1. Let p and q be two propositions. Then, the contrapositive of the implication $p \rightarrow q$ is

    a. $\sim q \rightarrow \sim p$

    b. $p \rightarrow q$

    c. $\sim p \rightarrow \sim q$

    d. $p \leftrightarrow q$

2. $\sim (\sim p) \leftrightarrow p$ is

    a. none of these

    b. a tautology

    c. neither a contradiction nor a tautology

    d. a contradiction

3. The statement $p \Rightarrow p \vee q$ is

    a. neither a contradiction nor a tautology

    b. a contradiction

    c. both a tautology and contradiction

    d. a tautology

4. Let p and q be two prepositions given by p: It is hot, q: He wants water Then, the verbal meaning of $p \rightarrow q$ is

    a. it is hot or he wants water.

    b. it is hot and he wants water.

    c. If it is hot, then he wants water.

    d. If and only if it is hot, he wants water.

5. Which of the following is logically equivalent to $p \wedge q$?

    a. $\sim (\sim p \wedge \sim q)$

    b. none of these

    c. $(p \wedge \sim q)$

    d. $\sim (p \rightarrow \sim q)$

6. Fill in the blanks:

    The _____ of a conjunction p $\wedge$ q is the disjunction of the negation of p and the negation of q.

7. Fill in the blanks:

    The truth value of the negation of the statement "r : 2 + 3 < 4" is _____.

8. Find out below sentence is a statement or not. justify your answer.
    The cat pussy is black

9. Write the component statements of the compound statements and check whether the compound statement is true or false:
    The sand heats up quickly in the sun and does not cool down fast at night.

10. Determine the statement is whether an inclusive OR or exclusive OR is used. Give reasons for your answer.
    "To enter a country, you need a passport or a voter registration card"

11. Write the negation of the statement 7 is greater than 4 or 6 is less than 7.

12. Write the negation of the following statements.

    i. If I become a doctor, then I will open a hospital.
    ii. If 2 + 3 = 5, then 5 is an odd number.

13. Show that the following statement is true by the method of contrapositive:

    p: If x is an integer and x^2 is even, then x is also even.

14. By giving a counter example, show that the following statements are not true.
    (i) p: "If all the angles of a triangle are equal then the triangle is an obtuse-angled triangle."

    (ii) q : "The equation x^2 - 1 = 0 does not have a root lying between 0 and 2".

15. Are the following pairs of statements negations of each other:
    (i) The number x is not a rational number.
    The number x is not an irrational number.
    (ii) The number x is a rational number.
    The number x is an irrational number.

Solution

1. (a) $\sim q \rightarrow \sim p$

   Explanation:

   the contrapositive of $p \rightarrow q \; is \; \sim q \rightarrow \sim p$

2. (b) a tautology

   Explanation:

   $\sim (\sim p) \equiv p$

   $T \leftrightarrow T \equiv T$

   $F \leftrightarrow F \equiv T$

3. (d) a tautology

   Explanation:

p	q	$p \vee q$	$p \Rightarrow p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

4. (c) If it is hot, then he wants water.

   Explanation:

   $\rightarrow$ symbol is replaced by if p then q

5. (d) $\sim (p \rightarrow \sim q)$

   Explanation:

   $\sim (\sim p \vee q)$ since $p \rightarrow q \equiv \sim p \vee q$

$p \wedge q$ since $\sim (p \vee q) \equiv \sim p \wedge \sim q$

6. negation

7. True

8. A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.
   Some cats are black and some not. So, the given sentence may or may not be true. Hence, it is not a statement.

9. The component statements of the compound statement, "The sand heats up quickly in the sun and does not cool down fast at night" are:
   p: The sand heats up quickly in the sun.
   q: The sand does not cool down fast at night.
   The compound statement p and q is false because p is true and q is false.

10. **In the statement:** "To enter a country, you need a passport or a voter registration card"
    An inclusive "OR" is used because a person can have both a passport and a voter registration card to enter a country.

11. Write p : 7 is greater than 4.
    and q: 6 is less than 7.
    Then, the disjunction is $p \vee q$.
    Now ~p : 7 is not greater than 4.
    and ~q : 6 is not less than 7.
    So, a negation of $p \vee q$ is given by
    $-(p \vee q) = 7$ is not greater than 4 and 6 is not less than 7.

12. i. Let p and q be the component statements.
       p : I become a doctor.
       q : I will open a hospital.
       The given proposition is $p \Rightarrow q$
       Now, $\sim (p \Rightarrow q) \equiv p \wedge (\sim q)$
       Therefore, the negation of the given proposition is 'I will become a doctor and I

will not open a hospital.'

ii. Similarly, we get, p:2 + 3 = 5

and q:5 is an odd number.

$$\therefore \sim (p \Rightarrow q) \equiv p \wedge \sim q$$

So, 2 + 3 = 5 and 5 is an even number.

13. Let q and r be the statements given by,

q: If x is an integer and x^2 is even

r : x is an even integer.

Then, p : "If q, then r".

If possible, let r be false. Then,

r is false.

$\Rightarrow$ x is not an even integer

$\Rightarrow$ x is an odd integer

$\Rightarrow$ x = (2n + 1) for some integer n

$\Rightarrow x^2 = 4n^2 + 4n + 1$

$\Rightarrow x^2 = 4n (n + 1) + 1$

$\Rightarrow x^2$ is an odd integer [$\because$ 4n (n + 1) is even]

$\Rightarrow$ q is false.

Thus, r is false $\Rightarrow$ q is false.

Hence, p : "If q, then r" is a true statement.

14. (i) Since the triangle is obtuse-angled triangle then $\theta > 90°$

Let $\theta = 100°$

Also all the angles of the triangle are equal.

$\therefore$ Sum of all angles of the triangle are 300° which is not possible.

Thus the given compound statement is not true.

(ii) We see that x = 1 is a root of the equation x^2 - 1 = 0, which lies between 0 and 2.

Thus the given compound statement is not true.

15. Let p: The number x is not a rational number.

q : The number x is not an irrational number.

Now ~ p: The number x is a rational number.

~ q : The number x is a irrational number.

∴ ~ p = q and ~ q = p

Thus p and q are negations of each other.

(ii) Let p: The number x is a rational number.

q: The number x is an irrational number.

Now ~ p: The number x is not a rational number.

~ p: The number x is not an irrational number.

∴ ~ p = q and ~ q = p

Thus p and q are negations of each other.