

Topics : Sequence & Series, Application of Derivatives, Limits, Continuity & Derivability

Type of Questions

Single choice Objective (no negative marking) Q. 1,2,3,4,5 (3 marks, 3 min.)

Subjective Questions (no negative marking) Q. 6,7,8 (4 marks, 5 min.)

M.M., Min.

[15, 15]

[12, 15]

1. If a, b, c, d, e are five positive numbers, then

(A) $\left(\frac{a}{b} + \frac{b}{c}\right)\left(\frac{c}{d} + \frac{d}{e}\right) \geq 4\sqrt{\frac{a}{e}}$

(B) $\frac{b}{a} + \frac{c}{b} + \frac{d}{c} + \frac{e}{d} + \frac{a}{e} \geq \frac{1}{5}$

(C) $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{e} + \frac{e}{a} < 5$

(D) None of these

2. Set of all possible values of a such that $f(x) = e^{2x} - (a + 1)e^x + 2x$ is monotonically increasing for all $x \in \mathbb{R}$, is

(A) (3, 4)

(B) $(-\infty, 0)$

(C) $(-\infty, 3]$

(D) (3, ∞)

3. If at each point of the curve $y = x^3 - ax^2 + x + 1$, tangent is inclined at an acute angle with the positive direction of the x-axis then

(A) $a > 0$

(B) $a \leq \sqrt{3}$

(C) $-\sqrt{3} < a < \sqrt{3}$

(D) none of these

4. If $f(x)$ is differentiable for all $x \in \mathbb{R}$ so that $f(2) = 4$ and $f'(x) \geq 5$ for all $x \in [2, 6]$, then $f(6)$

(A) ≥ 24

(B) ≤ 24

(C) ≥ 9

(D) none of these

5. Let $U_n = \frac{n!}{(n+2)!}$ where $n \in \mathbb{N}$. If $S_n = \sum_{n=1}^n U_n$, then $\lim_{n \rightarrow \infty} S_n$ equals

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) non existent

6. If the equation $x^2 e^x = k$ possess three real roots then the range of values of k is _____

7. Find value of a, b, c such that curves $y = x^2 + ax + b$ and $y = cx - x^2$ will touch each other at the point (1, 0).

8. If $f(x)$ and $g(x)$ are continuous functions in $[a, b]$ and they are differentiable in (a, b) then prove that

$$\left| \frac{f(a) - f(b)}{g(a) - g(b)} \right| = (b - a) \left| \frac{f'(c)}{g'(c)} \right| \text{ where } a < c < b.$$

Answers Key

1. (A) 2. (C) 3. (C) 4. (A)
5. (C) 6. $k \in (0, 4e^{-2})$ 7. $a = -3, b = 2, c = 1$