Trigonometry

Exercise – 8.1

Solution 1:

$$\sin X = \frac{\text{opposite side of the angle X}}{\text{hypotenuse}} = \frac{YZ}{ZX}$$

$$\cos X = \frac{\text{adjacent side of the angle X}}{\text{hypotenuse}} = \frac{XY}{ZX}$$

$$\tan X = \frac{\text{opposite side of the angle X}}{\text{adjacent side of the angle X}} = \frac{YZ}{XY}$$

$$\cot X = \frac{\text{adjacent side of the angle X}}{\text{opposite side of the angle X}} = \frac{XY}{YZ}$$

$$\sec X = \frac{\text{hypotenuse}}{\text{adjacent side of the angle X}} = \frac{ZX}{XY}$$

$$\text{cosec} X = \frac{\text{hypotenuse}}{\text{opposite side of the angle X}} = \frac{ZX}{YZ}$$

Similarly, for angle Z,

$$\sin Z = \frac{XY}{ZX};\ \cos Z = \frac{YZ}{ZX};\ \tan Z = \frac{XY}{YZ};\ \cot Z = \frac{YZ}{XY};$$

$$\sec Z = \frac{ZX}{YZ};\ \text{cosec} Z = \frac{ZX}{XY}.$$

Solution 2:

In ΔABC, $m\angle B = 90^\circ$, $\angle ACB = x$.

$\therefore \angle BAC = 90 - x$.

In ΔADC, $m\angle D = 90^\circ$, $\angle ACD = y$.

$\therefore m\angle DAC = 90^\circ - y$.

$$\tan x = \frac{AB}{BC};$$

$$\cot(90^\circ - y) = \cot\angle DAC = \frac{AD}{DC};$$

$$\sec y = \frac{AC}{CD};$$

$$\sin(90^\circ - x) = \frac{BC}{AC};$$

$$\text{cosec}(90^\circ - y) = \text{cosec}\angle BAC = \frac{AC}{DC};$$

$$\cos(90^\circ - x) = \cos\angle BAC = \frac{AB}{AC}.$$

Solution 3:

In ΔLMT, $m\angle T = 90°$ and $m\angle M = 50°$.

$\therefore$ $m\angle MLT = 40°$.

ML is the hypotenuse.

In ΔLNT, $m\angle T = 90°$, $m\angle TLN = 30°$.

$\therefore$ $m\angle N = 60°$.

LN is the hypotenuse.

$\tan 50° = \frac{LT}{MT}$; $\sec 40° = \frac{LM}{LT}$; $\sin 50° = \frac{LT}{LM}$;

$\cos 60° = \frac{TN}{LN}$; $\cot 60° = \frac{TN}{LT}$; $\text{cosec } 40° = \frac{LM}{MT}$;

$\cos 30° = \frac{LT}{LN}$.

Solution 4:

ΔUVW is a right angled triangle.

$\therefore$ By Pythagoras' Theorem,

$UW^2 = UV^2 + VW^2$

$\therefore (8)^2 = (6)^2 + VW^2$

$\therefore VW^2 = (8)^2 - (6)^2 = 64 - 36 = 28$

$\therefore VW = 2\sqrt{7}$

$\sin W = \frac{UV}{UW} = \frac{6}{8} = \frac{3}{4}$

$\cos W = \frac{VW}{UW} = \frac{2\sqrt{7}}{8} = \frac{\sqrt{7}}{4}$

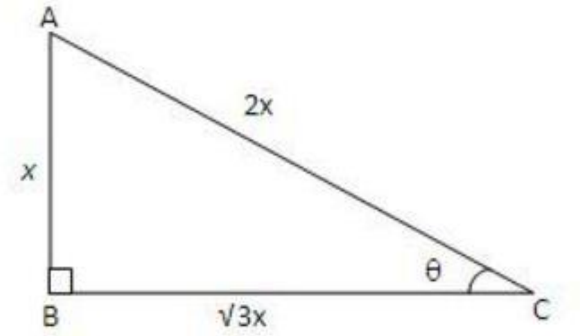
$\tan W = \frac{UV}{VW} = \frac{6}{2\sqrt{7}} = \frac{3}{\sqrt{7}}$

$\cot W = \frac{VW}{UV} = \frac{2\sqrt{7}}{6} = \frac{\sqrt{7}}{3}$

$\sec W = \frac{UW}{VW} = \frac{8}{2\sqrt{7}} = \frac{4}{\sqrt{7}}$

$\text{cosec } W = \frac{UW}{UV} = \frac{8}{6} = \frac{4}{3}$

Solution 5:



In the figure, ΔABC,

$m\angle B = 90^\circ, \angle C = \theta$

$\cos\theta = \frac{\sqrt{3}}{2}$ ...(Given)

From the figure, $\cos\theta = \frac{BC}{AC}$.

$\therefore BC = \sqrt{3}x$ and $AC = 2x$ ....(x is a constant, $x > 0$)

In the right angled ΔABC, by Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

$\therefore (2x)^2 = AB^2 + (\sqrt{3}x)^2$

$\therefore AB^2 = (2x)^2 - (\sqrt{3}x)^2$

$\therefore AB^2 = x^2$

$\therefore AB = x$

$\sin\theta = \frac{AB}{AC} = \frac{x}{2x} = \frac{1}{2}$

$\cos\theta = \frac{BC}{AC} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{AB}{BC} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}}$

$\cot\theta = \frac{BC}{AB} = \frac{\sqrt{3}x}{x} = \sqrt{3}$

$\sec\theta = \frac{AC}{BC} = \frac{2x}{\sqrt{3}x} = \frac{2}{\sqrt{3}}$

$\operatorname{cosec}\theta = \frac{AC}{AB} = \frac{2x}{x} = 2$

Solution 6:

i. We know that $\sin\theta \times \text{cosec}\,\theta = 1$

$$\therefore \frac{2}{7} \times \text{cosec}\,\theta = 1$$

$$\therefore \text{cosec}\,\theta = 1 \times \frac{7}{2} = \frac{7}{2}$$

ii. We know that $\tan\theta \times \cot\theta = 1$

$$\therefore \frac{2}{5} \times \cot\theta = 1$$

$$\therefore \cot\theta = 1 \times \frac{5}{2} = \frac{5}{2}$$

Solution 7(i):

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \tan\theta = \frac{\frac{\sqrt{2}}{3}}{\frac{1}{3}} = \sqrt{2} \qquad \ldots..(1)$$

$$\text{Now, } \tan\theta \times \cot\theta = 1$$

$$\therefore \sqrt{2} \times \cot\theta = 1 \qquad \ldots.[\text{From}(1)]$$

$$\therefore \cot\theta = \frac{1}{\sqrt{2}}.$$

$$\cos\theta \times \sec\theta = 1$$

$$\therefore \frac{1}{3} \times \sec\theta = 1$$

$$\therefore \sec\theta = 3$$

$$\sin\theta \times \operatorname{cosec}\theta = 1$$

$$\therefore \frac{\sqrt{2}}{3} \times \operatorname{cosec}\theta = 1$$

$$\therefore \operatorname{cosec}\theta = \frac{3}{\sqrt{2}}$$

Hence,

$$\tan\theta = \sqrt{2};\ \cot\theta = \frac{1}{\sqrt{2}};\ \sec\theta = 3;\ \operatorname{cosec}\theta = \frac{3}{\sqrt{2}}$$

Solution 7(ii):

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\therefore \sin\theta = \tan\theta \times \cos\theta = \frac{\sqrt{7}}{2} \times \frac{2}{\sqrt{11}}$$

$$\therefore \sin\theta = \frac{\sqrt{7}}{\sqrt{11}}$$

Exercise – 8.2

Solution 1:

(i) $\tan 45^\circ = 1$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$ and $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\therefore 2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2$$

(ii) $\cot 45^\circ = 1$, $\sec 60^\circ = 2$, $\operatorname{cosec} 30^\circ = 2$ and $\cot 90^\circ = 0$

$$\therefore 4\cot^2 45^\circ - \sec^2 60^\circ + \operatorname{cosec}^2 30^\circ + \cot 90^\circ$$

$$= 4(1)^2 - (2)^2 + (2)^2 + 0$$

$$= 4$$

Solution 2:

$$\text{L.H.S.} = \sin 90^\circ = 1$$

$$\text{R.H.S.} = 2\cos 45^\circ \times \sin 45^\circ$$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= 1.$$

$$\text{L.H.S.} = \text{R.H.S.}$$

$$\sin 90^\circ = 2\cos 45^\circ \times \sin 45^\circ.$$

Solution 3(i):

$$\sin 30^\circ = \frac{1}{2}$$
$$\cos(40^\circ + x) = \sin 30^\circ$$
$$\therefore \cos(40^\circ + x) = \frac{1}{2}$$
$$\text{But } \cos 60^\circ = \frac{1}{2}$$
$$\therefore \cos(40^\circ + x) = \cos 60^\circ$$
$$\therefore 40^\circ + x = 60^\circ$$
$$\therefore x = 60^\circ - 40^\circ = 20^\circ$$
$$x = 20^\circ$$

Solution 3(ii):

$$\tan y = \sin 45^\circ \times \cos 45^\circ + \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$
$$= \frac{1}{2} + \frac{1}{2} = 1$$
$$\therefore \tan y = 1$$
$$\text{But } \tan 45^\circ = 1$$
$$\therefore \tan y = \tan 45^\circ$$
$$\therefore y = 45^\circ$$

Solution 4:

$$\text{L.H.S.} = \sin A = \sin 30^\circ = \frac{1}{2}$$

$$\text{R.H.S.} = \sqrt{\frac{1-\cos 2A}{2}}$$

$$= \sqrt{\frac{1-\cos 60^\circ}{2}} \quad \ldots(\angle A = 30^\circ \quad \therefore 2A = 60^\circ)$$

$$= \sqrt{\frac{1-\frac{1}{2}}{2}}$$

$$= \sqrt{\frac{\frac{1}{2}}{2}}$$

$$= \sqrt{\frac{1}{4}}$$

$$= \frac{1}{2}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$\therefore \sin A = \sqrt{\frac{1-\cos 2A}{2}}$$

Solution 5:

$$\sin B = \frac{1}{2} \qquad \ldots(\text{Given})$$

$$\text{but } \sin 30^\circ = \frac{1}{2}$$

$$\therefore B = 30^\circ \quad (1)$$

$$\sin(A + B) = \frac{\sqrt{3}}{2} \qquad \ldots(\text{Given})$$

$$\text{but } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore A + B = 60^\circ$$

$$\therefore A + 30^\circ = 60^\circ \qquad \ldots[\text{From}(1)]$$

$$\therefore A = 30^\circ$$

$$\text{Thus } A = 30^\circ; B = 30^\circ$$

Exercise – 8.3

Solution 1(i):

$$\cos 38^\circ \times \cos 52^\circ - \sin 38^\circ \times \sin 52^\circ$$

$$= \sin(90^\circ - 38^\circ) \times \sin(90^\circ - 52^\circ) - \sin 38^\circ \times \sin 52^\circ$$

$$= \sin 38^\circ \times \sin 52^\circ - \sin 38^\circ \times \sin 52^\circ$$

$$= 0$$

Solution 1(ii):

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \times \operatorname{cosec} 31^\circ$$
$$= \frac{\cos 80^\circ}{\cos(90^\circ - 10^\circ)} + \sin(90^\circ - 59^\circ) \times \frac{1}{\sin 31^\circ}$$
$$= \frac{\cos 80^\circ}{\cos 80^\circ} + \sin 31^\circ \times \frac{1}{\sin 31^\circ}$$
$$= 1 + 1$$
$$= 2$$

Solution 1(iii):

$$\frac{2\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$$
$$= \frac{2\tan 53^\circ}{\tan(90^\circ - 37^\circ)} - \frac{\tan(90^\circ - 80^\circ)}{\tan 10^\circ}$$
$$= \frac{2\tan 53^\circ}{\tan 53^\circ} - \frac{\tan 10^\circ}{\tan 10^\circ}$$
$$= 2 - 1$$
$$= 1$$

Solution 2:

$$\tan 2A = \cot(A - 18^\circ)$$
$$\therefore \cot(90^\circ - 2A) = \cot(A - 18^\circ)$$
$$\therefore 90^\circ - 2A = A - 18^\circ$$
$$\therefore 3A = 108^\circ$$
$$\therefore A = 36^\circ$$
$$\therefore 2A = 72^\circ$$
$$\therefore (A - 18^\circ) = 36^\circ - 18^\circ = 18^\circ$$

Solution 3:

$$\sin 3\theta = \cos(\theta - 6^\circ)$$
$$\therefore \cos(90^\circ - 3\theta) = \cos(\theta - 6^\circ)$$
$$\therefore 90^\circ - 3\theta = \theta - 6^\circ$$
$$\therefore 4\theta = 96^\circ$$
$$\therefore \theta = 24^\circ$$
$$\therefore 3\theta = 72^\circ$$
$$\therefore (\theta - 6) = 24^\circ - 6^\circ = 18^\circ$$

Solution 4:

$$\sin A = \cos B$$
$$\therefore \sin A = \sin(90^\circ - B)$$
$$\therefore A = 90^\circ - B$$
$$\therefore A + B = 90^\circ$$