons, the properties of nuclei should, in principle, be deducible from the properties of quarks and their interactions, i.e. from the standard model. In practice, however, this is far beyond present calculational techniques and sometimes nuclear and particle physics are treated as two almost separate subjects. However, there are many connections between them and in introductory treatments it is still useful to present both subjects together.

The remaining sections of this chapter are devoted to introducing some of the basic theoretical tools needed to describe the phenomena of both nuclear and particle physics, starting with a key concept: antiparticles.

1.2 Relativity and Antiparticles

Elementary particle physics is also called high-energy physics. One reason for this is that if we wish to produce new particles in a collision between two other particles, then because of the relativistic mass–energy relation $E = mc^2$, energies are needed at least as great as the rest masses of the particles produced. The second reason is that to explore the structure of a particle requires a probe whose wavelength λ is smaller than the structure to be explored. By the de Broglie relation $\lambda = h/p$, this implies that the momentum p of the probing particle, and hence its energy, must be large. For example, to explore the internal structure of the proton using electrons requires wavelengths that are much smaller than the

¹⁸Note that the word ‘electric’ has been used when talking about charge. This is because the weak and strong interactions also have associated ‘charges’ which determine the strengths of the interactions, just as the electric charge determines the strength of the electromagnetic interaction. This will be discussed in more detail in later chapters.

¹⁹The magnetic moments of the proton and neutron do not obey the prediction of the Dirac equation and this is evidence that nucleons have structure and are not elementary. The proton magnetic moment was first measured by Otto Stern using a molecular beam method that he developed and for this he received the 1943 Nobel Prize in Physics.

classical radius of the proton, which is roughly 10^{-15} m. This in turn requires electron energies that are greater than 10^3 times the rest energy of the electron, implying electron velocities very close to the speed of light. Hence any explanation of the phenomena of elementary particle physics must take account of the requirements of the theory of special relativity, in addition to those of quantum theory. There are very few places in particle physics where a non-relativistic treatment is adequate, whereas the need for a relativistic treatment is far less in nuclear physics.

Constructing a quantum theory that is consistent with special relativity leads to the conclusion that for every particle of nature, there must exist an associated particle, called an *antiparticle*, with the same mass as the corresponding particle. This important theoretical prediction was first made by Dirac and follows from the solutions of the equation he first wrote down to describe relativistic electrons.²⁰ The *Dirac equation* is of the form

$$i\hbar \frac{\partial \Psi(\mathbf{x}, t)}{\partial t} = H(\mathbf{x}, \hat{\mathbf{p}}) \Psi(\mathbf{x}, t), \quad (1.1)$$

where $\hat{\mathbf{p}} = -i\hbar \nabla$ is the usual quantum mechanical momentum operator and the Hamiltonian was postulated by Dirac to be

$$H = c\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta mc^2. \quad (1.2)$$

The coefficients $\boldsymbol{\alpha}$ and β are determined by the requirement that the solutions of Equation (1.1) are also solutions of the *Klein–Gordon equation*²¹

$$-\hbar^2 \frac{\partial^2 \Psi(\mathbf{x}, t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \Psi(\mathbf{x}, t) + m^2 c^4 \Psi(\mathbf{x}, t). \quad (1.3)$$

This leads to the conclusion that $\boldsymbol{\alpha}$ and β cannot be simple numbers; their simplest forms are 4×4 matrices. Thus the solutions of the Dirac equation are four-component wavefunctions (called *spinors*) with the form²²

$$\Psi(\mathbf{x}, t) = \begin{pmatrix} \Psi_1(\mathbf{x}, t) \\ \Psi_2(\mathbf{x}, t) \\ \Psi_3(\mathbf{x}, t) \\ \Psi_4(\mathbf{x}, t) \end{pmatrix}. \quad (1.4)$$

²⁰Paul Dirac shared the 1933 Nobel Prize in Physics with Erwin Schrödinger. The somewhat cryptic citation stated ‘for the discovery of new productive forms of atomic theory’.

²¹This is a relativistic equation, which is ‘derived’ by starting from the relativistic mass–energy relation $E^2 = p^2 c^2 + m^2 c^4$ and using the usual quantum mechanical operator substitutions, $\hat{\mathbf{p}} = -i\hbar \nabla$ and $E = i\hbar \partial / \partial t$.

²²The details may be found in most quantum mechanics books, for example, pp. 475–477 of Sc68.

The interpretation of Equation (1.4) is that the four components describe the two spin states of a negatively charged electron with positive energy and the two spin states of a corresponding particle having the same mass but with negative energy. Two spin states arise because in quantum mechanics the projection in any direction of the spin vector of a spin- $\frac{1}{2}$ particle can only result in one of the two values $\pm\frac{1}{2}$, called ‘spin up’ and ‘spin down’, respectively. The two energy solutions arise from the two solutions of the relativistic mass–energy relation $E = \pm\sqrt{p^2c^2 + m^2c^4}$. The latter states can be shown to behave in all respects as *positively* charged electrons (called *positrons*), but with positive energy. The positron is referred to as the antiparticle of the electron. The discovery of the positron by Anderson in 1933, with all the predicted properties, was a spectacular verification of the Dirac prediction.

Although Dirac originally made his prediction for electrons, the result is general and is true whether the particle is an elementary particle or a hadron. If we denote a particle by P , then the antiparticle is in general written with a bar over it, i.e. $\bar{P}$. For example, the antiparticle of the proton is the antiproton $\bar{p}$,²³ with negative electric charge; and associated with every quark, q , is an antiquark, $\bar{q}$. However, for some very common particles the bar is usually omitted. Thus, for example, in the case of the positron e^+ , the superscript denoting the charge makes explicit the fact that the antiparticle has the opposite electric charge to that of its associated particle. Electric charge is just one example of a *quantum number* (spin is another) that characterizes a particle, whether it is elementary or composite (i.e. a hadron).

Many quantum numbers differ in sign for particle and antiparticle, and electric charge is an example of this. We will meet others later. When brought together, particle–antiparticle pairs, each of mass m , can annihilate, releasing their combined rest energy $2mc^2$ as photons or other particles. Finally, we note that there is symmetry between particles and antiparticles, and it is a convention to call the electron the particle and the positron its antiparticle. This reflects the fact that the normal matter contains electrons rather than positrons.

1.3 Symmetries and Conservation Laws

Symmetries and the invariance properties of the underlying interactions play an important role in physics. Some lead to conservation laws that are universal. Familiar examples are translational invariance, leading to the conservation of linear momentum; and rotational invariance, leading to conservation of angular momentum. The latter plays an important role in nuclear and particle physics as it leads to a scheme for the classification of states based, among other quantum

²³Carl Anderson shared the 1936 Nobel Prize in Physics for the discovery of the positron. The 1958 Prize was awarded to Emilio Segrè and Owen Chamberlain for their discovery of the antiproton.

numbers, on their spins.²⁴ Another very important invariance that we have briefly mentioned is gauge invariance. This fundamental property of all three interactions restricts the forms of the interactions in a profound way that initially is contradicted by experiment. This is the prediction of zero masses for all elementary particles, mentioned earlier. There are theoretical solutions to this problem whose experimental verification (or otherwise) is probably the most eagerly awaited result in particle physics today.

In nuclear and particle physics we need to consider additional symmetries of the Hamiltonian and the conservation laws that follow and in the remainder of this section we discuss two of the most important of these that we will need later – *parity* and *charge conjugation*.

1.3.1 Parity

Parity was first introduced in the context of atomic physics by Eugene Wigner in 1927.²⁵ It refers to the behaviour of a state under a spatial reflection, i.e. $\mathbf{x} \rightarrow -\mathbf{x}$. If we consider a single-particle state, represented for simplicity by a non-relativistic wavefunction $\psi(\mathbf{x}, t)$, then under the parity operator, $\hat{P}$,

$$\hat{P}\psi(\mathbf{x}, t) \equiv P\psi(-\mathbf{x}, t). \quad (1.5)$$

Applying the operator again, gives

$$\hat{P}^2\psi(\mathbf{x}, t) = P\hat{P}\psi(-\mathbf{x}, t) = P^2\psi(\mathbf{x}, t), \quad (1.6)$$

implying $P = \pm 1$. If the particle is an eigenfunction of linear momentum $\mathbf{p}$, i.e.

$$\psi(\mathbf{x}, t) \equiv \psi_{\mathbf{p}}(\mathbf{x}, t) = \exp[i(\mathbf{p} \cdot \mathbf{x} - Et)], \quad (1.7)$$

then

$$\hat{P}\psi_{\mathbf{p}}(\mathbf{x}, t) = P\psi_{\mathbf{p}}(-\mathbf{x}, t) = P\psi_{-\mathbf{p}}(\mathbf{x}, t) \quad (1.8)$$

and so a particle at rest, with $\mathbf{p} = \mathbf{0}$, is an eigenstate of parity. The eigenvalue $P = \pm 1$ is called the *intrinsic parity*, or just the *parity*, of the state. By considering a multiparticle state with a wavefunction that is the product of single-particle wavefunctions, it is clear that parity is a multiplicative quantum number.

The strong and electromagnetic interactions, but not the weak interactions, are invariant under parity, i.e. the Hamiltonian of the system remains unchanged under

²⁴These points are explored in more detail in, for example, Chapter 4 of Ma97.

²⁵Eugene Wigner shared the 1963 Nobel Prize in Physics, principally for his work on symmetries.

a parity transformation on the position vectors of all particles in the system. Parity is therefore conserved, by which we mean that the total parity quantum number remains unchanged in the interaction. Compelling evidence for parity conservation in the strong and electromagnetic interactions comes from the absence of transitions between nuclear states and atomic states, respectively, that would violate parity conservation. The evidence for non-conservation of parity in the weak interaction will be discussed in detail in Chapter 6.

There is also a contribution to the total parity if the particle has an orbital angular momentum l . In this case its wave function is a product of a radial part R_{nl} and an angular part $Y_l^m(\theta, \phi)$:

$$\psi_{lmn}(\mathbf{x}) = R_{nl} Y_l^m(\theta, \phi), \quad (1.9)$$

where n and m are the principal and magnetic quantum numbers and $Y_l^m(\theta, \phi)$ is a spherical harmonic. It is straightforward to show from the relations between Cartesian (x, y, z) and spherical polar co-ordinates (r, θ, ϕ) , i.e.

$$x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta, \quad (1.10)$$

that the parity transformation $\mathbf{x} \rightarrow -\mathbf{x}$ implies

$$r \rightarrow r, \quad \theta \rightarrow \pi - \theta, \quad \phi \rightarrow \pi + \phi, \quad (1.11)$$

and from this it can be shown that

$$Y_l^m(\theta, \phi) \rightarrow Y_l^m(\pi - \theta, \pi + \phi) = (-)^l Y_l^m(\theta, \phi). \quad (1.12)$$

Equation (1.12) may easily be verified directly for specific cases; for example, for the first three spherical harmonics,

$$Y_0^0 = \left(\frac{1}{4\pi}\right)^{\frac{1}{2}}, \quad Y_1^0 = \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \cos \theta, \quad Y_1^{\pm 1} = \left(\frac{3}{8\pi}\right)^{\frac{1}{2}} \sin \theta e^{\pm i\phi}. \quad (1.13)$$

Hence

$$\hat{P}\psi_{lmn}(\mathbf{x}) = P\psi_{lmn}(-\mathbf{x}) = P(-)^l \psi_{lmn}(\mathbf{x}), \quad (1.14)$$

i.e. $\psi_{lmn}(\mathbf{x})$ is an eigenstate of parity with eigenvalue $P(-1)^l$.

An analysis of the Dirac Equation (1.1) for relativistic electrons, shows that it is invariant under a parity transformation only if $P(e^+e^-) = -1$. This is a general result for all fermion–antifermion pairs, so it is a convention to assign $P = +1$ to all leptons and $P = -1$ to their antiparticles. We will see in Chapter 3 that in strong interactions quarks can only be created as part of a quark–antiquark pair, so the intrinsic parity of a single quark cannot be measured. For this reason, it is also

a convention to assign $P = +1$ to quarks. Since quarks are fermions, it follows from the Dirac result that $P = -1$ for antiquarks. The intrinsic parities of hadrons then follow from their structure in terms of quarks and the orbital angular momentum between the constituent quarks, using Equation (1.14). This will be explored in Chapter 3 as part of the discussion of the quark model.

1.3.2 Charge conjugation

Charge conjugation is the operation of changing a particle into its antiparticle. Like parity, it gives rise to a multiplicative quantum number that is conserved in strong and electromagnetic interactions, but violated in the weak interaction. In discussing charge conjugation, we will need to distinguish between states such as the photon γ and the neutral pion π^0 that do not have distinct antiparticles and those such as the π^+ and the neutron, which do. Particles in the former class we will collectively denote by a and those of the latter type will be denoted by b . It is also convenient at this point to extend our notation for states. Thus we will represent a state of type a having a wavefunction ψ_a by $|a, \psi_a\rangle$ and similarly for a state of type b . Then under the charge conjugation operator, $\hat{C}$,

$$\hat{C}|a, \psi_a\rangle = C_a|a, \psi_a\rangle, \quad (1.15a)$$

and

$$\hat{C}|b, \psi_b\rangle = |\bar{b}, \psi_{\bar{b}}\rangle, \quad (1.15b)$$

where C_a is a phase factor analogous to the phase factor in Equation (1.5).²⁶ Applying the operator twice, in the same way as for parity, leads to $C_a = \pm 1$. From Equation (1.15a), we see that states of type a are eigenstates of $\hat{C}$ with eigenvalues ± 1 , called their *C-parities*. States with distinct antiparticles can only form eigenstates of $\hat{C}$ as linear combinations.

As an example of the latter, consider a $\pi^+\pi^-$ pair with orbital angular momentum L between them. In this case

$$\hat{C}|\pi^+\pi^-; L\rangle = (-1)^L|\pi^+\pi^-; L\rangle, \quad (1.16)$$

because interchanging the pions reverses their relative positions in the spatial wavefunction. The same factor occurs for spin- $\frac{1}{2}$ fermion pairs $f\bar{f}$, but in addition there are two other factors. The first is $(-1)^{S+1}$, where S is the total spin of the pair.

²⁶A phase factor could have been inserted in Equation (1.15b), but it is straightforward to show that the relative phase of the two states b and $\bar{b}$ cannot be measured and so a phase introduced in this way would have no physical consequences.

This follows directly from the structure of the spin wavefunctions:

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \uparrow_1 \uparrow_2 \\ \uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2 \\ \downarrow_1 \downarrow_2 \end{pmatrix} \begin{matrix} S_z = 1 \\ S_z = 0 \\ S_z = -1 \end{matrix} \left. \vphantom{\begin{pmatrix} \uparrow_1 \uparrow_2 \\ \uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2 \\ \downarrow_1 \downarrow_2 \end{pmatrix}} \right\} S = 1 \quad (1.17a)$$

and

$$\frac{1}{\sqrt{2}} (\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2) \quad S_z = 0 \quad S = 0, \quad (1.17b)$$

where $\uparrow_i$ ($\downarrow_i$) represents particle i having spin ‘up’ (‘down’) in the z -direction. A second factor (-1) arises whenever fermions and antifermions are interchanged. This has its origins in quantum field theory.²⁷ Combining these factors, finally we have

$$\hat{C}|f\bar{f}; J, L, S\rangle = (-1)^{L+S}|f\bar{f}; J, L, S\rangle, \quad (1.18)$$

for fermion–antifermion pairs having total, orbital and spin angular momentum quantum numbers J , L and S , respectively.

1.4 Interactions and Feynman Diagrams

We now turn to a discussion of particle interactions and how they can be described by the very useful pictorial methods of Feynman diagrams.

1.4.1 Interactions

Interactions involving elementary particles and/or hadrons are conveniently summarized by ‘equations’ by analogy with chemical reactions, in which the different particles are represented by symbols which usually – but not always – have a superscript to denote their electric charge. In the interaction

$$\nu_e + n \rightarrow e^- + p, \quad (1.19)$$

for example, an electron neutrino ν_e collides with a neutron n to produce an electron e^- and a proton p , whereas the equation

$$e^- + p \rightarrow e^- + p \quad (1.20)$$

²⁷See, for example, pp. 249–250 of Go86.

represents an electron and proton interacting to give the same particles in the final state, but in general travelling in different directions. In such equations, conserved quantum numbers must have the same total values in initial and final states.

Particles may be transferred from initial to final states and vice versa, when they become antiparticles. Thus the process

$$\pi^- + p \rightarrow \pi^- + p, \quad (1.21a)$$

also implies the reaction

$$p + \bar{p} \rightarrow \pi^+ + \pi^-, \quad (1.22)$$

which is obtained by taking the proton from the final state to an antiproton in the initial state and the negatively charged pion in the initial state to a positively charged pion in the final state.

The interactions in Equations (1.20) and (1.21a), in which the particles remain unchanged, are examples of *elastic scattering*, in contrast to the reactions in Equations (1.19) and (1.22), where the final-state particles differ from those in the initial state. Collisions between a given pair of initial particles do not always lead to the same final state, but can lead to different final states with different probabilities. For example, the collision of a negatively charged pion and a proton can give rise to elastic scattering (Equation (1.21a)) and a variety of other reactions, such as

$$\pi^- + p \rightarrow n + \pi^0 \quad \text{and} \quad \pi^- + p \rightarrow p + \pi^- + \pi^- + \pi^+, \quad (1.21b)$$

depending on the initial energy. In particle physics it is common to refer (rather imprecisely) to such interactions as ‘inelastic’ scattering.

Similar considerations apply to nuclear physics, but the term *inelastic scattering* is reserved for the case where the final state is an excited state of the parent nucleus A , that subsequently decays, for example via photon emission, i.e.

$$a + A \rightarrow a + A^*; \quad A^* \rightarrow A + \gamma, \quad (1.23)$$

where a is a projectile and A^* is an excited state of A . A useful shorthand notation used in nuclear physics for the general reaction $a + A \rightarrow b + B$ is $A(a, b)B$. It is usual in nuclear physics to further subdivide types of interactions according to the underlying mechanism that produced them. We will return to this in Section 2.9, as part of a more general discussion of nuclear reactions.

Finally, many particles are unstable and spontaneously decay to other, lighter (i.e. having less mass) particles. An example of this is the free neutron (i.e. one not bound in a nucleus), which decays by the β -decay reaction

$$n \rightarrow p + e^- + \bar{\nu}_e, \quad (1.24)$$

with a mean lifetime of about 900 s .²⁸ The same notation can also be used in nuclear physics. For example, many nuclei decay via the β -decay reaction. Thus, denoting a nucleus with Z protons and N nucleons as (Z, N) , we have

$$(Z, N) \rightarrow (Z - 1, N) + e^+ + \nu_e. \quad (1.25)$$

This is also a weak interaction. This reaction is effectively the decay of a proton bound in a nucleus. Although a *free* proton cannot decay by the β -decay $p \rightarrow n + e^+ + \nu_e$ because it violates energy conservation (the final-state particles have greater total mass than the proton), a proton bound in a nucleus can decay because of its binding energy. This will be explained in Chapter 2.

1.4.2 Feynman diagrams

The forces producing all the above interactions are due to the exchange of particles and a convenient way of illustrating this is to use *Feynman diagrams*. There are mathematical rules and techniques associated with these that enable them to be used to calculate the quantum mechanical probabilities for given reactions to occur, but in this book Feynman diagrams will only be used as a convenient very useful pictorial description of reaction mechanisms.

We first illustrate them at the level of elementary particles for the case of electromagnetic interactions, which arise from the emission and/or absorption of photons. For example, the dominant interaction between two electrons is due to the exchange of a single photon, which is emitted by one electron and absorbed by the other. This mechanism, which gives rise to the familiar Coulomb interaction at large distances, is illustrated in the Feynman diagram Figure 1.1(a).

In such diagrams, we will use the convention that particles in the initial state are shown on the left and particles in the final state are shown on the right. (Some authors take time to run along the y -axis.) Spin- $\frac{1}{2}$ fermions (such as the electron)

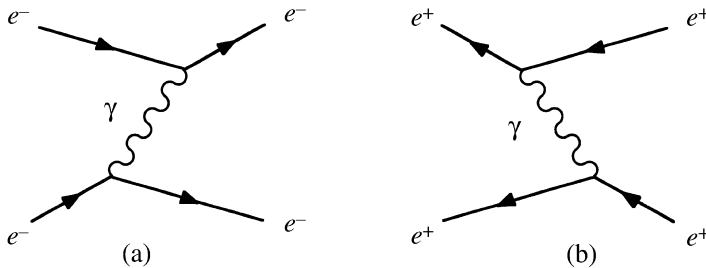


Figure 1.1 One-photon exchange in (a) $e^- + e^- \rightarrow e^- + e^-$ and (b) $e^+ + e^+ \rightarrow e^+ + e^+$

²⁸The reason that this decay involves an antineutrino rather than a neutrino will become clear in Chapter 3.

are drawn as solid lines and photons are drawn as wiggly lines. Arrowheads pointing to the right indicate that the solid lines represent electrons. In the case of photon exchange between two positrons, which is shown in Figure 1.1(b), the arrowheads on the antiparticle (positron) lines are conventionally shown as pointing to the left. In interpreting these diagrams, it is important to remember: (1) that the direction of the arrows on fermion lines does *not* indicate the particle's direction of motion, but merely whether the fermions are particles or antiparticles, and (2) that particles in the initial state are always to the left and particles in the final state are always to the right.

A feature of the above diagrams is that they are constructed from combinations of simple three-line vertices. This is characteristic of electromagnetic processes. Each vertex has a line corresponding to a single photon being emitted or absorbed, while one fermion line has the arrow pointing toward the vertex and the other away from the vertex, guaranteeing charge conservation at the vertex, which is one of the rules of Feynman diagrams.²⁹ For example, a vertex like Figure 1.2 would correspond to a process in which an electron emitted a photon and turned into a positron. This would violate charge conservation and is therefore forbidden.

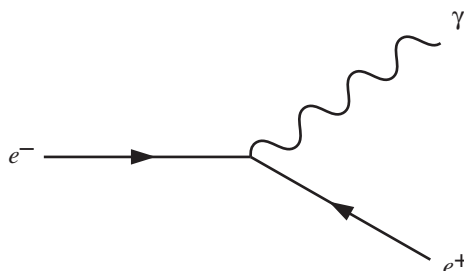


Figure 1.2 The forbidden vertex $e^- \rightarrow e^+ + \gamma$

Feynman diagrams can also be used to describe the fundamental weak and strong interactions. This is illustrated by Figure 1.3(a), which shows contributions to the elastic weak scattering reaction $e^- + \nu_e \rightarrow e^- + \nu_e$ due to the exchange of a Z^0 , and by Figure 1.3(b), which shows the exchange of a gluon g (represented by a coiled line) between two quarks, which is a strong interaction.

Feynman diagrams can also be drawn at the level of hadrons. As an illustration, Figure 1.4 shows the exchange of a charged pion (shown as a dashed line) between a proton and a neutron. We shall see later that this mechanism is a major contribution to the strong nuclear force between a proton and a neutron.

We turn now to consider in more detail the relation between exchanged particles and forces.

²⁹ Compare Kirchhoff's laws in electromagnetism.

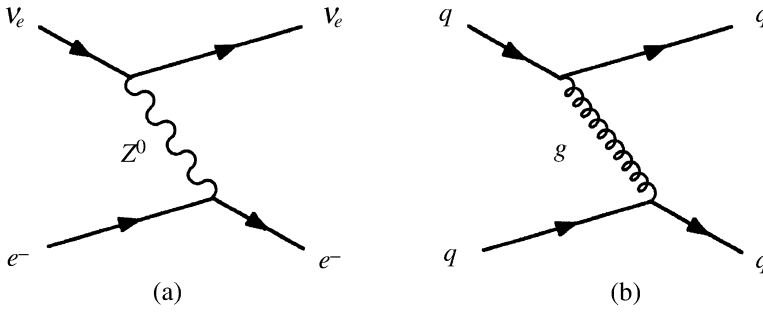


Figure 1.3 (a) Contributions of (a) Z^0 exchange to the elastic weak scattering reaction $e^- + \nu_e \rightarrow e^- + \nu_e$, and (b) gluon exchange contribution to the strong interaction $q + q \rightarrow q + q$

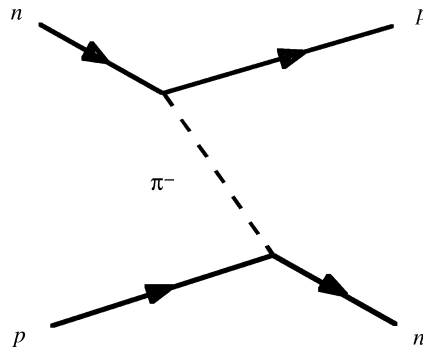


Figure 1.4 Single-pion exchange in the reaction $p + n \rightarrow n + p$

1.5 Particle Exchange: Forces and Potentials

This section starts with a discussion of the important relationship between forces and particle exchanges and then relates this to potentials. Although the idea of a potential has its greatest use in non-relativistic physics, nevertheless it is useful to illustrate concepts and is used in later sections as an intermediate step in relating theoretical Feynman diagrams to measurable quantities. The results can easily be extended to more general situations.

1.5.1 Range of forces

At each vertex of a Feynman diagram, charge is conserved by construction. We will see later that, depending on the nature of the interaction (strong, weak or electromagnetic), other quantum numbers are also conserved. However, it is easy to show that energy and momentum *cannot* be conserved simultaneously.

Consider the general case of a reaction $A + B \rightarrow A + B$ mediated by the exchange of a particle X , as shown in Figure 1.5. In the rest frame of the incident

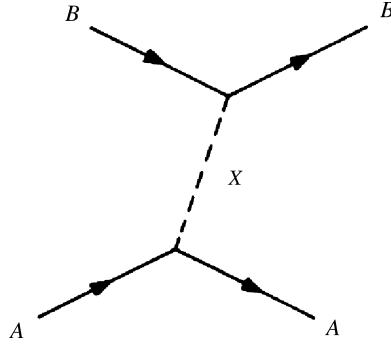


Figure 1.5 Exchange of a particle X in the reaction $A + B \rightarrow A + B$

particle A , the lower vertex represents the *virtual* process (‘virtual’ because X does not appear as a real particle in the final state),

$$A(M_A c^2, \mathbf{0}) \rightarrow A(E_A, \mathbf{p}_A c) + X(E_X, -\mathbf{p}_A c), \quad (1.26)$$

where E_A is the *total* energy of particle A and $\mathbf{p}_A$ is its three-momentum.³⁰ Thus, if we denote by P_A the four-vector for particle A ,

$$P_A = (E_A/c, \mathbf{p}_A) \quad (1.27)$$

and

$$P_A^2 = E_A^2/c^2 - \mathbf{p}_A^2 = M_A^2 c^2. \quad (1.28)$$

Applying this to the diagram and imposing momentum conservation, gives

$$E_A = (p^2 c^2 + M_A^2 c^4)^{1/2}, \quad E_X = (p^2 c^2 + M_X^2 c^4)^{1/2}, \quad (1.29)$$

where $p = |\mathbf{p}|$. The energy difference between the final and initial states is given by

$$\begin{aligned} \Delta E = E_X + E_A - M_A c^2 &\rightarrow 2pc, & p &\rightarrow \infty \\ &\rightarrow M_X c^2, & p &\rightarrow 0 \end{aligned} \quad (1.30)$$

and thus $\Delta E \geq M_X c^2$ for all p , i.e. energy is not conserved. However, by the Heisenberg uncertainty principle, such an energy violation is allowed, but only for a time $\tau \leq \hbar/\Delta E$, so we immediately obtain

$$r \leq R \equiv \hbar/M_X c \quad (1.31)$$

³⁰A resumé of relativistic kinematics is given in Appendix B.

as the maximum distance over which X can propagate before being absorbed by particle B . This maximum distance is called the *range* of the interaction and this was the sense of the word used in Section 1.1.1.

The electromagnetic interaction has an infinite range, because the exchanged particle is a massless photon. In contrast, the weak interaction is associated with the exchange of very heavy particles – the W and Z bosons. These lead to ranges that from Equation (1.31) are of the order of $R_{W,Z} \approx 2 \times 10^{-18}$ m. The fundamental strong interaction has infinite range because, like the photon, gluons have zero mass. On the other hand, the strong nuclear force, as exemplified by Figure 1.4 for example, has a much shorter range of approximately $(1-2) \times 10^{-15}$ m. We will comment briefly on the relation between these two different manifestations of the strong interaction in Section 7.1.

1.5.2 The Yukawa potential

In the limit that M_A becomes large, we can regard B as being scattered by a static potential of which A is the source. This potential will in general be spin dependent, but its main features can be obtained by neglecting spin and considering X to be a spin-0 boson, in which case it will obey the Klein–Gordon equation

$$-\hbar^2 \frac{\partial^2 \phi(\mathbf{x}, t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \phi(\mathbf{x}, t) + M_X^2 c^4 \phi(\mathbf{x}, t). \quad (1.32)$$

The static solution of this equation satisfies

$$\nabla^2 \phi(\mathbf{x}) = \frac{M_X^2 c^4}{\hbar^2} \phi(\mathbf{x}), \quad (1.33)$$

where $\phi(\mathbf{x})$ is interpreted as a static potential. For $M_X = 0$ this equation is the same as that obeyed by the electrostatic potential, and for a point charge $-e$ interacting with a point charge $+e$ at the origin, the appropriate solution is the Coulomb potential

$$V(r) = -e\phi(r) = -\frac{e^2}{4\pi\epsilon_0} \frac{1}{r}, \quad (1.34)$$

where $r = |\mathbf{x}|$ and ϵ_0 is the dielectric constant. The corresponding solution in the case where $M_X^2 \neq 0$ is easily verified by substitution to be

$$V(r) = -\frac{g^2}{4\pi} \frac{e^{-r/R}}{r}, \quad (1.35)$$

where R is the range defined earlier and g , the so-called *coupling constant*, is a parameter associated with each vertex of a Feynman diagram and represents the

basic strength of the interaction.³¹ For simplicity, we have assumed equal strengths for the couplings of particle X to the particles A and B .

The form of $V(r)$ in Equation (1.35) is called a *Yukawa potential*, after the physicist who first introduced the idea of forces due to massive particle exchange in 1935.³² As $M_X \rightarrow 0$, $R \rightarrow \infty$ and the Coulomb potential is recovered from the Yukawa potential, while for very large masses the interaction is approximately point-like (zero range). It is conventional to introduce a dimensionless parameter α_X by

$$\alpha_X = \frac{g^2}{4\pi\hbar c}, \quad (1.36)$$

that characterizes the strength of the interaction at short distances $r \leq R$. For the electromagnetic interaction this is the *fine structure constant*

$$\alpha \equiv e^2/4\pi\epsilon_0\hbar c \approx 1/137 \quad (1.37)$$

that governs the splittings of atom energy levels.³³

The forces between hadrons are also generated by the exchange of particles. Thus, in addition to the electromagnetic interaction between charged hadrons, all hadrons, whether charged or neutral, experience a strong *short-range* interaction, which in the case of two nucleons, for example, has a range of about 10^{-15} m, corresponding to the exchange of a particle with an effective mass of about $\frac{1}{7}$ th the mass of the proton. The dominant contribution to this force is the exchange of a single pion, as shown in Figure 1.4. This nuclear strong interaction is a complicated effect that has its origins in the fundamental strong interactions between the quark distributions within the two hadrons. Two neutral atoms also experience an electromagnetic interaction (the van der Waals force), which has its origins in the fundamental Coulomb forces, but is of much shorter range. Although an analogous mechanism is not in fact responsible for the nuclear strong interaction, it is a useful reminder that the force between two *distributions* of particles can be much more complicated than the forces between their components. We will return to this point when we discuss the nature of the nuclear potential in more detail in Section 7.1.

1.6 Observable Quantities: Cross Sections and Decay Rates

We have mentioned earlier that Feynman diagrams can be turned into probabilities for a process by a set of mathematical rules (the *Feynman Rules*) that can be

³¹Although we call g a (point) coupling *constant*, in general it will have a dependence on the momentum carried by the exchanged particle. We ignore this in what follows.

³²For this insight, Hideki Yukawa received the 1949 Nobel Prize in Physics.

³³Like g , the coupling α_X will in general have a dependence on the momentum carried by particle X . In the case of the electromagnetic interaction, this dependence is relatively weak.

derived from the quantum theory of the underlying interaction.³⁴ We will not pursue this in detail in this book, but rather will show in principle their relation to *observables*, i.e. things that can be measured, concentrating on the cases of two-body scattering reactions and decays of unstable states.

1.6.1 Amplitudes

The intermediate step is the *amplitude* f , the modulus squared of which is directly related to the probability of the process occurring. It is also called the *invariant amplitude* because, as we shall show, it is directly related to observable quantities and these have to be the same in all inertial frames of reference. To get some qualitative idea of the structure of f , we will use non-relativistic quantum mechanics and assume that the coupling constant g^2 is small compared with $4\pi\hbar c$, so that the interaction is a small perturbation on the free particle solution, which will be taken as plane waves.

If we expand the amplitude in a power series in g^2 and keep just the first term (i.e. lowest-order perturbation theory), then the amplitude for a particle in an initial state with momentum $\mathbf{q}_i$ to be scattered to a final state with momentum $\mathbf{q}_f$ by a potential $V(\mathbf{x})$ is proportional to

$$f(\mathbf{q}^2) = \int d^3\mathbf{x} V(\mathbf{x}) \exp[i\mathbf{q} \cdot \mathbf{x}/\hbar], \quad (1.38)$$

i.e. the Fourier transform of the potential, where $\mathbf{q} \equiv \mathbf{q}_i - \mathbf{q}_f$ is the momentum transfer.³⁵

The integration may be done using polar co-ordinates. Taking $\mathbf{q}$ in the x -direction, gives

$$\mathbf{q} \cdot \mathbf{x} = |\mathbf{q}|r \cos \theta \quad (1.39)$$

and

$$d^3\mathbf{x} = r^2 \sin \theta d\theta dr d\phi, \quad (1.40)$$

where $r = |\mathbf{x}|$. For the Yukawa potential, the integral in Equation (1.38) gives

$$f(\mathbf{q}^2) = \frac{-g^2\hbar^2}{\mathbf{q}^2 + M_X^2 c^2}. \quad (1.41)$$

³⁴In the case of the electromagnetic interaction, the theory is called Quantum Electrodynamics (QED) and is spectacularly successful in explaining experimental results. Richard Feynman shared the 1965 Nobel Prize in Physics with Sin-Itiro Tomonaga and Julian Schwinger for their work on formulating quantum electrodynamics. The Feynman Rules are discussed in an accessible way in Gr87.

³⁵See, for example, Chapter 11 of Ma92.

This amplitude corresponds to the exchange of a single particle, as shown for example in Figures 1.3 and 1.4. The structure of the amplitude, which is quite general, is a numerator proportional to the product of the couplings at the two vertices (or equivalently α_X in this case), and a denominator that depends on the mass of the exchanged particle and its momentum transfer squared. The denominator is called the *propagator* for particle X . In a relativistic calculation, the term $\mathbf{q}^2$ becomes q^2 , where q is the *four-momentum* transfer.

Returning to the zero-range approximation, one area where it is used extensively is in weak interactions, particularly applied to nuclear decay processes. In these situations, $M_X = M_{W,Z}$ and $f \rightarrow -G_F$, where G_F , the so-called *Fermi coupling constant*, is given from Equation (1.36) by

$$\frac{G_F}{(\hbar c)^3} = \frac{4\pi\alpha_W}{(M_W c^2)^2} = 1.166 \times 10^{-5} \text{GeV}^{-2}. \quad (1.42)$$

The numerical value is obtained from analyses of decay processes, including that of the neutron and a heavier version of the electron called the *muon*, whose properties will be discussed in Chapter 3.

All the above is for the exchange of a single particle. It is also possible to draw more complicated Feynman diagrams that correspond to the exchange of more than one particle. An example of such a diagram for elastic e^-e^- scattering, where two photons are exchanged, is shown in Figure 1.6.

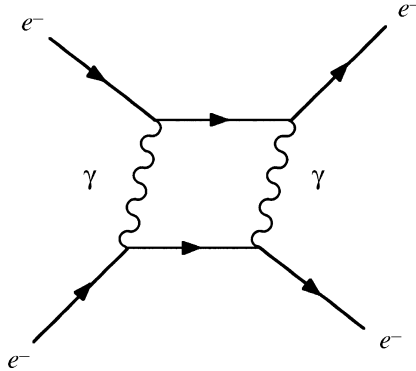


Figure 1.6 Two-photon exchange in the reaction $e^- + e^- \rightarrow e^- + e^-$

The number of vertices in any diagram is called the *order* n , and when the amplitude associated with any given Feynman diagram is calculated, it always contains a factor of $(\sqrt{\alpha})^n$. Since the probability is proportional to the square of the modulus of the amplitude, the former will contain a factor α^n . The probability associated with the single-photon exchange diagrams of Figure 1.1 thus contain a factor of α^2 and the contribution from two-photon exchange is of the order of α^4 .

As $\alpha \sim 1/137$, the latter is usually very small compared with the contribution from single-photon exchange.

This is a general feature of electromagnetic interactions: because the fine structure constant is very small, in most cases only the lowest-order diagrams that contribute to a given process need be taken into account, and more complicated higher-order diagrams with more vertices can (to a good approximation) be ignored in many applications.

1.6.2 Cross-sections

The next step is to relate the amplitude to measurables. For scattering reactions, the appropriate observable is the *cross-section*. In a typical scattering experiment, a beam of particles is allowed to hit a target and the rates of production of various particles in the final state are counted.³⁶ It is clear that the rates will be proportional to: (a) the number N of particles in the target illuminated by the beam, and (b) the rate per unit area at which beam particles cross a small surface placed in the beam at rest with respect to the target and perpendicular to the beam direction. The latter is called the *flux* and is given by

$$J = n_b v_i, \quad (1.43)$$

where n_b is the number density of particles in the beam and v_i their velocity in the rest frame of the target. Hence the rate W_r at which a specific reaction r occurs in a particular experiment can be written in the form

$$W_r = JN\sigma_r, \quad (1.44a)$$

where σ_r , the constant of proportionality, is called the *cross-section* for reaction r . If the beam has a cross-sectional area S , its intensity is $I = JS$ and so an alternative expression for the rate is

$$W_r = N\sigma_r I/S = I\sigma_r n_t t, \quad (1.44b)$$

where n_t is the number of target particles per unit volume and t is the thickness of the target. If the target consists of an isotopic species of atomic mass M_A (in atomic mass units—defined in Section 1.7 below), then $n_t = \rho N_A/M_A$, where ρ is the density of the target and N_A is Avogadro's constant. Thus, Equation (1.44b) may be written

$$W_r = I\sigma_r(\rho t)N_A/M_A, \quad (1.44c)$$

³⁶The practical aspects of experiments are discussed in Chapter 4.

where (ρt) is a measure of the amount of material in the target, expressed in units of mass per unit area. The form of Equation (1.44c) is particularly useful for the case of thin targets, commonly used in experiments. In the above, the product JN is called the *luminosity* L , i.e.

$$L \equiv JN \quad (1.45)$$

and contains all the dependencies on the densities and geometries of the beam and target. The cross-section is independent of these factors.

It can be seen from the above equations that the cross-section has the dimensions of an area; the rate per target particle $J\sigma_r$ at which the reaction occurs is equal to the rate at which beam particles would hit a surface of area σ_r , placed in the beam at rest with respect to the target and perpendicular to the beam direction. Since the area of such a surface is unchanged by a Lorentz transformation in the beam direction, the cross-section is the same in all inertial frames of reference, i.e. it is a Lorentz invariant.

The quantity σ_r is better named the *partial cross-section*, because it is the cross-section for a particular reaction r . The *total cross-section* σ is defined by

$$\sigma \equiv \sum_r \sigma_r. \quad (1.46)$$

Another useful quantity is the *differential cross-section*, $d\sigma_r(\theta, \phi)/d\Omega$, which is defined by

$$dW_r \equiv JN \frac{d\sigma_r(\theta, \phi)}{d\Omega} d\Omega, \quad (1.47)$$

where dW_r is the measured rate for the particles to be emitted into an element of solid angle $d\Omega = d\cos\theta d\phi$ in the direction (θ, ϕ) , as shown in Figure 1.7.

The total cross-section is obtained by integrating the partial cross-section over all angles, i.e.

$$\sigma_r = \int_0^{2\pi} d\phi \int_{-1}^1 d\cos\theta \frac{d\sigma_r(\theta, \phi)}{d\Omega}. \quad (1.48)$$

The final step is to write these formulae in terms of the scattering amplitude $f(\mathbf{q}^2)$ appropriate for describing the scattering of a non-relativistic spinless particle from a potential.

To do this it is convenient to consider a single beam particle interacting with a single target particle and to confine the whole system in an arbitrary volume V (which cancels in the final result). The incident flux is then given by

$$J = n_b v_i = v_i/V \quad (1.49)$$

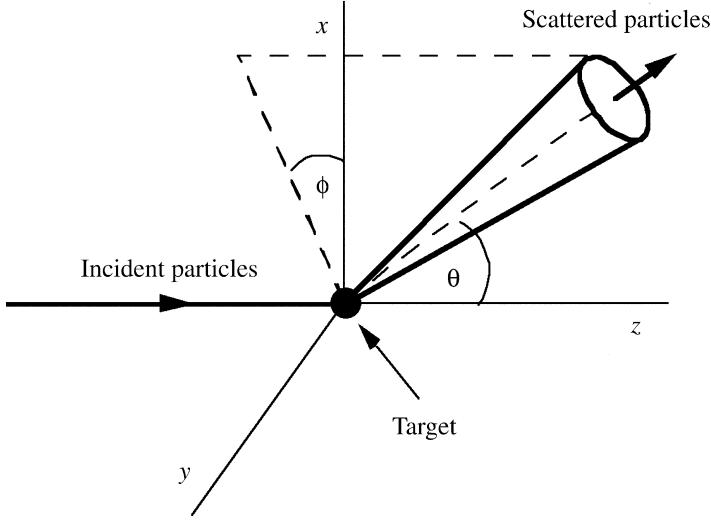


Figure 1.7 Geometry of the differential cross-section: a beam of particles is incident along the z-axis and collides with a stationary target at the origin; the differential cross-section is proportional to the rate for particles to be scattered into a small solid angle $d\Omega$ in the direction (θ, ϕ)

and since the number of target particles is $N = 1$, the differential rate is

$$dW_r = \frac{v_i}{V} \frac{d\sigma_r(\theta, \phi)}{d\Omega} d\Omega. \quad (1.50)$$

In quantum mechanics, provided the interaction is not too strong, the transition rate for any process is given in perturbation theory by the Born approximation³⁷

$$dW_r = \frac{2\pi}{\hbar} \left| \int d^3\mathbf{x} \psi_f^* V(\mathbf{x}) \psi_i \right|^2 \rho(E_f). \quad (1.51)$$

The term $\rho(E_f)$ is the *density-of-states factor* (see below) and we take the initial and final state wavefunctions to be plane waves:

$$\psi_i = \frac{1}{\sqrt{V}} \exp[i\mathbf{q}_i \cdot \mathbf{x}/\hbar], \quad \psi_f = \frac{1}{\sqrt{V}} \exp[i\mathbf{q}_f \cdot \mathbf{x}/\hbar], \quad (1.52)$$

³⁷This equation is a form of the *Second Golden Rule*. It is discussed in Appendix A.

where the final momentum $\mathbf{q}_f$ lies within a small solid angle $d\Omega$ located in the direction (θ, ϕ) (see Figure 1.7.). Then, by direct integration,

$$dW_r = \frac{2\pi}{\hbar V^2} |f(\mathbf{q}^2)|^2 \rho(E_f), \quad (1.53)$$

where $f(\mathbf{q}^2)$ is the scattering amplitude defined in Equation (1.38).

The density of states $\rho(E_f)$ that appears in Equation (1.51) is the number of possible final states with energy lying between E_f and $E_f + dE_f$ and is given by³⁸

$$\rho(E_f) = \frac{V}{(2\pi\hbar)^3} \frac{q_f^2}{v_f} d\Omega. \quad (1.54)$$

If we use this and Equation (1.53) in Equation (1.50), we have

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2\hbar^4} \frac{q_f^2}{v_i v_f} |f(\mathbf{q}^2)|^2. \quad (1.55)$$

Although this result has been derived in the laboratory system, because we have taken a massive target it is also valid in the centre-of-mass system. For a finite mass target it would be necessary to make a Lorentz transformation on Equation (1.55). The expression is also true for the general two-body relativistic scattering process $a + b \rightarrow c + d$.

All the above is for spinless particles, so finally we have to generalize Equation (1.55) to include the effects of spin. Suppose the initial-state particles a and b , have spins s_a and s_b and the final-state particles c and d have spins s_c and s_d . The total numbers of spin substates available to the initial and final states are g_i and g_f , respectively, given by

$$g_i = (2s_a + 1)(2s_b + 1) \quad \text{and} \quad g_f = (2s_c + 1)(2s_d + 1). \quad (1.56)$$

If the initial particles are unpolarized (which is the most common case in practice), then we must average over all possible initial spin configurations (because each is equally likely) and sum over the final configurations. Thus, Equation (1.55) becomes

$$\frac{d\sigma}{d\Omega} = \frac{g_f}{4\pi^2\hbar^4} \frac{q_f^2}{v_i v_f} |\overline{\mathcal{M}_{fi}}|^2, \quad (1.57)$$

where

$$|\overline{\mathcal{M}_{fi}}|^2 \equiv \overline{|f(\mathbf{q}^2)|^2} \quad (1.58)$$

and the bar over the amplitude denotes a spin-average of the squared matrix element.

³⁸The derivation is given in detail in Appendix A.

1.6.3 Unstable states

In the case of an unstable state, the observable of interest is its *lifetime at rest* τ , or equivalently its *natural decay width*, given by $\Gamma = \hbar/\tau$, which is a measure of the rate of the decay reaction. In general, an initial unstable state will decay to several final states and in this case we define Γ_f as the *partial width* for channel f and

$$\Gamma = \sum_f \Gamma_f \quad (1.59)$$

as the *total decay width*, while

$$B_f \equiv \Gamma_f/\Gamma \quad (1.60)$$

is defined as the *branching ratio* for decay to channel f .

The energy distribution of an unstable state to a final state f has the *Breit–Wigner* form

$$N_f(W) \propto \frac{\Gamma_f}{(W - M)^2 c^4 + \Gamma^2/4}, \quad (1.61)$$

where M is the mass of the decaying state and W is the invariant mass of the decay products.³⁹ The Breit–Wigner formula is shown in Figure 1.8 and is the same formula that describes the widths of atomic and nuclear spectral lines. (The overall factor depends on the spins of the particles involved.) It is a symmetrical bell-shaped curve with a maximum at $W = M$ and a full width Γ at half the maximum height of the curve. It is proportional to the number of events with invariant mass W .

If an unstable state is produced in a scattering reaction, then the cross section for that reaction will show an enhancement described by the same Breit–Wigner formula. In this case we say we have produced a *resonance state*. In the vicinity of a resonance of mass M , and width Γ , the cross-section for the reaction $i \rightarrow f$ has the form

$$\sigma_{fi} \propto \frac{\Gamma_i \Gamma_f}{(E - Mc^2)^2 + \Gamma^2/4}, \quad (1.62)$$

where E is the total energy of the system. Again, the form of the overall constant will depend on the spins of the particles involved. Thus, for example, if the resonance particle has spin j and the spins of the initial particles are s_1 and s_2 , then

$$\sigma_{fi} = \frac{\pi \hbar^2}{q_i^2} \frac{2j+1}{(2s_1+1)(2s_2+1)} \frac{\Gamma_i \Gamma_f}{(E - Mc^2)^2 + \Gamma^2/4}. \quad (1.63)$$

³⁹This form arises from a state that decays exponentially with time, although a proper proof of this is quite lengthy. See, for example, Appendix B of Ma97.

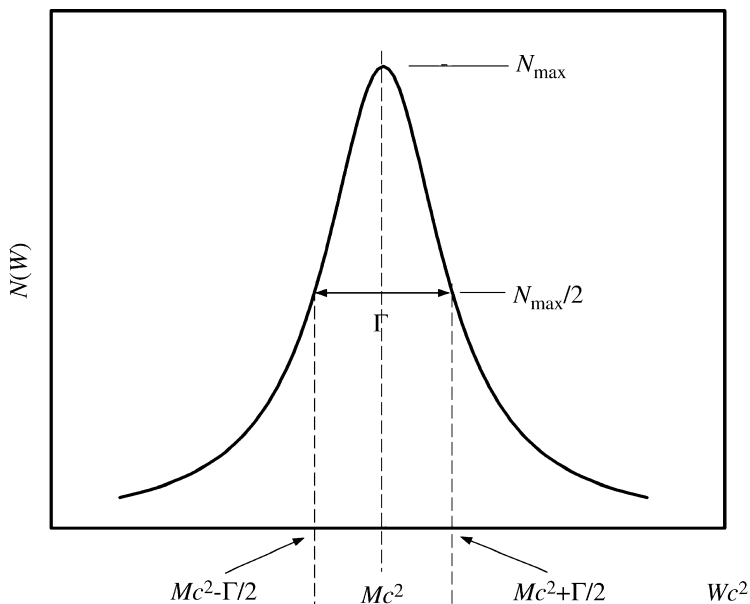


Figure 1.8 The Breit-Wigner formula (Equation (1.61))

In practice there will also be kinematical and angular momentum effects that will distort this formula from its perfectly symmetric shape.

An example of resonance formation in π^-p interactions is given in Figure 1.9, which shows the π^-p total cross-section in the centre-of-mass energy range

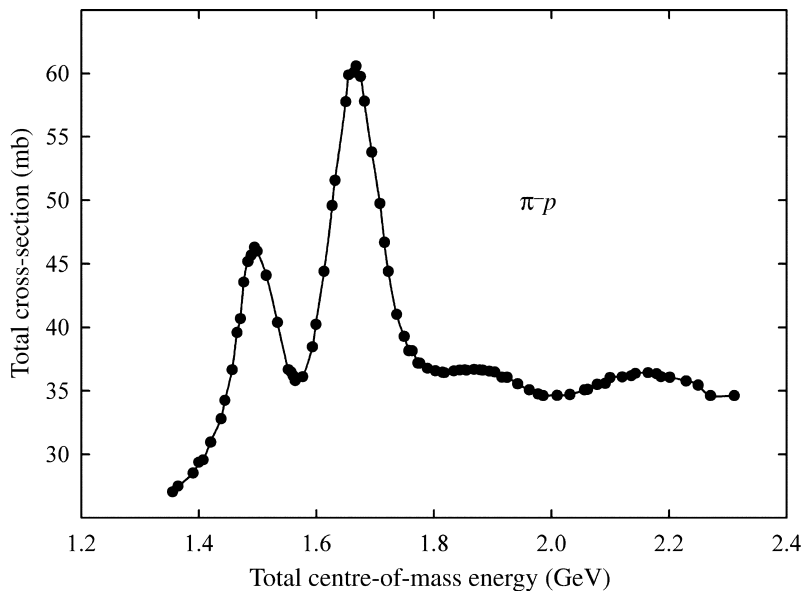


Figure 1.9 Total cross-sections for π^-p interactions (data from Ca68)

1.2–2.4 GeV. (The units used in the plots will become clear after the next section.) Two enhancements can be seen that are of the approximate Breit–Wigner resonance form and there are two other maxima at higher energies. In principle, the mass and width of a resonance may be obtained by using a Breit–Wigner formula and varying M and Γ to fit the cross-section in the region of the enhancement. In practice, more sophisticated methods are used that fit a wide range of data, including differential cross-sections, simultaneously and also take account of non-resonant contributions to the scattering. The widths obtained from such analyses are of the order of 100 MeV, with corresponding interaction times of order 10^{-23} s, which is consistent with the time taken for a relativistic pion to transit the dimension of a proton. Resonances are also a feature of interactions in nuclear physics and we will return to this in Section 2.9 when we discuss nuclear reaction mechanisms.

1.7 Units: Length, Mass and Energy

Most branches of science introduce special units that are convenient for their own purposes. Nuclear and particle physics are no exceptions. Distances tend to be measured in femtometres (fm) or, equivalently *fermis*, with $1 \text{ fm} \equiv 10^{-15} \text{ m}$. In these units, the radius of the proton is about 0.8 fm. The range of the strong nuclear force between protons and neutrons is of the order of 1–2 fm, while the range of the weak force is of the order of 10^{-3} fm. For comparison, the radii of atoms are of the order of 10^5 fm. A common unit for area is the *barn* (b) defined by $1 \text{ b} = 10^{-28} \text{ m}^2$. For example, the total cross-section for pp scattering (a strong interaction) is a few tens of millibarns (mb) (compare also the π^-p total cross-section in Figure 1.9), whereas the same quantity for νp scattering (a weak interaction) is a few tens of femtobarns (fb), depending on the energies involved. Nuclear cross-sections are very much larger and increase approximately like $A^{2/3}$, where A is the total number of nucleons in the nucleus.

Energies are invariably specified in terms of the electron volt (eV) defined as the energy required to raise the electric potential of an electron or proton by 1 V. In SI units, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. The units $1 \text{ keV} = 10^3 \text{ eV}$, $1 \text{ MeV} = 10^6 \text{ eV}$, $1 \text{ GeV} = 10^9 \text{ eV}$ and $1 \text{ TeV} = 10^{12} \text{ eV}$ are also in general use. In terms of these units, atomic ionization energies are typically a few eV, the energies needed to bind nucleons in heavy nuclei are typically 7–8 MeV per particle, and the highest particle energies produced in the laboratory are of the order of 1 TeV for protons. Momenta are specified in eV/c, MeV/c, etc..

In order to create a new particle of mass M , an energy at least as great as its rest energy Mc^2 must be supplied. The rest energies of the electron and proton are 0.51 MeV and 0.94 GeV respectively, whereas the W and Z^0 bosons have rest energies of 80 GeV and 91 GeV, respectively. Correspondingly, their masses are conveniently measured in MeV/c^2 or GeV/c^2 , so that, for example,

$$\begin{aligned} M_e &= 0.51 \text{ MeV}/c^2, & M_p &= 0.94 \text{ GeV}/c^2, \\ M_W &= 80.3 \text{ GeV}/c^2, & M_Z &= 91.2 \text{ GeV}/c^2 \end{aligned} \quad (1.64)$$

In SI units, $1 \text{ MeV}/c^2 = 1.78 \times 10^{-30} \text{ kg}$. In nuclear physics it is also common to express masses in *atomic mass units* (u), where $1 u = 1.661 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2$.

Although practical calculations are expressed in the above units, it is usual in particle physics to make theoretical calculations in units chosen such that $\hbar \equiv h/2\pi = 1$ and $c = 1$ (called *natural units*) and many books do this. However, as this book is about both nuclear and particle physics, only practical units will be used.

A table giving numerical values of fundamental and derived constants, together with some useful conversion factors is given on page XV.

Problems

- 1.1 ‘Derive’ the Klein–Gordon equation using the information in Footnote 21 and verify that Equation (1.35) is a static solution of the equation.
- 1.2 Verify that the spherical harmonic $Y_1^1 = \sqrt{\frac{3}{8}} \sin \theta e^{i\phi}$ is an eigenfunction of parity with eigenvalue $P = -1$.
- 1.3 A proton and antiproton at rest in an S-state annihilate to produce $\pi^0 \pi^0$ pairs. Show that this reaction cannot be a strong interaction.
- 1.4 Suppose that an intrinsic C -parity factor is introduced into Equation (1.15b), which then becomes $\hat{C}|b, \psi_b\rangle = C_b|\bar{b}, \psi_{\bar{b}}\rangle$. Show that the eigenvalue corresponding to any eigenstate of $\hat{C}$ is independent of C_b , so that C_b cannot be measured.
- 1.5 Consider the reaction $\pi^- d \rightarrow nn$, where d is a spin-1 S-wave bound state of a proton and a neutron called the deuteron and the initial pion is at rest. Deduce the intrinsic parity of the negative pion.
- 1.6 Write down equations in symbol form that describe the following interactions:
 - (a) elastic scattering of an electron antineutrino and a positron;
 - (b) inelastic production of a pair of neutral pions in proton–proton interactions;
 - (c) the annihilation of an antiproton with a neutron to produce three pions.
- 1.7 Draw a lowest-order Feynman diagram for the following processes:
 - (a) $\nu_e \nu_\mu$ elastic scattering;
 - (b) $n \rightarrow p + e^- + \bar{\nu}_e$;

(c) $e^+e^- \rightarrow e^+e^-$;

(d) a fourth-order diagram for the reaction $\gamma + \gamma \rightarrow e^+ + e^-$.

- 1.8** Calculate the energy-momentum transfer between two particles equivalent to a distance of approach of (a) 1 fm and (b) 10^{-3} fm. Assuming that the intrinsic strengths of the fundamental weak and electromagnetic interactions are approximately equal, compare the relative sizes of the invariant (scattering) amplitudes for weak and electromagnetic processes at these two energy-momentum transfers.
- 1.9** Verify by explicit integration that $f(\mathbf{q}) = -g^2\hbar^2[\mathbf{q}^2 + m^2c^2]^{-1}$ is the amplitude corresponding to the Yukawa potential

$$V(r) = -\frac{g^2}{4\pi} \frac{e^{-r/R}}{r},$$

where $R = \hbar/mc$ is the range and $r = |\mathbf{x}|$.

- 1.10** Two beams of particles consisting of n bunches with N_i ($i = 1, 2$) particles in each, traverse circular paths and collide ‘head-on’. Show that in this case the general expression Equation (1.45) for the luminosity reduces to $L = nN_1N_2f/A$, where A is the cross-sectional area of the beam and f is the frequency, i.e. $f = 1/T$, where T is the time taken for the particles to make one traversal of the ring.
- 1.11** A thin (density 1 mg cm^{-2}) target of ^{24}Mg ($M_A = 24.3$ atomic mass units) is bombarded with a 10 nA beam of alpha particles. A detector subtending a solid angle of 2×10^{-3} sr, records 20 protons/s. If the scattering is isotropic, determine the cross-section for the $^{24}\text{Mg}(\alpha, p)$ reaction.