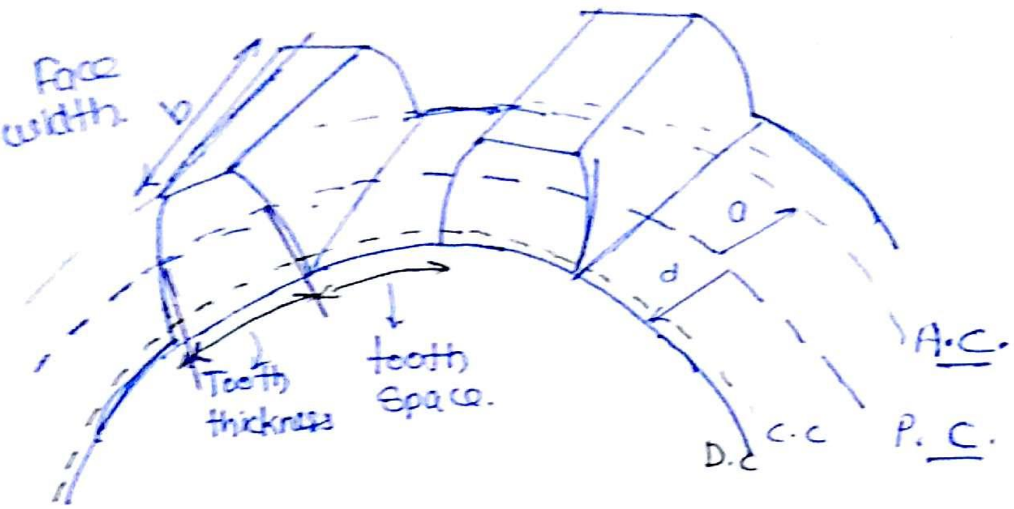
"Gear" (Spur Gear)



Tooth space − tooth thickness = backlash
[for thermal expansion]

Tooth space ≃ tooth thickness.

- 'a' Addendum = 1 M
- 'd' Dedendum = 1.157 M
- 'c' Clearance = 0.157 M

M = module

The aim of design is to determine module of Gear

- $P_c = \pi M$
- $P_d = \frac{1}{m}$

$P_c \cdot P_d = \pi$

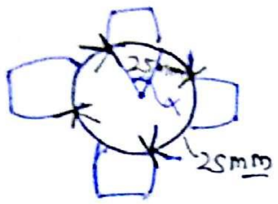
- Face width = 10 M
- $m = \frac{D}{Z}$ (Z → tooth)
- T = torque.

Circular Pitch :- (P_c)

$$P_c = \pi M$$

$$P_c = \frac{\pi D}{Z}$$

eg $\pi D = 100mm$, $z = 4$



$P_c = 25mm$

tooth space + tooth thickness = P_c

$$\boxed{\text{tooth space} = \text{tooth thickness} = \frac{P_c}{2}}$$

$$\alpha = \frac{360^\circ}{2Z}$$

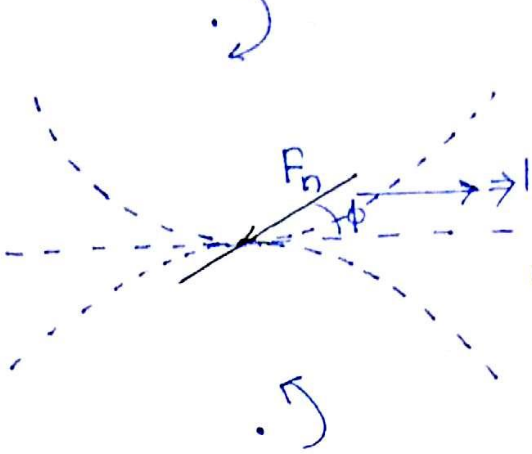
↳ Angle cover by tooth space/thickness on centre

* When two Gear's are messing together their circular pitch must be equal hence module also be equal

$$P_{c_1} = P_{c_2}$$

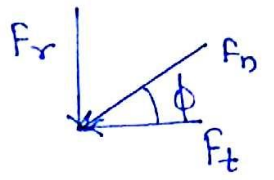
$$\pi m_1 = \pi m_2 \Rightarrow \boxed{m_1 = m_2}$$

Force analysis Used for Gears:-



Common tengent to pitch circle

→ line of action
or
common normal to point of contact
or
commom tengent to pitch circle.



ϕ = pressure angle

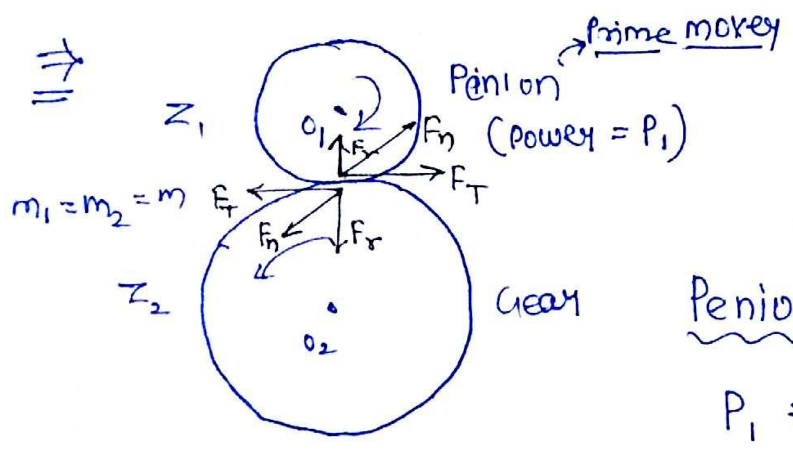
$$F_t = F_n \cos\phi \qquad T = F_t \times R$$

$$F_n = \frac{F_t}{\cos\phi} \qquad F_t = \frac{2T}{D}, \quad D = \text{pitch dia}$$

$$F_r = F_n \sin\phi$$

$$F_r = F_t \tan\phi$$

⇛



Penion (small) is Generally prime mover because it have low inertial

Penion:-

$$P_1 = \frac{2\pi N_1 T_1}{60} \Rightarrow ?$$

$$F_t = \frac{T_1}{R_1} = \frac{2T_1}{D_1}$$

$$F_t = \frac{2T_1}{D_1} \quad , \; D_1 = m z_1$$

F_t = known.

$$F_n = \frac{F_t}{\cos\phi} \Rightarrow F_r = F_t \tan\phi$$

Gear: F_n, F_r are known.

$$T_2 = F_t \times \frac{D_2}{2}$$

$$T_2 = F_t \times \frac{m z_2}{2}$$

T_2 = known.

Gear ratio = $G \geq 1$

$$\boxed{G = \frac{z_2}{z_1} = \frac{D_2}{D_1} = \frac{N_1}{N_2}} = \text{Constant}$$

law of Gearing.

Case ① if $\eta_m = 100\%$ (efficiency)

$$P_2 = P_1$$

$$\frac{2\pi N_2 T_2}{60} = \frac{2\pi N_1 T_1}{60}$$

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \Rightarrow \eta_m = 100\%$$

$$\boxed{G = \frac{z_2}{z_1} = \frac{D_2}{D_1} = \frac{N_1}{N_2} = \frac{T_2}{T_1}}$$

only when $\eta_m = 100\%$.

tooths   speed   torque.

Case ② if $\eta_m \neq 100$

$$P_2 = \eta_m P_1$$

$$\frac{2\pi N_2 T_2}{60} = \eta_m \frac{2\pi N_1 T_1}{60} \qquad \left\{ \because T_2 = \text{Actual torque} \right.$$

$$\frac{N_1}{N_2} = \frac{T_2}{\eta_m \cdot T_1}$$

$$G.R. = \frac{Z_2}{Z_1} = \frac{D_2}{D_1} = \frac{N_1}{N_2} = \frac{T_2}{\eta_m T_1}$$

T_2 = Actual but

but $T_2 \neq F_t \cdot \frac{D_2}{2}$

$$\boxed{\text{Torque loss} = F_t \cdot \frac{D_2}{2} - T_2}$$

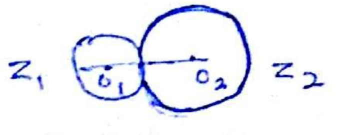
Resultant force on Gear = F_n

Resultant force on Pinion = F_n

S. 3

Concept of centre distance :-

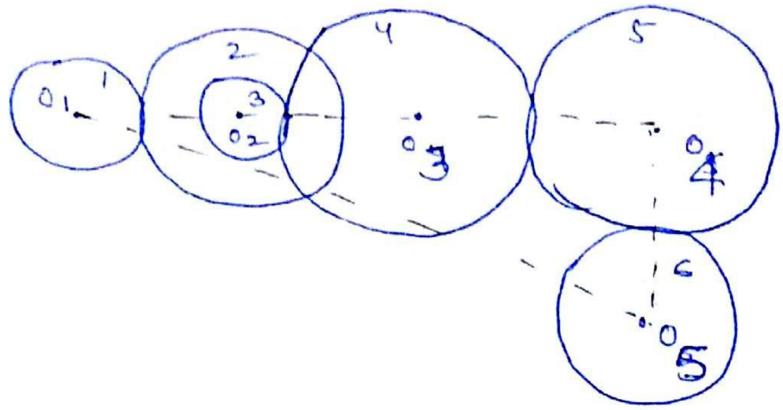
Centre distance (CD) = $O_1 O_2$



$$O_1 O_2 = r_1 + r_2$$

$$CD = \frac{mz_1}{2} + \frac{mz_2}{2} = \frac{m(z_1+z_2)}{2}$$

eg



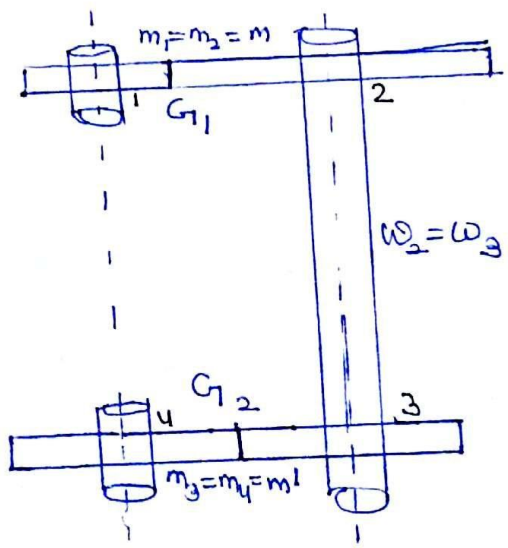
$$CD = O_1O_4 = r_1 + r_2 + r_2 + 2r_4 + r_5$$

$$CD = m\left(\frac{z_1 + z_2}{2}\right) + m'\frac{(z_3 + 2z_4 + z_5)}{2}$$

$$O_4O_5 = \frac{m'(z_5 + z_6)}{2}$$

$$\therefore O_1O_5 = \sqrt{(O_1O_4)^2 + (O_4O_5)^2}$$

eg Reverted Gear train:



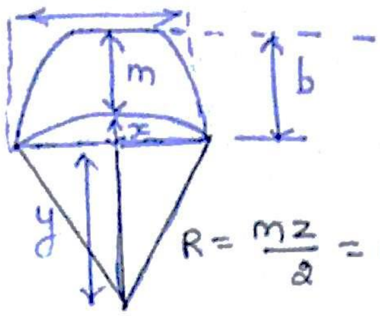
$$G_{total} = G_1 \times G_2$$

$$CD = r_1 + r_2 = r_3 + r_4$$

$$CD = \frac{m(z_1+z_2)}{2} = \frac{m'(z_3+z_4)}{2}$$

Gate Book

Q. 5.22



$R = \frac{mz}{2} = 64\,mm$

$a = \frac{p_c}{2}$

$a = \frac{4\pi}{2} = 6.28\,mm$

$b = m + x$

$x = R - y$

$y = \sqrt{(64)^2 - (3.14)^2}$

$y = 63.92$

$x = 64 - 63.92$

$x = 0.08$

$b = 4.08 \simeq 4.1\,mm$

*

Q. 5.11

Gate book

There is maximum chance of interference in rack and pinion because the radius of addendum of rack is ∞, So always design a gear by assuming rack & pinion. to avoid Interference

$$Z_{min} = \frac{2a_r}{\sin^2 20} = \frac{2 \times 1}{\sin^2 20}$$

$Z_{min} = 17.09$

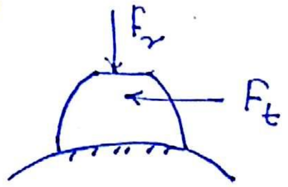
$Z = 18$ Ans

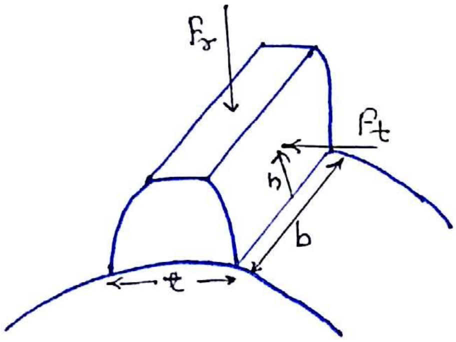
$a_r = 1 \rightarrow$ Full depth

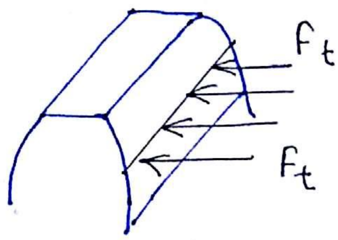
$a_r = 0.8 \rightarrow$ Stub tooth.

$$\left\{ \begin{array}{lll} \text{Full depth}, & \phi = 20^\circ & T_{min} = 18 \text{ teeth} \\ \text{Stub tooth}, & \phi = 20^\circ & T_{min} = 14 \text{ teeth} \\ \text{Full depth}, & \phi = 14\tfrac{1}{2}^\circ & T_{min} = 32 \text{ teeth.} \\ \text{Stub tooth}, & \phi = 14\tfrac{1}{2}^\circ & T_{min} = 26 \text{ teeth.} \end{array} \right\}$$

Design of spur Gear (Lewis eqn)







$F_r \Rightarrow$ ACL

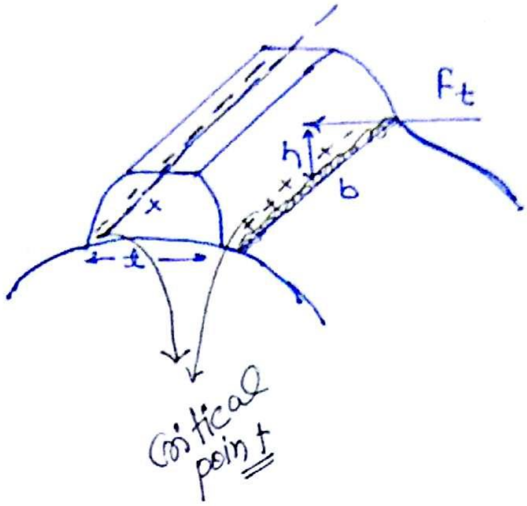
$F_t \Rightarrow$ TSL $\rightarrow$ V

⇒ Due to axial compresive force F_r, Gear tooth is subjected to compressive stress.

⇒ Due to constant shear force F_t, Gear tooth is subjected to direct shear stress. (Effect of TSL)

⇒ Due to variable moment '$F_t \cdot x$' Gear tooth is subjected to variable bending stress.

⇒ For the safe design of Gear tooth, the effect of direct shear stress & compressive stress neglected only bending stress will be considered.



$$(\sigma_b)_{max} = \frac{M_{max}\, y_{max}}{I_{N.A.}}$$

$$M_{max} = F_t \cdot h$$

$$I_{NA} = \frac{bt^3}{12}$$

$$y_{max} = t/2$$

$$(\sigma_b)_{max} = \frac{6 F_t \cdot h}{bt^2}$$

safe Cond^n

$$(\sigma_b)_{max} \leq \sigma_{per.}$$

$$\frac{6 F_t \cdot h}{bt^2} \leq \sigma_{per.}$$

$$(F_t)_{max} = \frac{bt^2}{6h} \sigma_{per.}$$

$$F_t = \frac{bt^2 m}{6hm} \sigma_{per.} \qquad \frac{t^2}{6hm} = Y = \text{lewi's form factor.}$$

**

Beam strength of Gear tooth $\boxed{(F_t)_{max} = bmY[\sigma_{per}]}$

safe Cond^n $F_{act} \leq (F_t)_{max}$
(acting)

Beam strength:- It is defined as the maximum Value to tangential force that can gear tooth wear without any bending failure.

Lewi's Form Factor (Tooth Geomentry Factor) (Y)

$$Y = \pi y$$

y = tooth form factor

$$y = \left(0.154 - \frac{0.192}{Z}\right)$$

for full depth, $\phi = 20°$

$$Y = \pi\left[0.154 - \frac{0.192}{Z}\right]$$ for full depth, $\phi = 20°$

* lewi's form factor depands upon no. of tooth, Geomentry of tooth profile and pressure angle.

$$Z_P < Z_G$$
$$\Downarrow$$
$$Y_P < Y_G$$

* Doesn't depant on module (m).

20° stub tooth

- small addendum
- small dedendum
- Greater bending strength
- Small Interference
- strong tooth
- low cost
- Gear width small teeth

$$(F_t)_{max} = bmY[\sigma_P]_{per}$$

→ Weaker gear is a gear which has min. value of beam strength and always design for weaker gear.

→ When pinion and Gear both are made of same material

$$[\sigma_P]_P = [\sigma_P]_G \quad , \quad Y_P < Y_G$$

$$(F_t)_{\substack{max \\ P}} > (F_t)_{\substack{max \\ G}}$$

Hence pinion is weaker so design for pinion in this case.

→ When pinion and Gear ~~both~~ are made of different material so design for gear which has minimum value of product $[Y\sigma_b]$.

Actual load :- Power = known

rpm known

$$P = \frac{2\pi NT}{60}$$

T = known

$$F_t = \frac{2T}{D} = \frac{2T}{mz}$$

static load F_t = known

safe Cond^n

$$F_{act} \le (F_t)_{max}$$

$$F_{actual} = F_{dynamic}$$

$$F_d \le (F_t)_{max}$$

$F_{dynamic} > F_{static}$

$F_{dynamic} = C_v . S . F_t$

** imp

$$\boxed{F_t . C_v . S = b m Y [\sigma_b]_{per.}}$$

take C_v & $S > 1$
if given C_v & $S < 1$
then use $1/C_v$ & $1/S$

⇒ C_v = Velocity Factor
⇒ S = service/over load Factor.

$$C_v = \frac{3+V}{3} \quad \text{when, } V \le 10\ m/s$$

$$C_v = \frac{6+V}{6} \quad \text{when, } V > 10\ m/s.$$

Q.5.20

$m = 3$ mm
$z = 16$
$b = 36$ mm

$\phi = 20°$
$P = 3$ kW
$N = 20$ rev/s.

$C_v = 1.5$
$S . = 0.3$

$$P = \frac{2\pi N T}{}$$

$$3 \times 10^3 = 2\pi \times 20 \times T$$

$$T = 23.88$$

$m = \frac{D}{Z}$

$$F_t . r = T$$

$$T = F_t . \frac{mz}{2}$$

$$F_t = \frac{23.8 \times 2}{3 \times 16 \times 10^{-3}}$$

$$F_t = 994.58\ N$$

$$F_t = 994.58$$

$$F_t C_v S = m b Y [\sigma_b]$$

$$994.58 \times 1.5 = 36 \times 3 \times 0.3 [\sigma_b] \times 10^{-3}$$

$$\sigma_b = 46 \text{ MPa}$$

Q. 5.24

$$Y = 0.32 \qquad F_t = 3552 \text{ N}$$

$$F_t C_v = m b Y [\sigma_b]$$

$$355 \times 1.5 = 25 \times 4 \times 0.32 [\sigma_b] \times 10^{-4}$$

$$\sigma_b = 166.5 \text{ MPa}$$

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limiting ~~Geared~~ wear load $\propto \left(\frac{1}{E_p} + \frac{1}{E_g}\right)$

Wear strength of Gear tooth:→

It is defined as the maximum value of load that a gear tooth can wear without any wear failure.

* Wear strength is always ~~wh~~ check for pinion only because pinion is subject to more wear. (Small size).

$$\boxed{F_w = D_p \cdot b \cdot Q \cdot K}$$

- D_p - pitch dia. of pinion
- b - face width
- Q - Ratio factor
- $Q = \dfrac{2G}{G \pm 1}$ (+) external (-) Internal.

G = Gear ratio.

External Gear has more wear as compared to internal gear

- K = material combination constant

$$K = \frac{\sigma_{es}^2 \sin\phi \left[\frac{1}{E_p} + \frac{1}{E_G}\right]}{1.4}$$, ϕ = pressure angle

- $K \propto (SUt)^2$, $K \propto (BHN)^2$

σ_{es} = Surface endurance limit

E_p, E_G = Youngs modulus of pinion & Gear.

Safe Condition: $\boxed{F_{act} \le F_{wear}}$

$S . C_v . F_t \le D_p . b . Q . K$

Hence safe for wear.

~~Ea~~ Practical case

$$\boxed{F_{wear} \ge (F_t)_{max} \ge F_{act}}$$

Imp eq:

$F_W = 10 KN$, $(F_t)_{max} = 15 KN$

Power - ?

$F_{act} \le 10 KN$ $F_t = ?$

$F_t . C_v . S \le 10.$

Assumption made in lewis's equation:-

1. Gear tooth as a cantilever beam fixed at root portion
2. effect of direct shear & comp. stress are neglected.
3. Gear tooth assumed as prismatic throughout.
4. Effect of stress consentration factor are neglibale.
5. error in tooth manufacturing & spacing are neglected
6. Inertia of rotating part neglected.
7. Deflection of Gear tooth under load is neglected.
8. Contact ~~rati~~ ratio assume as 1.

All these assumption are the reason for dynamic load.

type of wear:-

Abrasive wear:- these type wear occurs b/w meshing gear surface due to persence of ~~forgein~~ foragain material, by the lubricant or due to dust deposite.
These type of wear occurs in open Gear.

Scoring/ scuffing Gear:- These type of wear occurs between meshing gear surface, due to failure of lubrican.
scoring - scrathes in sliding direction.
scuffing - welding due to heating.

Coresive wear:- occur b/w meshing Gear surface due to chemical reaction b/w lubricant and gear surface.

Pitting wear:- occurs b/w meshing Gear Surface due to repeated stess under cycling loading.