# MISCELLANEOUS EXERCISE
## CHAPTER 10.

**Q No 1:** Find the value of $k$ for which the line $(k-3)x-(4-k^2)y+(k^2-7k+6)=0$ is
(i) Parallel to $x$-axis (ii) Parallel to $y$-axis (iii) Passes through origin

**Soln:** (i) For line to be parallel to $x$-axis will be of for $y=b$
ie Coeff of $x$ is zero.
ie $k-3=0$ or $k=3$

(ii) Line parallel to $y$-axis is of form $x=p$
ie Coeff of $y$ is zero. ie $4-k^2=0$ ie $k^2=4$
or $k=\pm 2$.

(iii) Since given line passes through origin $O(0,0)$
$\therefore$ $O(0,0)$ will satisfy eqn. of line.
ie $0+0+k^2-7k+6=0 \Rightarrow k=\frac{-(-7)\pm\sqrt{(-7)^2-4\times1\times6}}{2}$

$$k=\frac{7\pm\sqrt{49-24}}{2}=\frac{7\pm5}{2}\Rightarrow\frac{12}{2},\frac{2}{2} \text{ ie } 6,1.$$

**Q No 2:** Find the value of $\theta$ and $p$ if the equation $x\cos\theta+y\sin\theta=p$ is normal form of line. $\sqrt{3}x+y+2=0$

**Soln:** The given line is $\sqrt{3}x+y+2=0$
or $-\sqrt{3}x-y=2$
Dividing both sides by $\sqrt{(-\sqrt{3})^2+(-1)^2}=\sqrt{3+1}=2$

$$-\frac{\sqrt{3}}{2}x-\frac{1}{2}y=\frac{2}{2}$$

Comparing $\cos\theta=-\frac{\sqrt{3}}{2}$, $\sin\theta=-\frac{1}{2}$, $p=1$

$$\Rightarrow \theta=\frac{7\pi}{6} \text{ and } p=1$$

**Q No 3:** Find the eqn of lines, which cut-off intercepts on axes whose sum and product are 1 and $-6$ respectively.

**Soln:** - Let the intercepts be $a$ and $b$
ATQ $a+b=1$ and $ab=-6$

$\Rightarrow a(1-a)=-6 \Rightarrow a-a^2=-6 \Rightarrow a^2-a-6=0$

$\Rightarrow a^2-3a+2a-6=0 \Rightarrow (a-3)(a+2)=0$

$\Rightarrow a=3,-2$

when $a=3$; $b=1-a=1-3=-2$

when $a=-2$; $b=1-(-2)=1+2=3$

$\therefore$ Reqd lines are $\frac{x}{3}-\frac{y}{2}=1$ and. $\frac{x}{-2}+\frac{y}{3}=1$

**Q No 4**: what are the points on the y-axis whose distance from line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units.

Sol: Let Any point on y-axis is $A(0, y_1)$

$\therefore$ ⊥r distance of $A(0,y_1)$ from $\frac{x}{3}+\frac{y}{4}=1$ or $4x+3y-12=0$

$$\frac{|0+3y_1-12|}{\sqrt{(4)^2+(3)^2}}=4. \Rightarrow |3y_1-12|=5\times4$$

$$\Rightarrow 3y_1-12=\pm20 \Rightarrow 3y_1-12=+20 \text{ or } 3y_1-12=-20$$

$$\Rightarrow 3y_1=32 \text{ or } 3y_1=-8$$

$$\Rightarrow y_1=\frac{32}{3} \text{ or } y_1=-\frac{8}{3}$$

$\therefore$ Reqd pts are $(0,\frac{32}{3})$ ; $(0,-\frac{8}{3})$

**Q No 5**: Find the ⊥r distance from the origin to the line joining the pts $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$

Soln: Eqn of line through given pts will be.

$$y-\sin\theta=\frac{\sin\phi-\sin\theta}{\cos\phi-\cos\theta}(x-\cos\theta) \quad \left[\text{Two-point form}\right]$$

ie $$y-\sin\theta=\frac{2\cos\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}}{-2\sin\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}}(x-\cos\theta) \quad \left[\text{Using C-D formulae}\right]$$

ie $$y-\sin\theta=-\frac{\cos\frac{\theta+\phi}{2}}{\sin\frac{\theta+\phi}{2}}(x-\cos\theta)$$

or $$\sin\left(\frac{\theta+\phi}{2}\right)(y-\sin\theta)+\cos\left(\frac{\theta+\phi}{2}\right)(x-\cos\theta)=0$$

or. $$\sin\left(\frac{\theta+\phi}{2}\right)y+\cos\left(\frac{\theta+\phi}{2}\right)x-\left(\sin\frac{\theta+\phi}{2}\sin\theta+\cos\frac{\theta+\phi}{2}\cos\theta\right)=0$$

or. $$\cos\left(\frac{\theta+\phi}{2}\right)x+\sin\left(\frac{\theta+\phi}{2}\right)y-\cos\left(\frac{\theta+\phi}{2}-\theta\right)=0$$

or $$\cos\left(\frac{\theta+\phi}{2}\right)x+\sin\left(\frac{\theta+\phi}{2}\right)y-\cos\left(\frac{\phi-\theta}{2}\right)=0.$$

$\therefore$ Length of perpendicular from the origin to the line(1)

$$=\frac{\left|0+0-\cos\left(\frac{\phi-\theta}{2}\right)\right|}{\sqrt{\cos^2\left(\frac{\theta+\phi}{2}\right)+\sin^2\left(\frac{\theta+\phi}{2}\right)}}=\frac{\left|\cos\left(\frac{\theta-\phi}{2}\right)\right|}{1.}$$

$$= \left| \frac{2\sin\frac{\theta-\phi}{2}\cos\frac{\theta-\phi}{2}}{2\sin\frac{\theta-\phi}{2}} \right| = \frac{|\sin\theta-\phi|}{2|\sin\frac{\theta-\phi}{2}|}$$

**Q No 6**: Find the eqn of line ∥ to y-axis and drawn through the point of intersection of the lines $x-7y+5=0$ and $3x+y=0$

**Sol**: Solving $x-7y+5=0$ and $3x+y=0$ for point of intersection.

$$x-7(-3x)+5=0 \Rightarrow x+21x+5=0 \Rightarrow 22x=-5$$

$$\Rightarrow x = -\frac{5}{22}.$$

Now Any line ∥ to y-axis is $x = x_1$

Here $x = -\frac{5}{22}$.

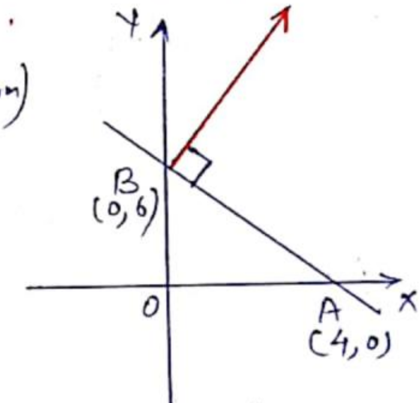
$\therefore$ Reqd eqn is $x + \frac{5}{22} = 0$ or $22x+5=0$

**Q No 7**: Find the eqn of line drawn ⊥r. to the line $\frac{x}{4}+\frac{y}{6}=1$ through the point, where it meets y-axis.

**Soln**: The given line is $\frac{x}{4}+\frac{y}{6}=1$. (Intercept form)

$\therefore$ It meets x axis at $A(4,0)$

and y-axis at $B(0,6)$




$\therefore$ Slope of $AB = \frac{0-6}{4-0} = -\frac{3}{2}$.

$\therefore$ slope of line ⊥r to $AB = \frac{2}{3}$.

$\therefore$ Eqn of line passing through $(0,6)$ having slope $\frac{2}{3}$ will be.

$$y-6 = \frac{2}{3}(x-0)$$

$$\Rightarrow 3y-18 = 2x \quad \text{or} \quad 2x-3y+18=0$$

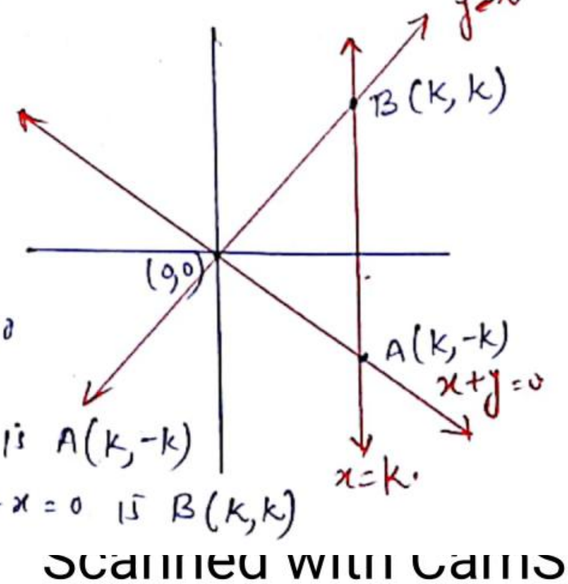
**Q No. 8**: Find the area of Δ formed by lines $y-x=0$; $x+y=0$ and $x-k=0$

**Soln**: Solving $y-x=0$

$x+y=0$

$x-k=0$ for

points of intersection we get




The point of intersection of $y-x=0$ and $x+y=0$ is $O(0,0)$

The point of intersection of $x+y=0$ and $x=k$ is $A(k,-k)$

and The point of intersection of $x=k$ and $y-x=0$ is $B(k,k)$

$\therefore$ Vertices of required $\Delta$ are $O(0,0)$, $A(k,-k)$, $B(k,k)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\begin{vmatrix} 0 & 0 \\ k & -k \\ k & k \\ 0 & 0 \end{vmatrix} = \frac{1}{2}\left[0+0+k^2+k^2+0+0\right] = \frac{1}{2}\left[2k^2\right]$

$= k^2$ square units.

**Q No 9**: Find the value of $p$ so that the three lines $3x+y-2=0$, $px+2y-3=0$ and $2x-y-3=0$ may intersect at one point.

**Sol.**: Solving $3x+y-2=0$ and $2x-y-3=0$ for point of intersection.

$3x+y=2 \quad -(1)$

$2x-y=3 \quad -(2)$

Adding $5x=5 \Rightarrow x=1$.

Put $x=1$ in $3x+y=2$ we get $3(1)+y=2$ or $y=2-3=-1$

$\therefore$ (1) and (2) intersect at point $(1,-1)$

This point will lie on the line $px+2y-3=0$

$\Rightarrow \quad p\times 1+2\times(-1)-3=0$

$\Rightarrow \quad p-2-3=0 \Rightarrow p-5=0$ or $p=5$.

**Q No 10**: If three lines whose eqns are $y=m_1x+c_1$, $y=m_2x+c_2$ and $y=m_3x+c_3$ are concurrent, then show that

$$m_1(c_2-c_3)+m_2(c_3-c_1)+m_3(c_1-c_2)=0$$

**Soln**: The given lines are $y=m_1x+c_1 \quad -(1)$

$y=m_2x+c_2 \quad -(2)$

$y=m_3x+c_3 \quad -(3)$

Subtracting (2) from (1) we get

$0=(m_1-m_2)x+(c_1-c_2)$

$\Rightarrow \quad x=-\frac{c_1-c_2}{m_1-m_2}=\frac{c_2-c_1}{m_1-m_2}$

Putting in (1) we get

$y=m_1\left(\frac{c_2-c_1}{m_1-m_2}\right)+c_1=\frac{m_1(c_2-c_1)+c_1(m_1-m_2)}{m_1-m_2}$

$=\frac{m_1c_2-\cancel{m_1c_1}+\cancel{m_1c_1}-c_1m_2}{m_1-m_2}=\frac{m_1c_2-m_2c_1}{m_1-m_2}$

$\therefore$ lines (1) and (2) intersect at point $\left(\frac{c_2-c_1}{m_1-m_2}, \frac{m_1c_2-m_2c_1}{m_1-m_2}\right)$

Since the lines (1), (2) and (3) meet in a point

$\therefore$ Pt $\left(\frac{c_2-c_1}{m_1-m_2}, \frac{m_1c_2-m_2c_1}{m_1-m_2}\right)$ lies on (3)

$$\therefore \quad \frac{m_1c_2-m_2c_1}{m_1-m_2} = m_3\left(\frac{c_2-c_1}{m_1-m_2}\right)+c_3$$

$$\therefore \quad m_1c_2-m_2c_1 = m_3c_2-m_3c_1+m_1c_3-m_2c_3$$

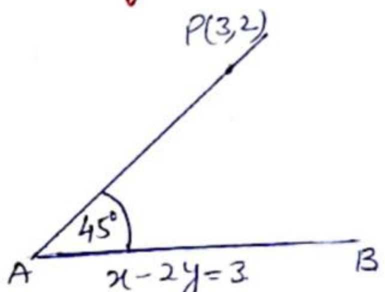
$$\therefore \quad m_1c_2-m_1c_3+m_2c_3-m_2c_1+m_3c_1-m_3c_2=0$$

$$\therefore \quad m_1(c_2-c_3)+m_2(c_3-c_1)+m_3(c_1-c_2)=0$$

which is required condition.

**Q.No 11.** Find the eqn. of the lines through the point $(3,2)$ which make an angle of $45^\circ$ with the line $x-2y=3$

Soln: Slope of line $x-2y=3$ is $-\frac{1}{-2}=\frac{1}{2}$.




Let slope of required line be $m$.

Now Angle between lines $=45^\circ$

$$\therefore \quad \tan 45^\circ = \pm\frac{m-\frac{1}{2}}{1+m\times\frac{1}{2}} \qquad \left[\because \tan\theta = \pm\frac{m_1-m_2}{1+m_1m_2}\right]$$

$$\therefore \quad 1 = \pm\frac{2m-1}{2+m}$$

$$\Rightarrow \frac{2m-1}{2+m}=1 \quad ; \quad \frac{2m-1}{2+m}=-1$$

$$\Rightarrow 2m-1=2+m \quad ; \quad 2m-1=-2-m$$

$$\Rightarrow m=3 \quad ; \quad 3m=-1$$

$$\Rightarrow m=3 \quad ; \quad m=-\frac{1}{3}.$$

$\therefore$ When $m=3$, eqn of line through $P(3,2)$ will be

$y-2=3(x-3)$ ie $y-2=3x-9$ ie $3x-y-7=0$

When $m=-\frac{1}{3}$, eqn of line through $P(3,2)$ will be

$y-2=-\frac{1}{3}(x-3)$ ie $3y-6=-x+3$ ie $x+3y-9=0$

**Q.No 12:** Find the equation of line passing through the point of intersection of lines $4x+7y-3=0$ and $2x-3y+1=0$ has that has equal intercepts on axes.

**Soln:** Solving $4x+7y-3=0$ — ①
$2x-3y+1=0$ — ② for point of intersection

$$\begin{array}{r} 4x+7y-3=0 \\ \cancel{4x}-6y+2=0 \\ \underset{-}{\;} \quad \underset{+}{\;} \quad \underset{-}{\;} \\ \hline 13y-5=0 \end{array} \quad (\text{multiplying ② by 2})$$

$\Rightarrow y = \frac{5}{13}$.

Putting $y=\frac{5}{13}$ in ②

$$2x - 3\times\frac{5}{13} + 1 = 0$$

or $26x-15+13=0$ or $26x-2=0$ or $26x=2 \Rightarrow x=\frac{2}{26}=\frac{1}{13}$

Now let the required line make equal intercept a on both axes.

$\therefore$ Intercept form of line is $\frac{x}{a}+\frac{y}{a}=1$ or $x+y=a$.

Since it passes through intersection of ① and ② ie $\left(\frac{1}{13},\frac{5}{13}\right)$

$$\therefore \quad \frac{1}{13}+\frac{5}{13}=a \quad \Rightarrow a=\frac{6}{13}$$

$\therefore$ Required line is $x+y=\frac{6}{13}$ or $13x+13y=6$.

**Q No 13:** Show that the equation of line passing through the origin and making an angle $\theta$ with the line $y=mx+c$ is

$$\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}.$$

**Sol:** The eqn of any line through origin $O(0,0)$ is
$y = Mx$.

$\because$ It makes an angle $\theta$ with the line $y=mx+c$

$$\tan\theta = \pm \frac{M-m}{1+Mm}.$$

$\Rightarrow \quad \tan\theta(1+Mm) = M-m, \; -M+m.$

$\Rightarrow \quad \tan\theta + Mm\tan\theta = M-m, \quad \tan\theta + Mm\tan\theta = -M+m.$

$\Rightarrow \quad M - Mm\tan\theta = m+\tan\theta, \quad M + Mm\tan\theta = m - \tan\theta.$

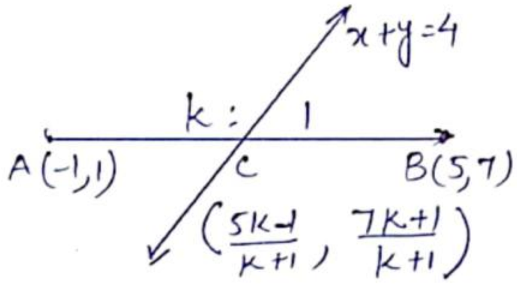
$$\Rightarrow \quad M = \frac{m+\tan\theta}{1-m\tan\theta}, \quad M = \frac{m-\tan\theta}{1+m\tan\theta}.$$

$\therefore$ eqn of line is $y = \dfrac{m + \tan\theta}{1 - m\tan\theta}x$ , $y = \dfrac{m - \tan\theta}{1 + m\tan\theta}x$.

or $\dfrac{y}{x} = \dfrac{m \pm \tan\theta}{1 \mp m\tan\theta}$.

**QNo. 14:** In what ratio, the line joining $(-1, 1)$ and $(5, 7)$ is divided by the line $x + y = 4$.

**Soln:** Let the line $x + y = 4$ divides the line joining $A(-1, 1)$ and $B(5, 7)$ in ratio $k:1$ at point C.




$\therefore$ Coordinates of C are

$$C\left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right) \quad \text{[Using Section formula]}$$

Now Since C lies on $x + y = 4$

$$\Rightarrow \frac{5k-1}{k+1} + \frac{7k+1}{k+1} = 4$$

$$\Rightarrow \frac{5k-1+7k+1}{k+1} = 4 \Rightarrow \frac{12k}{k+1} = 4$$

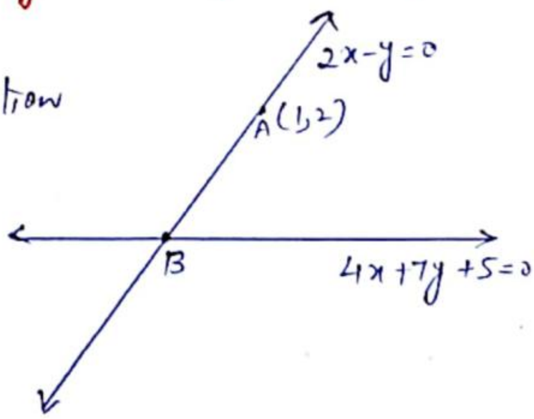
$$\Rightarrow 12k = 4(k+1) \Rightarrow 12k = 4k + 4$$

$$\Rightarrow 8k = 4 \Rightarrow k = \frac{1}{2}$$

$\therefore$ Ratio is $\frac{1}{2} : 1$ or $1:2$.

**QNo 15:** Find the distance of line $4x + 7y + 5 = 0$ from the point $(1, 2)$ along the line $2x - y = 0$

**Sol:** Let B be the point of intersection of lines $2x - y = 0$ — ①
and $4x + 7y + 5 = 0$ — ②




Solving ① and ②

$$4x + 7(2x) + 5 = 0$$

$$\Rightarrow 4x + 14x + 5 = 0$$

$$\Rightarrow 18x + 5 = 0 \Rightarrow x = -\frac{5}{18}$$

$$\therefore y = 2x = 2 \times \frac{-5}{18} = \frac{-10}{18}.$$

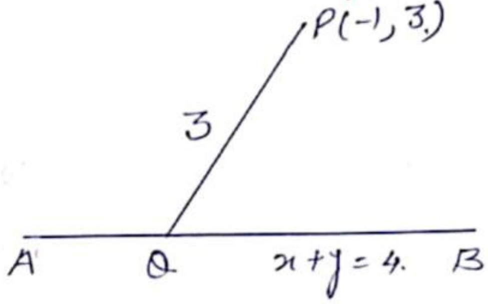
$\therefore$ B is $B\left(\frac{-5}{18}, \frac{-10}{18}\right)$

$\therefore$ Distance of $4x+7y+5=0$ from $A(1,2)$ along $2x-y=0$

$= |AB| = \sqrt{\left(1+\frac{5}{18}\right)^2+\left(2+\frac{10}{18}\right)^2}$

$= \sqrt{\left(\frac{23}{18}\right)^2+\left(\frac{46}{18}\right)^2} = \sqrt{\frac{(23)^2}{(18)^2}\left[1+(2)^2\right]} = \sqrt{\left(\frac{23}{18}\right)^2(5)} = \frac{23}{18}\sqrt{5}$ units

**Q No. 16.** Find the direction in which a straight line must be drawn through the point $(-1,2)$ so that its point of intersection with the line $x+y=4$ may be at a distance of 3 units from this point.




Sol. Let the equation of AB be

$x+y=4$ — (1)

Let $m$ be the slope of line through $P(-1,2)$ and meeting AB in Q

$\Rightarrow$ n of PQ is $y-2=m(x+1)$

or $y = mx+m+2$ — (2)

To find ~~quad~~ coordinates of Q, we solve (1) and (2)

From (1) and (2) $x+mx+m+2=4$

$\Rightarrow x(1+m) = 2-m$

$\Rightarrow x = \frac{2-m}{1+m}$

$\therefore y = 4-x = 4-\frac{2-m}{1+m} = \frac{4+4m-2+m}{1+m}$

ie $y = \frac{5m+2}{1+m}$

$\therefore$ Q is $Q\left(\frac{2-m}{1+m}, \frac{5m+2}{1+m}\right)$

Now $PQ = 3$

$\Rightarrow PQ^2 = 9$

$\Rightarrow \left(\frac{2-m}{1+m}+1\right)^2+\left(\frac{5m+2}{1+m}-3\right)^2 = 9$ [Distance formula]

$\Rightarrow \left(\frac{2-m+1+m}{1+m}\right)^2+\left(\frac{5m+2-3-3m}{1+m}\right)^2 = 9$

$$\Rightarrow \frac{9}{(1+m)^2} + \frac{9m^2}{(1+m)^2} = 9$$

$$\Rightarrow \frac{9(1+m^2)}{(1+m)^2} = 9 \Rightarrow 1+m^2 = (1+m)^2$$

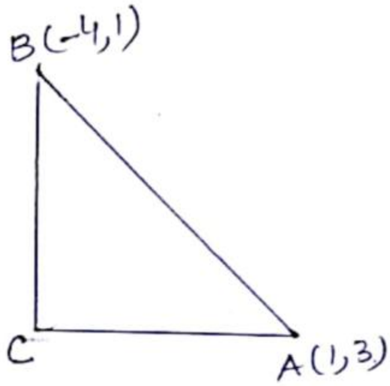
$$\Rightarrow 1+m^2 = 1+m^2+2m.$$

$$\Rightarrow m = 0.$$

$\therefore$ slope of line is $m = 0$

$\Rightarrow$ line is parallel to $x$-axis.

QNo 17: The hypotenuse of a st ∠d Δ has its ends at $(1,3)$ and $(-4,1)$ Find eqn of legs of Δ.




Soln: Let Δ ACB be st ∠d Δ so that $\angle C = 90°$.

Let slope of $AC = m$

$\therefore$ slope of $BC = -\frac{1}{m}$.

Now Eqn of AC will be

$$y-3 = m(x-1)$$

And equation of B is $y-1 = -\frac{1}{m}(x+4)$

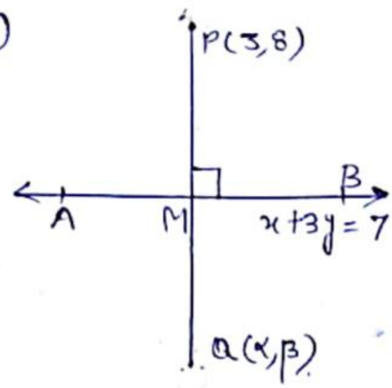
Here $m$ can assume many values. and so there will be many equations of legs AC and BC.

QNo 18. Find the image of point $(3,8)$ with respect to line $x+3y=7$. assuming the line to be a plane mirror.

Soln: Let eqn of line AB is $x+3y=7$ —(1)

From $P(3,8)$ draw $PM \perp AB$ and produce it to Q so that M is mid-point of PQ.

So Q is image of P.




Now slope of $AB = -\frac{1}{3}$.

Since $PM \perp AB$.

$\therefore$ slope of $PM = 3$.

$\therefore$ Equation of PM is

$$y-8 = 3(x-3)$$

$$\Rightarrow \quad 3x-y-1=0 \quad \text{—} \; (2)$$

Solving (1) and (2) we get

$$\frac{x}{-3-7} = \frac{y}{-21+1} = \frac{1}{-1-9}$$

$$\Rightarrow \quad \frac{x}{-10} = \frac{y}{-20} = \frac{1}{-10}$$

$$\Rightarrow \quad x = \frac{-10}{-10} \; ; \; y = \frac{-20}{-10}$$

$$\Rightarrow \quad x = 1 \; ; \; y = 2.$$

$\therefore$ M is $M(1,2)$ [Point of intersection of (1) and (2)]

Let Q be $Q(\alpha, \beta)$

Since M is mid point of PQ.

$\therefore \quad \frac{\alpha+3}{2} = 1 \qquad \frac{\beta+8}{2} = 2$ [Mid point formula]

$\Rightarrow \quad \alpha = -1 \; , \; \beta = -4$

$\therefore$ Image of $P(3,8)$ is $Q(-1,-4)$

<u>Q No 19</u>: If the lines $y=3x+1$ and $2y=x+3$ are equally inclined to the line $y=mx+4$. find value of $m$.

<u>Soln</u>: The equations of given lines are.

$$y = 3x+1 \quad \text{—} \; (1)$$

$$2y = x+3 \quad \text{or} \quad y = \frac{1}{2}x + \frac{3}{2} \quad \text{—} \; (2)$$

$$y = mx+4. \quad \text{—} \; (3)$$

Slopes of (1), (2), (3) are $3, \frac{1}{2}, m$ respectively.

Since (3) is equally inclined to (1) and (2)

$$\therefore \quad \left| \frac{m-3}{1+3m} \right| = \left| \frac{m-\frac{1}{2}}{1+\frac{1}{2}m} \right|$$

$$\therefore \quad \left| \frac{m-3}{1+3m} \right| = \left| \frac{2m-1}{2+m} \right|$$

$$\therefore \quad \frac{m-3}{1+3m} = \pm \frac{2m-1}{m+2}.$$

When $\dfrac{m-3}{1+3m} = \dfrac{2m-1}{m+2}$

$$(m-3)(m+2) = (2m-1)(1+3m)$$

$$\Rightarrow m^2 - m - 6 = 6m^2 - m - 1$$

$$\Rightarrow 5m^2 = -5 \Rightarrow m^2 = -1$$ which is not possible.

When $\dfrac{m-3}{1+3m} = -\dfrac{2m-1}{m+2}$

$$(m-3)(m+2) = -(2m-1)(1+3m)$$

$$\therefore m^2 - m - 6 = -6m^2 + m + 1$$

$$\Rightarrow 7m^2 - 2m - 7 = 0$$

$$\Rightarrow m = \frac{2 \pm \sqrt{(-2)^2 + 4\times(-7)(7)}}{14}$$

$$= \frac{2 \pm \sqrt{4+196}}{14} = \frac{2 \pm \sqrt{200}}{14} = \frac{2 \pm 10\sqrt{2}}{14}$$

$$m = \frac{1 \pm 5\sqrt{2}}{7}$$

QNo 20: If sum of $\perp r$ distances of of a variable point $P(x,y)$ from lines $x+y-5=0$ and $3x-2y+7=0$ is always 10. Show that P must move on a line.

Sol: The equations of given line are

$$x+y-5=0 \quad \cdots (1)$$
$$3x-2y+7=0 \quad \cdots (2)$$

Let M and M be the feet of $\perp rs$ from $P(x,y)$ on (1) and (2) resp.

From given condition $PM + PN = 10$

$$\frac{x+y-5}{\sqrt{1+1}} + \frac{3x-2y+7}{\sqrt{9+4}} = 10$$

or $$\frac{x+\ -5}{\sqrt{2}} + \frac{3x-2y+7}{\sqrt{13}} = 10$$

or $$\sqrt{13}(x+y-5) + \sqrt{2}(3x-2y+7) = 10\sqrt{2}\sqrt{13}$$

or $$\sqrt{13}x + \sqrt{13}y - 5\sqrt{13} + 3\sqrt{2}x - 2\sqrt{2}y + 7\sqrt{2} = 10\sqrt{26}$$

$\therefore \quad (\sqrt{13}+3\sqrt{2})x+(\sqrt{13}-2\sqrt{2})y+(7\sqrt{2}-5\sqrt{13}-10\sqrt{26})=0$

which being a linear equation in $x$ and $y$, always represent a straight line.

**Q No 21:** Find the eqn of line which is equidistant from parallel lines $9x+6y-7=0$ and $3x+2y+6=0$

**Soln:** The eqns of given lines are.

$$9x+6y-7=0 \quad —(1)$$

and $$3x+2y+6=0$$

or. $$9x+6y+18=0 \quad —(2)$$

Lines (1) and (2) are parallel.

$\therefore$ Line which is equidistance (at same distance) from (1) and (2) must be parallel to them.

$\therefore$ Eqn of reqd line is of form

$$9x+6y+k=0$$

Now distance between (1) and (3) $= \dfrac{|k+7|}{\sqrt{(9)^2+(6)^2}} = \dfrac{|k+7|}{\sqrt{117}}$

Distance between (2) and (3) $= \dfrac{|k-18|}{\sqrt{(9)^2+(6)^2}} = \dfrac{|k-18|}{\sqrt{117}}$

Now ATQ.
$$\frac{|k+7|}{\sqrt{117}} = \frac{|k-18|}{\sqrt{117}}$$

$$\Rightarrow \quad k+7 = \pm\, k-18$$

$\Rightarrow \quad k+7 = k-18 \quad$ or $\quad k+7 = -(k-18)$

$\Rightarrow \quad 7 = -18$ which is impossible

$\therefore \quad k+7 = -k+18$

$\Rightarrow \quad 2k = 18-7$

$\Rightarrow \quad k = \frac{11}{2}$

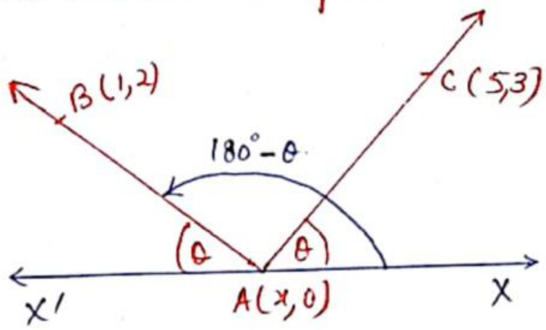
$\therefore$ (3) becomes $\quad 9x+6y+\frac{11}{2}=0$

or $\quad 18x+12y+11=0$

which is required eqn.

**Q No. 22**: A ray of light passing through the point (1, 2) reflects on x-axis at point A and the reflected ray passes through the point (5, 3) find the coordinates of A.

**Sol.**: Let $A(x, 0)$ be a point on x-axis at which the ray BA is reflected to AC such that




$\angle XAC = \angle X'AB = \theta$ (say)

$\therefore \angle BAX = 180° - \theta$

Now slope of $AC = \frac{3-0}{5-x} = \frac{3}{5-x}$

$\tan\theta = \frac{3}{5-x}$ —(1)

Slope of $AB = \frac{2-0}{1-x} = \frac{2}{1-x}$

$\tan(180° - \theta) = \frac{2}{1-x}$

$\Rightarrow -\tan\theta = \frac{2}{1-x}$

or $\tan\theta = -\frac{2}{1-x}$ —(2)

from (1) and (2)

$\frac{3}{5-x} = -\frac{2}{1-x}$

or $3(1-x) = -2(5-x)$

$\Rightarrow 3 - 3x = -10 + 2x$

$\Rightarrow 13 = 5x \Rightarrow x = \frac{13}{5}$

$\therefore$ A is $\left(\frac{13}{5}, 0\right)$.

**Q No 23**: Prove that the product of lengths of ⊥rs drawn from the points $(\sqrt{a^2-b^2}, 0)$ and $(-\sqrt{a^2-b^2}, 0)$ to the line

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is $b^2$.

**Sol**: The eqn of given line is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta - 1 = 0$ —(1)

Let $p_1$ be the length of ⊥r from $(\sqrt{a^2-b^2}, 0)$ to the line (1.)

$$\therefore p_1 = \frac{\frac{\sqrt{a^2-b^2}}{a}\cos\theta + 0 - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} = \frac{\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{a^2}}}$$

Let $p_2$ be length of $\perp r$ from $(-\sqrt{a^2-b^2}, 0)$ to the line ①

$$p_2 = \frac{-\frac{\sqrt{a^2-b^2}}{a}\cos\theta + 0 - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} = \frac{-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}$$

Now
$$p_1 p_2 = \frac{-\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta + 1\right)}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}$$

$$= -\frac{\left(\frac{a^2-b^2}{a^2}\cos^2\theta - 1\right)}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}} \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right].$$

$$= -\frac{\frac{\left[(a^2-b^2)\cos^2\theta - a^2\right]}{a^2}}{\frac{b^2\cos^2\theta + a^2\sin^2\theta}{a^2 b^2}} = \frac{-b^2\left((a^2-b^2)\cos^2\theta - a^2\right)}{b^2\cos^2\theta + a^2\sin^2\theta}$$

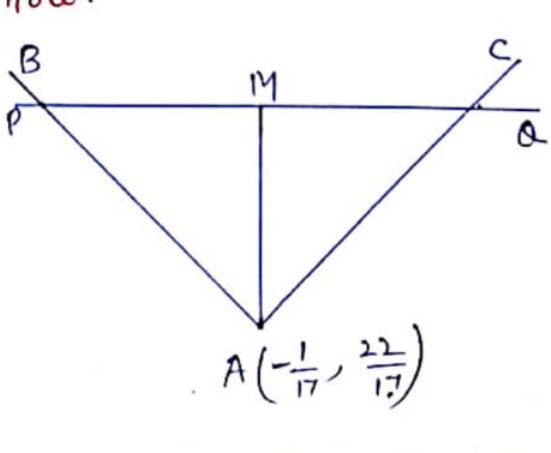
$$= -\frac{b^2\left[-a^2(1-\cos^2\theta) - b^2\cos^2\theta\right]}{b^2\cos^2\theta + a^2\sin^2\theta} = \frac{b^2\left[a^2\sin^2\theta + b^2\cos^2\theta\right]}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= b^2.$$

$$\therefore \quad p_1 p_2 = b^2.$$

Q No 24: A person standing at junction (crossing) of two straight paths represented by equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find the eqn of path that he should follow.

Sol: Consider the paths AB, AC with equations




$$2x - 3y + 4 = 0 \quad -(1)$$
$$3x + 4y - 5 = 0 \quad -(2)$$

Solving for $x$ and $y$.

$$\frac{x}{15-16} = \frac{y}{12+10} = \frac{1}{8+9}$$

$$\Rightarrow \frac{x}{-1} = \frac{y}{22} = \frac{1}{17}$$

$$\therefore x = -\frac{1}{17}, \quad y = \frac{22}{17}$$

$$\therefore \quad A \text{ is } A\left(-\frac{1}{17}, \frac{22}{17}\right)$$

Let the third path to reach be PQ with eqn.

$$6x - 7y + 8 = 0 \quad -(3)$$

Now since the shortest distance from A to (3) be ⊥r. distance and let M be the foot of ⊥r from A to PQ

Now slope of PQ is $= -\frac{6}{-7} = \frac{6}{7}$

$\therefore$ slope of AM $= -\frac{7}{6}$

$\therefore$ Eqn of AM will be

$$y - \frac{22}{17} = -\frac{7}{6}\left(x + \frac{1}{17}\right)$$

ie $\quad 17y - 22 = -\frac{7}{6}(17x + 1)$

ie $\quad 6(17y - 22) = -7(17x + 1)$

ie $\quad 102y - 132 = -119x - 7$

or $\quad 119x + 102y - 125 = 0$

Which is the reqd eqn of path.

#.

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