DETERMINANTS (XII, R. S. AGGARWAL)

EXERCISE 6A (Pg. No.: 258)

Very short –Answer Questions

1. If A is a 2×2 matrix such that $|A|\neq 0$ and $|A|=5$, write the value of $|4A|$

Sol. $|4A|=4^2\cdot|A|$ $\{\because A$ is a 2×2 matrix

$=16\times5=80$

2. If A is a 3×3 matrix such that $|A|\neq 0$ and $|3A|=k|A|$ then write the value of k

Sol. $\because |3A|=K|A|$

$\Rightarrow 3^2|A|=K|A|$ $\{\because A$ is a 3×3 matrix

$\Rightarrow K=27$

3. Let A be a square matrix of order 3, write the value of $|2A|$, where $|A|=4$

Sol. $|2A|=2^3\cdot|A|$ $\{\because$ order of matrix A is 3×3

$=8\times4=32$

4. If A_{ij} is the cofactor of the element a_{ij} of $\begin{vmatrix}2 & -3 & 5\\6 & 0 & 4\\1 & 5 & -7\end{vmatrix}$ then write the value of $(a_{32}A_{32})$

Sol. Here $a_{32}=5$

$A_{32}=(-1)^{3+2}\begin{vmatrix}2 & 5\\6 & 4\end{vmatrix}=-(8-30)=22$ $\therefore a_{32}\cdot A_{32}=5\times22=110$

5 Evaluate $\begin{vmatrix}x^2-x+1 & x-1\\x+1 & x+1\end{vmatrix}$

Sol. $\begin{vmatrix}x^2-x+1 & x-1\\x+1 & x+1\end{vmatrix}$

$=(x+1)(x^2-x+1)-(x-1)(x+1)$

$=x^3+1-(x^2-1)$ $=x^3+1-x^2+1=x^3-x^2+2$

6. Evaluate $\begin{vmatrix}a+ib & c+id\\-c+id & a-ib\end{vmatrix}$

Sol. $\begin{vmatrix}a+ib & c+id\\-c+id & a-ib\end{vmatrix}$

$=(a+ib)(a-ib)-(c+id)(-c+id)$

$=(a+ib)(a-ib)+(c+id)(c-id)$ $=a^2+b^2+c^2+d^2$

7. If $\begin{vmatrix}3x & 7\\-2 & 4\end{vmatrix}=\begin{vmatrix}8 & 7\\6 & 4\end{vmatrix}$ find the value of x

Sol. Given $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

$\Rightarrow 4\times 3x - 7\times(-2) = 8\times 4 - 7\times 6$

$\Rightarrow 12x + 14 = 32 - 42 \Rightarrow 12x + 14 = -10 \Rightarrow 12x = -10 - 14$

$\Rightarrow 12x = -24 \Rightarrow x = -2$

8. if $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ write the value of x

Sol. Given $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow 2x^2 - 40 = 18 + 14 \Rightarrow 2x^2 - 40 = 32 \Rightarrow 2x^2 = 72 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

9. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 5 \end{vmatrix}$ find the value of x

Sol. Given $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 5 \end{vmatrix}$

$\Rightarrow 2x\cdot(x+1) - 2(x+3)(x+1) = 5 - 15$

$\Rightarrow 2x^2 + 2x - 2x^2 - 8x - 6 = -10$

$\Rightarrow -6x - 6 = -10$

$\Rightarrow -6x = -10 + 6$

$\Rightarrow -6x = -4$

$\Rightarrow x = \frac{4}{6} \Rightarrow x = \frac{2}{3}$

10. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, find the value of $3|A|$

Sol. $\because A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$

$= 3\times 4 - 4\times 1 = 6 - 4 = 2$

$\therefore 3|A| = 3\times 2 = 6$

11. Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$

Sol. $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$

$= 2(35 - 20) = 2\times 15 = 30$

12. Evaluate $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix}$

Sol. $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix} = \sqrt{6}\times\sqrt{24}-\sqrt{5}\times\sqrt{20}$

$= \sqrt{6}\times\sqrt{6}\times\sqrt{4}-\sqrt{5}\times\sqrt{5}\times\sqrt{4} \ = 6\times 2-5\times 2 = 2$

13. Evaluate $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

Sol. $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$= 2\cos^2\theta + 2\sin^2\theta \ = 2\left(\cos^2\theta+\sin^2\theta\right) = 2$

14. Evaluate $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$

Sol. $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$

$= \cos\alpha\cdot\cos\alpha - \sin\alpha\cdot\left(-\sin\alpha\right) \ = \cos^2\alpha+\sin^2\alpha \ = 1$

15. Evaluate $\begin{vmatrix} \sin 60^\circ & \cos 60^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{vmatrix}$

Sol. $\begin{vmatrix} \sin 60^\circ & \cos 60^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{vmatrix}$

$= \sin 60^\circ\cdot\cos 30 - \cos 60^\circ\cdot\left(-\sin 30\right)$

$= \sin 60^\circ\cdot\cos 30^\circ + \cos 60^\circ\cdot\sin 30^\circ$

$= \sin\left(60^\circ+30^\circ\right) \ = \sin 90^\circ = 1$

16. Evaluate $\begin{vmatrix} \cos 65^\circ & \sin 65^\circ \\ \sin 25^\circ & \cos 25^\circ \end{vmatrix}$

Sol. $\begin{vmatrix} \cos 65^\circ & \sin 65^\circ \\ \sin 25^\circ & \cos 25^\circ \end{vmatrix}$

$= \cos 65^\circ\cdot\cos 25^\circ - \sin 65^\circ\cdot\sin 25^\circ$

$= \cos\left(65^\circ+25^\circ\right) = \cos 90^\circ = 0$

17. Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

Sol. $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

$= \cos 75^\circ\cdot\cos 15^\circ - \sin 75^\circ\cdot\sin 15^\circ$

$= \cos\left(75^\circ+15^\circ\right) = \cos 90^\circ = 0$

18. Evaluate $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$

Sol. $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$

$= -2\begin{vmatrix} 2 & 4 \\ 4 & 6 \end{vmatrix}$ $\quad \{$expard by C_2

$= -2(12-16) = -2\times(-4) = 8$

19. Without expanding the determinate prove that $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = 0$

Sol. **L.H.S.** $= \begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$

$= \begin{vmatrix} 41-40 & 1 & 5 \\ 79-72 & 7 & 9 \\ 29-24 & 5 & 3 \end{vmatrix}$ $\quad \{ C_1 \to C_1 - 8C_3$

$= \begin{vmatrix} 1 & 1 & 5 \\ 7 & 7 & 9 \\ 5 & 5 & 3 \end{vmatrix}$

$= 0$ $\{\because C_1$ identical C_2

20. For what value of x, the given matrix $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix ?

Sol. If a be a singular matrix

$|A| = 0$

$\Rightarrow \begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix} = 0$

$\Rightarrow 4(3-2x) - 2(x+1) = 0$

$\Rightarrow 12 - 8x - 2x - 2 = 0$

$\Rightarrow 14 - 10x = 0 \Rightarrow 10x = 14$

$\Rightarrow x = \frac{14}{10} \Rightarrow x = \frac{7}{5}$

21. $\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix}$

Sol. Let $\Delta = \begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix} \Rightarrow \Delta = \{14(-7) - 9(-8)\} \Rightarrow \Delta = (-98+72) = -26$

22. $\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix}$

Sol. Let $\Delta = \begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix} \Rightarrow \Delta = \{3\times3 - \sqrt{5}(-\sqrt{5})\} \Rightarrow \Delta = 9+5 = 14 \quad \therefore \Delta = 14$

EXERCISE 6B (Pg. No.: 260)

Evaluates:-

1. $\begin{vmatrix} 67 & 18 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$

Sol. $\Delta = \begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix} \Rightarrow \Delta = \begin{vmatrix} 67-39 & 19-13 & 21-14 \\ 39-81 & 13-24 & 14-26 \\ 81 & 24 & 26 \end{vmatrix}$ (Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$)

$\Rightarrow \Delta = \begin{vmatrix} 28 & 6 & 7 \\ -42 & -11 & -12 \\ 81 & 24 & 26 \end{vmatrix} \Rightarrow \Delta = 28\begin{vmatrix} -11 & -12 \\ 24 & 26 \end{vmatrix} - 6\begin{vmatrix} -42 & -12 \\ 81 & 26 \end{vmatrix} + 7\begin{vmatrix} -42 & -11 \\ 81 & 24 \end{vmatrix}$

$\Rightarrow \Delta = 28(-286+288) - 6(-1092+972) + 7(-1008+891)$

$\Rightarrow \Delta = 28(2) - 6(-120) + 7(-117) \quad \Rightarrow \Delta = 56 + 720 - 819 \quad \Rightarrow \Delta = -43$

2. $\begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$

Sol. Let $\Delta = \begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix} \Rightarrow \Delta = 29\begin{vmatrix} 31 & 27 \\ 54 & 46 \end{vmatrix} - 26\begin{vmatrix} 25 & 27 \\ 63 & 46 \end{vmatrix} + 22\begin{vmatrix} 25 & 31 \\ 63 & 54 \end{vmatrix}$

$\Rightarrow \Delta = 29(1426-1458) - 26(1150-1701) + 22(1350-1953)$

$\Rightarrow \Delta = -998 + 14326 - 13266 \quad \therefore \Delta = 132$

3. $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Sol. $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 & 0 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} \{ R_1 \to R_1 - 6R_3$

$= 0$

4. $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

Sol. $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 9 \\ 0 & -7 & -20 \\ 0 & -20 & -46 \end{vmatrix}$

$$= \begin{vmatrix} -7 & -20 \\ -20 & -46 \end{vmatrix}$$

$$= (-7)\times(-46) - (-20)\times(-20)$$

$$= 322 - 400 = -78$$

Using properties of determinates prove that

5. $$\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 4a+b & 6a+b \end{vmatrix}$$

Sol. Let $A = \begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 4a+b & 6a+b \end{vmatrix}$

Applying $C_3 \to C_3 - C_2,\ C_2 \to C_2 - C_1,\ A = \begin{vmatrix} a+b & a & a \\ 2a+b & a & a \\ 4a+b & 0 & 2a \end{vmatrix} = a.a \begin{vmatrix} a+b & 1 & 1 \\ 2a+b & 1 & 1 \\ 4a+b & 0 & 2 \end{vmatrix}$

Applying $C_3 \to C_3 - C_2,\quad A = a^2 \begin{vmatrix} a+2b & 1 & 0 \\ 2a+b & 1 & 0 \\ 4a+b & 0 & 2 \end{vmatrix} = 2a^2 \begin{vmatrix} a+2b & 1 \\ 2a+b & 1 \end{vmatrix} = 2a^2(a+2b-2a-b) = 2a^2(b-a$

6. $$\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Sol. L.H.S. $= \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$

$$= \begin{vmatrix} 1-1 & (b+c)-(c+a) & (b^2+c^2)-(c^2+a^2) \\ 1-1 & (c+a)-(a+b) & (c^2+a^2)-(a^2+b^2) \\ 1 & a+b & a^2+b^2 \end{vmatrix} \begin{cases} R_1 \to R_1 - R_2 \\ R_2 + R_2 - R_3 \end{cases}$$

$$= \begin{vmatrix} 0 & b-a & b^2-a^2 \\ 0 & c-b & c^2-b^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = \begin{vmatrix} 0 & b-a & (b-a)(b+a) \\ 0 & (c-b) & (c-b)(c+b) \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$

$$= (b-a)(c-b) \begin{vmatrix} 0 & 1 & b+a \\ 0 & 1 & c+b \\ 1 & a+b & a^2+b^2 \end{vmatrix} \begin{cases} \text{taking common } (b-a)\&(c-b) \\ \text{from } R_1 \text{ and } R_2 \text{ respectively} \end{cases}$$

$$(a-b)(b-c) \begin{vmatrix} 1 & b+a \\ 1 & c+b \end{vmatrix} \{ \text{exparding by } C_1$$

$$= (a-b)(b-c)(c+b-b-a)$$

$= (a-b)(b-c)(c-a) =$ R.H.S. proved

7. $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$

Sol. $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1+p+q \\ 1 & 3 & 1+3p+2q \\ 3 & 6 & 1+6p+3q \end{vmatrix} + \begin{vmatrix} 1 & p & 1+p+q \\ 2 & 2p & 1+3p+2q \\ 3 & 3p & 1+6p+3q \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1+p+q \\ 1 & 3 & 1+3p+2q \\ 3 & 6 & 1+6p+3q \end{vmatrix} + 0 \quad \{\because C_1 \to pC_1$

$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 6 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & p \\ 2 & 3 & 3p \\ 3 & 6 & 6p \end{vmatrix} + \begin{vmatrix} 1 & 1 & q \\ 2 & 3 & 2q \\ 3 & 6 & 3q \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 6 & 1 \end{vmatrix} + 0 + 0$

$= \begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & 1 \\ 2 & 5 & 1 \end{vmatrix} \begin{cases} C_1 \to C_1 - C_3 \\ C_2 \to C_2 - C_3 \end{cases}$

$= \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} \quad \{\text{Expending bt } R_1$

$= 5 - 4 = 1$

8. $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$

Sol. L.H.S. $= \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

$= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$

$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$

$$=(a+x+y+z)\begin{vmatrix}0 & -a & 0\\ 0 & a & -a\\ 1 & y & a+z\end{vmatrix}\begin{cases}R_1\to R_1-R_2\\ R_2\to R_2-R_3\end{cases}$$

Expanding by C_1

$$=(a+x+y+z)\begin{vmatrix}-a & 0\\ a & -a\end{vmatrix}$$

$$=a^2(a+x+y+z)=R.H.S$$

9. $\begin{vmatrix}x & a & a\\ a & x & a\\ a & a & x\end{vmatrix}=(x+2a)(x-a)^2$.

Sol. Let $\Delta=\begin{vmatrix}x & a & a\\ a & x & a\\ a & a & x\end{vmatrix}\Rightarrow\Delta=\begin{vmatrix}x+a+a & a & a\\ a+x+a & x & a\\ a+a+x & a & x\end{vmatrix}$ (Applying $C_1\to C_1+C_2+C_3$)

$$\Rightarrow\Delta=\begin{vmatrix}x+2a & a & a\\ x+2a & x & a\\ x+2a & a & x\end{vmatrix}\Rightarrow\ \Delta=(x+2a)\begin{vmatrix}1 & a & a\\ 1 & x & a\\ 1 & a & x\end{vmatrix}$$

$$\Rightarrow\Delta=(x+2a)\begin{vmatrix}1-1 & a-x & a-a\\ 1-1 & x-a & a-x\\ 1 & a & x\end{vmatrix}$$ (Applying $R_1\to R_1-R_2, R_2\to R_2-R_3$)

$$\Rightarrow\Delta=(x+2a)\begin{vmatrix}0 & a-x & 0\\ 0 & x-a & a-x\\ 1 & a & x\end{vmatrix}\Rightarrow\Delta=(x+2a)\begin{vmatrix}0 & a-x & 0\\ 0 & x-a & -(x-a)\\ 1 & a & x\end{vmatrix}$$ Taking out (x-a) from R_1 and R_2

$$\Rightarrow\Delta=(x+2a)(x-a)\begin{vmatrix}0 & -(x-a) & 0\\ 0 & 1 & -1\\ 1 & a & x\end{vmatrix}\Rightarrow\Delta=(x+2a)(x-a)(x-a)\begin{vmatrix}0 & -1 & 0\\ 0 & 1 & -1\\ 1 & a & x\end{vmatrix}$$

$$\Rightarrow\Delta=(x+2a)(x-a)(x-a)\{1(1-0)\}\quad\therefore\ \Delta=(x+2a)(x-a)^2$$

10. $\begin{vmatrix}x+4 & 2x & 2x\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix}=(5x+4)(x-4)^2$.

Sol. Let $\Delta=\begin{vmatrix}x+4 & 2x & 2x\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix}$

$$\Rightarrow\Delta=\begin{vmatrix}x+4+2x+2x & 2x & 2x\\ 2x+x+4+2x & x+4 & 2x\\ 2x+2x+x+4 & 2x & x+4\end{vmatrix}$$ (Applying $c_1\to c_1+c_2+c_3$)

$$\Rightarrow\Delta=\begin{vmatrix}5x+4 & 2x & 2x\\ 5x+4 & x+4 & 2x\\ 5x+4 & 2x & x+4\end{vmatrix}\Rightarrow\Delta=(5x+4)\begin{vmatrix}1 & 2x & 2x\\ 1 & x+4 & 2x\\ 1 & 2x & x+4\end{vmatrix}$$

$$\Rightarrow \Delta = (5x+4)\begin{vmatrix} 1-1 & 2x-x-4 & 2x-2x \\ 1-1 & x+4-2x & 2x-x-4 \\ 1 & 2x & x+4 \end{vmatrix} \quad (\text{Applying } R_1 \to R_1 - R_2,\ R_1 \to R_2 - R_3)$$

$$\Rightarrow \Delta = (5x+4)\begin{vmatrix} 0 & x-4 & 0 \\ 0 & -x+4 & x-4 \\ 1 & 2x & x+4 \end{vmatrix} \text{ expanding along } R_3$$

$$\Rightarrow \Delta = (5x+4)\begin{vmatrix} x-4 & 0 \\ -x+4 & x-4 \end{vmatrix} \quad \Rightarrow \Delta = (5x+4)(x-4)^2 \quad \therefore \Delta = (5x+4)(x-4)^2$$

11. $\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2$

Sol. $\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$

$$= \begin{vmatrix} 5x+\lambda & 2x & 2x \\ 5x+\lambda & x+\lambda & 2x \\ 5x+\lambda & 2x & 2x \end{vmatrix} \quad \{C_1 \to C_1 + C_2 + C_3$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+\lambda & 2x \\ 1 & 2x & x+\lambda \end{vmatrix}$$

$$= (5x+\lambda)\begin{vmatrix} 0 & 0 & x-\lambda \\ 1 & x+\lambda & 2x \\ 1 & 2x & x+\lambda \end{vmatrix} \{R_1 \to R_1 - R_3$$

12. $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (1-a)^3$

Sol. Let $\Delta = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} \quad \Rightarrow \Delta = \begin{vmatrix} 1 & 2a+1 & a^2+2a \\ 1 & a+2 & 2a+1 \\ 1 & 3 & 3 \end{vmatrix} \quad [C_1 \leftrightarrow C_3]$

$$\Rightarrow \Delta = \begin{vmatrix} 1-1 & 2a+1-a-2 & a^2+2a-2a-1 \\ 1-1 & a+2-3 & 2a+1-3 \\ 1 & 3 & 3 \end{vmatrix} \quad (\text{Applying } R_1 \to R_1 - R_2, R_2 \to R_2 - R_3)$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & a-1 & a^2-1 \\ 0 & a-1 & 2a-2 \\ 1 & 3 & 3 \end{vmatrix} \quad \Rightarrow \Delta = \begin{vmatrix} 0 & a-1 & (a+1)(a-1) \\ 0 & a-1 & 2(a-1) \\ 1 & 3 & 3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a-1 & (a+1)(a-1) \\ a-1 & 2(a-1) \end{vmatrix} \quad \Rightarrow \Delta = (a-1)^2\left[1\begin{vmatrix} 1 & a+1 \\ 1 & 2 \end{vmatrix}\right]$$

$$\Rightarrow \Delta = (a-1)^2\left[1(2-a-1)\right] \quad \Rightarrow \Delta = (a-1)^2(-a+1) \quad \therefore \Delta = (1-a)^3$$

13. $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$

Sol. L.H.S $= \begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$$= \begin{vmatrix} 3x+3y & x+y & x+2y \\ 3x+3y & x & x+y \\ 3x+3y & x+2y & x \end{vmatrix} \{C_1 \to C_1 + C_2 + C_3$$

$$= (3x+3y)\begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix} \{C_1 \to C_1 + C_2 + C_3$$

$$= 3(x+y)\begin{vmatrix} 0 & y & y \\ 0 & -2y & y \\ 1 & x+2y & x \end{vmatrix} \begin{cases} R_1 + R_1 - R_2 \\ R_2 + R_2 - R_3 \end{cases}$$

$$= 3(x+y)\begin{vmatrix} y & y \\ -2y & y \end{vmatrix}$$

$$= 3(x+y)\{y^2 + 2y^2\}$$

$$= 3(x+y)\cdot 3y^2 = 9y^2(x+y) = R.H.S$$

14. $\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$

Sol. $\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$

$$= \begin{vmatrix} 3x-x+y-x+z & -x+y & -x+z \\ x-y+3y+z-y & 3y & z-y \\ x-z+y-z+3z & y-Z & 3z \end{vmatrix} \quad \{C_1 \to C_1 + C_2 + C_3$$

$$= \begin{vmatrix} x+y+z & -x+z & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & -x+y & z-x \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix} \{\text{tacing}(x+y+z) \text{ common from } x+y+z$$

$$= (x+y+z)\begin{vmatrix} 0 & -x-2y & -x+y \\ 0 & 2y+z & -y-2z \\ 1 & y-z & 3z \end{vmatrix} \begin{cases} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{cases}$$

$$=(x+y+z)\begin{vmatrix}-x-2y & -x+y \\ 2y+z & -y-2z\end{vmatrix} \text{ expanding by } C_1$$

$$=(x+y+z)\begin{vmatrix}-3x & -x+y \\ -3z & -y-2z\end{vmatrix} \{ C_1 \to C_1 - 2C_2$$

$$=(x+y+z)\{-3x(-y-2z)-(-3z)(-x+y)\}$$

$$=(x+y+z)\{3xy+6xz-3zx+3yz\}$$

$$=(x+y+z)\{3xy+3yz+3zx\} = 3(x+y+z)(xy+yz+2x) = \text{R.H.S}$$

15. $$\begin{vmatrix}x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3\end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

Sol. $$\text{L.H.S} = \begin{vmatrix}x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3\end{vmatrix}$$

$$=xyz\begin{vmatrix}1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2\end{vmatrix}$$ {taking x, y and Z common from C_1, C_2 and C_3 respectively

$$=x\cdot y\cdot z\begin{vmatrix}0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2\end{vmatrix} \begin{cases}C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3\end{cases}$$

$$=xyz\begin{vmatrix}0 & 0 & 1 \\ x-y & y-z & z \\ (x-y)(x+y) & (y-z)(y+z) & z^2\end{vmatrix}$$

{taking $(x-y),(y-z)$ and $z-x$ common resection $C_1, C_2 \& C_3$

$$=xyz(x-y)(y-z)\begin{vmatrix}0 & 0 & 1 \\ 1 & 1 & z \\ x+y & y+z & z^2\end{vmatrix}$$

$$=xyz(x-y)(y-z)\begin{vmatrix}1 & 1 \\ x+y & y+z\end{vmatrix}$$

$$=x\cdot y\cdot z(x-y)(y-z)(y+z-x-y) = xyz(x-y)(y-z)(z-x) = R.H.S$$

16. $$\begin{vmatrix}b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c\end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

Sol. $$\text{L.H.S.} = \begin{vmatrix}b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c\end{vmatrix}$$

$$=\begin{vmatrix} 2(a+b+c) & 0 & a+b+c \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} \{R_1 \to R_1+R_2+R_3$$

$$=(a+b+c)\begin{vmatrix} 2 & 0 & 1 \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} \text{ \{taking } (a+b+c) \text{ common from } R_1$$

$$=(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ c+a-2b & b-c & b \\ a+b-2c & c-a & c \end{vmatrix} \{C_1 \to C_1-2C_3$$

$$=(a+b+c)\begin{vmatrix} c+a-2b & b-c \\ a+b-2c & c-a \end{vmatrix} \text{ \{expanding by } R_1$$

$$=(a+b+c)\{(c-a)(c+a-2b)-(b-c)(a+b-2c)\}$$

$$=(a+b+c)\{c^2+ca-2bc-ca-a^2+2ab-ab-b^2+2bc+ac+bc-2c^2\}$$

$$=(a+b+c)\{-a^2-b^2-c^2+ab+bc+ac\}$$

$$=-(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$$

$$=-(a^3+b^3+c^3-3abc)=3abc-a^3-b^3-c^3=R.H.S$$

17. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}=4abc$

Sol. L.H.S. $=\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ Applying $R_1 \to R_1-R_2-R_3$

$$=\begin{vmatrix} b+c-b-c & a-c-a-c & a-b-a-b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$=\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Expanding determinant along R_1

$$=0\begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix}-(-2c)\begin{vmatrix} b & b \\ c & a+b \end{vmatrix}-2b\begin{vmatrix} b & c+a \\ c & 0 \end{vmatrix}$$

$$=0-(-2c)\begin{vmatrix} b & b \\ c & a+b \end{vmatrix}-2b\begin{vmatrix} b & c+a \\ c & 0 \end{vmatrix}$$

$$=0+2c\{b(a+b)-2c\}-2b\{cb-c(c+a)\}$$

$$=2c(ab+b^2-bc)-2b(bc-c^2-ca)$$

$$=2abc+2b^2c-2bc^2-2b^2c+2bc^2+2abc$$

$$=4abc=R.H.S$$

18. $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = -a^3$

Sol. L.H.S. $= \begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$

$= \begin{vmatrix} a & a+2b & a+2b+3c \\ 0 & a & 2a+b \\ 0 & 3a & 5a+3b \end{vmatrix} \begin{Bmatrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 6R_1 \end{Bmatrix}$

Expanding by C_1

$= a\begin{vmatrix} a & 2a+b \\ 3a & 5a+3b \end{vmatrix}$

$= a\left\{\left(5a^2+3ab\right)-\left(6a^2+3ab\right)\right\}$

$= a\left\{5a^2+3ab-6a^2-3ab\right\}$

$= a\times\left(-a^2\right) = -a^3 = R.H.S$

19. $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$

Sol. L.H.S. $= \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$= \begin{vmatrix} a+b & -b-c & -b \\ a+b & b+c & -a \\ -a-b & b+c & a+b+c \end{vmatrix} \begin{Bmatrix} C_1 \to C_1 + C_2 \\ C_2 \to C_2 + C_3 \end{Bmatrix}$

$= (a+b)(b+c)\begin{vmatrix} 1 & -1 & -b \\ 1 & 1 & -a \\ -1 & 1 & a+b+c \end{vmatrix}$ {taking $(a+b)\,\&\,(b+c)$ common from $C_1\,\&\,C_2$

$= (a+b)(b+c)\begin{vmatrix} 0 & 0 & a+c \\ 1 & 1 & -a \\ -1 & 1 & a+b+c \end{vmatrix}$ $\{R_1 \to R_1 + R_3$

$= (a+b)(b+c)(a+c)\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$ {expanding by R_1

$= (a+b)(b+c)(a+c)(1+1) = 2(a+b)(b+c)(c+a) = R.H.S$

20. $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = \left(b^2-ac\right)\left(ax^2+2bxy+cy^2\right)$

Sol. Let $\Delta = \begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$

Applying $R_1 \to xR_1$ and $R_2 \to yR_2$, we get

$$\Delta = \frac{1}{xy}\begin{vmatrix} ax & bx & ax^2+bxy \\ by & cy & bxy+cy^2 \\ ax+by & bx+cy & 0 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1 - R_2$

$$\Delta = \frac{1}{xy}\begin{vmatrix} ax & bx & ax^2+bxy \\ by & cy & bxy+cy^2 \\ 0 & 0 & -\left(ax^2+2bxy+cy^2\right) \end{vmatrix}$$

Now expanding along R_3, we get

$$\Delta = \frac{1}{xy}\times\left\{-\left(ax^2+2bxy+cy^2\right)\right\}\begin{vmatrix} ax & bx \\ by & cy \end{vmatrix}$$

$$= -\frac{1}{xy}\cdot\left(ax^2+2bxy+cy^2\right)\left(acxy-b^2xy\right)$$

$$= \left(ax^2+2bxy+cy^2\right)\left(b^2-ac\right)$$

21. $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = -4(a-b)(b-c)(c-a)$

Sol. Let $\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2-a^2 & (b+1)^2-b^2 & (c+1)^2-c^2 \\ (a-1)^2-a^2 & (b-1)^2-b^2 & (c-1)^2-c^2 \end{vmatrix} \quad \text{(Applying } R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1\text{)}$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ 2a+1 & 2b+1 & 2c+1 \\ -2a+1 & -2b+1 & -2c+1 \end{vmatrix} \Rightarrow \Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ 2a+1 & 2b+1 & 2c+1 \\ 2 & 2 & 2 \end{vmatrix} \quad \text{(Applying } R_3 \to R_3 + R_2\text{)}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} a^2 & b^2 & c^2 \\ 2a+1 & 2b+1 & 2c+1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} a^2-b^2 & b^2-c^2 & c^2 \\ 2a+1-2b-1 & 2b+1-2c-1 & 2c+1 \\ 1-1 & 1-1 & 1 \end{vmatrix} \quad \text{(Applying } C_1 \to C_1 - C_2, C_2 \to C_2 - C_3\text{)}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} (a-b)(a+b) & (b-c)(b+c) & c^2 \\ 2(a-b) & 2(b-c) & 2c+1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = 4(a-b)(b-c)\left[1\begin{vmatrix} a+b & b+c \\ 1 & 1 \end{vmatrix}\right] \quad \Rightarrow \Delta = 4(a-b)(b-c)[a+b-b-c]$$

$$\Rightarrow \Delta = 4(a-b)(b-c)(a-c) \quad \therefore \Delta = -4(a-b)(b-c)(c-a)$$

22. $$\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix} = -8$$

Sol. Let $$\Delta = \begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (x-2)^2+x^2 & (x+1)^2 & x^2 \\ (x-1)^2+(x+1)^2 & x^2 & (x+1)^2 \\ x^2+(x+2)^2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$ [Applying $C_1 \to C_1 + C_3$]

$$= \begin{vmatrix} 2x^2-4x+4 & (x-1)^2 & x^2 \\ 2x^2+2 & x^2 & (x+1)^2 \\ 2x^2+4x+4 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} x^2-2x+2 & (x-1)^2 & x^2 \\ x^2+1 & x^2 & (x+1)^2 \\ x^2+2x+2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$ [Taking common 2 from C_1]

$$= 2\begin{vmatrix} 1 & (x-1)^2 & x^2 \\ 1 & x^2 & (x+1)^2 \\ 1 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$ [Applying $C_1 \to C_1 - C_2$]

$$= 2\begin{vmatrix} 1 & (x-1)^2 & x^2 \\ 0 & x^2-(x-1)^2 & (x+1)^2-x^2 \\ 0 & (x+1)^2-(x-1)^2 & (x+2)^2-x^2 \end{vmatrix}$$ [Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$$= 2\begin{vmatrix} x^2-(x-1)^2 & (x+1)^2-x^2 \\ (x+1)^2-(x-1)^2 & (x+2)^2-x^2 \end{vmatrix} = 2\begin{vmatrix} (2x-1) & 2x+1 \\ 4x & 4x+4 \end{vmatrix} = 8\begin{vmatrix} 2x-1 & 2x+1 \\ x & x+1 \end{vmatrix}$$

$$= 8\left|2x^2+2x-x-1-2x^2-x\right| = 8\times(-1) = -8$$

23. $$\begin{vmatrix}(m+n)^2 & l^2 & mn\\(n+l)^2 & m^2 & ln\\(l+m)^2 & n^2 & lm\end{vmatrix}=(l-m)(m-n)(n-l)(l+m+n)(l^2+m^2+n^2)$$

Sol. Let $\Delta=\begin{vmatrix}(m+n)^2 & l^2 & mn\\(n+l)^2 & m^2 & ln\\(l+m)^2 & n^2 & lm\end{vmatrix} \Rightarrow \Delta=\begin{vmatrix}(m+n)^2+l^2-2mn & l^2 & mn\\(n+l)^2+m^2-2ln & m^2 & ln\\(l+m)^2+n^2-2lm & n^2 & lm\end{vmatrix}$ (Applying $c_1\to c_1+c_2-2c_3$)

$$\Rightarrow\Delta=\begin{vmatrix}l^2+m^2+n^2 & l^2 & mn\\l^2+m^2+n^2 & m^2 & ln\\l^2+m^2+n^2 & n^2 & lm\end{vmatrix} \Rightarrow\Delta=(l^2+m^2+n^2)\begin{vmatrix}1 & l^2 & mn\\1 & m^2 & ln\\1 & n^2 & lm\end{vmatrix}$$

$$\Rightarrow\Delta=(l^2+m^2+n^2)\begin{vmatrix}1-1 & l^2-m^2 & mn-ln\\1-1 & m^2-n^2 & ln-lm\\1 & n^2 & lm\end{vmatrix}$$ (Applying $R_1\to R_1-R_2$ & $R_2\to R_2-R_3$)

$$\Rightarrow\Delta=(l^2+m^2+n^2)\begin{vmatrix}0 & (l-m)(l+m) & -n(l-m)\\0 & (m-n)(m+n) & -l(m-n)\\1 & n^2 & ml\end{vmatrix}$$

$$\Rightarrow\Delta=(l^2+m^2+n^2)(l-m)(m-n)\begin{vmatrix}0 & l+m & -n\\0 & m+n & -l\\1 & n^2 & ml\end{vmatrix}$$

$$\Rightarrow\Delta=(l^2+m^2+n^2)(l-m)(m-n)\left\{1\begin{vmatrix}l+m & -n\\m+n & -l\end{vmatrix}\right\}$$

$$\Rightarrow\Delta=(l^2+m^2+n^2)(l-m)(m-n)(-l^2-ml+mn+n^2)$$

$$\Rightarrow\Delta=(l^2+m^2+n^2)(l-m)(m-n)(n^2-l^2+mn-ml)$$

$$\Rightarrow\Delta=(l^2+m^2+n^2)(l-m)(m-n)\{n(n-l)+m(n-l)\}$$

$$\therefore\ \Delta=(l-m)(m-n)(n-l)(l+m+n)(l^2+m^2+n^2)$$

24. $$\begin{vmatrix}(b+c)^2 & a^2 & bc\\(c+a)^2 & b^2 & ca\\(a+b)^2 & c^2 & ab\end{vmatrix}=(a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$$

Sol. L.H.S. $=\begin{vmatrix}(b+c)^2 & a^2 & bc\\(c+a)^2 & b^2 & ca\\(a+b)^2 & c^2 & ab\end{vmatrix}$

$$=\begin{vmatrix}b^2+c^2+2bc & a^2 & bc\\c^2+a^2+2ac & b^2 & ca\\a^2+b^2+2ab & c^2 & ab\end{vmatrix}$$

$$=\begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^3 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix} \{C_1+C_1+C_2-2C_3$$

$$=\left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)\begin{vmatrix} 0 & a^2-b^2 & bc-ca \\ 0 & b^2-c^2 & ca-ab \\ 1 & c^2 & ab \end{vmatrix} \qquad \begin{cases} R_1+R_1-R_2 \\ R_2+R_2-R_3 \end{cases}$$

$$=\left(a^2+b^2+c^2\right)\begin{vmatrix} 0 & (a-b)(a+b) & -c(a-b) \\ 0 & (-c)(b+c) & -a(b-c) \\ 1 & c^2 & ab \end{vmatrix}$$

$$=(a-b)(b-c)\left(a^2+b^2+c^2\right)\begin{vmatrix} 0 & a+b & -c \\ 0 & b+c & -a \\ 1 & c^2 & ab \end{vmatrix}$$ { taking $(a-b)(b-c)$ common from R_1 & R_2

$$=(a-b)(b-c)\left(a^2+b^2+c^2\right)\begin{vmatrix} a+b & -c \\ b+c & -a \end{vmatrix}$$ { expanding by C_1

$$=(a-b)(b-c)\left(a^2+b^2+c^2\right)\left(-a^2-ab+bc+c^2\right)$$

$$=(a-b)(b-c)\left(a^2+b^2+c^2\right)\left(c^2-a^2+bc-ab\right)$$

$$=(a-b)(b-c)\left(a^2+b^2+c^2\right)\{(v-a)(c+a)+b(c-a)\}$$

$$=(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)(c+a+b)$$

$$=(a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)=R.H.S$$

25. $$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}=4a^2b^2c^2$$

Sol. L.H.S = $$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

26. $$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}=\left(1+a^2+b^2\right)^3$$

Sol. Replace C_1 by C_1-bC_3, C_2 by C_2+aC_3 and take $\left(1+a^2+b^2\right)$ common from each C_1 and C_2, so that $\Delta=\left(1+a^2+b^2\right)^2\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$

Replace R_3 by $R_3 - bR_1$ to get, $\Delta = \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$

$\Rightarrow \Delta = \left(1+a^2+b^2\right)^2 \left(1-a^2+b^2+2a^2\right) = \left(1+a^2+b^2\right)^3$.

27. $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)\left(a^2+b^2+c^2\right)$

Sol. Let $\Delta = \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix}$

$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} a^2 & b^2-bc & c^2+bc \\ a^2+ac & b^2 & c^2-ac \\ a^2-ab & ab+b^2 & c^2 \end{vmatrix}$ (Multiply a, b, c in c_1, c_2 & c_3 and dividing by abc)

$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} a^2+b^2+c^2 & b^2-bc & c^2+bc \\ a^2+b^2+c^2 & b^2 & c^2-ac \\ a^2+b^2+c^2 & ab+c^2 & c^2 \end{vmatrix}$ (Applying $c_1 \to c_1+c_2+c_3$)

$\Rightarrow \Delta = \frac{1}{abc}\left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & b^2-bc & c^2+bc \\ 1 & b^2 & c^2-ac \\ 1 & ab+b^2 & c^2 \end{vmatrix}$

$\Rightarrow \Delta = \frac{1}{abc}\left(a^2+b^2+c^2\right)\begin{vmatrix} 1-1 & b^2-bc-b^2 & c^2+bc-c^2+ac \\ 1-1 & b^2-ab-b^2 & c^2-ac-c^2 \\ 1 & ab+b^2 & c^2 \end{vmatrix}$ (Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$)

$\Rightarrow \Delta = \frac{1}{abc}\left(a^2+b^2+c^2\right)\begin{vmatrix} 0 & -bc & bc+ac \\ 0 & -ab & -ac \\ 1 & ab+b^2 & c^2 \end{vmatrix}$ $\Rightarrow \Delta = \frac{1}{abc}\left(a^2+b^2+c^2\right)\left[1\begin{vmatrix} -bc & bc+ac \\ -ab & -ac \end{vmatrix}\right]$

$\Rightarrow \Delta = \frac{1}{abc}\left(a^2+b^2+c^2\right)\left[1(-bc)(-ac)+ab(bc+ac)\right]$

$\Rightarrow \Delta = \frac{1}{abc}\left(a^2+b^2+c^2\right)\left[abc^2+ab^2c+a^2bc\right]$ $\Rightarrow \Delta = \frac{1}{abc}\left(a^2+b^2+c^2\right)(abc)(c+b+a)$

$\therefore \Delta = (a+b+c)\left(a^2+b^2+c^2\right)$

28. $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

Sol. Let $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$. Multiplying a, b, c with R_1, R_2 & R_3.

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & abc & b(c+a) \\ ca^2b^2 & abc & c(a+b) \end{vmatrix} = \frac{(abc)abc}{abc}\begin{vmatrix} bc & 1 & a(b+c) \\ ca & 1 & b(c+a) \\ ab & 1 & c(a+b) \end{vmatrix} \quad \text{(Applying } c_1 \to c_1 + c_3\text{)}$$

$$\Rightarrow \Delta = abc\begin{vmatrix} ab+bc+ca & 1 & a(b+c) \\ ab+bc+ca & 1 & b(c+a) \\ ab+bc+ca & 1 & c(a+b) \end{vmatrix} \quad \Rightarrow \Delta = abc(ab+bc+ca)\begin{vmatrix} 1 & 1 & a(b+c) \\ 1 & 1 & b(c+a) \\ 1 & 1 & c(a+b) \end{vmatrix} = 0$$

29. $$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

Sol. $$\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix}$$

$$= abc\begin{vmatrix} \frac{(b+c)^2}{a} & b & c \\ a & \frac{(a+c)^2}{b} & c \\ a & b & \frac{(a+b)^2}{c} \end{vmatrix} \quad \{\text{taking } a, b \,\&\, c \text{ common from } R_1, R_2 \,\&\, R_3 \text{ respectively}$$

$$= \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} \quad \{R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$$

Replace C_2 by $C_2 - C_1$ and C_3 by $C_3 - C_1$ so that $\Delta = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (c+a)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix}$

Take $(a+b+c)$ common from C_2 and C_3 to get

$$\Delta = (a+b+c)^2\begin{vmatrix} b^2+c^2+2bc & a-b-c & a-b-c \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Replace R_1 by $R_1 - R_2 - R_3$ to get $\Delta = (a+b+c)^2\begin{vmatrix} 2bc & -2c & -2b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$

Replace C_2 by $C_2 + \frac{1}{b}C_1$ and C_3 by $C_3 + \frac{1}{c}C_1$ to get

$$\Delta=(a+b+c)^2\begin{vmatrix}2bc & 0 & 0\\ b^2 & c+a & \frac{b^2}{c}\\ c^2 & \frac{c^2}{b} & a+b\end{vmatrix}=2bc(a+b+c)^2\{(a+c)(a+b)-bc\}$$

$$=2bc(a+b+c)^2(a^2+ab+ac+bc-bc)=2abc(a+b+c)^3$$

30. $$\begin{vmatrix}b^2-ab & b-c & bc-ac\\ ab-a^2 & a-b & b^2-ab\\ bc-ac & c-a & ab-a^2\end{vmatrix}=0$$

Sol. Let $\Delta=\begin{vmatrix}b^2-ab & b-c & bc-ac\\ ab-a^2 & a-b & b^2-ab\\ bc-ac & c-a & ab-a^2\end{vmatrix}$ $\Rightarrow \Delta=\begin{vmatrix}b(b-a) & b-c & c(b-a)\\ a(b-a) & a-b & b(b-a)\\ c(b-a) & c-a & a(b-a)\end{vmatrix}$

$\Rightarrow \Delta=(b-a)^2\begin{vmatrix}b & b-c & c\\ a & a-b & b\\ c & c-a & a\end{vmatrix}$ $\Rightarrow \Delta=(b-a)^2\begin{vmatrix}b-c & b-c & c\\ a-b & a-b & b\\ c-a & c-a & a\end{vmatrix}$ (Applying $C_1\to C_1-C_3$)

$\Rightarrow \Delta=(b-a)^2.0$ $(\because C_1\,\&\,C_2$ are identical$)$

$\therefore\ \Delta=0$

31. $$\begin{vmatrix}-a(b^2+c^2-a^2) & 2b^3 & 2c^3\\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3\\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2)\end{vmatrix}=abc(a^2+b^2+c^2)^3$$

Sol. Let $\Delta=\begin{vmatrix}-a(b^2+c^2-a^2) & 2b^3 & 2c^3\\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3\\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2)\end{vmatrix}$

$$\Rightarrow \Delta=abc\begin{vmatrix}-(b^2+c^2-a^2) & 2b^2 & 2c^2\\ 2a^2 & -(c^2+a^2-b^2) & 2c^2\\ 2a^2 & 2b^2 & -(a^2+b^2-c^2)\end{vmatrix}$$

$\Rightarrow \Delta=(abc)\begin{vmatrix}a^2+b^2+c^2 & 2b^2 & 2c^2\\ a^2+b^2+c^2 & -a^2+b^2-c^2 & 2c^2\\ a^2+b^2+c^2 & 2b^2 & c^2-a^2-b^2\end{vmatrix}$ (Applying $c_1\to c_1+c_2+c_3$)

$$\Rightarrow \Delta=(abc)(a^2+b^2+c^2)\begin{vmatrix}1 & 2b^2 & 2c^2\\ 1 & b^2-a^2-c^2 & 2c^2\\ 1 & 2b^2 & c^2-a^2-b^2\end{vmatrix}$$

$\Rightarrow \Delta=(abc)(a^2+b^2+c^2)\begin{vmatrix}1-1 & 2b^2-b^2+a^2+c^2 & 2c^2-2c^2\\ 1-1 & b^2-a^2-c^2-2b^2 & 2c^2-c^2+a^2+b^2\\ 1 & 2b^2 & c^2-a^2-b^2\end{vmatrix}$ $\left[\begin{array}{l}\text{Applying } R_1\to R_1-R_2\\ \&\ R_2\to R_2-R_3\end{array}\right]$

$$\Rightarrow \Delta = (abc)(a^2+b^2+c^2)\begin{vmatrix} 0 & a^2+b^2+c^2 & 0 \\ 0 & -(a^2+b^2+c^2) & a^2+b^2+c^2 \\ 1 & 2b^2 & c^2-a^2-b^2 \end{vmatrix}$$

32. $\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0$ where α, β, γ are in AP

Sol. L.H.S $= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$

$$= \begin{vmatrix} 1 & x-y & x-\alpha \\ 1 & x-3 & x-\beta \\ 1 & x-2 & x-\gamma \end{vmatrix} \{ C_1 \to C_1 - C_2$$

$$= \begin{vmatrix} 0 & -1 & \beta-\alpha \\ 0 & -1 & \gamma-\beta \\ 1 & x-2 & x-y \end{vmatrix} \begin{cases} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{cases}$$

$$= \begin{vmatrix} -1 & \beta-\alpha \\ -1 & \gamma-\beta \end{vmatrix} \quad \{\text{expanding by } C_1$$

$$= -y+\beta+\beta-\alpha = 2\beta-\alpha-\gamma$$

$$= 2\beta-(\alpha+\gamma)$$

$= 2\beta - 2\beta \quad \{\because \alpha, \beta$ and y are in AP $\therefore 2\beta\alpha+\alpha+\gamma$

$$= 0 = R.H.S$$

33. $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$

Sol. L.H.S $= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$

$$= \begin{vmatrix} (a+1)(a+2) & (a+2) & 1 \\ (a+2)(a+3)-(a+1)(a+2) & (a+3)-(a-2) & 1-1 \\ (a+2)(a+4)-(a+2)(a+3) & (a+4)-(a+3) & 1-1 \end{vmatrix} \begin{cases} R_3 \to R_3 - R_2 \\ R_2 \to R_2 - R_1 \end{cases}$$

$$= \begin{vmatrix} (a+1)(a+2) & (a+2) & 1 \\ (a+2)\{a+3-a-1\} & a+3-a-2 & 0 \\ (a+3)\{a+4-a-2\} & a+4-a-3 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 2(a+3) & 1 & 0 \end{vmatrix}$$

$$=\begin{vmatrix} 2(a+2) & 1 \\ 2(a+3) & 1 \end{vmatrix} \quad \{\text{expanding by } C_3$$

$$= 2(a+2)-2(a+3) \quad = 2a+4-2a-6$$

$$= -2 = R.H.S$$

34. If $x \neq y \neq z$ and $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0$ prove that $xyz(xy+yz+zx) = (x+y+z)$

Sol. $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \Delta_1 - \Delta_2 = 0 \quad (\text{let}) \qquad \ldots \text{(i)}$$

Now, $\Delta_1 = \begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix}$

$$= x\cdot y\cdot z \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$

$$= x\cdot y\cdot z \begin{vmatrix} 0 & x^2-y^2 & x^3-y^3 \\ 0 & y^2-z^2 & y^3-z^3 \\ 1 & z^2 & z^3 \end{vmatrix} \qquad \begin{cases} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{cases}$$

$$= x\cdot y\cdot z \begin{vmatrix} 0 & (x-y)(x+y) & (x-y)(x^2+xy+y^2) \\ 0 & (y-z)(y+z) & (y-z)(y^2+yz+z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

$$= xyz(x-y)(y-z) \begin{vmatrix} 0 & x+y & x^2+xy+y^2 \\ 0 & y+z & y^2+yz+z^2 \\ 1 & z^2 & z^3 \end{vmatrix}$$

{taking $(x-y)$ and $(y-z)$ common from R_1 and R_2 respectively

$$= xyz(x-y)(y-z) \begin{vmatrix} x+y & x^2+xy+y^2 \\ y+z & y^2+yz+z^2 \end{vmatrix} \quad \{\text{expanding by } C_1$$

$$= xyz(x-y)(y-z)\left\{(x+y)(y^2+yz+z^2)-(y+z)(x^2+xy+y^2)\right\}$$

$$= xyz(x-y)(y-z)\left\{xy^2+xyz+xz^2+y^3+y^2z+yz^2-x^2y-xy^2-y^3-x^2z-xyz-y^2z\right\}$$

$$= xyz(x-y)(y-z)\left\{xz^2-x^2z+yz^2-yx^2\right\}$$

$$= xyz(x-y)(y-z)\left\{xz(z-x)+y\left(z^2-x^2\right)\right\}$$

$$= xyz(x-y)(y-2)\left\{xz(z-x)+y(z-x)(z+x)\right\}$$

$$= xyz(x-y)(y-z)(z-x)\{xz+yz+xy\}$$

$$= xyz(x-y)(y-z)(z-x)(xy+yz+zx) \qquad \ldots \text{(i)}$$

$$\Delta_2 = \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta_2 = \begin{vmatrix} x-y & x^3-y^3 & 1-1 \\ y-z & y^3-z^3 & 1-1 \\ z & z^3 & 1 \end{vmatrix} \begin{cases} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{cases}$$

$$\Rightarrow \Delta_2 = \begin{vmatrix} x-y & (x-y)\left(x^2+xy+y^2\right) & 0 \\ y-z & (y-z)\left(y^2+yz+z^2\right) & 0 \\ z & z^3 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta_2 = (x-y)(y-z)\begin{vmatrix} 1 & x^2+xy+y^2 & 0 \\ 1 & y^2+yz+z^2 & 0 \\ z & z^3 & 1 \end{vmatrix} \quad \{\text{takin } (x-y) \text{ and } (y-z) \text{ common from } R_1 \text{ and } R_2$$

$$\Rightarrow \Delta_2 = (x-y)(y-z)\begin{vmatrix} 1 & x^2+xy+y^2 \\ 1 & y^2+yz+z^2 \end{vmatrix} \quad \{\text{taking by } C_3$$

$$\Rightarrow \Delta_2 = (x-y)(y-z)\left\{y^2+yz+z^2-x^2-xy-y^2\right\}$$

$$\Rightarrow \Delta_2 = (x-y)(y-z)\{(z-x)(z+x)+y(z-x)\}$$

$$\Rightarrow \Delta_2 = (x-y)(y-z)(z-x)(x+y+z) \qquad \ldots. \text{(iii)}$$

From (i), (ii) and (iii) we have

$$xyz(x-y)(y-z)(z-x)(xy+yz+zx)-(x-y)(y-z)(z-x)(x+y+z)=0$$

$$\Rightarrow (x-y)(y-z)(z-x)\left[xyz(xy+yz+zx)-(x+y+z)\right]=0$$

$$\Rightarrow xyz(xy+yz+zx)-(x+y+z)=0$$

$$\Rightarrow xyz(xy+yz+zx)=x+y+z$$

35. Prove that $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$

Sol. L.H.S. $= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$

$$= \begin{vmatrix} 0 & a^2-b^2+bc-ca & a^3-b^3 \\ 0 & b^2-c^2+ca-ab & b^3-c^3 \\ 0 & c^2+ab & c^3 \end{vmatrix} \qquad \begin{cases} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{cases}$$

$$=\begin{vmatrix} 0 & (a-b)(a+b)-c(a-b) & (a-b)(a^2+ab+b^2) \\ 0 & (b-c)(b+c)-a(b-c) & (b-c)(b^2+bc+c^2) \\ 1 & c^2+ab & c^3 \end{vmatrix} \left\{\begin{array}{l}\text{tyaking } (a-b) \text{ and } (b-c) \\ \text{common from } R_1 \text{ and } R_2\end{array}\right.$$

$$=(a-b)(b-c)\begin{vmatrix} 0 & a+b-c & a^2+ab+b^2 \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

$$=(a-b)(b-c)\begin{vmatrix} a+b-c & a^2+ab+b^2 \\ b+c-a & b^2+bc+c^2 \end{vmatrix}$$

$$=(a-b)(b-c)\left\{(a+b-c)(b^2+bc+c^2)-(b+c-a)(a^2+ab+b^2)\right\}$$

$$=(a-b)(b-c)\left\{\begin{array}{l} ab^2+abc+ac^2+b^3+b^2c+bc^2-b^2c \\ -bc^2-c^3-a^2b-ab^2-b^3-ca^2-abc-b^2c+a^3+a^2b+ab^2 \end{array}\right\}$$

$$=(a-b)(b-c)\left\{ac^2-c^3-ca^2-b^2c+a^3+ab^2\right\}$$

$$=(a-b)(b-c)\left\{-c^2(c-a)-a^2(c-a)-b^2(c-a)\right\}$$

$$=(a-b)(b-c)(c-a)(-a^2-b^2-c^2)$$

$$=-(a-b)(b-c)(c-a)(a^2+b^2+c^2)=R.H.S$$

Without expanding the determinant prove that

36. $$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Sol. L.H.S. $$=\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$=\frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix} \left\{\begin{array}{l} R_1 \to aR_1 \\ R_2 \to bR_2 \\ R_3 \to cR_3 \end{array}\right.$$

$$=\frac{abc}{abc}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \{\text{taking abc common from } C_3$$

$$=\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = -\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} \{R_2 \leftrightarrow R_3$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = R.H.S$$

37. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix}$

Sol. $\Delta = \begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix}$

Applying $C_3 \to C_3 - (a+b+c)C_1$, we get

$$\Delta = \begin{vmatrix} 1 & bc & -a \\ 1 & ca & -b \\ 1 & ab & -c \end{vmatrix}$$

Applying $C_2 \leftrightarrow C_3$ we get

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Applying $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$

$$\Delta = \frac{1}{abc} \cdot \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$$

Taking *abc* common from C_3, we have

$$\Delta = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Hence, $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix}$ Proved

38. Show that $x = 2$ is a root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix} = 0$

Sol. Let $\Delta = \begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$

Putting $x = 2$ in $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$ we have

$$\Delta = \begin{vmatrix} 2 & -6 & -1 \\ 2 & -3\times 2 & 2-3 \\ -3 & 2\times 2 & 2+2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 2 & -6 & -1 \\ 2 & -6 & -1 \\ -3 & 4 & 4 \end{vmatrix}$$

Here R_1 identical R_2

$\Rightarrow \Delta = 0$

Hence $x = 2$ is a root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix} = 0$

Solve that following equation

39. $\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = 0$

Sol. $\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$

$$= \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & b^3-x^3 \\ 0 & c-x & c^3-x^3 \end{vmatrix} \begin{cases} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{cases}$$

$$= \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & (b-x)(b^2+bx+x^2) \\ 0 & c-x & (c-x)(c^2+cx+x^2) \end{vmatrix}$$

$$= (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & (b^2+bx+x^2) \\ 0 & 1 & (c^2+cx+x^2) \end{vmatrix} \{\text{taking } (b-x) \text{ and } (c-x) \text{ common from } R_2 \,\&\, R_3 \text{ res}$$

$$= (b-x)(c-x)\begin{vmatrix} 1 & b^2+bx+x^2 \\ 1 & c^2+cx+x^2 \end{vmatrix} \{\text{expanding by } C_1$$

$$= (b-x)(c-x)(c^2+cx+x^2-b^2-bx-x^2)$$

$$= (b-x)(c-x)\left[(c-b)(c+b)+x(c-b)\right]$$

$$= (b-x)(c-x)(c-b)[c+b+x]$$

$$\because \begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = 0$$

$$\Rightarrow (b-x)(c-x)(c-b)(c+b+x) = 0$$

$= x = b$ or $x = c$ or $x = -(b+c)$

40. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ b & b & x+c \end{vmatrix} = 0$

Sol. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ 0 & b & x+c \end{vmatrix}$

$= \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} \{C_1 \to C_1 + C_2 + C_3$

$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$ taking $(x+a+b+c)$ common from C_1

$= (x+a+b+c)\begin{vmatrix} 0 & 0 & -x \\ 1 & x+b & x+c \\ 1 & b & x+c \end{vmatrix} \{R_3 \to R_3 - R_1$

$= (x+a+b+c)(-x)\begin{vmatrix} 1 & x+b \\ 1 & b \end{vmatrix}$

$= (x+a+b+c)\{-x(b-x-b)\} = x^2(x+a+b+c)$

$\therefore \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

$\Rightarrow x^2(x+a+b+c) = 0$

$\Rightarrow x = 0$ or $x = -(a+b+c)$

41. $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Sol. $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}$

$= \begin{vmatrix} 3x-2 & 3x-2 & 3x-2 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} \{R_1 \to R_1 + R_2 + R_3$

$= (3x-2)\begin{vmatrix} 1 & 1 & 1 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}$ {taking $(3x-2)$ common from R_1

$$=(3x-2)\begin{vmatrix} 0 & 1 & 1 \\ 0 & 3x-8 & 3 \\ -3x+11 & 3 & 3x-8 \end{vmatrix} \{C_1 \to C_1 - C_3$$

$$=(3x-2)(-3x+11)\begin{vmatrix} 1 & 1 \\ 3x-8 & 3 \end{vmatrix} \text{ \{expanding by } C_1$$

$$=(3x-2)(11-3x)(3-3x+8)$$

$$=(3x-2)(11-3x)(11-3x)$$

$$=(3x-2)(11-3x)^2$$

$$\because \begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\therefore (3x-2)(11-3x)^2 = 0$$

$$\Rightarrow x = \frac{2}{3} \text{ or } x = \frac{11}{3}$$

42. $$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Sol. $$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} \text{t}|$$

$$=\begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix} \{C_1 \to C_1 + C_2 + C_3$$

$$=(x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix}$$

$$=(x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 1 & 3 & x+4 \end{vmatrix} \{R_2 \to R_2 - R_1$$

$$=(x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} \{R_3 \to R_3 - R_1$$

$$=(x+9)(x-1)^2 \text{ \{expanding by } C_1$$

$$\because \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow (x+9)(x-1)^2 = 0$$

$\Rightarrow x=-9$ or $x=1$

43. $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0$

Sol. $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$

$$=\begin{vmatrix} 9+x & 9+x & 9+x \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

$$=(9+x)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} \{R_1 \to R_1+R_2+r_3$$

$$=(9+x)\begin{vmatrix} 0 & 1 & 1 \\ 0 & x & 2 \\ 7-x & 6 & x \end{vmatrix} \{C_1 \to C_1-C_3$$

$$=(9+x)(7-x)\begin{vmatrix} 1 & 1 \\ x & 2 \end{vmatrix} \text{ \{expanding by } C_1$$

$$=(9+x)(7-x)(2-x)$$

$$\because \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0$$

$$\Rightarrow (9+x)(7-x)(2-x)=0$$

$$\Rightarrow x=2 \text{ or } x=7 \text{ or } x=-9$$

44. $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}=0$

Sol. Let $\Delta=\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}$

$$\Rightarrow \Delta=\begin{vmatrix} x-2 & 3x-6 & 3-x-1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} \{R_1 \to R_1-R_2$$

$$\Rightarrow \Delta=\begin{vmatrix} x-2 & 3(x-2) & -(x-2) \\ 2 & -3x & (x-3) \\ -3 & 2x & (x+2) \end{vmatrix}$$

$$\Delta=(x-2)\begin{vmatrix}1 & 3 & -1\\ 2 & -3x & x-3\\ -3 & 2x & x+2\end{vmatrix} \quad \{\text{taking } (x-2) \text{ common from } R_1$$

$$\Rightarrow \Delta=(x-2)\begin{vmatrix}1 & 0 & 0\\ 2 & -3x-6 & x-1\\ -3 & 2x+9 & x-1\end{vmatrix} \{C_3 \to C_3+C_1$$

$$\Rightarrow \Delta=(x-2)(x-1)\begin{vmatrix}1 & 0 & 0\\ 2 & -(3x+6) & 1\\ -3 & 2x+9 & 1\end{vmatrix} \quad \{\text{taking } (x-1) \text{ common from } C_3$$

$$\Rightarrow \Delta=(x-2)(x-1)\begin{vmatrix}-3x-6 & 1\\ 2x+9 & 1\end{vmatrix}$$

$$\Rightarrow \Delta=(x-2)(x-1)(-3x-6-2x-9)$$

$$\Rightarrow \Delta=(x-2)(x-1)(-5x-15)$$

$$\Rightarrow \Delta=-5(x-2)(x-1)(x+3)$$

$$\because \begin{vmatrix}x & -6 & -1\\ 2 & -3x & x-3\\ -3 & 2x & x+2\end{vmatrix}=0$$

$$\therefore -5(x-2)(x-1)(x+3)=0$$

$$\Rightarrow x=1 \text{ or } x=2 \text{ or } x=-3$$

45. Prove that $\begin{vmatrix}a & b-c & c+b\\ a+c & b & c-a\\ a-b & b+a & c\end{vmatrix}=(a+b+c)(a^2+b^2+c^2)$

Sol. Let $\Delta=\begin{vmatrix}a & b-c & c+b\\ a+c & b & c-a\\ a-b & a+b & c\end{vmatrix}$

Apply $C_1=aC_1;\ C_2=bC_2;\ C_3=cC_3$ and divide the Δ by abc, we get

$$\Delta=\frac{1}{abc}\begin{vmatrix}a^2 & b^2-bc & c^2+bc\\ a^2+ac & b^2 & c^2-ac\\ a^2-ab & ab+b^2 & c^2\end{vmatrix}$$

Applying $C_1=C_1+C_2+C_3,$ we get

$$\Delta=\frac{1}{abc}\begin{vmatrix}a^2+b^2+c^2 & b^2-bc & c^2+bc\\ a^2+b^2+c^2 & b^2 & c^2-ac\\ a^2+b^2+c^2 & ab+b^2 & c^2\end{vmatrix}$$

$$\Rightarrow \Delta=\frac{a^2+b^2+c^2}{abc}\begin{vmatrix}1 & b^2-bc & c^2+bc\\ 1 & b^2 & c^2-ac\\ 1 & ab+b^2 & c^2\end{vmatrix}$$

Applying $R_1=R_1-R_2$ and $R_2=R_2-R_3,$ we get

$$\Rightarrow \Delta = \frac{a^2+b^2+c^2}{abc}\begin{vmatrix} 0 & -bc & bc+ac \\ 0 & -bc & -ac \\ 1 & ab+b^2 & c^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{a^2+b^2+c^2}{abc}\left[1\left\{abc^2+ab\left(bc+ac\right)\right\}\right]$$

$$\Rightarrow \Delta = \frac{a^2+b^2+c^2}{abc}\left[abc^2+ab^2c+a^2bc\right]$$

$$\Rightarrow \Delta = \frac{a^2+b^2+c^2}{abc}\times abc\left(c+b+a\right)$$

$$\Rightarrow \Delta = \left(a^2+b^2+c^2\right)\left(a+b+c\right)$$

EXERCISE 6 C (Pg.No.: 269)

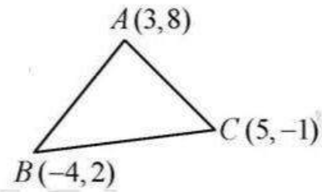
1. Find the area of the triangles whose vertices are:

(i) $A(3,8), B(-4,2)$ and $C(5,-1)$ (ii) $A(-2,4), B(2,-6)$ and $C(5,4)$

(iii) $A(-8,-2), B(-4,-6)$ and $C(-1,5)$ (iv) $P(0,0), Q(6,0)$ and $C(4,3)$

(v) $P(1,1), Q(2,7)$ and $R(10,8)$

Sol. **(i)** $A(3,8), B(-4,2)$ and $C(5,-1)$

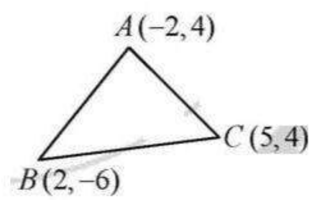


$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$,

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4-3 & 2-8 & 0 \\ 5-3 & -1-8 & 0 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -7 & -6 \\ 2 & -9 \end{vmatrix} = \frac{1}{2}(63+12) = \frac{1}{2}(75) = 37.5 \text{ sq. units.}$$

(ii) $A(-2,4), B(2,-6)$ and $C(5,4)$

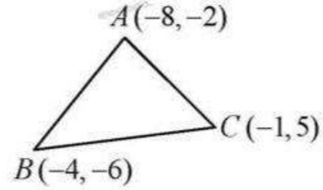


$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2+2 & -6-4 & 0 \\ 5+2 & 4-4 & 0 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 4 & -10 \\ 7 & 0 \end{vmatrix} = \frac{1}{2}(0+70) = 35 \text{ sq. units.}$$

(iii) $A(-8,-2), B(-4,-6), C(-1,5)$

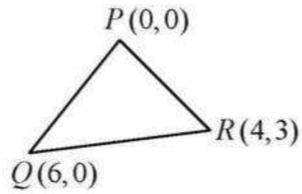


$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -8 & -2 & 1 \\ -4 & -6 & 1 \\ -1 & 5 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$,

$$\Delta=\frac{1}{2}\begin{vmatrix}-8 & -2 & 1\\ -4+8 & -6+2 & 0\\ -1+8 & 5+2 & 0\end{vmatrix}=\frac{1}{2}\begin{vmatrix}4 & -4\\ 7 & 7\end{vmatrix}=\frac{1}{2}(28+28)=28 \text{ sq. units.}$$

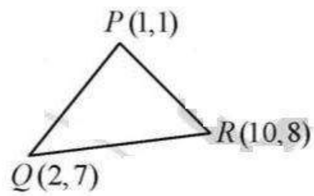
(iv) $P(0,0), Q(6,0)$ and $C(4,3)$



$$\text{Area of } \Delta=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 6 & 0 & 1\\ 4 & 3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}6 & 0\\ 4 & 3\end{vmatrix}=\frac{1}{2}(18-0)=9 \text{ sq. units.}$$

(v) $P(1,1), Q(2,7)$ and $R(10,8)$



$$\text{Area of } \Delta=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\frac{1}{2}\begin{vmatrix}1 & 1 & 1\\ 2 & 7 & 1\\ 10 & 8 & 1\end{vmatrix}$$

Applying $R_2\to R_2-R_1,\ R_3\to R_3-R_1$

$$\Delta=\frac{1}{2}\begin{vmatrix}1 & 1 & 1\\ 1 & 6 & 0\\ 9 & 7 & 0\end{vmatrix}=\frac{1}{2}\begin{vmatrix}1 & 6\\ 9 & 7\end{vmatrix}=\frac{1}{2}(7-54)=\frac{1}{2}(-47) \quad \therefore \text{ Area of } \Delta=\frac{47}{2} \text{ sq. units.}$$

2. Use determinants to show that the following points are collinear.

(i) $A(2,3), B(-1,-2)$ and $C(5,8)$ (ii) $A(3,8), B(-4,2)$ and $C(10,14)$

(iii) $P(-2,5), Q(-6,-7)$ and $R(-5,-4)$

Sol. **(i)** $A(2,3), B(-1,-2)$ and $C(5,8)$

$$\Delta=\frac{1}{2}\begin{vmatrix}2 & 3 & 1\\ -1 & -2 & 1\\ 5 & 8 & 1\end{vmatrix}$$

Applying $R_2\to R_2-R_1,\ R_3\to R_3-R_1$, $\Delta=\frac{1}{2}\begin{vmatrix}2 & 3 & 1\\ -3 & -5 & 0\\ 3 & 5 & 0\end{vmatrix}=\frac{1}{2}\begin{vmatrix}-3 & -5\\ 3 & 5\end{vmatrix}=0$

$\therefore\ \Delta=0$. Hence, the given points are collinear.

(ii) $A(3,8), B(-4,2)$ and $C(10,14)$

$$\Delta=\frac{1}{2}\begin{vmatrix}3 & 8 & 1\\ -4 & 2 & 1\\ 10 & 14 & 1\end{vmatrix}$$

Applying $R_2\to R_2-R_1,\ R_3\to R_3-R_1$,

$\Delta=\frac{1}{2}\begin{vmatrix}3 & 8 & 1\\ -7 & -6 & 0\\ 7 & 6 & 0\end{vmatrix}=\frac{1}{2}\begin{vmatrix}-7 & -6\\ 7 & 6\end{vmatrix}=\frac{1}{2}(-42+42)=0$. So, the given points are collinear.

(iii) $P(-2, 5), Q(-6, -7), R(-5, -4)$

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 5 & 1 \\ -6 & -7 & 1 \\ -5 & -4 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

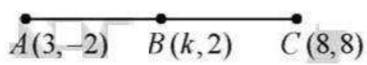
$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 5 & 1 \\ -4 & -12 & 0 \\ -3 & -9 & 0 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -4 & -12 \\ -3 & -9 \end{vmatrix} = \frac{1}{2}(36-36) = 0$$. Hence, the given points are collinear.

3. Find the value of k for which the points $A(3,-2), B(k,2)$ and $C(8,8)$ are collinear.

Sol. Since, the given points are collinear.

$\therefore \quad \Delta = 0$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$



Applying $R_2 \to R_2 - R_1, \; R_3 \to R_3 - R_1$

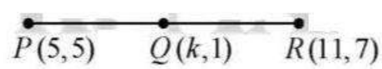
$$\begin{vmatrix} 3 & -2 & 1 \\ k-3 & 4 & 0 \\ 5 & 10 & 0 \end{vmatrix} = 0 \;\Rightarrow \begin{vmatrix} k-3 & 4 \\ 5 & 10 \end{vmatrix} = 0 \;\Rightarrow 10(k-3) - 20 = 0 \;\Rightarrow k-3-2=0 \;\Rightarrow k=5$$

4. Find the value of k for which the points $P(5,5), Q(k,1)$ and $R(11,7)$ are collinear.

Sol. Since the given points are collinear.

$\therefore \quad \Delta = 0$

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix} = 0$$



$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

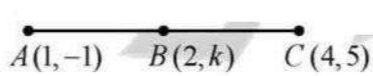
$$\begin{vmatrix} 5 & 5 & 1 \\ k-5 & -4 & 0 \\ 6 & 2 & 0 \end{vmatrix} = 0 \;\Rightarrow \begin{vmatrix} k-5 & -4 \\ 6 & 2 \end{vmatrix} = 0 \;\Rightarrow 2k-10+24=0 \;\Rightarrow 2k+14=0 \;\Rightarrow k=-7$$

5. Find the value of k for which the points $A(1,-1), B(2,k)$ and $C(4,5)$ are collinear.

Sol. Since the given points are collinear.

$\therefore \quad \Delta = 0$

$$\frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & k & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0$$



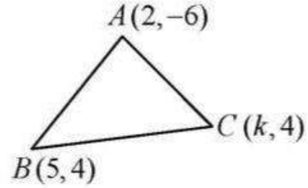
Applying $R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & k+1 & 0 \\ 3 & 6 & 0 \end{vmatrix} = 0 \;\Rightarrow \begin{vmatrix} 1 & k+1 \\ 3 & 6 \end{vmatrix} = 0 \;\Rightarrow 6-3(k+1)=0 \;\Rightarrow k+1=2 \;\Rightarrow k=1$$

6. Find the value of k for which the area of ΔABC having vertices $A(2,-6), B(5,4)$ and $C(k,4)$ is 35 sq units.

Sol. $\Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1,\quad R_3 \to R_3 - R_1$



$$\pm 35 = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 3 & 10 & 0 \\ k-2 & 10 & 0 \end{vmatrix} \Rightarrow \pm 35 \times 2 = \begin{vmatrix} 3 & 10 \\ k-2 & 10 \end{vmatrix} = 10\begin{vmatrix} 3 & 1 \\ k-2 & 1 \end{vmatrix}$$

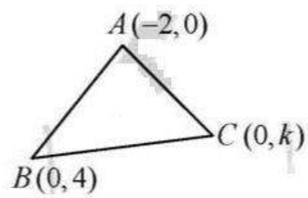
$\Rightarrow 35 \times 2 = 10(3-k+2) \quad \Rightarrow \pm 7 = 5-k$

Taking +ve sign, $7 = 5-k \quad \Rightarrow k = -2$; Taking –ve sign, $-7 = 5-k \quad \Rightarrow k = 12$.

Hence, $k = -2, 12$

7. If $A(-2,0), B(0,4)$ and $C(0,k)$ be three points such that area of ΔABC is 4 sq. units, find the value of k.

Sol. $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$



$$\Rightarrow \pm 4 = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} \Rightarrow \pm 8 = -2\begin{vmatrix} 4 & 1 \\ k & 1 \end{vmatrix} \quad \Rightarrow \pm 4 = -(4-k)$$

Taking +ve sign, $4 = -4 + k \quad \Rightarrow k = 8$

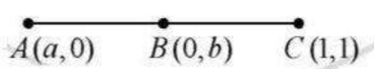
Taking –ve sign, $-4 = -4 + k \quad \Rightarrow k = 0$. Hence, $k = 0, 8$

8. If the points $A(a,0), B(0,b)$ and $C(1,1)$ are collinear, prove that $\frac{1}{a}+\frac{1}{b}=1$.

Sol. Since the given points are collinear.

$\therefore \quad \Delta = 0$

$$\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$



Applying $R_2 \to R_2 - R_1,\quad R_3 \to R_3 - R_1$

$$\begin{vmatrix} a & 0 & 1 \\ -a & b & 0 \\ 1-a & 1 & 0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} -a & b \\ 1-a & 1 \end{vmatrix} = 0 \quad \Rightarrow -a - b(1-a) = 0 \;\Rightarrow\; -a - b + ab = 0 \;\Rightarrow a + b = ab$$

Dividing both side by ab, $\frac{a}{ab}+\frac{b}{ab}=1 \quad \Rightarrow \frac{1}{b}+\frac{1}{a}=1$. Hence, $\frac{1}{a}+\frac{1}{b}=1$ proved.