CONTINIUTY and DIFFERENTIABILITY
Multiple Choice Questions

1 If $f(x) = \begin{cases} kx^2, & x \le 2 \\ 3, & x > 2 \end{cases}$ is continuous then value of k is :

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

2 If $f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ is continuous then value of k is :

(a) $\frac{9}{5}$ (b) $\frac{5}{9}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

3 If $f(x) = \begin{cases} mx-1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ is continuous then value of m is :

(a) $\frac{11}{5}$ (b) $\frac{5}{11}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

4 If $f(x) = \begin{cases} mx^2, & x \le 5 \\ 6x-5, & x > 5 \end{cases}$ is continuous then value of m is :

(a)- 1 (b)4 (c)3 (d)1

5 If $f(x) = \begin{cases} \frac{\sin 2x}{3x}, & x \ne 0 \\ m-1, & x = 0 \end{cases}$ is continuous then value of m is :

(a)2/3 (b)3/2 (c)3/5 (d)5/3

6 If $f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x+5, & x > 5 \end{cases}$ is continuous then value of k is :

(a) $\frac{19}{5}$ (b) $\frac{5}{9}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

7 If $f(x) = \begin{cases} kx-1, & x \le 5 \\ 3x+5, & x > 5 \end{cases}$ is continuous then value of k is :

(a) $\frac{21}{5}$ (b) $\frac{5}{19}$ (c) $\frac{5}{21}$ (d) $\frac{19}{5}$

8 If $f(x) = \begin{cases} \frac{\sin 7x}{3x}, & x \ne 0 \\ m, & x = 0 \end{cases}$ is continuous at $x = 0$ then value of m is

(a) $\frac{3}{7}$ (b) $\frac{4}{7}$ (c) $\frac{7}{4}$ (d) $\frac{7}{3}$

9 If $y = \log[x + \sqrt{x^2+1}\,]$ then $\frac{dy}{dx}$ is

(a) $\sqrt{x^2+1}$ (b) $\frac{1}{\sqrt{x^2+1}}$ (c) $\frac{x}{\sqrt{x^2+1}}$ (d)none of these

10 If $f(x) = \begin{cases} \frac{x^3-8}{x-2}, & x \ne 2 \\ k, & x = 2 \end{cases}$ is continuous at $x = 2$ then value of k is

(a)8 (b)2 (c)6 (d)12

11 $\frac{d}{dx}\{\tan^{-1}(e^x)\}$ is equal to :

(a) $e^x \tan^{-1} e^x$ (b) $\frac{e^x}{1+e^{2x}}$ (c) 0 (d) $e^x \sec^{-1} x$

12 If $y = \sin x$ then at $x = \frac{\pi}{2}$, y_2 is equal to :

(a) -1 (b) 1 (c) 0 (d) $\frac{1}{2}$

13 If $x = 2at, y = at^2$ then $\frac{dy}{dx}$ is equal to:

(a)2 (b) $2a$ (c) $2at$ (d) t

14 If $y = \cos^{-1}(e^x)$ then $\frac{dy}{dx}$ is equal to:

(a) $e^x \sin^{-1}(e^x)$ (b) $e^x \cos^{-1}(e^x)$ (c) $\frac{-e^x}{\sqrt{1-e^{2x}}}$ (d) $\frac{e^x}{\sqrt{1-e^{2x}}}$

15 If $y = \sin^{-1}(e^x)$ then $\frac{dy}{dx}$ is equal to:

(a) $e^x \sin^{-1}(e^x)$ (b) $e^x \cos^{-1}(e^x)$ (c) $\frac{-e^x}{\sqrt{1-e^{2x}}}$ (d) $\frac{e^x}{\sqrt{1-e^{2x}}}$

16 $\frac{d}{dx}\{\cot^{-1}(e^x)\}$ is equal to :

(a) $e^x \tan^{-1} e^x$ (b) $\frac{e^x}{1+e^{2x}}$ (c) $\frac{-e^x}{1+e^{2x}}$ (d) $e^x \sec^{-1} x$

17 If $y = x^2$ then $y_1(5)$ is equal to :

(a)10 (b)25 (c)32 (d) none of these

18 If $y = \log(\sin x)$ then at $x = \frac{\pi}{4}, \frac{dy}{dx}$ is

(a)0 (b) -1 (c)1 (d) $\sqrt{2}$

19 If $y = e^{\log x}$ then $\frac{dy}{dx}$ is

(a)$\log x - x$ (b)$xe^{\log x}$ (c)1 (d)$e^{\log x}\log x$

20 If $y = \log(\sec x)$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is

(a)1 (b)-1 (c)0 (d)none of these

True/False

1) If $y = 10x$ then $\frac{dy}{dx} = 0$.
2) If $y = 500$ then $\frac{dy}{dx} = 0$
3) If $y = \tan x$ then $\frac{dy}{dx} = \sin x$
4) If $y = \cot x$ then $\frac{dy}{dx} = \log(\cos x)$
5) If $y = \tan 2x$ then $\frac{dy}{dx} = 2\sec^2 2x$
6) Trigonometric functions are differentiable functions in their respective domains.
7) The composition of two continuous functions is continuous.
8) Every differentiable function is a continuous function.
9) Role's theorem holds for the functions where Lagrange's mean value theorem holds.
10) Logarithmic differentiation is essential for the function f when $f(x) = \left(p(x)\right)^{q(x)}$

2 and 4 Marks Questions

1. Examine the continuity of the following functions f at the indicated points:

(i) $f(x) = \begin{cases} 2x+1, & x \le 2 \\ 3x-1, & x > 2 \end{cases}$ at $x = 2$.

(ii) $f(x) = \begin{cases} 3x+1, & x \le 2 \\ 4x-1, & x > 2 \end{cases}$ at $x = 2$.

(iii) $f(x) = \begin{cases} 2x-1, & x < 2 \\ \frac{3x}{2}, & x \ge 2 \end{cases}$ at $x = 2$.

(iv) $f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & x \ne 3 \\ 5, & x = 3 \end{cases}$ at $x = 3$.

(v) $f(x) = \begin{cases} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sin x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$ at $x = 0$.

(vi) $f(x) = \begin{cases} \frac{\sqrt{1+2x}-\sqrt{1-2x}}{\sin x}, & x \ne 0 \\ 2, & x = 0 \end{cases}$ at $x = 0$.

(vii) $f(x) = \begin{cases} \frac{\sqrt{1+3x}-\sqrt{1-3x}}{\sin x}, & x \ne 0 \\ 3, & x = 0 \end{cases}$ at $x = 0$.

2. In the following, determine the value of constant/scalar so that given function f is continuous at the indicated points :

(i) $f(x) = \begin{cases} \frac{\sin 2x}{x}, & x \ne 0 \\ m, & x = 0 \end{cases}$ at $x = 0$.

(ii) $f(x) = \begin{cases} \frac{\sin 5x}{3x}, & x \ne 0 \\ m, & x = 0 \end{cases}$ at $x = 0$.

(iii) $f(x) = \begin{cases} \frac{\tan 2x}{7x}, & x \ne 0 \\ m, & x = 0 \end{cases}$ at $x = 0$.

(iv) $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & x \ne 0 \\ k, & x = 0 \end{cases}$ at $x = 0$.

(v) $f(x)=\begin{cases}\frac{1-\cos 2x}{2x^2}, & x\neq 0\\ k, & x=0\end{cases}$ at $x=0$

(vi) $f(x)=\begin{cases}\frac{x^2-9}{x-3}, & x\neq 3\\ k, & x=3\end{cases}$ at $x=3$

(vii) $f(x)=\begin{cases}\frac{x^2-25}{x-5}, & x\neq 5\\ k, & x=5\end{cases}$ at $x=5$

3. Determine the value of constants a & b so that function f defined bellow is continuous everywhere:

(i) $f(x)=\begin{cases}x+2, & x\leq 2\\ ax+b, & 2<x<5\\ 3x-2, & x\geq 5\end{cases}$

(ii) $f(x)=\begin{cases}ax^2+b, & x>2\\ 2, & x=2\\ 2ax-b, & x<2\end{cases}$

(iii) $f(x)=\begin{cases}3ax+b, & x>1\\ 11, & x=1\\ 5ax-2b, & x<1\end{cases}$

4. Differentiate the following w.r.t. x

(i) $\left(\frac{3x-1}{3x+1}\right)^2$ (ii) $\left(\frac{4x-1}{4x+1}\right)^2$ (iii) $\sin x^2$

(iv) $\sin(x^2+5)$ (v) $\sqrt{15x^2-x+1}$

5. Find $\frac{dy}{dx}$ in each of the following :

(i) $ax+by^2=\cos y$ (ii) $xy+y^2=\tan x+y$ (iii) $\sin^2 y+\cos xy=10$

(iv) $\sqrt{x}+\sqrt{y}=35$ (v) $y(y+1)=x(x+1)(x+2)$ (vi) $y^2=4ax$

(vii) $y=\frac{4}{3}x^{3/4}$ (viii) $x^{2/3}+y^{2/3}=2$

6. Differentiate the following w.r.t. x :

(i) $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ (ii) $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ (iii) $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ (iv) $\sin^{-1}(1-2x^2)$

(v) $\sin^{-1}(3x-4x^3)$ (vi) $\cos^{-1}(4x^3-3x)$ (vii) $\tan^{-1}\left(\frac{x^{\frac{2}{3}}+a^{\frac{2}{3}}}{1-x^{\frac{2}{3}}a^{\frac{2}{3}}}\right)$ (viii) $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

(ix) $\cos^{-1}\sqrt{\frac{1+x}{2}}$ (x) $\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x}\right)$ (xi) $\tan^{-1}\left(\frac{\sqrt{a^2x^2+1}-1}{ax}\right)$ (xii) $\cot^{-1}\left(\frac{\sqrt{x^2+1}-1}{x}\right)$

(xiii) $\cot^{-1}\left(\frac{1+x}{1-x}\right)$ (xiv) $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ (xv) $\cot^{-1}\left(\sqrt{1+x^2}-x\right)$ (xvi) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

(xvii) $\tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$ (xviii) $\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$ (xix) $e^{\sin\sqrt{x}}$ (xx) $e^{\cos^{-1}(x+1)}$

(xxi) $\log\cos 5x$ (xxii) $\log\left(x+3+\sqrt{x^2+6x+3}\right)$ (xxiii) $\log\left(x+\sqrt{x^2+1}\right)$

(xxiv) $\log\left(\frac{1+x}{1-x}\right)$ (xxv) $\log\{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\}$ (xxvi) $\log\left(\frac{x+\sqrt{x^2-a^2}}{x-\sqrt{x^2-a^2}}\right)$

7. If $e^{x+y}=xy$ then show that $\frac{dy}{dx}=\frac{y(1-x)}{x(y-1)}$.

8. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ then prove that $(1-x^2)\frac{dy}{dx}-xy=1$.

9. Find $\frac{dy}{dx}$ for the following parametric functions :

(i) $x=a\cos^2\theta,\ y=b\sin^2\theta$

(ii) $x=2\sin^2\theta,\ y=2\cos^2\theta$

(iii) $x=a(\theta-\sin\theta),\ y=b(1+\cos\theta)$

(iv) $x=a(\theta+\sin\theta),\ y=b(1+\cos\theta)$

10. (i) If $x=2\cos\theta-\cos 2\theta$, $y=2\sin\theta-\sin 2\theta$ then find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{2}$.

(ii)If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$, $y=a\sin\theta$ then find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$.

11. If $x=\frac{1-t^2}{1+t^2}$, $y=\frac{2t}{1+t^2}$ then prove that $\frac{dy}{dx}+\frac{x}{y}=0.$

12. Differentiate the following w.r.t. as indicated :

(i) $\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$ $w.r.t.$ $\tan^{-1}x$

(ii) $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ $w.r.t.$ $\sin^{-1}x$

(iii) $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $w.r.t.$ $\tan^{-1}x$

(iv) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ $w.r.t.$ $\tan^{-1}x$

(v) $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ $w.r.t.$ $\tan^{-1}x$

(vi) $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ $w.r.t.$ $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

(vii) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ $w.r.t.$ $\tan^{-1}x$

(viii) $\tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$ $w.r.t.$ $\tan^{-1}ax$

(ix) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ $w.r.t.$ $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(x) $\cos^2 x$ $w.r.t.$ $e^{\sin x}$

13. Differentiate the following w.r.t. x :

(i) $(\sin x)^x$ (ii) $x^{\sin^{-1}x}$ (iii) $(\cos x)^{\sin x}$

(iv) $x^x\sin^{-1}\sqrt{x}$ (v) $\cos x^x$

14. Differentiate the following w.r.t. :

(i) $x^{\sin x}+(\sin x)^x$ (ii) $x^{\log x}+(\log x)^x$ (iii) $x^{\tan x}+(\tan x)^x$

(iv) $x^{\cos x}+(\sin x)^{\tan x}$ (v) $x^x+(\sin x)^x$ (vi) $x^{\sin^{-1}x}+(\sin^{-1}x)^x$

15. (i) If $(\sin x)^y=(\sin y)^x$, find $\frac{dy}{dx}$.

(ii) If $(\sin x)^y=(\cos y)^x$, find $\frac{dy}{dx}$.

(iii) If $x^p y^q=(x+y)^{p+q}$ show that $\frac{dy}{dx}=\frac{y}{x}$.

(iv) If $y=x^y$ show that $\frac{dy}{dx}=\frac{y^2}{x(1-y\log x)}$.

(v) If $x^y+y^x=\log a$, find $\frac{dy}{dx}$.

16. If $y=\sqrt{x+\sqrt{x+\sqrt{x+\cdots\ldots\ldots to\ \infty}}}$ then prove that $(2y-1)\frac{dy}{dx}=1.$

17. If $y=\sqrt{3^x+\sqrt{3^x+\sqrt{3^x+\cdots\ldots\ldots\ldots\infty}}}$ then prove that $(2y-1)\frac{dy}{dx}=3^x\log 3$.

18. If $y=\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+\cdots\ldots\ldots\infty}}}$ then prove that $(2y-1)\frac{dy}{dx}=\sec^2 x$.

19. If $y=(\sin x)^{(\sin x)^{(\sin x)\ldots\ldots\infty}}$ then show that $\frac{dy}{dx}=\frac{y^2\cot x}{1-y\log\sin x}$.

20. If $y=(\tan x)^{(\tan x)^{(\tan x)\ldots\ldots\infty}}$ prove that $\frac{dy}{dx}=2$ at $x=\frac{\pi}{4}$.

21. If $y=\sin^{-1}x$ then show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$.

22. Find $\frac{d^2y}{dx^2}$ in the following :

(i) $x=a\cos\theta$, $y=b\sin\theta$ (ii) $x=a\cos^3\theta$, $y=a\sin^3\theta$ (iii) $x=a\cos^3\theta$, $y=b\sin^3\theta$

23. If $y=(\sin^{-1}x)^2$ then prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$.

24. If $y=(\tan^{-1}x)^2$ then show that $(1+x^2)^2y_2+2x(1+x^2)y_1-2=0$.

25. If $y=\log\left(x+\sqrt{x^2+1}\right)$ then show that $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$.

26. If $y=\left(x+\sqrt{x^2+1}\right)^m$ then prove that $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2y=0$.

27. If $y=\cos(2\cos^{-1}x)$ then prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$.

28. If $y = \sin(m\sin^{-1}x)$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

29. If $y = e^{m\sin^{-1}x}$ then show that $(1-x^2)y_2 - xy_1 - m^2y = 0$.

30. If $y = e^{m\tan^{-1}x}$ prove that

(i) $(1+x^2)\frac{d^2y}{dx^2} + (2x-m)\frac{dy}{dx} = 0$ (ii) $(1+x^2)^2y_2 + 2x(1+x^2)y_1 - m^2y = 0$.

31. Verify Rolle's theorem for the following functions :

(i) $f(x) = x^3 - 4x$ on $[-2, 2]$
(ii) $f(x) = x^3 + 3x^2 - 24x - 80$ on $[-4, 5]$
(iii) $f(x) = x^3 - 6x^2 + 11x - 6$ on $[1, 3]$
(iv) $f(x) = (x+1)(x-4)$ on $[-1, 4]$
(v) $f(x) = (x-3)(x-5)^2$ on $[3, 5]$
(vi) $f(x) = \sin 2x$ on $[0, \frac{\pi}{2}]$
(vii) $f(x) = \cos x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$
(viii) $f(x) = \sin^2 x$ on $[0, \pi]$
(ix) $f(x) = \cos^2 x$ on $[0, \pi]$
(x) $f(x) = \sin x + \cos x$ on $[0, 2\pi]$

32. Verify Lagrange's Mean Value theorem for the following functions :

(i) $f(x) = x^3 - 2x^2 - x + 3$ in $[0, 1]$
(ii) $f(x) = x^3 - 5x^2 - 3x$ in $[1, 3]$
(iii) $f(x) = (x-1)(x-2)(x-3)$ in $[1, 4]$
(iv) $f(x) = \cos x$ in $[0, \frac{\pi}{2}]$

33. Verify LMVT for $f(x) = x(x-1)(x-2)$ in $[0, \frac{1}{2}]$.

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