Statistics

Quick Revision

Measures of Dispersion

The dispersion is the measure of variations in the values of the variable. It measures the degree of scatterdness of the observation in a distribution around the central value.

Range

Range is defined as the difference between two extreme observations of the distribution.

Range of distribution = Maximum value of observation − Minimum value of observation

Mean Deviation

Mean deviation is defined as the arithmetic mean of the absolute deviations of all the values taken about any central value 'a' (mean or median). The mean deviation from 'a' is denoted as MD(a).

$$\therefore \text{MD}(a) = \frac{\text{Sum of absolute values of deviations from 'a'}}{\text{Number of observations}}$$

(i) **Mean deviation for ungrouped data** Let n observations be $x_1, x_2, x_3, \ldots, x_n$, then mean deviation about their mean or median is given by

$$\text{MD} = \frac{\Sigma |x_i - A|}{n}$$

where, A = mean or median

(ii) **Mean deviation for discrete frequency distribution**

Let the given data consist of discrete observations $x_1, x_2, x_3, \ldots, x_n$ occurring with frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively, then

$$\text{MD} = \frac{\Sigma f_i |x_i - A|}{\Sigma f_i} = \frac{\Sigma f_i |x_i - A|}{N}$$

where, A = mean or median

(iii) **Mean deviation for continuous frequency distribution** Here, the procedure is same as for a discrete frequency distribution. The only difference is that here we have to obtain the mid-points of the various classes and take the deviations of these mid-points from the given central value.

Note Median $= l + \dfrac{\frac{N}{2} - cf}{f} \times h$

where, l = lower limit, f = frequency
h = width of median class
and cf = cumulative frequency of class just preceding the median class.

Variance

Variance is the arithmetic mean of the square of the deviations about mean $\bar{x}$. Let $x_1, x_2, \ldots, x_n$ be n observations with $\bar{x}$ as the mean, then the variance denoted by σ^2, is given by $\sigma^2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n}$.

Standard Deviation

If σ^2 is the variance, then σ is called the standard deviation which is given by

$$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}.$$

Thus, standard deviation (SD) $= \sqrt{\text{Variance}}$

(i) **Standard deviation for ungrouped data**
SD of n observations $x_1, x_2, \ldots, x_n$ is given by

$$\sigma = \sqrt{\frac{\Sigma(x_1 - \bar{x})^2}{n}}.$$

(ii) **Standard deviation of a discrete frequency distribution** Let the discrete frequency distribution be $x_i : x_1, x_2, \ldots, x_n$ and $f_i : f_1, f_2, \ldots, f_n$, then

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}} \text{ or } \sigma = \frac{1}{N}\sqrt{N\Sigma f_i x_i^2 - (\Sigma f_i x_i)^2}$$

where, f_i's are the frequency of x_i's and $N = \sum_{i=1}^{n} f_i$.

Also, by shortcut method,

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i d_i^2 - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

where, $d_i = x_i - a$, a = assumed mean

Standard Deviation of a continuous frequency distribution

If there is a frequency distribution of n classes and each class defined by its mid-point x_i, with corresponding frequency f_i, then

$$\sigma = \frac{1}{N}\sqrt{N\Sigma f_i x_i^2 - (\Sigma f_i x_i)^2}$$

Objective Questions

Multiple Choice Questions

1. The mean deviation from the mean of the set of observations –1, 0 and 4 is
(a) 3 (b) 1
(c) –2 (d) 2

2. When tested, the lives (in hours) of 5 bulbs were noted as follows
1357, 1090, 1666, 1494, 1623
The mean deviations (in hours) from their mean is
(a) 178 (b) 179
(c) 220 (d) 356

3. Mean deviation about the median for the data
3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21 is
(a) 4.27 (b) 5.24
(c) 5.27 (d) 4.24

4. The mean deviation about the median for the data
32, 34, 36, 38, 40, 42, 44, 46, 48, 50 is
(a) 4.5 (b) 4
(c) 5.5 (d) 5

5. Consider the following data

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Then, the mean deviation about the median for the data is
(a) 5 (b) 5.3
(c) 5.1 (d) 5.2

6. Consider the following data

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Then, the mean deviation about the median for the data is
(a) 3.15 (b) 3.23
(c) 3.21 (d) 3.17

7. The mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is
(a) 2 (b) 2.57
(c) 3 (d) 3.75

8. Following are the marks obtained by 9 students in a mathematics test
50, 69, 20, 33, 53, 39, 40, 65, 59
The mean deviation from the median is
(a) 9 (b) 10.5
(c) 12.67 (d) 14.76

9. The mean deviation about the median for the data 34, 66, 30, 38, 44, 50, 40, 60, 42, 51 is
(a) 8.7 (b) 7.7
(c) 87 (d) 77

10. Consider the following data

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of students	2	3	8	14	8	3	2

Then, the mean deviation about the mean is

(a) 20 (b) 10
(c) 30 (d) 15

11. The scores of a batsman in 10 innings are 48, 80, 58, 44, 52, 65, 73, 56, 64, 54, then the mean deviation from the median is

(a) 7.6 (b) 8.6
(c) 9.6 (d) 10.1

12. 6, 5, 5.25, 5.5, 4.75, 4.5, 6.25, 7.75, 9.

The mean deviation from the mean for the given data is

(a) 1.1 (b) 2.1
(c) 4.1 (d) 5.1

13. 38, 70, 48, 40, 42, 55, 63, 46, 54, 44.

The mean deviation about the mean for the given data is

(a) 8.4 (b) 7.4
(c) 6.3 (d) 4

14.

Age (in years)	10	12	15	18	21	23
Frequency	3	5	4	10	8	4

The mean deviation about the median of the given frequency distribution is (in years)

(a) 3.24 (b) 2.24
(c) 8.1 (d) 7.2

15.

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

The mean deviation about the mean for the given data is

(a) 2.1 (b) 2.2
(c) 2.3 (d) 2.4

16. Consider the following data

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Then, the mean deviation about the median for the data is

(a) 6
(b) 8
(c) 7
(d) None of the above

17. Mean deviation about the median for the data

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17 is

(a) 2.44 (b) 2.33
(c) 1.44 (d) 1.33

18. Variance of the data 2, 4, 5, 6, 8, 17 is 23.33. Then, variance of 4, 8, 10, 12, 16, 34 will be

(a) 23.33 (b) 25.33
(c) 46.66 (d) 48.66

19. Consider the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. If 1 is added to each number the variance of the numbers, so obtained is

(a) 6.5 (b) 2.87
(c) 3.87 (d) 8.25

20. Consider the following data

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

If 2 is added to each number, then variance of the numbers so obtained is

(a) 6.5 (b) 2.87
(c) 3.87 (d) 8.25

21. Find the variance of the following data

Class interval	4-8	8-12	12-16	16-20
Frequency	3	6	4	7

(a) 13 (b) 18
(c) 19 (d) 20

22. The mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12 respectively are

(a) 9, 9.50
(b) 8, 8.50
(c) 9, 9.25
(d) 8, 8.25

23. Find the variance of the following data

Class interval	4-8	8-12	12-16	16-20
Frequency	3	6	4	7

(a) 13 (b) 18 (c) 19 (d) 20

24. Following are the marks obtained by 9 students in a mathematics test

50, 69, 20, 33, 53, 39, 40, 65, 59

The mean deviation from the median is

(a) 9 (b) 10.5
(c) 12.67 (d) 14.76

25. The standard deviation of data 6, 5, 9, 13, 12, 8 and 10 is

(a) $\sqrt{\frac{52}{7}}$ (b) $\frac{52}{7}$ (c) $\sqrt{6}$ (d) 6

26. The standard deviation for the data 6, 7, 10, 12, 13, 4, 8, 12 is

(a) $\sqrt{8.25}$ (b) $\sqrt{7.25}$
(c) $\sqrt{9.25}$ (d) $\sqrt{10.25}$

27. If mean and standard deviation of 100 items are 50 and 4 respectively, then the sum of all the item and the sum of the squares of item is

(a) 5000, 251600 (b) 5000, 256100
(c) 4000, 215600 (d) 4000, 255600

28. 6, 7, 10, 12, 13, 4, 8, 12

The variance for the given data is

(a) 8.25 (b) 9.25
(c) 10.5 (d) 8.9

29. 45, 60, 62, 60, 50, 65, 58, 68, 44, 48

The variance for the given data is

(a) 56.2 (b) 66.2
(c) 65.2 (d) 55

30.

x_i	10	15	18	20	25
f_i	3	2	5	8	2

The variance for the data given distribution is

(a) 16 (b) 15 (c) 11 (d) 17

Assertion-Reasoning MCQs

Directions (Q. Nos. 31-45) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation of A.
(b) A is true, R is true; R is not a correct explanation of A.
(c) A is true; R is false
(d) A is false; R is true.

31. **Assertion** (A) In order to find the dispersion of values of x from mean $\bar{x}$, we take absolute measure of dispersion.

Reason (R) Sum of the deviations from mean $(\bar{x})$ is zero.

32. **Assertion** (A) The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3.

Reason (R) The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

33. **Assertion** (A) Consider the following data.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Then, the mean deviation about the mean is 6.32.

Reason (R) Consider the following data.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Then, the mean deviation about the mean is 15.

34. **Assertion** (A) The mean deviation about median calculated for series, where variability is very high, cannot be fully relied.

Reason (R) The median is not a representative of central tendency for the series where degree of variability is very high.

35. **Assertion** (A) The mean deviation about the mean to find measure of dispersion has certain limitations.

Reason (R) The sum of deviations from the mean is more than the sum of deviations from median. Therefore, the mean deviation about the mean is not very scientific, where degree of variability is very high.

36. **Assertion** (A) The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is 80%.

Reason (R) Mean marks scored by the students of a class is 53. The mean marks of the girls is 55 and the mean marks of the boys is 50. The percentage of girls in the class is 64%.

37. **Assertion** (A) The weights (in kg) of 15 students are as follows

31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30

If the weight 44 kg is replaced by 46 kg and 27 kg is by 25 kg, then new median is 35.

Reason (R) The mean deviation from the median of the weights (in kg) 54, 50, 40, 42, 51, 45, 47, 57 is 4.78.

38. **Assertion** (A) The proper measure of dispersion about the mean of a set of observations i.e. standard deviation is expressed as positive square root of the variance.

Reason (R) The units of individual observations x_i and the unit of their mean are different that of variance. Since, variance involves sum of squares of $(x - \bar{x})$.

39. Consider the following data

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Assertion (A) The variance of the data is 45.8.

Reason (R) The standard deviation of the data is 6.77.

40. Consider the following data

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Assertion (A) The mean of the data is 19.

Reason (R) The variance of the data is 43.4.

41. Consider the following data

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Assertion (A) The mean of the data using shortcut method is 32.

Reason (R) The standard deviation of the data using shortcut method is 1.69.

42. **Assertion** (A) If each of the observations $x_1, x_2, \ldots, x_n$ is increased by a, where a is a negative or positive number, then the variance remains unchanged.

Reason (R) Adding or subtracting a positive or negative number to (or from) each observation of a group does not affect the variance.

43. If for a distribution $\Sigma(x-5)=3$, $\Sigma(x-5)^2 = 43$ and the total number of items is 18.

Assertion (A) Mean of the distribution is 4.1666.

Reason (R) Standard deviation of the distribution is 1.54.

44. **Assertion** (A) The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Reason (R) The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

45. **Assertion** (A) If the mean of n observations $1^2, 2^2, 3^2, \ldots, n^2$ is $\frac{46n}{11}$, then n is equal to 11.

Reason (R) For two data sets each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively.

The variance of combined data set is $\frac{11}{2}$.

Case Based MCQs

46. For a group of 200 candidates, the mean and the standard deviation of scores were found to be 40 and 15, respectively. Later on it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively.

Student	Eng	Hind	Social	Science	Maths
Ramu	39	59	84	80	41
Rajitha	79	92	68	38	75
Komala	41	60	38	71	82
Patil	77	77	87	75	42
Pursi	72	65	69	83	67
Gayathri	46	96	53	71	39

Answer the following questions on the basis of above information.

(i) Find the sum of correct scores.

(a) 7991 (b) 8000
(c) 8550 (d) 6572

(ii) Find the correct mean.

(a) 42.924 (b) 39.955
(c) 38.423 (d) 41.621

(iii) The formula of variance is

(a) $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$ (b) $\sum_{i=1}^{n}(x_i-\bar{x})^2$

(c) $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\Sigma f_i}$ (d) $\sum_{i=1}^{n}f_i(x_i-\bar{x})^2$

(iv) Find the correct variance.

(a) 280.3 (b) 235.6
(c) 224.143 (d) 226.521

(v) Find the correct standard deviation.

(a) 14.971 (b) 11.321
(c) 16.441 (d) 12.824

47. You are given some observations as 34, 66, 30, 38, 44, 50, 40, 60, 42, 51.

Based on these observations, answer the following questions.

(i) The mean of the given data is

(a) 40.5 (b) 45.0
(c) 45.5 (d) 50.5

(ii) The mean deviation about the mean is

(a) 10.0 (b) 9.5
(c) 9.1 (d) 9.0

(iii) The median of the given data is

(a) 41 (b) 42
(c) 43 (d) 44

(iv) The mean deviation about the median is

(a) 8.0 (b) 8.3 (c) 8.5 (d) 8.7

(v) The difference between mean deviation about the mean and mean deviation about the median is

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

48. You are given the following grouped data.

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

Based on these data, answer the following questions.

(i) Mean of the grouped data is

(a) 7.0 (b) 7.5
(c) 8.0 (d) 8.5

(ii) Mean deviation about the mean is

(a) 2.1 (b) 2.2
(c) 2.3 (d) 2.4

(iii) The value of median is

(a) 5 (b) 6
(c) 7 (d) 8

(iv) Mean deviation about the median is

(a) 1.9 (b) 2.0
(c) 2.2 (d) 2.3

(v) The difference between mean and median is

(a) 0.9 (b) 0.7
(c) 0.5 (d) 0.3

49. Consider the data

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Based on above information answer the following questions.

(i) Mean is calculated by using the formula

(a) $\bar{x} = \frac{\Sigma f_i x_i}{N}$ (b) $\bar{x} = \Sigma f_i x_i$

(c) $\bar{x} = \frac{\Sigma f_i x_i^2}{N}$ (d) None of these

(ii) Variance is calculated by using the formula

(a) $\sigma^2 = \frac{1}{N}\Sigma f_i (x_i - \bar{x})^2$ (b) $\sigma^2 = \frac{1}{N}\Sigma f_i (x_i + \bar{x})^2$

(c) $\sigma^2 = \frac{1}{N}\Sigma (x_i - \bar{x})$ (d) None of these

(iii) Mean of the given data is

(a) 10 (b) 12 (c) 14 (d) 15

(iv) Variance of the given data is

(a) 40 (b) 45.8
(c) 41.5 (d) 39.8

(v) Standard deviation of the given data is

(a) 6.77 (b) 5
(c) 4.8 (d) 3.19

50. Consider the data

Class	Frequency
0-10	6
10-20	7
20-30	15
30-40	16
40-50	4
50-60	2

Based above information answer the following questions.

(i) Median is calculated by using the formula

(a) $M = l + \frac{\frac{N}{2} - cf}{f} \times h$ (b) $M = l + \frac{\frac{N}{2} + cf}{f} \times h$

(c) $M = l - \frac{\frac{N}{2} - cf}{f} \times h$ (d) None of these

(ii) Mean deviation about median is calculated by using the formula

(a) $MD = \frac{\Sigma f_i |x_i + M|}{N}$ (b) $MD = \frac{\Sigma f_i |x_i - M|}{N}$

(c) $MD = \frac{\Sigma |x_i - M|}{N}$ (d) None of these

(iii) Total frequency of the given data is

(a) 10 (b) 20
(c) 50 (d) 60

(iv) Median of the given data is

(a) 28 (b) 20
(c) 18 (d) 8

(v) Mean deviation about median is

(a) 10.16 (b) 15
(c) 9.16 (d) 8.5

ANSWERS

Multiple Choice Questions

1. (d)	*2. (a)*	*3. (c)*	*4. (d)*	*5. (c)*	*6. (b)*	*7. (b)*	*8. (c)*	*9. (a)*	*10. (b)*
11. (b)	*12. (a)*	*13. (a)*	*14. (a)*	*15. (c)*	*16. (c)*	*17. (b)*	*18. (c)*	*19. (d)*	*20. (d)*
21. (c)	*22. (c)*	*23. (c)*	*24. (c)*	*25. (a)*	*26. (c)*	*27. (a)*	*28. (b)*	*29. (b)*	*30. (d)*

Assertion-Reasoning MCQs

31. (a)	*32. (c)*	*33. (c)*	*34. (a)*	*35. (a)*	*36. (c)*	*37. (b)*	*38. (a)*	*39. (b)*	*40. (b)*
41. (d)	*42. (a)*	*43. (d)*	*44. (d)*	*45. (b)*					

Case Based MCQs

46. (i) - (a); (ii) - (b); (iii) - (a); (iv) - (c); (v) - (a)

47. (i) - (c); (ii) - (d); (iii) - (c); (iv) - (d); (v) - (c)

48. (i) - (b); (ii) - (c); (iii) - (c); (iv) - (d); (v) - (c)

49. (i) - (a); (ii) - (a); (iii) - (c); (iv) - (b); (v) - (a)

50. (i) - (a); (ii) - (b); (iii) - (c); (iv) - (a); (v) - (a)

SOLUTIONS

1. Mean $(\bar{x}) = \dfrac{-1 + 0 + 4}{3} = 1$

$$\therefore \text{MD}(\bar{x}) = \frac{\Sigma|x_i - \bar{x}|}{n}$$

$$= \frac{|-1-1| + |0-1| + |4-1|}{3} = 2$$

2. Since, the lives of 5 bulbs are 1357, 1090, 1666, 1494 and 1623.

$$\therefore \text{ Mean} = \frac{1357 + 1090 + 1666 + 1494 + 1623}{5}$$

$$\Rightarrow \bar{x} = \frac{7230}{5}$$

$$\Rightarrow \bar{x} = 1446$$

$$\therefore \text{ MD}(\bar{x}) = \frac{|1357-1446| + |1090-1446| + |1666-1446| + |1494-1446| + |1623-1446|}{5}$$

$$= \frac{89 + 356 + 220 + 48 + 177}{5}$$

$$= 178$$

3. Arranging the data into ascending order, we have

3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21.

Now, median $= \left(\dfrac{11+1}{2}\right)$th observation $= 9$

Now, $|x_i - M|$ are 6, 6, 5, 4, 2, 0, 1, 3, 9, 10, 12.

Therefore, $\displaystyle\sum_{i=1}^{11} |x_i - M| = 58$

and $\text{MD}(M) = \dfrac{1}{11}\displaystyle\sum_{i=1}^{11} |x_i - M|$

$$= \frac{1}{11} \times 58 = 5.27$$

4. The given data

32, 34, 36, 38, 40, 42, 44, 46, 48, 50

are in ascending order.

Here, total number of observations are 10 i.e. $n = 10$, which is even.

$$\therefore \text{ Median}, (M) = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{5\text{th observation} + 6\text{th observation}}{2}$$

$$= \frac{40 + 42}{2} = \frac{82}{2} = 41$$

$|x_i - M|$ are 9, 7,5, 3, 1, 1, 3, 5, 7, 9.

Therefore, $\displaystyle\sum_{i=1}^{10} |x_i - M| = 50$

and $\text{MD}(M) = \dfrac{1}{10}\displaystyle\sum_{i=1}^{10} |x_i - M| = \dfrac{50}{10} = 5$

5.

x_i	f_i	cf	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
15	3	3	$\lvert 15-30 \rvert = 15$	45
21	5	8	$\lvert 21-30 \rvert = 9$	45
27	6	14	$\lvert 27-30 \rvert = 3$	18
30	7	21	$\lvert 30-30 \rvert = 0$	0
35	8	29	$\lvert 35-30 \rvert = 5$	40
Total	$\Sigma f_i = 29$			$\Sigma f_i \lvert x_i - M \rvert = 148$

Here, $N = \Sigma f = 29$ (odd)

$\therefore$ Median $M = \left(\frac{N+1}{2}\right)$th observation

$= \left(\frac{29+1}{2}\right)$th observation

$= 15$th observation

$\Rightarrow \quad M = 30$

$\therefore$ Mean deviation about median

$$= \frac{\Sigma f_i \lvert x_i - M \rvert}{\Sigma f_i} = \frac{148}{29} = 5.1$$

6.

x_i	f_i	cf	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
5	8	8	$\lvert 5-7 \rvert = 2$	16
7	6	14	$\lvert 7-7 \rvert = 0$	00
9	2	16	$\lvert 9-7 \rvert = 2$	04
10	2	18	$\lvert 10-7 \rvert = 3$	06
12	2	20	$\lvert 12-7 \rvert = 5$	10
15	6	26	$\lvert 15-7 \rvert = 8$	48
Total	$\Sigma f_i = 26$			$\Sigma f_i \lvert x_i - M \rvert = 84$

Here, $N = \Sigma f_i = 26$ (even)

$$\text{Median } (M) = \frac{\left(\frac{N}{2}\right)\text{th observation} + \left(\frac{N}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\left(\frac{26}{2}\right)\text{th observation} + \left(\frac{26}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{13\text{th observation} + 14\text{th observation}}{2}$$

$$= \frac{7+7}{2} = \frac{14}{2} = 7$$

$\therefore$ Mean deviation about median

$$= \frac{\Sigma f_i \lvert x_i - M \rvert}{\Sigma f_i}$$

$$= \frac{84}{26} = 3.23$$

7. Given, observations are 3, 10, 10, 4, 7, 10 and 5.

$$\therefore \quad \bar{x} = \frac{3+10+10+4+7+10+5}{7}$$

$$= \frac{49}{7} = 7$$

x_i	$d_i = \lvert x_i - \bar{x} \rvert$
3	4
10	3
10	3
4	3
7	0
10	3
5	2
Total	$\Sigma d_i = 18$

Now, $\text{MD} = \frac{\Sigma d_i}{N}$

$$= \frac{18}{7} = 2.57$$

8. Since, marks obtained by 9 students in Mathematics are
50, 69, 20, 33, 53, 39, 40, 65 and 59.
Rewrite the given data in ascending order.
20,33, 39, 40, 50, 53, 59, 65, 69

Here, $n = 9$ [odd]

$\therefore$ Median $= \left(\frac{9+1}{2}\right)$ term $= 5$th term

Median $= 50$

x_i	$d_i = \lvert x_i - Me \rvert$
20	30
33	17
39	11
40	10
50	0
53	3
59	9
65	15
69	19
$N = 2$	$\Sigma d_i = 114$

$\therefore \quad \text{MD} = \frac{114}{9} = 12.67$

9. The given data can be arranged in ascending order as 30, 34, 38, 40, 42, 44, 50, 51, 60, 66. Here, total number of observations are 10. i.e. $n = 10$, which is even.

$\therefore$ Median,

$$(M) = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\left(\frac{10}{2}\right)\text{th observation} + \left(\frac{10}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{(5\text{th observation} + 6\text{th observation})}{2}$$

$$= \frac{42+44}{2} = \frac{86}{2} = 43$$

Let us make the table for absolute deviation

x_i	$\lvert x_i - M \rvert$
30	$\lvert 30 - 43 \rvert = 13$
34	$\lvert 34 - 43 \rvert = 9$
38	$\lvert 38 - 43 \rvert = 5$
40	$\lvert 40 - 43 \rvert = 3$
42	$\lvert 42 - 43 \rvert = 1$
44	$\lvert 44 - 43 \rvert = 1$
50	$\lvert 50 - 43 \rvert = 7$
51	$\lvert 51 - 43 \rvert = 8$
60	$\lvert 60 - 43 \rvert = 17$
66	$\lvert 66 - 43 \rvert = 23$
Total	$\sum_{i=1}^{10} \lvert x_i - M \rvert = 87$

Now, mean deviation about median,

$$\text{MD} = \frac{\sum_{i=1}^{10} \lvert x_i - M \rvert}{10}$$

$$= \frac{87}{10}$$

$$= 8.7$$

10. Take the assumed mean $a = 45$ and $h = 10$, and form the following table

Marks obtained	No. of students (f_i)	Mid value (x_i)	$d_i = \frac{x_i - 45}{10}$	$f_i d_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - x \rvert$
10-20	2	15	– 3	– 6	30	60
20-30	3	25	– 2	– 6	20	60
30-40	8	35	– 1	– 8	10	80
40-50	14	45	0	0	0	0
50-60	8	55	1	8	10	80
60-70	3	65	2	6	20	60
70-80	2	75	3	6	30	60
	40			0		400

Therefore, $\bar{x} = a + \frac{\sum_{i=1}^{7} f_i d_i}{N} \times h$

$$= 45 + \frac{0}{40} \times 10$$

$$= 45$$

and $MD(\bar{x}) = \frac{1}{N} \sum_{i=1}^{7} f_i \lvert x_i - \bar{x} \rvert$

$$= \frac{400}{40} = 10$$

11. Arranging the data in ascending order, we have

44, 48, 52, 54, 56, 58, 64, 65, 73, 80

Here, $n = 10$.

So, median is the mean of 5th and 6th terms.

$\therefore$ Median $(M) = \left(\frac{56+58}{2}\right) = 57$

We make the table from the given data.

Scores (x_i)	Deviation from median $(x_i - M)$	$\|x_i - M\|$
44	$44 - 57 = -13$	13
48	$48 - 57 = -9$	9
52	$52 - 57 = -5$	5
54	$54 - 57 = -3$	3
56	$56 - 57 = -1$	1
58	$58 - 57 = 1$	1
64	$64 - 57 = 7$	7
65	$65 - 57 = 8$	8
73	$73 - 57 = 16$	16
80	$80 - 57 = 23$	23
Total		86

$$\therefore \text{Mean deviation} = \frac{\sum |x_i - M|}{n} = \frac{86}{10} = 8.6$$

Hence, the mean deviation from the median is 8.6.

12. Let $\bar{x}$ be the mean of given data.

$$\text{Then, } \bar{x} = \frac{6.5 + 5 + 5.25 + 5.5 + 4.75 + 4.5 + 6.25 + 7.75 + 8.5}{9}$$

$$= \frac{54}{9} = 6$$

Let us make the table for deviation and absolute deviation.

x_i	$x_i - \bar{x}$	$\|x_i - \bar{x}\|$
6.5	0.5	0.50
5.0	-1	1.00
5.25	-0.75	0.75
5.5	-0.5	0.50
4.75	-1.25	1.25
4.5	-1.50	1.50
6.25	0.25	0.25
7.75	1.75	1.75
8.5	2.5	2.50
Total		$\sum_{i=1}^{9} \|x_i - \bar{x}\| = 10.00$

$\therefore$ Mean deviation about mean,

$$\text{MD}(\bar{x}) = \frac{\sum_{i=1}^{9} |x_i - \bar{x}|}{9} = \frac{10}{9} = 1.1$$

Hence, the mean deviation about mean is 1.1.

13. Given observations are

38, 70, 48, 40, 42, 55, 63, 46, 54 and 44

Here, number of observations, $n = 10$

$$\therefore \text{Mean, } \bar{x} = \frac{(38 + 70 + 48 + 40 + 42 + 55 + 63 + 46 + 54 + 44)}{10}$$

$$= \frac{500}{10} = 50$$

Let us make the table for deviation and absolute deviation

x_i	$x_i - \bar{x}$	$\|x_i - \bar{x}\|$
38	$38 - 50 = -12$	12
70	$70 - 50 = 20$	20
48	$48 - 50 = -2$	2
40	$40 - 50 = -10$	10
42	$42 - 50 = -8$	8
55	$55 - 50 = 5$	5
63	$63 - 50 = 13$	13
46	$46 - 50 = -4$	4
54	$54 - 50 = 4$	4
44	$44 - 50 = -6$	6
Total		$\sum_{i=1}^{10} \|x_i - \bar{x}\| = 84$

$$\text{Now, } MD = \frac{\sum_{i=1}^{10} |x_i - \bar{x}|}{10}$$

$$= \frac{84}{10} = 8.4$$

14. The given observations are already in ascending order.

Now, let us make the cumulative frequency.

Age (x_i)	Frequency (f_i)	cf
10	3	3
12	5	8
15	4	12
18	10	22
21	8	30
23	4	34
Total	$N = 34$	

Here, $\Sigma f_i = N = 34$, which is even.

∴ Median

$$= \frac{\text{Value of } \left(\frac{34}{2}\right)\text{th observation} + \text{Value of } \left(\frac{34}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\text{Value of 17th observation} + \text{Value of 18th observation}}{2}$$

$$= \frac{18+18}{2} = 18$$

[∵ both of these observation lies in the cumulative frequency 22 and its corresponding observation is 18.]

Now, let us make the following table from the given data.

$\lvert x_i - 18\rvert$	8	6	3	0	3	5	Total
$f_i\lvert x_i - 18\rvert$	24	30	12	0	24	20	110

$$= \frac{\Sigma f_i \lvert x_i - M\rvert}{\Sigma f_i}$$

$$= \frac{110}{34} = 3.24 \text{ yr}$$

15. Let us make the following table from the given data.

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
Total	40	300		92

Here, $N = \Sigma f_i = 40$, $\Sigma f_i x_i = 300$

Now, mean $(\bar{x}) = \frac{1}{N} \Sigma f_i x_i = \frac{1}{40} \times 300 = 7.5$

∴ Mean deviation about the mean,

$$\text{MD}(\bar{x}) = \frac{1}{N} \Sigma f_i \lvert x_i - \bar{x}\rvert$$

$$= \frac{1}{40} \times 92 = 2.3$$

Hence, the mean deviation about mean is 2.3.

16. The given data is 36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Arranging the data in ascending order,

36, 42, 45, 46, 46, 49, 51, 53, 60, 72

Number of observations = 10 (even)

$$\text{Median } M = \frac{\left(\frac{N}{2}\right)\text{th observation} + \left(\frac{N}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\left(\frac{10}{2}\right)\text{th observation} + \left(\frac{10}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\text{5th observation} + \text{6th observation}}{2}$$

$$= \frac{46+49}{2} = 47.5$$

x_i	$\lvert x_i - M\rvert$
36	$\lvert 36 - 47.5\rvert = 11.5$
42	$\lvert 42 - 47.5\rvert = 5.5$
45	$\lvert 45 - 47.5\rvert = 2.5$
46	$\lvert 46 - 47.5\rvert = 1.5$
46	$\lvert 46 - 47.5\rvert = 1.5$
49	$\lvert 49 - 47.5\rvert = 1.5$
51	$\lvert 51 - 47.5\rvert = 3.5$
53	$\lvert 53 - 47.5\rvert = 5.5$
60	$\lvert 60 - 47.5\rvert = 12.5$
72	$\lvert 72 - 47.5\rvert = 24.5$
	$\Sigma\lvert x_i - M\rvert = 70$

∴ Mean deviation about median

$$= \frac{\Sigma \lvert x_i - M\rvert}{n}$$

$$= \frac{70}{10} = 7$$

17. The given data is 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Arranging in ascending order,

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

Number of observations = 12 (even)

$$\text{Median } M = \frac{\left(\frac{N}{2}\right)\text{th observation} + \left(\frac{N}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\left(\frac{12}{2}\right)\text{th observation} + \left(\frac{12}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{6\text{th observation} + 7\text{th observation}}{2}$$

$$= \frac{13+14}{2} = \frac{27}{2}$$

$\Rightarrow \quad M = 13.5$

x_i	$\lvert x_i - M \rvert$
10	$\lvert 10 - 13.5 \rvert = 3.5$
11	$\lvert 11 - 13.5 \rvert = 2.5$
11	$\lvert 11 - 13.5 \rvert = 2.5$
12	$\lvert 12 - 13.5 \rvert = 1.5$
13	$\lvert 13 - 13.5 \rvert = 0.5$
13	$\lvert 13 - 13.5 \rvert = 0.5$
14	$\lvert 14 - 13.5 \rvert = 0.5$
16	$\lvert 16 - 13.5 \rvert = 2.5$
16	$\lvert 16 - 13.5 \rvert = 2.5$
17	$\lvert 17 - 13.5 \rvert = 3.5$
17	$\lvert 17 - 13.5 \rvert = 3.5$
18	$\lvert 18 - 13.5 \rvert = 4.5$
	$\Sigma \lvert x_i - M \rvert = 28$

$\therefore$ Mean deviation about median

$$= \frac{\Sigma \lvert x_i - M \rvert}{n} = \frac{28}{12} = 2.33$$

18. When each observation is multiplied by 2, then variance is also multiplied by 2.

We are given, 2, 4, 5, 6, 8, 17.

When each observation multiplied by 2, we get 4, 8, 10, 12, 16, 34.

$\therefore$ Variance of new series

$= 2 \times$ Variance of given data

$= 2 \times 23.33 = 46.66$

19. Given numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.

If 1 is added to each number, then observations will be 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11.

$$\therefore \quad \Sigma x_i = 2 + 3 + 4 + \ldots + 11$$
$$= \frac{10}{2}[2 \times 2 + 9 \times 1]$$
$$= 5[4 + 9] = 65$$

$$\text{and} \quad \Sigma x_i^2 = 2^2 + 3^2 + 4^2 + 5^2 + \ldots + 11^2$$
$$= (1^2 + 2^2 + 3^2 + \ldots + 11^2) - (1^2)$$
$$= \frac{11 \times 12 \times 23}{6} - 1$$
$$= \frac{11 \times 12 \times 23 - 6}{6}$$
$$= 505$$

$$\text{Variance } (\sigma^2) = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$
$$= \frac{505}{10} - \left(\frac{65}{10}\right)^2$$
$$= 50.5 - 42.25 = 8.25$$

20. We have the following numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

If 2 is added to each number, we get

3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Sum of these numbers,

$$\Sigma x_i = 3 + \ldots + 11 + 12 = 75$$

Sum of squares of these numbers,

$$\Sigma x_i^2 = 3^2 + \ldots + 11^2 + 12^2 = 645$$

$$\text{Variance } (\sigma^2) = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$
$$= \frac{645}{10} - (7.5)^2$$
$$= 64.5 - 56.25$$
$$= 8.25$$

21.

Class interval	Mid-value (x_i)	f_i
4-8	6	3
8-12	10	6
12-16	14	4
16-20	18	7

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{3 \times 6 + 6 \times 10 + 4 \times 14 + 7 \times 18}{20} = 13$$

$$\text{Variance } (\sigma^2) = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$$

$$= \frac{3(-7)^2 + 6(-3)^2 + 4(1)^2 + 7(5)^2}{20} = 19$$

22.

x_i	x_i^2
6	36
7	49
10	100
12	144
13	169
4	16
8	64
12	144
Total = 72	722

$$\text{Mean} = \frac{\Sigma x_i}{n} = \frac{72}{8} = 9$$

$$\text{Variance} = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$= \frac{722}{8} - \left(\frac{72}{8}\right)^2$$

$$= 90.25 - 81$$

$$= 9.25$$

23.

Class interval	Mid value (x_i)	f_i
4-8	6	3
8-12	10	6
12-16	14	4
16-20	18	7

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{3 \times 6 + 6 \times 10 + 4 \times 14 + 7 \times 18}{20}$$

$$= 13$$

$$\text{Variance } (\sigma^2) = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$$

$$= \frac{3(-7)^2 + 6(-3)^2 + 4(1)^2 + 7(5)^2}{20}$$

$$= \frac{147 + 54 + 4 + 175}{20} = 19$$

24. Since, marks obtained by 9 students in Mathematics are 50, 69, 20, 33, 53, 39, 40, 65 and 59.

Rewrite the given data in ascending order.

20,33, 39, 40, 50, 53, 59, 65, 69,

Here, $n = 9$ [odd]

$\therefore$ Median $= \left(\frac{9+1}{2}\right)$ term $= 5$ th term

$Me = 50$

x_i	$d_i = \lvert x_i - Me \rvert$
20	30
33	17
39	11
40	10
50	0
53	3
59	9
65	15
69	19
$N = 2$	$\Sigma d_i = 114$

$\therefore$ $\text{MD} = \frac{114}{9} = 12.67$

25. Given, data are 6, 5, 9, 13, 12, 8, and 10.

x_i	x_i^2
6	36
5	25
9	81
13	169
12	144
8	64
10	100
$\Sigma x_i = 63$	$\Sigma x_i^2 = 619$

$$\therefore \text{ SD} = \sigma = \sqrt{\frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2}$$

$$= \sqrt{\frac{619}{7} - \left(\frac{63}{7}\right)^2} = \sqrt{\frac{7 \times 619 - 3969}{49}}$$

$$= \sqrt{\frac{4333 - 3969}{49}} = \sqrt{\frac{364}{49}} = \sqrt{\frac{52}{7}}$$

26.

x_i	x_i^2
6	36
7	49
10	100
12	144
13	169
4	16
8	64
12	144
$\Sigma x_i = 72$	$\Sigma x_i^2 = 722$

$$\therefore \text{ SD}, \sigma = \sqrt{\frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2}$$

$$= \sqrt{\frac{722}{8} - \left(\frac{72}{8}\right)^2}$$

$$= \sqrt{90.25 - 81}$$

$$= \sqrt{9.25}$$

27. Here, $\bar{x} = 50$, $n = 100$ and $\sigma = 4$

$$\therefore \quad \frac{\Sigma x_i}{100} = 50$$

$$\Rightarrow \quad \Sigma x_i = 5000$$

and $$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\Rightarrow \quad (4)^2 = \frac{\Sigma x_i^2}{100} - (50)^2$$

$$\Rightarrow \quad 16 = \frac{\Sigma x_i^2}{100} - 2500$$

$$\Rightarrow \quad \frac{\Sigma x_i^2}{100} = 16 + 2500 = 2516$$

$$\therefore \quad \Sigma x_i^2 = 251600$$

Hence, required answer is 5000, 251600.

28. Given observations are 6, 7, 10, 12, 13, 4, 8, 12.

Number of observations = 8

$$\therefore \text{Mean}, (\bar{x}) = \frac{6 + 7 + 10 + 12 + 13 + 4 + 8 + 12}{8}$$

$$= \frac{72}{8} = 9$$

Now, let us make the following table for deviation.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	– 3	9	13	4	16
7	– 2	4	4	– 5	25
10	1	1	8	– 1	1
12	3	9	12	3	9
Total		74	Total		74

$\therefore$ Sum of squares of deviations

$$= \sum_{i=1}^{8} (x_i - \bar{x})^2 = 74$$

$$\text{Variance}, \sigma^2 = \frac{\sum_{i=1}^{8} (x_i - \bar{x})^2}{n} = \frac{74}{8} = 9.25$$

29. Let $\bar{x}$ be the mean of the given set of observations.

Number of observations = 10

$$\therefore \quad \bar{x} = \frac{\left[\begin{array}{r} 45 + 60 + 62 + 60 + 50 + 65 + 58 \\ + 68 + 44 + 48 \end{array}\right]}{10}$$

$$= \frac{560}{10} = 56$$

Make a table from the given data.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
45	45 – 56 = – 11	121
60	60 – 56 = 4	16
62	62 – 56 = 6	36
60	60 – 56 = 4	16
50	50 – 56 = – 6	36
65	65 – 56 = 9	81
58	58 – 56 = 2	4
68	68 – 56 = 12	144
44	44 – 56 = – 12	144
48	48 – 56 = – 8	64
Total		662

We have, $n = 10$ and $\Sigma (x_i - \bar{x})^2 = 662$

$$\therefore \text{Variance}, \sigma^2 = \frac{1}{n} \Sigma (x_i - \bar{x})^2 = \frac{662}{10} = 66.2$$

30.

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$ $=(x_i - 18)$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
10	3	30	-8	64	192
15	2	30	-3	9	18
18	5	90	0	0	0
20	8	160	2	4	32
25	2	50	7	49	98
Total	$N = \Sigma f_i = 20$	$\Sigma f_i x_i = 360$			$\Sigma f_i (x_i - \bar{x})^2 = 340$

Here, $N = 20$ and $\Sigma f_i x_i = 360$

$\therefore$ Mean, $(\bar{x}) = \frac{1}{N} \Sigma f_i x_i = \frac{360}{20} = 18$

Now, variance, $(\sigma^2) = \frac{1}{N} \Sigma f_i (x_i - \bar{x})^2$

$$= \frac{1}{20} \times 340 = 17$$

31. **Assertion** The deviation of an observation x from a fixed value 'a' is the difference $(x - a)$. In order to find the dispersion of values of x from a central value a, we find the deviations about a. An absolute measure of dispersion is the mean of these deviations.

Reason To find the mean, we must obtain the sum of the deviations. But, we know that a measure of central tendency lies between the maximum and the minimum values of the set of observations.

Therefore, some of the deviations will be negative and some positive. Thus, the sum of deviations may vanish. Moreover, the sum of the deviations from mean $(\bar{x})$ is zero. Also,

Mean of deviations

$$= \frac{\text{Sum of deviations}}{\text{Number of observations}} = \frac{0}{n} = 0$$

Thus, finding the mean of deviations about mean is not of any use for us, as far as the measure of dispersion is concerned.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

32. **Assertion** Mean of the given series

$$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\Sigma x_i}{n}$$

$$= \frac{4 + 7 + 8 + 9 + 10 + 12 + 13 + 17}{8} = 10$$

x_i	$\lvert x_i - \bar{x} \rvert$
4	$\lvert 4 - 10 \rvert = 6$
7	$\lvert 7 - 10 \rvert = 3$
8	$\lvert 8 - 10 \rvert = 2$
9	$\lvert 9 - 10 \rvert = 1$
10	$\lvert 10 - 10 \rvert = 0$
12	$\lvert 12 - 10 \rvert = 2$
13	$\lvert 13 - 10 \rvert = 3$
17	$\lvert 17 - 10 \rvert = 7$
$\Sigma x_i = 80$	$\Sigma \lvert x_i - \bar{x} \rvert = 24$

$\therefore$ Mean deviation about mean

$$= \frac{\Sigma \lvert x_i - \bar{x} \rvert}{n} = \frac{24}{8} = 3$$

Reason Mean of the given series

$$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\Sigma x_i}{n}$$

$$= \frac{38 + 70 + 48 + 40 + 42 + 55 + 63 + 46 + 54 + 44}{10} = 50$$

x_i	$\lvert x_i - \bar{x} \rvert$
38	$\lvert 38 - 50 \rvert = 12$
70	$\lvert 70 - 50 \rvert = 20$
48	$\lvert 48 - 50 \rvert = 02$
40	$\lvert 40 - 50 \rvert = 10$
42	$\lvert 42 - 50 \rvert = 08$
55	$\lvert 55 - 50 \rvert = 05$
63	$\lvert 63 - 50 \rvert = 13$
46	$\lvert 46 - 50 \rvert = 04$
54	$\lvert 54 - 50 \rvert = 04$
44	$\lvert 44 - 50 \rvert = 06$
$\Sigma x_i = 500$	$\Sigma \lvert x_i - \bar{x} \rvert = 84$

$\therefore$ Mean deviation about mean

$$= \frac{\Sigma \lvert x_i - \bar{x} \rvert}{n}$$

$$= \frac{84}{10} = 8.4$$

Hence, Assertion is true and Reason is false.

33. **Assertion**

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
5	7	35	$\lvert 5-14 \rvert = 9$	63
10	4	40	$\lvert 10-14 \rvert = 4$	16
15	6	90	$\lvert 15-14 \rvert = 1$	06
20	3	60	$\lvert 20-14 \rvert = 6$	18
25	5	125	$\lvert 25-14 \rvert = 11$	55
Total	$\Sigma f_i = 25$	350		158

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{350}{25} = 14$$

∴ Mean deviation about mean

$$= \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{\Sigma f_i} = \frac{158}{25} = 6.32$$

Reason

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10	4	40	$\lvert 10 - 50 \rvert = 40$	160
30	24	720	$\lvert 30 - 50 \rvert = 20$	480
50	28	1400	$\lvert 50 - 50 \rvert = 00$	000
70	16	1120	$\lvert 70 - 50 \rvert = 20$	320
90	8	720	$\lvert 90 - 50 \rvert = 40$	320
Total	$\Sigma f_i = 80$	$\Sigma f_i x_i = 4000$		1280

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{4000}{80} = 50$$

∴ Mean deviation about mean

$$= \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{\Sigma f_i}$$

$$= \frac{1280}{80} = 16$$

Hence, Assertion is true and Reason is false.

34. **Assertion** In a series, where the degree of variability is very high, the median is not a representative central tendency. Thus, the mean deviation about median calculated for such series can not be fully relied.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

35. **Assertion** The sum of the deviations from the mean (minus signs ignored) is more than the sum of the deviations from median. Therefore, the mean deviation about the mean is not very scientific. Thus, in many cases, mean deviation may give unsatisfactory results. Also mean deviation is calculated on the basis of absolute values of the deviations and therefore, cannot be subjected to further algebraic treatment. This implies that we must have some other measure of dispersion. Standard deviation is such a measure of dispersion.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

36. **Assertion** Let the number of boys and girls be x and y.

∴ $52x + 42y = 50(x + y)$

⇒ $2x = 8y$

⇒ $x = 4y$

∴ Total number of students in the class

$$= x + y = 5y$$

∴ Required percentage of boys

$$= \frac{4y}{5y} \times 100\%$$

$$= 80\%$$

Reason Let the number of boys be x and number of girls be y.

∴ $53(x + y) = 55y + 50x$

⇒ $3x = 2y$

⇒ $x = \frac{2y}{3}$

∴ Total number of students

$$= x + y = \frac{2y}{3} + y = \frac{5}{3}y$$

Hence, required percentage

$$= \frac{y}{5y/3} \times 100\%$$

$$= \frac{3}{5} \times 100\% = 60\%$$

Hence, Assertion is true and Reason is false.

37. **Assertion** Since, 44 kg is replaced by 46 kg and 27 kg is replaced by 25 kg, then the given series becomes 31, 35, 25, 29, 32, 43, 37, 41, 34, 28, 36, 46, 45, 42, 30.

On arranging this series in ascending order, we get

25, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 45, 46.

Total number of students are 15, therefore middle term is 8th whose corresponding value is 35.

Reason On arranging the terms in increasing order of magnitude

40, 42, 45, 47, 50, 51, 54, 55, 57

Number of terms, $N = 9$

$\therefore$ Median $= \left(\frac{9+1}{2}\right)$th term = 5th term = 50 kg

Weight (in kg)	Deviation from median (d)	$\lvert d \rvert$
40	–10	10
42	–8	8
45	–5	5
47	–3	3
50	0	0
51	1	1
54	4	4
55	5	5
57	7	7
		$\lvert d \rvert = 43$

MD from median $= \frac{43}{9} = 4.78$ kg

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

38. **Assertion** In the calculation of variance, we find that the units of individual observations x_i and the unit of their mean $\bar{x}$ are different from that of variance, since variance involves the sum of squares of $(x_i - \bar{x})$.

For this reason, the proper measure of dispersion about the mean of a set of observations is expressed as positive square-root of the variance and is called standard deviation.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

39. **Assertion** Presenting the data in tabular form, we get

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	–10	100	300
8	5	40	–6	36	180
11	9	99	–3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	30	420			1374

$$N = 30, \sum_{i=1}^{7} f_i x_i = 420, \sum_{i=1}^{7} f_i (x_i - \bar{x})^2 = 1374$$

Therefore, $$\bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N} = \frac{1}{30} \times 420 = 14$$

$$\therefore \text{Variance } (\sigma^2) = \frac{1}{N} \sum_{i=1}^{7} f_i (x_i - \bar{x})^2$$

$$= \frac{1}{30} \times 1374 = 45.8$$

Reason Standard deviation

$$(\sigma) = \sqrt{45.8} = 6.77$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

40. **Assertion**

x_i	f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
6	2	36	12	72
10	4	100	40	400
14	7	196	98	1372
18	12	324	216	3888
24	8	576	192	4608
28	4	784	112	3136
30	3	900	90	2700
Total = 130	40		760	16176

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{760}{40} = 19$$

Reason Variance $= \dfrac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\dfrac{\Sigma f_i x_i}{\Sigma f_i}\right)^2$

$$= \frac{16176}{40} - \left(\frac{760}{40}\right)^2$$

$$= 404.4 - (19)^2$$

$$= 404.4 - 361 = 43.4$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

41. Assertion

Mid value (x_i)	Frequency (f_i)	Deviation from mean $d_i = x_i - A$, $A = 64$	d_i^2	$f_i d_i$	$f_i d_i^2$
60	2	−4	16	−8	32
61	1	−3	9	−3	9
62	12	−2	4	−24	48
63	29	−1	1	−29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
Total	100	0		0	286

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$(\bar{x}) = 64 + \frac{0}{100} = 64$$

Reason Standard deviation (σ)

$$= \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2}$$

$$= \sqrt{\frac{286}{100} - \left(\frac{0}{100}\right)^2} = \sqrt{2.86} = 1.69$$

Hence Assertion is false and Reason is true.

42. Assertion Let $\bar{x}$ be the mean of $x_1, x_2 \ldots, x_n$. Then, variance is given by

$$\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

If a is added to each observation, the new observations will be

$$y_i = x_i + a \quad \ldots\text{(i)}$$

Let the mean of the new observations be $\bar{y}$. Then,

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n}(x_i + a)$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} a\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{na}{n} = \bar{x} + a$$

i.e. $\quad \bar{y} = \bar{x} + a \quad \ldots$(ii)

Thus, the variance of the new observations is

$$\sigma_2^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + a - \bar{x} - a)^2$$

[using Eqs. (i) and (ii)]

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sigma_1^2$$

Thus, the variance of the new observations is same as that of the original observations.

Reason We may note that adding (or subtracting) a positive number to (or from) each observation of a group does not affect the variance.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

43. Assertion Given,

$$\Sigma(x - 5) = 3$$

$\therefore \quad \Sigma x - \Sigma 5 = 3$

$\Rightarrow \quad \Sigma x - 5 \times 18 = 3 \quad [\because n = 18]$

$\Rightarrow \quad \Sigma x = 3 + 90$

$\Rightarrow \quad \Sigma x = 93$

Now, $\quad \Sigma(x - 5)^2 = 43$

$\Rightarrow \quad \Sigma(x^2 + 25 - 10x) = 43$

$\Rightarrow \quad \Sigma x^2 + \Sigma 25 - 10\Sigma x = 43$

$\Rightarrow \quad \Sigma x^2 + 25 \times 18 - 10 \times 93 = 43$

$\Rightarrow \quad \Sigma x^2 = 43 + 930 - 450$

$\Rightarrow \quad \Sigma x^2 = 973 - 450$

$\Rightarrow \quad \Sigma x^2 = 523$

Now, mean $= \dfrac{\Sigma x}{n} = \dfrac{93}{18} = 5.16$

Reason SD $(\sigma) = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$= \sqrt{\frac{523}{18} - \left(\frac{93}{18}\right)^2}$

$= \sqrt{\frac{523 \times 18 - 93 \times 93}{18 \times 18}}$

$= \frac{1}{18}\sqrt{9414 - 8649}$

$= \frac{1}{18}\sqrt{765} = \frac{27.66}{18} = 1.54$

Hence Assertion is false and Reason is true.

44. Assertion Sum of n even natural numbers

$= n(n+1)$

Mean $(\bar{x}) = \frac{n(n+1)}{n} = n+1$

Variance $= \left[\frac{1}{n}\Sigma(x_i)^2\right] - (\bar{x})^2$

$= \frac{1}{n}[2^2 + 4^2 + \ldots + (2n)^2] - (n+1)^2$

$= \frac{1}{n}2^2[1^2 + 2^2 + \ldots + n^2] - (n+1)^2$

$= \frac{4}{n}\frac{n(n+1)(2n+1)}{6} - (n+1)^2$

$= \frac{(n+1)[2(2n+1) - 3(n+1)]}{3}$

$= \frac{(n+1)(n-1)}{3}$

$= \frac{n^2 - 1}{3}$

Hence Assertion is false and Reason is true.

45. Assertion Mean of $1^2, 2^2, 3^2, \ldots, n^2$ is

$\frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{n} = \frac{\Sigma n^2}{n}$

$\therefore \quad \frac{46n}{11} = \frac{n(n+1)(2n+1)}{6n}$

$\Rightarrow \quad 22n^2 + 33n + 11 - 276n = 0$

$\Rightarrow \quad (n-11)(22n-1) = 0$

$\Rightarrow \quad n = 11$ and $n \neq \frac{1}{22}$

Reason $\because \ \sigma_x^2 = 4$ and $\sigma_y^2 = 5$

Also, $\bar{x} = 2$ and $\bar{y} = 4$

Now, $\frac{\Sigma x_i}{5} = 2 \Rightarrow \Sigma x_i = 10$

$\frac{\Sigma y_i}{5} = 4$

$\Rightarrow \quad \Sigma y_i = 20$

Since, $\sigma_x^2 = \frac{1}{5}(\Sigma x_i^2) - (\bar{x})^2$

$\Rightarrow \quad \Sigma x_i^2 = 40$

Similarly, $\Sigma y_i^2 = 105$

$\therefore \quad \sigma_z^2 = \frac{1}{10}(\Sigma x_i^2 + \Sigma y_i^2) - \left(\frac{\bar{x} + \bar{y}}{2}\right)^2$

$= \frac{1}{10}(40 + 105) - 9$

$= \frac{55}{10} = \frac{11}{2}$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

46. (i) We have, $n = 200$, incorrect mean $= 40$ and incorrect standard deviation $= 15$

Now, incorrect mean $= 40$

$\Rightarrow \quad \frac{\text{Incorrect } \Sigma x_i}{200} = 40$

Incorrect $\Sigma x_i = 8000$

Correct $\Sigma x_i = 8000 - (34 + 53) + (43 + 35)$

$= 8000 - 87 + 78 = 7991$

(ii) Correct mean $= \frac{7991}{200} = 39.955$

(iii) $\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

(iv) Incorrect SD $= 15$

$\Rightarrow$ Incorrect variance $= (15)^2 = 225$

$\Rightarrow \frac{\text{Incorrect } \Sigma x_i^2}{200} - (\text{Incorrect mean})^2 = 225$

$\Rightarrow \frac{\text{Incorrect } \Sigma x_i^2}{200} - (40)^2 = 225$

$\Rightarrow$ Incorrect $\Sigma x_i^2 = 200(1600 + 225)$

$= 200 \times 1825$

$= 365000$

Now, Correct $\Sigma x_i^2 =$ Incorrect Σx_i^2

$-(34^2 + 53^2) + (43^2 + 35^2)$

$= 365000 - 3965 + 3074$

$= 364109$

So, correct variance

$$= \frac{1}{200}(\text{correct } \Sigma x_i^2) - (\text{correct mean})^2$$

$$= \frac{1}{200}(364109) - \left(\frac{7991}{200}\right)^2$$

$$= 1820.545 - 1596.402 = 224.143$$

(v) Correct standard deviation

$$= \sqrt{\text{correct variance}}$$

$$= \sqrt{224.143} \quad \text{[using part (iv)]}$$

$$= 14.971$$

47. (i) Given, observations are

34, 66, 30, 38, 44, 50, 40, 60, 42, 51.

Here, number of observations, $n = 10$

$$\therefore \text{ Mean, } \bar{x} = \frac{34 + 66 + 30 + 38 + 44 + 50 + 40 + 60 + 42 + 51}{10}$$

$$\Rightarrow \quad \bar{x} = \frac{455}{10} = 45.5$$

(ii) Let us make the table for absolute deviation

x_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x}\rvert$
34	$34 - 45.5 = -11.5$	11.5
66	$66 - 45.5 = 20.5$	20.5
30	$30 - 45.5 = -15.5$	15.5
38	$38 - 45.5 = -7.5$	7.5
44	$44 - 45.5 = -1.5$	1.5
50	$50 - 45.5 = 4.5$	4.5
40	$40 - 45.5 = -5.5$	5.5
60	$60 - 45.5 = 14.5$	14.5
42	$42 - 45.5 = -3.5$	3.5
51	$51 - 45.5 = -5.5$	5.5

$$\text{Now, } MD = \frac{\sum_{i=1}^{10} \lvert x_i - \bar{x}\rvert}{10} = \frac{90}{10} = 9.0$$

(iii) The given data can be arranged in ascending order as 30, 34, 38, 40, 42, 44, 50, 51, 60, 66.

Here, total number of observations are 10. i.e. $n = 10$, which is even.

$$\therefore \text{ Median } (M) = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{\left(\frac{10}{2}\right)\text{th observation} + \left(\frac{10}{2}+1\right)\text{th observation}}{2}$$

$$= \frac{(5\text{th observation} + 6\text{th observation})}{2}$$

$$= \frac{42 + 44}{2} = \frac{86}{2} = 43$$

(iv) Let us make the table for absolute deviation

x_i	$\lvert x_i - M\rvert$
30	$\lvert 30 - 43\rvert = 13$
34	$\lvert 34 - 43\rvert = 9$
38	$\lvert 38 - 43\rvert = 5$
40	$\lvert 40 - 43\rvert = 3$
42	$\lvert 42 - 43\rvert = 1$
44	$\lvert 44 - 43\rvert = 1$
50	$\lvert 50 - 43\rvert = 7$
51	$\lvert 51 - 43\rvert = 8$
60	$\lvert 60 - 43\rvert = 17$
66	$\lvert 66 - 43\rvert = 23$
Total	$\sum_{i=1}^{10} \lvert x_i - M\rvert = 87$

Now, mean deviation about median,

$$\text{MD} = \frac{\sum_{i=1}^{10} \lvert x_i - M \rvert}{10}$$

$$= \frac{87}{10} = 8.7$$

(v) The difference between mean deviation about the mean and mean deviation about the median $= 9.0 - 8.7$

$$= 0.3$$

48. (i) Let us make the following table from the given data.

x_i	f_i	$f_i x_i$	$\|x_i - \bar{x}\|$	$f_i\|x_i - \bar{x}\|$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
Total	40	300		92

Here, $N = \Sigma f_i = 40$, $\Sigma f_i x_i = 300$

Now, mean $(\bar{x}) = \frac{1}{N} \Sigma f_i x_i = \frac{1}{40} \times 300 = 7.5$

(ii) Mean deviation about the mean,

$\text{MD}(\bar{x}) = \frac{1}{N} \Sigma f_i |x_i - \bar{x}| = \frac{1}{40} \times 92 = 2.3$

Hence, the mean deviation about mean is 2.3.

(iii) The given observations are already in ascending order. Now, let us make the cumulative frequency.

x_i	f_i	cf
2	2	2
5	8	10
6	10	20
8	7	27
10	8	35
12	5	40
Total	$N = 40$	

Here, $\Sigma f_i = 40$, which is even.

$\therefore$ Median,

$$= \frac{\text{Value of } \left(\frac{40}{2}\right) \text{th observation} + \text{Value of } \left(\frac{40}{2} + 1\right) \text{the observation}}{2}$$

$$= \frac{\text{Value of 20th observation} + \text{Value of 21st observation}}{2}$$

$$= \frac{6+8}{2} = \frac{14}{2} = 7$$

[$\because$ 20th observation lies in the cumulative frequency 20 and its corresponding observation is 6 and the 21st observation lies in the cumulative frequency 27 where the corresponding observation is 8]

(iv)

	x_i	f_i	$\|x_i - 7\|$	$f_i\|x_i - 7\|$
	2	2	5	10
	5	8	2	16
	6	10	1	10
	8	7	1	7
	10	8	3	24
	12	5	5	25
Total				92

Here, $\Sigma f_i |x_i - M| = 92$

and $\Sigma f_i = 40$

$\therefore$ The required mean deviation

$$= \frac{\Sigma f_i |x_i - M|}{\Sigma f_i} = \frac{92}{40} = 2.3$$

(v) The difference between mean and median $= 7.5 - 7 = 0.5$

49. Let us make the following table from the given data.

x_i	f_i	$f_i x_i$	$x_i - \bar{x} = x_i - 14$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	−10	100	300
8	5	40	−6	36	180
11	9	99	−3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
Total	30	420			1374

Here, we have, $N = \Sigma f_i = 30$, $\Sigma f_i x_i = 420$

$$\bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N} = \frac{420}{30} = 14$$

(i) $\bar{x} = \frac{\Sigma f_i x_i}{N}$

(ii) $\sigma^2 = \frac{1}{N} \Sigma f_i (x_i - \bar{x})^2$

(iii) $\bar{x} = 14$

(iv) Variance $(\sigma^2) = \frac{1}{N} \sum_{i=1}^{7} f_i (x_i - \bar{x})^2$

$$\left[\because \sigma^2 = \frac{1}{N} \left\{ \sum_{i=1}^{n} f_i (x_i - \bar{x})^2 \right\} \right]$$

$$= \frac{1}{30} \times 1374 = 45.8$$

(v) Standard deviation,

$$\sigma = \sqrt{\sigma^2} = \sqrt{45.8} = 6.77$$

50. Let us make the following table from the given data.

Class	f_i	cf	Mid-point (x_i)	$\lvert x_i - M \rvert$, $M = 28$	$f_i \lvert x_i - M \rvert$
0-10	6	6	5	23	138
10-20	7	13	15	13	91
20-30	15	28	25	3	45
30-40	16	44	35	7	112
40-50	4	48	45	17	68
50-60	2	50	55	27	54
Total	50				508

Here, $\frac{N}{2} = \frac{50}{2} = 25$

which item lies in the cumulative frequency 28. Therefore, 20-30 is the median class.

So, we have, $l = 20$, $cf = 13$, $f = 15$, $h = 10$ and $N = 50$

Now, Median, $M = l + \frac{\frac{N}{2} - cf}{f} \times h$

$$= 20 + \frac{25 - 13}{15} \times 10$$

$$= 20 + 8 = 28$$

(i) Median, $M = l + \frac{\frac{N}{2} - cf}{f} \times h$

(ii) Mean deviation, MD $= \frac{\Sigma f_i \lvert x_i - M \rvert}{N}$

(iii) $N = \Sigma f_i = 50$

(iv) Median $= 28$

(v) The mean deviation about median is given by

$$\text{MD}(M) = \frac{1}{N} \sum_{i=1}^{6} f_i \lvert x_i - M \rvert$$

$$= \frac{1}{50} \times 508 = 10.16$$

Hence, the mean deviation about median is 10.16.