Exercise 5.4

Answer 1E.

(a)

Consider the weight of gorilla is $360-\text{lb}$.

Height of the tree is $20\,\text{ft}$.

Find the work done if the gorilla reaches that height in 10 seconds.

The work done is defined to be the product of the force *F* and the distance *d* is as follows:

$$\begin{aligned} W &= Fd \\ &= (360)(20) \\ &= 7200 \end{aligned}$$

Thus, work done if the gorilla reaches the height in 10 seconds is $\boxed{7200\text{ ft-lb}}$.

(b)

Consider the weight of gorilla is $360-\text{lb}$.

Height of the tree is $20\,\text{ft}$.

Find the work done if the gorilla reaches that height in 5 seconds.

The work done is defined to be the product of the force *F* and the distance *d* is as follows:

$$\begin{aligned} W &= Fd \\ &= (360)(20) \\ &= 7200 \end{aligned}$$

Thus, work done if the gorilla reaches the height in 5 seconds is $\boxed{7200\text{ ft-lb}}$.

Answer 2E.

Weight of the rock is $= 200\text{ - kg}$

The rock has been lifted to the height $h = 3\text{m}$

The force exerted is equal and opposite to that exerted by gravity, so

$$\begin{aligned} F &= mg \\ &= (200)(9.8) \\ &= 1960\text{ N} \end{aligned}$$

Work done is given by

$$\begin{aligned} W &= Fd \\ &= (1960)(3) \\ &= 5880 \end{aligned}$$

Thus the work done in lifting a 200-kg rock to a height of 3m is $= \boxed{5880\text{ J}}$

Thus the work done in lifting a 200-kg rock to a height of 3m is $= \boxed{5880\text{ J}}$

Answer 3E.

Force acting on the object is $F = 5x^{-2}$
We need to calculate the work done in moving the object from $x = 1$ ft to $x = 10$ ft

Now the work done in moving the object from $x = 1$ ft to $x = 10$ ft is given by

$$W = \int_1^{10} 5x^{-2} dx$$

$$= 5\int_1^{10} x^{-2} dx$$

$$= 5\left[\frac{x^{-1}}{-1}\right]_1^{10}$$

$$= 5\left[-\frac{1}{x}\right]_1^{10}$$

$$= 5\left(-\frac{1}{10}+1\right)$$

$$= 5\left(\frac{9}{10}\right)$$

$$= \frac{9}{2}$$

$$= 4.5$$

Thus the work done in moving the object from $x = 1$ ft to $x = 10$ ft is $= \boxed{4.5 \text{ ft-lb}}$

Answer 4E.

Force $= \cos\left(\frac{\pi x}{3}\right)$ N

Since particle is moving from $x = 1$ to another position $x = 2$, so work done is given by

$$W = \int_1^2 \cos\left(\frac{\pi x}{3}\right) dx$$

$$= \frac{3}{\pi}\left[\sin\left(\frac{\pi}{3}x\right)\right]_1^2$$

$$= \frac{3}{\pi}\left[\sin\frac{2\pi}{3} - \sin\frac{\pi}{3}\right]$$

$$= \frac{3}{\pi}\left[\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right] = 0 \text{ N-m}$$

Hence the work done in moving the particle from $x = 1$ to $x = 2$ is $\boxed{0 \text{ Joule}}$

Next let us find the work done in moving the particle from $x = 1$ to $x = 1.5$

$$W_1 = \int_1^{1.5} \cos\left(\frac{\pi x}{3}\right) dx$$

$$= \frac{3}{\pi}\left[\sin\frac{\pi x}{3}\right]_1^{3/2}$$

$$= \frac{3}{\pi}\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{3}\right]$$

$$= \frac{3}{\pi}\left[1 - \frac{\sqrt{3}}{2}\right] = \boxed{\frac{3}{2\pi}\left(2-\sqrt{3}\right) \text{ Joule}}$$

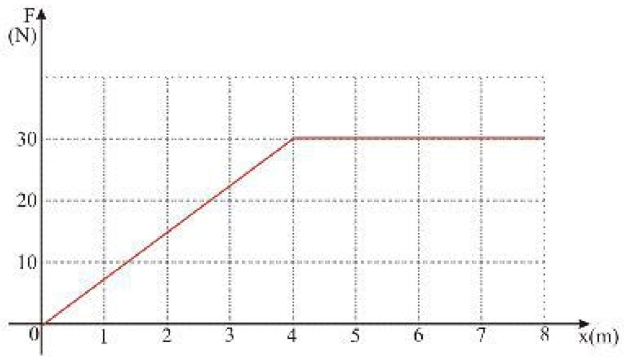
The work done in moving the particle form 1 to 2 is equal to sum of the works done in moving form 1 to 1.5 and 1.5 to 2, we get

$$W_1 + W_2 = \frac{3}{2\pi}\left(2-\sqrt{3}\right) + \frac{3}{2\pi}\left(\sqrt{3}-2\right) = 0 \text{ Joule},$$

This is equal to W, as evaluated earlier

Therefore we find that $\boxed{W = W_1 + W_2}$

Answer 5E.



We solve it for two intervals [0, 4] and [4, 8]

Now in the interval $0 \le x \le 4$

The slope of line segment is $= \frac{30}{4} = 7.5$

So equation of line is $y = 7.5x$

Or $F = 7.5x$

Then work done in moving form x = 0 to x = 4 is

$$W_1 = \int_0^4 F\,dx = \int_0^4 7.5(x)\,dx$$

$$= 7.5\left[\frac{x^2}{2}\right]_0^4 \qquad \text{[By FTC - 2]}$$

$$= 7.5 \times 8 = 60 \text{ J}$$

From x = 4 to x = 8, the fore is constant = 30N and distance is = 8 – 4 = 4m

Then work done is moving form x = 4 to x = 8 is

$$W_2 = Force \times distance$$

$$= 30 \times 4 \text{ N-m} = 120 \text{ J}$$

Then total work $W = W_1 + W_2 = (60 + 120)\text{J}$

Or $\boxed{W = 180 \text{ J}}$

Answer 6E.

The values of a force function $f(x)$, where x is measured in meters and $f(x)$ in Newton are given as

$x(m)$	4	6	8	10	12	14	16	18	20
$f(x)$ (N)	5	5.8	7.0	8.8	9.6	8.2	6.7	5.2	4.1

We have the interval $4 \le x \le 20$
If we divide this interval in to 4 sub intervals
So width of sub interval is $\Delta x = \dfrac{20-4}{4} = 4$
And sub intervals are [4, 8], [8, 12], [12, 16] and [16, 20]
The mid pints of these intervals are

$$x_1^\bullet = 6,\ x_2^\bullet = 10,\ x_3^\bullet = 14,\ x_4^\bullet = 18$$

Then we can estimate the work done by mid point rule as

$$W \approx \sum_{i=1}^{4} f\left(x_i^\bullet\right)\Delta x$$

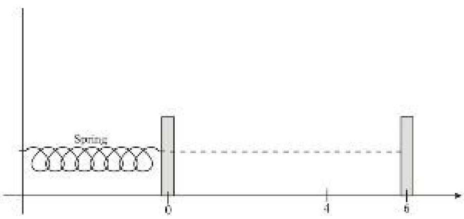
Where $x_i^\bullet$ is the mid point of the interval $\left[x_{i-1}, x_i\right]$

So work done $W \approx \Delta x\left[f\left(x_1^\bullet\right)+f\left(x_2^\bullet\right)+f\left(x_3^\bullet\right)+f\left(x_4^\bullet\right)\right]$

$$\approx 4\left[5.8+8.8+8.2+5.2\right] J$$

Or $\boxed{W \approx 112\ \ \mathrm{J}}$

Answer 7E.



Since $4 \text{ in} = \dfrac{4}{12} = \dfrac{1}{3}\text{ft}$

And $6 \text{ in} = \dfrac{1}{2}\text{ft}$

Now the required force to hold a spring stretched 4 in $= \dfrac{1}{3}$ ft, is given F = 10 lb

By Hooke's law, we have

$$F(x) = kx$$

When $x = \dfrac{1}{3}$ ft $\qquad f(x) = 10\text{ lb} \qquad$ (given)

So $\dfrac{1}{3}k = 10$

Or $\boxed{k = 30}$

Then the equation of force is $F(x) = 30x$

Then work done in stretching the spring x = 0 to $x = \dfrac{1}{2}$ ft is

$$W = \int_0^{1/2} 30x\,dx = 30\int_0^{1/2} x\,dx$$

$$= 30\left[\frac{x^2}{2}\right]_0^{1/2} \text{ ft-lb} \qquad \text{[By FTC - 2]}$$

$$= 30\left[\frac{1}{8}\right] \text{ft-lb}$$

Or $\boxed{W = \dfrac{15}{4}\ \text{ft-lb}}$

Answer 8E.

The spring is stretched from 20 cm to 30 cm

So amount stretched is $30 - 20 = 10\text{cm} = 0.1\text{m}$

And required force is $F(0.1) = 25N$ (given)

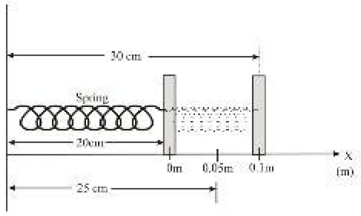
According to Hooke's law the force required to hold the spring stretched x m beyond its natural length is $F(x) = kx$

So we have $k.(0.1) = 25$

Or $k = \frac{25}{0.1}$

Or $\boxed{k = 250}$

Thus $F(x) = 250x$



The work done in stretching the spring form 20 cm to 25cm or 0.2m to 0.25m is

$$W = \int_0^{0.05} 250x\,dx$$

$$= 250\left[\frac{x^2}{2}\right]_0^{0.05} J$$

$$= \frac{250}{2}\left[(0.05)^2\right]$$

$$= 125 \times 0.0025$$

Or $\boxed{W = 0.3125 \text{ J}}$

Answer 9E.

According to Hooke's Law, the force required to hold the spring stretched x meters beyond its natural length is $f(x) = kx$

When the spring is stretched from 30cm to 42 cm, the amount stretched is $12cm = 0.12m$.

Thus 2 J and the work done in stretching the spring from 30cm to 42 cm

$$2 = \int_0^{0.12} f(x)dx$$

$$= \int_0^{0.12} kxdx$$

$$= k\int_0^{0.12} xdx$$

$$= k\left[\frac{x^2}{2}\right]_0^{0.12}$$

$$= k\left[\frac{(0.12)^2}{2} - \frac{(0)^2}{2}\right]$$

$$= k\left[\frac{(0.12)^2}{2}\right]$$

$$= k\left[\frac{0.0144}{2}\right]$$

So, $k = \frac{4}{0.0144}$

$$k = \frac{4}{\left(\frac{144}{10000}\right)}$$
$$= \frac{40000}{144}$$
$$= \frac{2500}{9}$$
$$= 277.8$$

(a)

When the starting spring is stretched from 30cm to 35cm, the amount stretched is $5cm = 0.05m$.

And the spring is stretched from 30cm to 40cm, the amount stretched is $10cm = 0.1m$

Thus $f(x) = 138.9x$ and the work done in stretching the spring from 35cm to 40cm is calculated as follows:

$$W = \int_{0.05}^{0.1} (277.8)\, x dx$$
$$= 277.8 \int_{0.05}^{0.1} x dx$$
$$= 277.8 \left[\frac{x^2}{2} \right]_{0.05}^{0.1}$$
$$= 277.8 \left[\frac{(0.1)^2}{2} - \frac{(0.05)^2}{2} \right]$$
$$= 277.8 \left[\frac{0.01}{2} - \frac{0.0025}{2} \right]$$
$$= 277.8 \left[\frac{0.0075}{2} \right]$$
$$= 277.8 \times 0.00375$$
$$= 1.04175$$

Therefore, the work done in stretching the spring from 35cm to 40cm is 1.04175 J

(b)

The force is 30N

But, applying Hooke's Law, the force required to hold the spring stretched x meters beyond its natural length is $f(x) = kx$

$$30\text{N} = kx$$
$$30\text{N} = \frac{2500}{9} x$$
$$x = \frac{30 \times 9}{2500}$$
$$= \frac{270}{2500}$$
$$= 0.108 \text{ m}$$

Distance stretched is $0.108\ m = 10.8\ cm$ from its natural length.

Answer 10E.

Given that the work required for stretching a spring 1 ft beyond its natural length is 12 ft-lb

So $\int_0^1 f(x)\,dx = 12$

By Hooke's law $f(x) = kx$, where $f(x)$ is force and k is any (spring) constant

So $\int_0^1 kx\,dx = 12$

Or $k\left[\frac{x^2}{2}\right]_0^1 = 12$

Or $\frac{k}{2} = 12$ or $\boxed{k = 24}$

Thus $f(x) = 24x$

Now we want to stretch the spring 9 in $= \frac{9}{12}$ ft from it's natural length so required work is

$$W = \int_0^{9/12} 24x\,dx$$

$$= 24\left[\frac{x^2}{2}\right]_0^{9/12}$$

$$= 12.\left(\frac{81}{144}\right) \text{ ft-lb}$$

Or $W = \frac{81}{12}$

Or $\boxed{W = \frac{27}{4} = 6.75 \text{ ft-lb}}$

Answer 11E.

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The length of a spring is 20 cm.

Since the works W_1 and W_2 are done in stretching the spring from 20 cm to 30 cm and 30 cm to 40 cm.

Need to compare the works W_1 and W_2.

By Hooke's law, the work done in stretching the spring from 20 cm to 30 cm is calculated as,

$$W_1 = \int_a^b F(x)dx$$

$$= \int_0^{(30-20)cm} kxdx \qquad \text{Use } F(x) = kx$$

$$= k\int_0^{10} xdx$$

$$= k\left(\frac{x^2}{2}\right)_0^{10}$$

$$= \frac{k}{2}(10^2)$$

$$= 50k$$

In the same manner, the work done in stretching the spring from 30 cm to 40 cm is calculated as,

$$W_2 = \int_a^b F(x)dx$$

$$= \int_{30-20}^{40-20} kxdx \qquad \text{Use } F(x) = kx$$

$$= k\int_{10}^{20} xdx$$

$$= k\left(\frac{x^2}{2}\right)_{10}^{20}$$

$$= \frac{k}{2}\left(20^2 - 10^2\right)$$

$$= \frac{k}{2}(300)$$

$$= 150k$$

Here, the value k is the spring constant.

Divide W_2 by W_1, this implies that

$$\frac{W_2}{W_1} = \frac{150k}{50k}$$

$$\frac{W_2}{W_1} = \frac{3}{1}$$

$$W_2 = 3W_1$$

Use the result; it confirms that the work W_2 is three times that of W_1.

Answer 12E.

Let natural length of spring is L m

Since 10cm = 0.1m

And 12cm = 0.12m

So we have, from $(0.1 - L) to (0.12 - L)$, the work required to stretch a spring is 6 J

So $\int_{(0.1-L)}^{(0.12-L)} kx dx = 6$

$$\Rightarrow k\left[\frac{x^2}{2}\right]_{(0.1-L)}^{(0.12-L)} = 6$$

$$\Rightarrow k\left[x^2\right]_{(0.1-L)}^{(0.12-L)} = 12$$

$$\Rightarrow k\left[(.12 - L)^2 - (0.1 - L)^2\right] = 12$$

$$\Rightarrow k\left[0.0144 + L^2 - 0.24L - 0.01 - L^2 + 0.2L\right] = 12$$

Or $k[0.0044 - 0.04L] = 12$

Or $0.0044k - 0.04Lk = 12$ --- (1)

Similarly we have form (0.12 – L) to (0.14 – L), the work required to stretch a spring is 10 J

So $\int_{(0.12-L)}^{(0.14-L)} kx\, dx = 10$

$$\Rightarrow k\left[\frac{x^2}{2}\right]_{(0.12-L)}^{(0.14-L)} = 10 \quad or \quad k\left[x^2\right]_{(0.12-L)}^{(0.14-L)} = 20$$

Or $k\left[(0.14 - L)^2 - (0.12 - L)^2\right] = 20$

$$\Rightarrow k\left[0.0196 + L^2 - 0.28L - 0.0144 - L^2 + 0.24L\right] = 20$$

$$\Rightarrow k[0.0052 - 0.4L] = 20$$

Or $0.0052k - 0.04Lk = 20$ --- (2)

Now we subtract equation (1) from equation (2)

$0.0008k = 8$

Or $k = 10000$

Putting this value of k in equation (1)

We have

$44 - 400L = 12$

Or $\boxed{L = 0.08}$ m

Or $\boxed{L = 8 \text{ cm}}$

So the natural length of the spring is 8 cm.

Answer 13E.

(A)

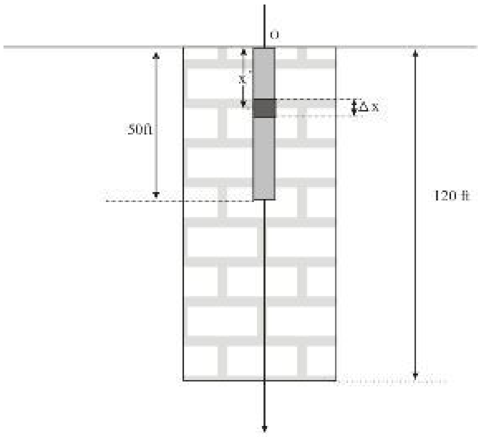


Fig.1

Let the top of building be at the origin and height of building be along x – axis.
If we divide the rope in to small parts of length Δx

So weight of the i[th] part of the rope is $= 0.5 \times \Delta x = \frac{1}{2}\Delta x$

And distance is $= x_i^*$ from the origin to i[th] part of rope

The work done on the i[th] part in ft-lb is

$$\frac{1}{2}\Delta x.\ x_i^* = \frac{1}{2}x_i^*\Delta x$$

So total work done

$$W = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{2}x_i^*\Delta x = \int_0^{50}\frac{1}{2}x\,dx$$

$$= \frac{1}{4}\left[x^2\right]_0^{50}$$

$$= \frac{1}{4}[2500]$$

Or $\boxed{W = 625 \text{ ft-lb}}$

(B) Now we want to pull half of the rope so work done is

$$W = \int_{25}^{50}\frac{x}{2}dx = \frac{1}{4}\left[x^2\right]_{25}^{50} = \frac{1}{4}[2500-625]$$

$$\boxed{W = \frac{1875}{4}\text{ft-lb}}$$

Answer 14E.

Total length of chain = 10 m
And mass = 80 kg
So weight is = 8 kg per meter
Let Δx be the length of i^{th} part of chain then weight or mass of the Δx is $= 8\Delta x$
And distance is $= x_i^*$

Force = mass × g $= 8\Delta x \times (9.8)$ $\quad (g = 9.8\ \text{m/s}^2)$ (acceleration due to gravity)

$= 78.4\Delta x$

Then work done = force × distance

$= 78.4\Delta x \times x_i^*$

$= 78.4 x_i^* \Delta x$

$$\text{Total work done } = \lim_{n\to\infty}\sum_{i=1}^{n} 78.4 x_i^* \Delta x$$
$$= \int_0^6 78.4\, x\, dx$$
$$= 78.4\int_0^6 x\, dx$$
$$= 78.4\left[\frac{x^2}{2}\right]_0^6$$
$$= 78.4\left[\frac{36}{2}\right]$$
$$\boxed{W = 1411.2\ J}$$

Answer 15E.

Weight of cable = 2 lb/ft
Weight of coal = 800 lb
Distance = 500 ft
Let i^{th} part of the cable has width Δx so the weight of i^{th} part of cable is $= 2\Delta x$
and the distance from mine shaft of the i^{th} part of the cable is $= x_i^*$.
So work done on the i^{th} part of the cable is $= 2x_i^*\Delta x$

So total work done for the cable

$$W_{ca} = \lim_{n\to\infty}\sum_{i=1}^{n} 2x_i^*\Delta x$$
$$= \int_0^{500} 2x\, dx$$
$$= \left[x^2\right]_0^{500}$$
$$= 250000 \text{ ft-lb}$$

Since weight of coal is = 800 lb
And distance is = 500 ft
So work done to lift the coal up to top of mine shaft is

$$W_{co} = 800 \times 500$$
$$= 400{,}000 \text{ ft-lb}$$

Total work done

$$W = W_{ca} + W_{co}$$
$$W = 250000 + 400000$$
$$\boxed{W = 650{,}000 \text{ ft-lb}}$$

Answer 16E.

Weight of the bucket = 4 lb
Weight of water which can be filled in the bucket = 40 lb
Total weight of bucket + water = 44 lb
Distance, the bucket full of water in to be pulled up = 80 ft
Now let at any time t, the bucket be at a distance x ft from the bottom of the well
Then $t = \frac{x}{2}.S$, where S is the rate at which water is leaking from the bucket
So at this time t, the amount of water which has leaked from the bucket

$$= \frac{x}{2}.(0.2) = 0.1x \quad \text{lb}$$

Answer 17E.

Weight of bucket = 10 kg
Length or distance = 12 m
Weight of rope = 0.8 kg/m
At any point x of the rope, the mass of the rope is

$$= (0.8)(12 - x) = (9.6 - 0.8x) \text{ kg}$$

Since the water finishes draining at 12 m level and total weight of water is $= 36$ kg

So rate of draining $= \frac{36}{12} = 3$ kg/m

At the point x, the mass of water is $= 3(12-x) = (36-3x)$ kg

$$\text{Total mass} = (9.6-0.8x)+(36-3x)+10$$
$$= (55.6-3.8x) \text{ kg}$$

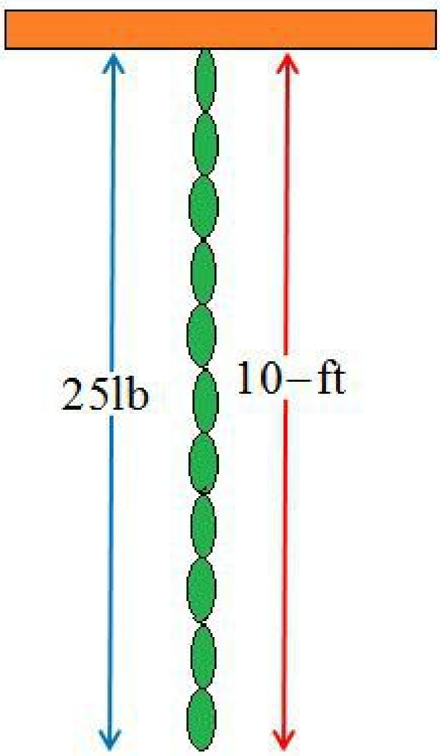
$$\text{Force} = (9.8)(55.6-3.8x)$$

$$\text{Work } W = \lim_{n\to\infty}\sum_{i=1}^{n}(9.8)(55.6-3.8x_i^*)\Delta x$$
$$= \int_0^{12}(9.8)(55.6-3.8x)\,dx$$
$$= 9.8\int_0^{12}(55.6-3.8x)\,dx$$
$$= 9.8\left[55.6x-1.9x^2\right]_0^{12}$$
$$= 9.8[667.2-273.6]$$
$$= 9.8\times 393.6$$

Or $\boxed{W = 3857.28\text{ J} \approx 3857\text{ J}}$

Answer 18E.

A 10-ft chain weighs 25 lb and hangs from a ceiling.



Let's place the origin at the top of the ceiling and the x – axis pointing downward as in the above figure.

Divide the chain into small parts with length Δx

If x^*_i is a point in the ith such interval, then all points in the interval are lifted by approximately the same amount, namely x^*_i

The chain weighs $\frac{25}{10} = 2.5$ pounds per foot, so the weight of the ith part is $2.5\Delta x$

Thus, the work done on the ith part, in foot-pounds, is $(2.5\Delta x) \times x^*_i = 2.5x^*_i \Delta x$

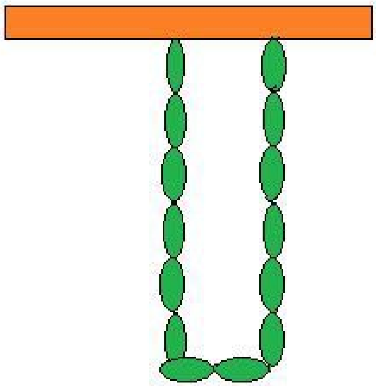
Get the total work done by adding all these approximations and letting the number of parts becomes large $(\text{so } \Delta x \to 0)$

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} 2x^*_i \Delta x$$

Lift the lower end of the chain to the ceiling, so that it is in level with the upper end.

That is the chain is half of the part.

So, the diagram is as follows:



Therefore calculate the work done as follows:

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} 2x^*_i \Delta x$$
$$= \frac{1}{2} \int_0^{10} 2.5x dx$$
$$= \frac{1}{2}(2.5)\left[\frac{x^2}{2}\right]_0^{10}$$
$$= \frac{2.5}{2} \times \frac{100}{2}$$
$$= \boxed{62.5 \text{ ft-lb}}$$

Answer 19E.

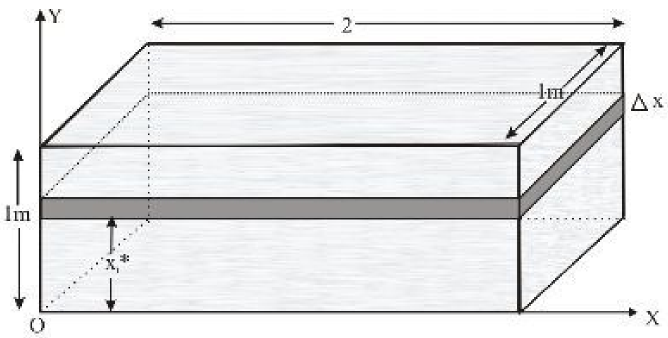


Fig.1

We divide the interval $\left[\frac{1}{2},1\right]$ in to n sub intervals with end points $x_0, x_1, ---x_n$ and choose x_i^* in the i^{th} subinterval. This divides half of the water in to n layers.

The width of the i^{th} layer is 1m and length is 2 m, and height is Δx

So volume of the i^{th} layer of water is

$$V_i = 1\times 2\times \Delta x = 2\Delta x$$

The force required to raise this layer must overcome the force of gravity so

$$F_i = m_i g = \text{density of water}\times\text{volume}\times g$$
$$\approx 1000\times 9.8\times 2\Delta x$$
$$\approx 19600\Delta x$$

This layer must cover a distance of approximately $(1-x_i^*)$

So work done to raise this layer to the top is

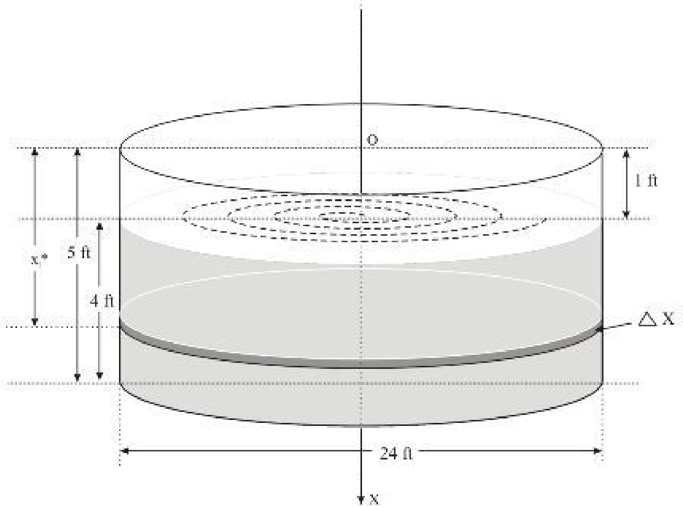
$$W_i \approx F_i\left(1-x_i^*\right)\approx 9800\times 2\left(1-x_i^*\right)\Delta x$$

So total work done is pumping half of the water out of the aquarium

$$W = \lim_{n\to\infty}\sum_{i=1}^{n} 19600\left(1-x_i^*\right)\Delta x$$
$$= \int_{1/2}^{1} 19600\,(1-x)\,dx$$
$$= 19600\left[x-\frac{x^2}{2}\right]_{1/2}^{1}$$
$$= 19600\left[1-\frac{1}{2}-\frac{1}{2}+\frac{1}{8}\right]$$

Or $\boxed{W = 2450\ J}$

Answer 20E.



Let's consider a vertical co-ordinate line and place the origin at the center of top of the pool. The total depth of water is 4 ft

So water extends from depth of 1 ft to depth of 5 ft

Now we divide the interval [1, 5] in to n sub intervals and choose x_i^* in the i^{th} sub interval. This divides the water in to n sub layers

The i^{th} layer is a circular cylinder with radius 12ft and height Δx

So the volume of i^{th} layer of water is

$$V_i \approx \pi(12)^2\Delta x = 144\pi\Delta x\ \text{ft}^3$$

And mass is $m_i = 62.5\times\text{volume}$

$$\approx 62.5\times 144\pi\Delta x = 9000\pi\Delta x$$

i^{th} layer of water must travel a distance of x_i^*. So work done to raise this layer to the top is $W_i \approx F_i x_i^* \approx 9000\pi x_i^*\Delta x$

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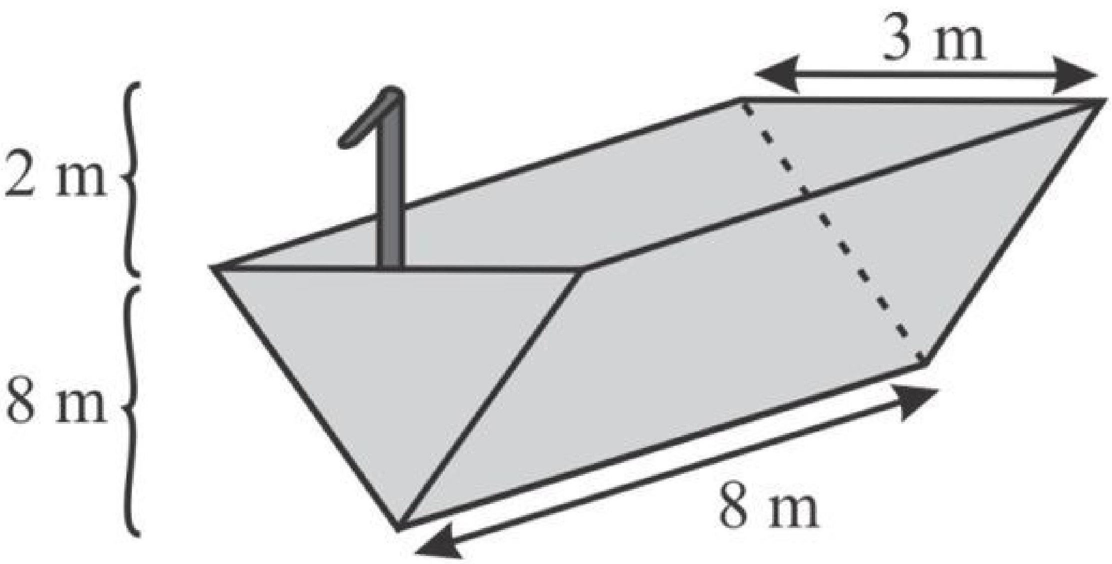
Answer 21E.

Consider the tank is full of water



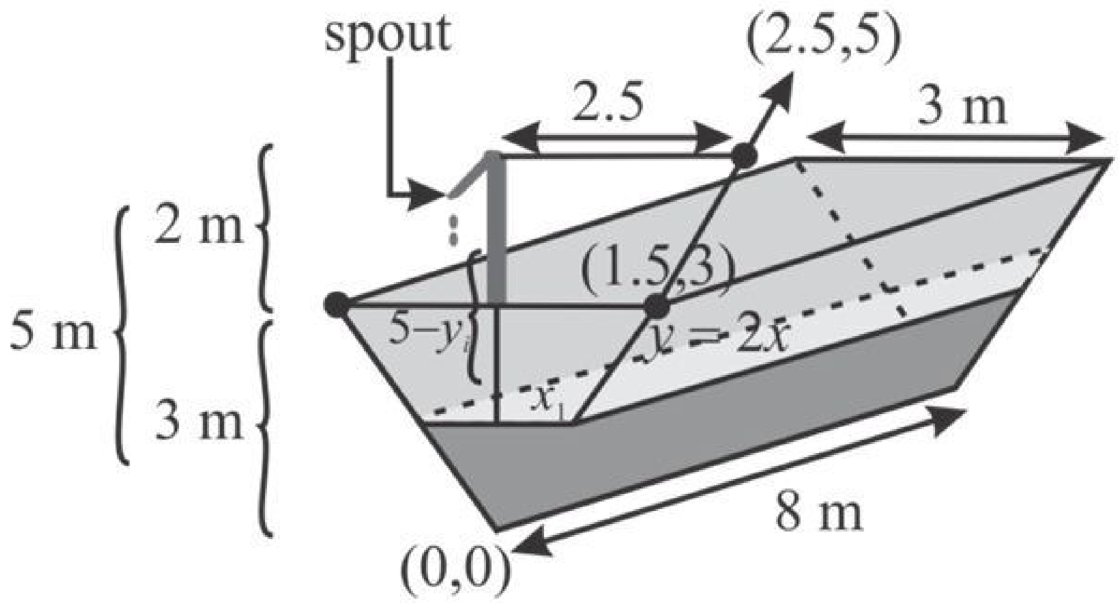
The objective is to find the work required to pump the water out of the spout.

Consider the height to be the sum of the heights of the tank and that of the sprout.

So, the height is 5m.

Assume that the water extends from a depth of 2m to a depth of 5m and so divide the interval [2, 5] into n subintervals with the endpoints $y_0, y_1, ..., y_n$ and choose y_i* in the i^{th} subinterval.

This divides the water into n layers.



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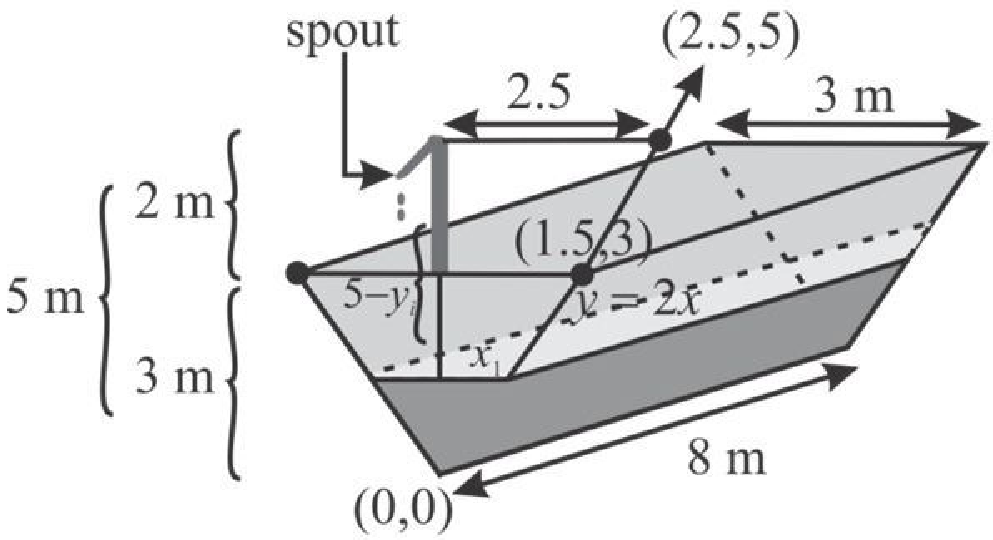


The force required to raise this layer must overcome the force of gravity and so

$$F_i = m_i g$$
$$\approx 9.8 \times 8000 \times (5 - y_i*)\Delta y$$

Each particle in the layer must travel a distance of approximately y_i*. The work W_i done to raise this layer to the top is approximately the product of the force F_i and the distance y_i*. Since the radius of the frustum of cone lower limit is 0 and the upper is 3

$$W_i = F_i y_i*$$
$$= 9.8 \times 8000 y_i* \times (5 - y_i*)\Delta y$$
$$= 78400(5y_i* - y_i*^2)\Delta y$$

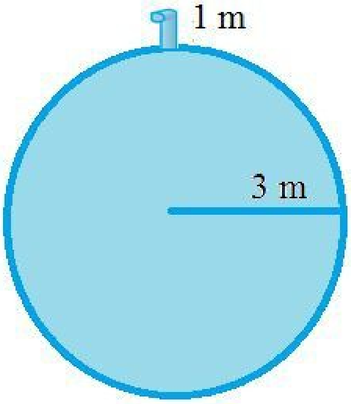
So,

$$W = \int_0^3 78400(5y - y^2)dy$$
$$= 78400\left(\frac{5y^2}{2} - \frac{y^3}{3}\right)_0^3$$
$$= 78400\left(\frac{5(3)^2}{2} - \frac{(3)^3}{3}\right)$$
$$= 78400\left(\frac{45}{2} - \frac{27}{3}\right)$$
$$= 78400(22.5 - 9)$$
$$= 78400(13.5)$$
$$= 1058.4 \times 10^3$$
$$\approx 1.06 \times 10^6 \text{ joules}$$

Hence, work required to pump the water out of the spout is $\boxed{1.06 \times 10^6 \text{ joules}}$.

Answer 22E.

A tank is full of water



We have to consider the horizontal cross section of the sphere in the form of Δy.

This has to be lifted to the vertical distance nothing but the height.

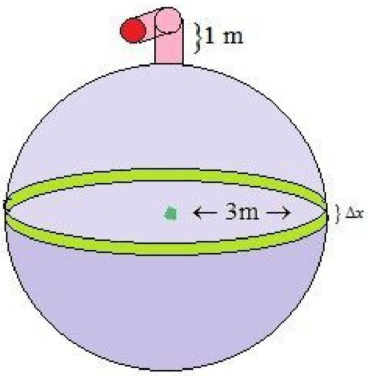
While the radius of the circular side is 3 units, assume the bottom of the tank is at (0, 0).

Then the centre of the tank is at (0, 0) and so, the equation of the circle is

$$(x-0)^2+(y-0)^2=3^2$$

Or, $x=\pm\sqrt{9-y^2}$

$-\sqrt{9-y^2}$ and $\sqrt{9-y^2}$



So, the width of the rectangular strip of depth in the tank is varying on the circular surface measuring

$$\sqrt{9-y^2}-\left(-\sqrt{9-y^2}\right)=2\sqrt{9-y^2}$$

Now, the area of the cross section of the sphere is

$$L(y)=2\sqrt{9-y^2}$$

This thin sheet of diesel has to be lifted to the height of the tank first and then to a height of 1 ft.

So, consider the height to which the diesel to be lifted =1 – *y*

That is the strip depth = 1 – *y*.

Also, the weight density of diesel is 62.5lb. /ft3.

Use these in the formula for force

$$F=\int_a^b w\cdot(\text{strip depth})\cdot L(y)\,dy$$

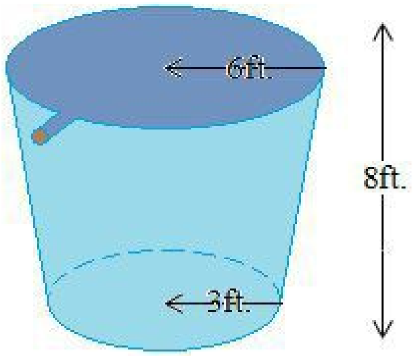
Therefore the work done is

$$=\int_{-3}^{3}62.5\left(2\sqrt{9-y^2}\right)(1-y)\,dy$$

$$=\boxed{1767.15\text{ J}}$$

Answer 23E.

A tank is full of water.



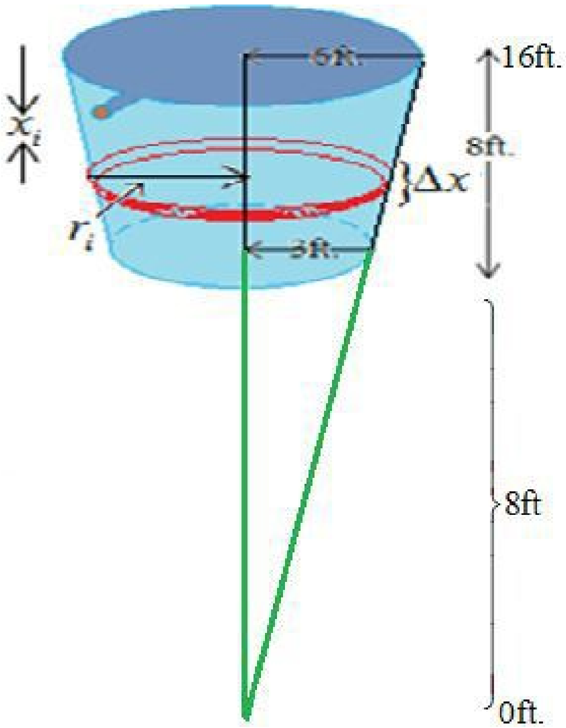
Observe that by going to a depth of 8ft., the radius of the conical tank is halved from 6 ft. to 3ft.

Since, the changes are linear, it can be easily seen that the radius becomes 0 when the conical shape has the height of 16 ft.

Let $x = x_i$ be measure depth in feet below the spout at the top of the tank.

A horizontal disk-shaped slice of water Δx ft thick and lying at coordinate $x = x_i$ has radius

So, the figure can conveniently be shown as



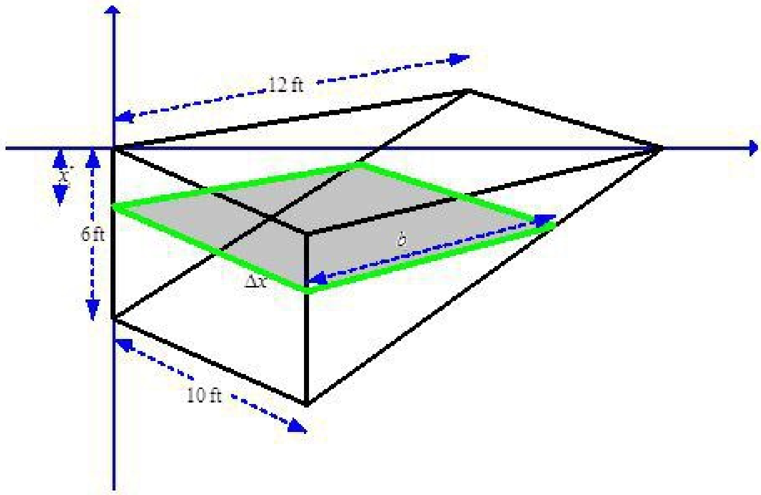
Answer 24E.

Given, that the tank has a base width of length of $10\,\text{ft}$ and a total depth of $6\,\text{ft}$.

The water extends from a depth of $0\,\text{ft}$ to a depth of $6\,\text{ft}$.

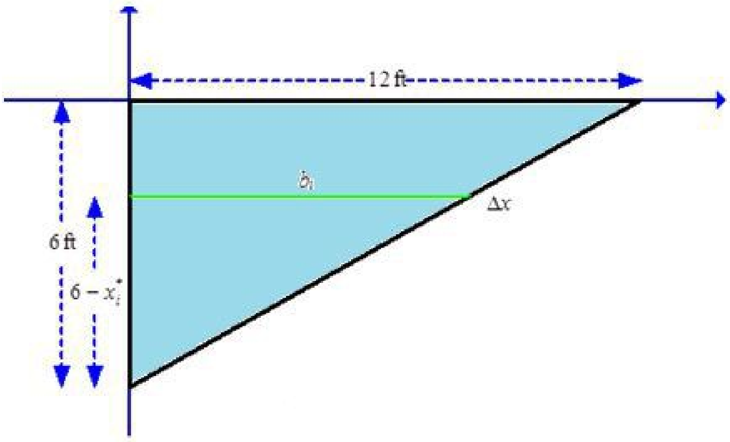
Find the work required to pump the water out of the spout.

Sketch the structure of the tank as shown below:



The water extends from a depth of $0\,\text{ft}$ to a depth of $6\,\text{ft}$. So, divide the interval $[0,6]$ into n subintervals with endpoints $x_0, x_1, \ldots, x_n$. This divides the water into n layers. Choose the i^{th} layer at a depth of x_i^*. The i^{th} layer is approximated by a rectangular plate with width b_i ft., length $10\,\text{ft}$, and height Δx.

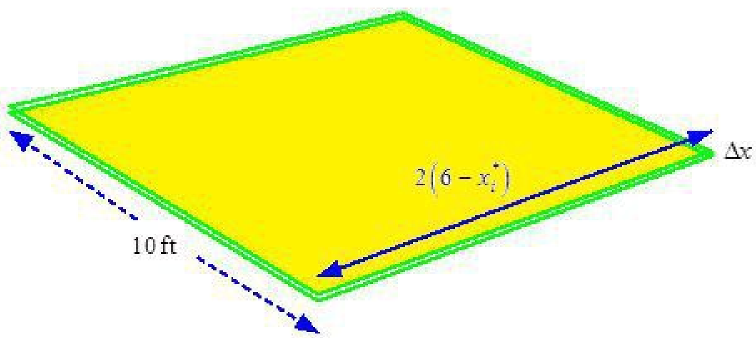
Compute b_i from similar triangles.



Using the conditions of similar triangles, the following is obtained:

$$\frac{b_i}{12} = \frac{\left(6 - x_i^*\right)}{6}$$
$$b_i = 2\left(6 - x_i^*\right)$$

Compute the volume of the i^{th} layer.



An approximation to the volume of the ith layer of water is computed as shown below:

$$\begin{aligned} V_i &\approx 10 \cdot 2\left(6 - x_i^*\right)\Delta x \\ &= 20\left(6 - x_i^*\right)\Delta x \end{aligned}$$

The force required to raise this layer is computed as under:

$$F_i = w \cdot V_i$$
$$\approx 62.5 \cdot 20\left(6 - x_i^*\right)\Delta x$$
$$= 1250\left(6 - x_i^*\right)\Delta x$$

Each particle in the layer must travel a distance of approximately x_i^*. The work W_i done to raise this layer to the top is approximately the product of the force F_i and distance x_i^*.

$$W_i \approx F_i x_i^*$$
$$\approx 1250 x_i^*\left(6 - x_i^*\right)\Delta x$$

Integrate to obtain the total work done.

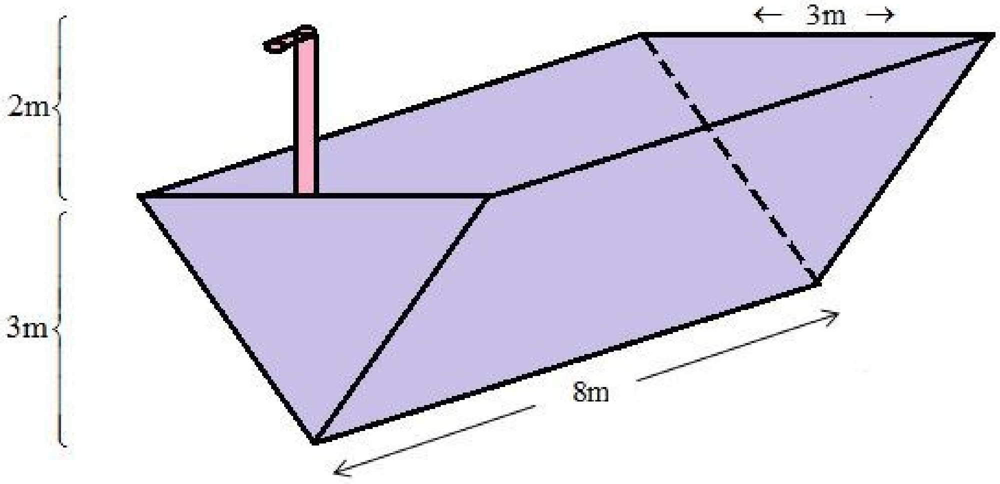
$$1250\int_0^6 \left(6x - x^2\right)dx = 1250\left[\frac{6}{2}x^2 - \frac{1}{3}x^3\right]_0^6$$ Since $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$= 1250\left(3(36) - \frac{216}{3}\right)$$
$$= 1250(108 - 72)$$
$$= 1250(36)$$
$$= 45000$$
$$\approx 4.5 \times 10^4 \ ft - lb$$

Therefore, $\boxed{\text{Total work} \approx 4.5 \times 10^4 \ ft - lb}$.

Answer 25E.

A tank is full of water.

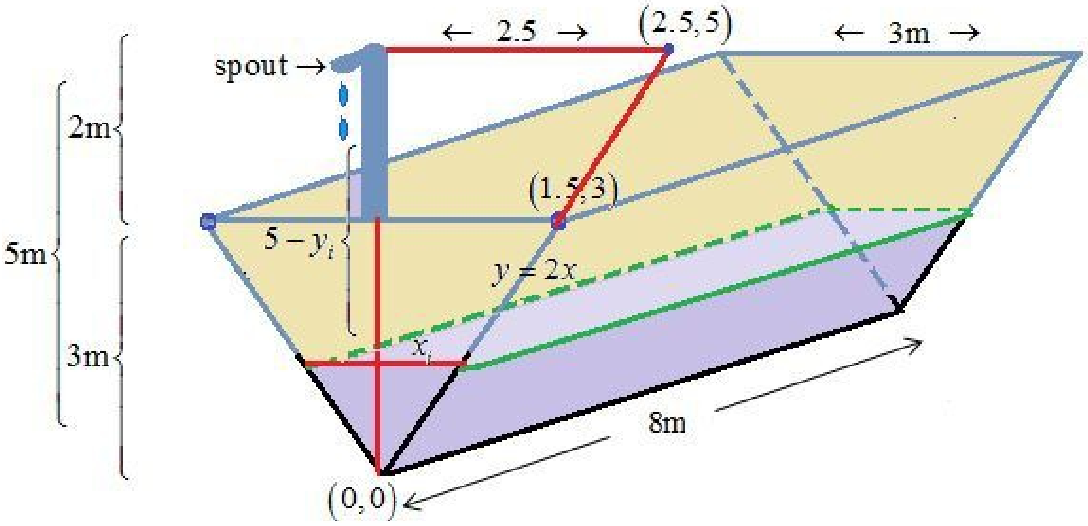


Consider the height to be the sum of the heights of the tank and that of the sprout.

So, the height is 5m.

Assume that the water extends from a depth of 2m to a depth of 5m. So, divide the interval [2, 5] into n subintervals with the endpoints, $y_0, y_1, ..., y_n$ and choose y_i* in the i^{th} subinterval.

This divides the water into n layers.



The i^{th} layer is approximated by a triangular surface on one side and lateral rectangular surface on the other as follows:

$$\frac{x_i}{5-y_i}=\frac{2.5}{5} \text{ Or, } r_i = x_i = \frac{5-y_i}{2}$$

Observe that the triangular side of the tank has the horizontal width $2x_i$ which is nothing but $5-y_i$ from the above equation.

So, volume of the i^{th} layer of the tank = 8($5-y_i*$) Δy.

Mass of i^{th} layer of water in the tank m_i = density $\times$ volume

$= 1000\times$ 8($5-y_i*$) Δy

The force required to raise this layer must overcome the force of gravity.

So, write as follows:

$$F_i = m_i g$$
$$\approx 9.8\times 8000\times(5-y_i*)\Delta y$$

Each particle in the layer must travel a distance of approximately, y_i*. The work W_i done to raise this layer to the top is approximately the product of the force F_i and the distance y_i*. Since the radius of the frustum of cone's lower limit is 0 and upper is 3

$$W_i = F_i y_i*$$
$$= 9.8\times 8000 y_i * \times(5-y_i*)\Delta y$$
$$= 78400\left(5y_i*-y_i*^2\right)\Delta y$$

So,

$$W = \int_0^3 78400\left(5y - y^2\right)dy$$

$$= 78400\left(\frac{5y^2}{2} - \frac{y^3}{3}\right)_0^3$$

$$= 78400\left(\frac{5(3)^2}{2} - \frac{(3)^3}{3}\right)$$

$$= 78400\left(\frac{45}{2} - \frac{27}{3}\right)$$

$$= 78400(22.5 - 9)$$

$$= 78400(13.5)$$

$$= 1058400$$

$$= 1058.4 \times 10^3$$

$$\approx 1.06 \times 10^6 \text{ joules}$$

But, the pump breaks down after 4.7×10^5 J of work is done.

Find the depth of the water remaining in the tank.

Let the water above a certain level, h be pumped out.

So that, $4.7 \times 10^5 = \int_h^3 78400\left(5y - y^2\right)dy$.

$$470000 = 78400\left(\frac{5y^2}{2} - \frac{y^3}{3}\right)_h^3$$

$$\frac{470000}{78400} = \left(\frac{5(3)^2}{2} - \frac{(3)^3}{3}\right) - \left(\frac{5(h)^2}{2} - \frac{(h)^3}{3}\right)$$

$$5.99 = (13.5) - \left(\frac{5(h)^2}{2} - \frac{(h)^3}{3}\right)$$

So,

$$W = \int_0^3 78400\left(5y - y^2\right)dy$$

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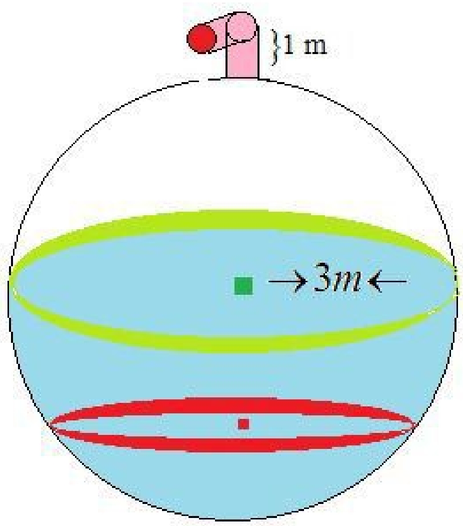
$$5.99 = (13.5) - \left(\frac{5(h)^2}{2} - \frac{(h)^3}{3}\right)$$

Answer 26E.

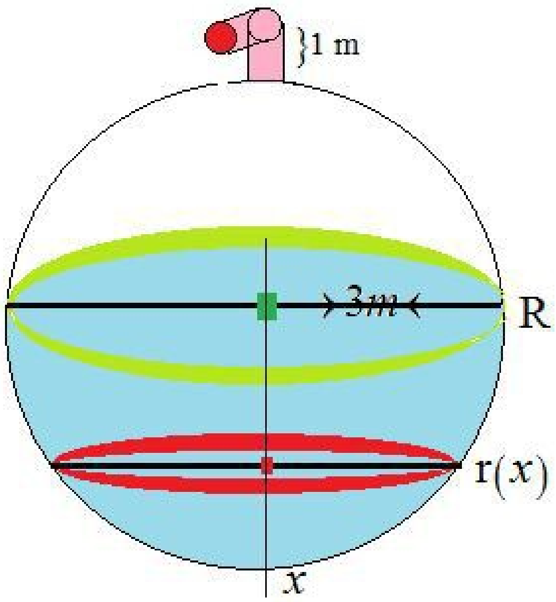
Consider the tank is half of oil that has a density of $900 kg / m^3$



Thus an approximation to the volume of the ith layer of water is $V_i \approx \pi r_i^2 \Delta x$

$$V_i = \pi\Delta x\left(R^2 - x_i^2\right)$$

$$m_i = V_i \text{density}$$



Distance travelled is $x+r+1$

$$r = x$$

$$x^2 + y^2 = R^2$$

$$x^2 + \left(r(x)\right)^2 = R^2$$

$$r(x) = \sqrt{R^2 - x^2}$$

To find the total work done with the tank is half full of oil.

$$W = \int_0^R (x+r+1)V(x)(\text{dinsity})g$$

$$= \int_0^R \pi(900)(9.8)(x+R+1)\left(R^2 - x^2\right)dx$$

$$= \pi(900)(9.8)\int_0^R (x+R+1)\left(R^2 - x^2\right)dx$$

$$= \pi(900)(9.8)\int_0^R \left(R^3 + R^2x + R^2 - Rx^2 - x^3 - x^2\right)dx$$

$$= \pi(900)(9.8)\left[\left(R^3 + R^2\right)x - \frac{(R+1)x^3}{3} + \frac{R^2}{2}x^2 - \frac{x^4}{4}\right]_0^R$$

Since $R = 3$ so that

$$W = \pi(900)(9.8)\left[\left(3^3 + 3^2\right)x - \frac{(3+1)x^3}{3} + \frac{3^2}{2}x^2 - \frac{x^4}{4}\right]_0^3$$

$$= \pi(900)(9.8)\left[\left(3^3 + 3^2\right)(3) - \frac{(3+1)3^3}{3} + \frac{3^2}{2}3^2 - \frac{3^4}{4}\right]$$

$$\approx 2.55 \times 10^6 \text{ J}$$

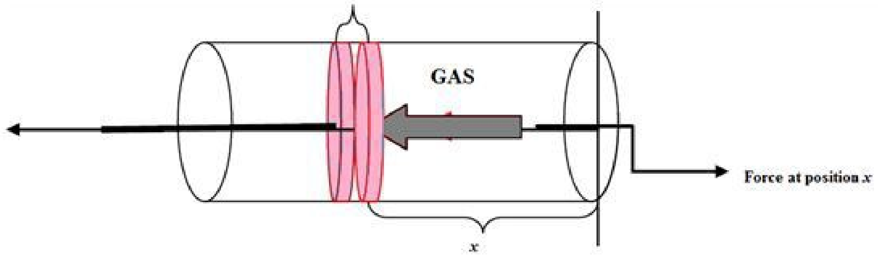
Answer 27E.

The radius of the cylinder is r.

The force exerted by the gas on the piston is,

$$F = \pi r^2 P$$

Here, P is the pressure.



The volume of the gas at the position x at time t is given by,

$$V(x) = \pi r^2 x$$

The force on the piston at position x must be,

$$\begin{aligned} F(V(x)) &= \pi r^2 P(V(x)) \\ &= \pi r^2 P \end{aligned}$$

Thus, if the piston changes from position $x = x_1$ to position $x = x_2$, then the volumes of the gas at these positions respectively,

$$V_1 = \pi r^2 x_1$$
$$V_2 = \pi r^2 x_2$$

Therefore, the work done on the position during small change Δx in position is,

$$\begin{aligned} \Delta_i W &= \text{Force} \times \text{Distance} \\ &= \pi r^2 P_i \Delta x \\ &= F_i \Delta x \\ &= F_i\left(\frac{1}{\pi r^2}\Delta V\right) \end{aligned}$$
$$= P_i \Delta V$$

Where, $x_1 < x_{i-1} \le \xi_i \le x_i < x_2$ and $V_1 < V_{i-1} \le V_i^* \le V_i < V_2$.

Then, the total work done of the piston by gas is then nearly,

$$\begin{aligned} \text{Total Work done} &\approx \sum_{i=1}^{n} F_i(V_i^*)\Delta x \\ &\approx \sum_{i=1}^{n} F_i(V_i^*)\frac{1}{\pi r^2}\Delta V \\ &\approx \sum_{i=1}^{n} P_i \Delta V \end{aligned}$$

Hence, the work done by the gas when the volume expands from volume expands from volume V_1 to volume V_2 is,

$$\begin{aligned} W &= \lim_{\max \Delta V \to 0} \sum_{i=1}^{n} P_i \Delta V \\ &= \int_{V_1}^{V_2} P(V)\,dV \\ &= \boxed{\int_{V_1}^{V_2} P\,dV} \end{aligned}$$

Answer 2E.

In a steam engine the pressure *P* and volume *V* of steam satisfy the equation,

$$PV^{1.4} = k$$

The work done by the gas when the volume expands from volume V_1 to volume V_2 is,

$$W = \int_{V_1}^{V_2} P(V)\,dV$$

Therefore,

$$\begin{aligned} W &= \int_{V_1}^{V_2} P(V)\,dV \\ &= \int_{V_1}^{V_2} \frac{k}{V^{1.4}}\,dV \qquad \left(\text{Since } PV^{1.4} = k\right) \\ &= k\left[-\frac{1}{0.4}V^{-0.4}\right]_{V_1}^{V_2} \\ &= 2.5k\left(V_1^{-0.4} - V_2^{-0.4}\right) \end{aligned}$$

The steam starts at pressure of $160\,\text{lb/in}^2$ and a volume of 100 in2 and expands to a volume of 800 in2

That is,

$$P_1 = 160 \text{ lb/in}^2$$
$$V_1 = 100 \text{ in}^3$$
$$V_2 = 800 \text{ in}^3$$

Now, find the work done using the above information.

$$\begin{aligned} W &= 2.5k\left(V_1^{-0.4} - V_2^{-0.4}\right) \\ &= 2.5P_1V_1^{1.4}\left(V_1^{-0.4} - V_2^{-0.4}\right) \qquad \left(PV^{1.4} = k\right) \\ &= 2.5(160)(100)^{1.4}\cdot\left(100^{-0.4} - 800^{-0.4}\right) \qquad \left(\text{Substitute values for } P_1, V_1, V_2\right) \\ &= (400)(630.9573)\cdot(0.1585 - 0.06899) \\ &= (252382.92)\cdot(0.0895) \\ &= \boxed{22590.79 \text{ lb-inch}} \end{aligned}$$

Answer 29E.

a)

Consider the two bodies with masses m_1 and m_2 attract each other with a force,

$$F = G\frac{m_1 m_2}{r^2}$$

Where, *r* is the distance between the bodies and *G* is the gravitational constant.

Then, the work done in moving object from *a* to *b* is,

$$\begin{aligned} W &= \lim_{n\to\infty}\sum_{i=1}^{n} F(x_i)\Delta x \\ &= \int_a^b F(x)\,dx \end{aligned}$$

Substitute expression for force in the above equation.

$$\begin{aligned} \text{Work} &= \int_a^b G\frac{m_1 m_2}{r^2}\,dr \\ &= Gm_1m_2\left[-\frac{1}{r}\right]_a^b \\ &= Gm_1m_2\left(\frac{1}{a} - \frac{1}{b}\right) \\ &= \boxed{\frac{Gm_1m_2(b-a)}{ab} \text{ Joules}} \end{aligned}$$

b)

A satellite of mass 1000 kg is to be launched at the vertical height of 1000km.

The mass of the earth and radius is given by,

$$m = 5.98\times10^{24}\,kg$$
$$r = 6.37\times10^{6}\,m$$

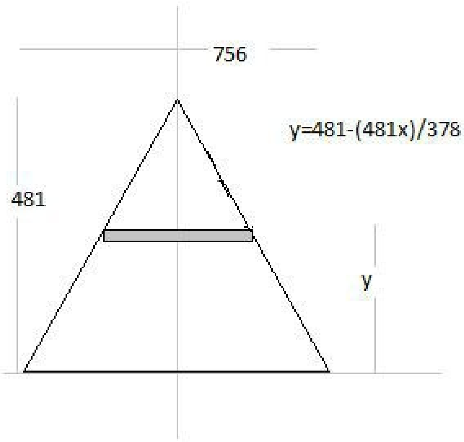
Then, the work required to launch the satellite is,

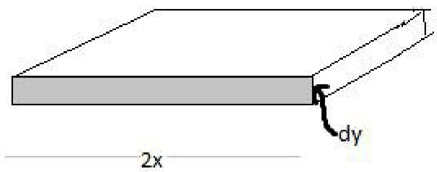
$$W = \frac{Gm_1m_2(b-a)}{ab}$$
$$= \frac{6.67\times10^{-11}\times1000\times5.98\times10^{24}\left(\left(1000\times10^{3}+6.37\times10^{6}\right)-6.37\times10^{6}\right)}{\left(6.37\times10^{6}\right)\left(1000\times10^{3}+6.37\times10^{6}\right)}$$
$$= \frac{39.89\times10^{16}\left(7.37\times10^{6}-6.37\times10^{6}\right)}{\left(7.37\times10^{6}\right)\left(6.37\times10^{6}\right)}$$
$$= \frac{39.89\times10^{16}\times10^{6}}{46.95\times10^{12}}$$
$$= 0.8491\times10^{10}$$
$$= \boxed{8.5\times10^{9}\ \text{J}}$$

Answer 30E.

The Great Pyramid of King Khufu was built of limestone in Egypt over a 20-year time period from 2580 BC to 2560 BC.

The base of the pyramid is square with side length 756ft and height 481ft.





The work done is given by,

$$W = Fd$$
$$dW = dF\,d$$
$$dW = (mg)(y)$$
$$dW = \rho gVy \qquad (m = \rho V) \qquad ......(1)$$

But the volume of the pyramid is,

$$V = \text{area}\times\text{thickness}$$
$$= (2x)^2\times dy$$

Here, $x = \frac{378}{481}(481-y)$.

Substitute expression for volume in equation (1).

$$\begin{aligned} dW &= \rho g(2x)^2(dy)y \\ &= 4\rho g x^2 y dy \\ &= 4\rho g\left(\frac{378}{481}\right)^2(481-y)^2 y dy \qquad \left(\text{since } x=\frac{378}{481}(481-y)\right) \\ &= \frac{571536}{231361}\rho g\left(y^2-962y+231361\right) y dy \end{aligned}$$

Integrate on both sides.

$$\begin{aligned} \int dW &= \frac{571536}{231361}\rho g\int_0^{481}\left(231361y+y^3-962y^2\right)dy \\ W &= \frac{571536}{231361}(150)(32.2)\left[\frac{231361y^2}{2}+\frac{y^4}{4}-\frac{962y^3}{3}\right]_0^{481} \\ &= \frac{571536}{231361}(150)(32.2)\left(\frac{53527912321}{12}\right) \\ &= \boxed{5.32\times10^{13}\ \text{ft-lb}} \end{aligned}$$

b)

If each laborer worked 10 hours a day for 20 years, for 340 days a year, then the work done by the each laborer is,

$$\begin{aligned} W_{each\ laborer} &= 200\frac{\text{ft-lb}}{\text{hr}}\left(\frac{10\text{hr}}{1\text{day}}\right)\left(\frac{340}{1\text{yr}}\right)(20) \\ &= 13600000\,\text{ft-lb} \end{aligned}$$

Hence, the number of workers needed to construct the pyramid is,

$$\begin{aligned} \text{Total workers} &= \frac{\text{Total work done}}{\text{work done by the laborers}} \\ &= \frac{W}{W_{each\ labor}} \\ &= \frac{5.23\times10^{13}}{13600000} \\ &= \boxed{3845588} \end{aligned}$$

Therefore, 3845588 workers are required to construct the pyramid.