# Stability in Time-Domain

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## Stability

A Linear Time Invariant (LTI) system is said to be stable if the following conditions are satisfied:

(i) If the system is excited by a bounded input, the output must be bounded.

(ii) If input to the system is zero, the output must be zero, irrespective of all the initial condition.

## Marginal Stable System

A LTI system is said to be marginal stable if for the bounded input, the output maintains the constant amplitude and frequency.

## Absolute Stable System

System is stable for all the value of system gain (K) from 0 to ∞.

## Conditional Stable System

System is stable for a certain range of K.

## Routh Hurwitz (R-H) Criteriaon

### Purpose

1. To find out closed loop system stability.
2. Number of closed loop pole in right side or left side of the s-plane.
3. Range of K for conditionally stable system.
4. Value of K required for marginal stability and thereby, determine the frequency of oscillations.

**Note:**

To find a closed loop system stability by using R-H criteria we need characteristic equation.

## Routh Array

- General $n^{th}$ order characteristics equation is
  $a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s^1 + a_n = 0$
- Routh array

$$\begin{array}{c|cccc} s^n & a_0 & a_2 & a_4 & \cdots \\ s^{n-1} & a_1 & a_3 & a_5 & \cdots \\ s^{n-2} & \dfrac{a_1 a_2 - a_0 a_3}{a_1} & \dfrac{a_1 a_4 - a_0 a_5}{a_1} & & \cdots \\ \vdots & \vdots & & & \\ s^0 & a_n & & & \end{array}$$

1. For closed loop system to be stable, all the coefficient in the first column must be positive and there should not be any missing coefficient in the first column.
2. If there is any sign change in the first column of the Routh array then system is unstable.
3. Number of sign changes is equal to number of roots lie in the right side of the s-plane.

**Note:**

- Magnitude and sign of every term in each row and column depend on first column element, therefore we always see first column term.
- Row zero occurs only when all the poles are located symmetrical about origin.
- The presence of a zero in the first column of the Routh's tabulation leads to following conclusions
  (i) Equal real roots with opposite signs.
  (ii) Pair of conjugate roots on imaginary axis.

## Difficulties and Limitations of Routh Array

1. The routh array is applicable to LTI system only.
2. It determines poles in LHS or RHS of s-plane but not their exact location.
3. Whenever any one coefficient is zero in first column then replace zero by smallest positive constant ($\epsilon$) and continue the Routh array, finally substitute $\epsilon = 0$ and check for the sign change.
4. When routh array ends abruptly, construct an auxiliary equation and differentiate it to get new coefficient to complete the routh array.
5. When the system is marginally stable, find the frequency of sustain oscillations form auxiliary equation. The auxiliary equation should be an even polynomial of order two only.

## Root Locus

Root locus is defined as the locus of closed loop poles obtained when system gain K varied from 0 to $\infty$.

## Angle Condition

Angle condition is used for checking whether certain points lie on root locus or not and hence, the validity of root locus for closed loop poles.

For a point to lie on root locus, the angle evaluated at that point must be an odd multiple of $\pm 180°$ i.e. $\pm(2q + 1)180°$.

## Magnitude Condition

The magnitude condition is used for finding the system gain K at any point on root locus.

$$|G(s)H(s)| = 1$$

## Construction Rules of Root Locus

1. Root locus is symmetrical about real axis.
2. Let P = Number of open loop poles
   Z = Number of open loop zeros
   If $p > z$
   then (a) The number of branches of root locus = P
   (b) The number of branches terminating at zero = Z
   (c) The number of branches terminating at infinity = P – Z
3. A point on real axis is said to be on root locus if, to the right side of this point, the sum of open loop poles and zeros is an odd number.
4. **Angle of Asymptotes**
   - Number of asymptotes = P – Z
   - Angle of asymptotes $= \dfrac{(2q+1)180°}{P-Z}$ (where, q = 0, 1, 2, ⋯)

**5. Centroid**

Centroid is the point of intersection of asymptote on the real axis.

$$\text{Centroid} = \frac{\Sigma\text{Real part of open loop poles} - \Sigma\text{Real part of open loop zeros}}{P - Z}$$

**6. Break away point**

They are the points where multiple roots of characteristic equation occurs.

**Procedure to find out break away point:**

(a) Construct characteristic equation i.e. $1 + G(s) H(s) = 0$.

(b) Write K in terms of s.

(c) Find $\frac{dK}{ds}$.

(d) The root of $\frac{dK}{ds} = 0$ give break away points.

**7. Intersection of root locus with imaginary axis**

Roots of auxiliary equation at $K = K_{(marginal)}$ from routh array gives intersection of root locus with imaginary axis.

**8. Angle of departure and arrival**

The angle of departure is tangent to the root locus at the complex poles.

$$\boxed{\phi_D = 180° + \phi}$$

The angle of arrival is tangent to the root locus at the complex zero.

$$\boxed{\phi_A = 180° - \phi}$$

$$\boxed{\phi = \phi_z - \phi_p}$$

where, $\phi_z$ = Sum of all the angles subtended by remaining zeros

$\phi_p$ = Sum of all the angle subtended by poles

**Note:**

**Whenever the system transfer function consist of the poles at origin then the root locus is nothing but angle of asymptotes.**

**Effect of Addition of Pole**

1. Operating range of K decreases for system to be stable.
2. Relative stability reduces.
3. System becomes more oscillatory.
4. The break away point shift towards the imaginary axis.
5. Damping factor decreases and system becomes unstable.
6. Settling time ($T_s$) and bandwidth increases.
7. Rise time ($T_r$) decreases.

**Effect of Addition of Zero**

1. System become stable.
2. Relative stability improved.
3. Range of K for stability increases.
4. System becomes less oscillatory.
5. Break away point shift towards the left side of the s-plane.
6. Damping factor increases.
7. Bandwidth decreases.

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