10. PRESTRESS CONCRETE

Prestress Concrete is one in which there have been introduced internal stresses of such magnitude and distribution that stresses resulting from given external loading is counter balanced to a desired degree.

> **Remember:** In case of prestress concrete very high strain steel and high strain concrete is used.

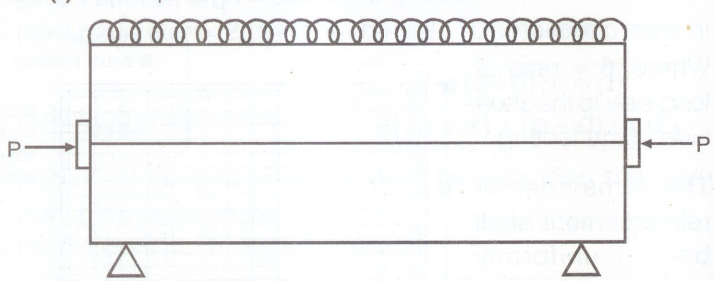
ANALYSIS OF PRESTRESS AND BENDING STRESS

- **Assumptions**
  - (i) Concrete is homogeneous elastic material.
  - (ii) Within the range of working stresses, both concrete and steel behave elastically i.e. Hooke's law is valid.
  - (iii) A transverse plane section before bending remain plain after bending.
- **Following are the three concepts of analysis**
  - (a) Stress concept analysis.
  - (b) Strength concept analysis.
  - (c) Load balancing method.

STRESS CONCEPT METHOD

Following are the two cases for analysis.

Case-(i) Beam provided with a concentric tendon:



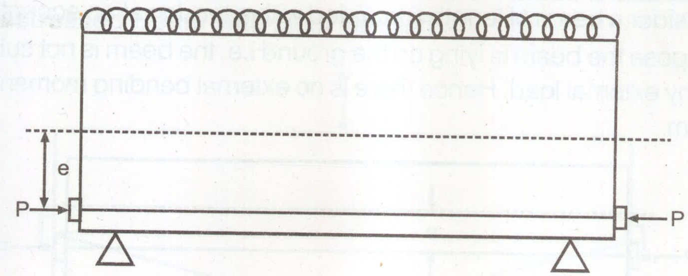
Let P be the prestressing force applied by the tendon. Due to this prestressing force, the direct compressive force induced is given by, $f_a = \frac{P}{A}$.

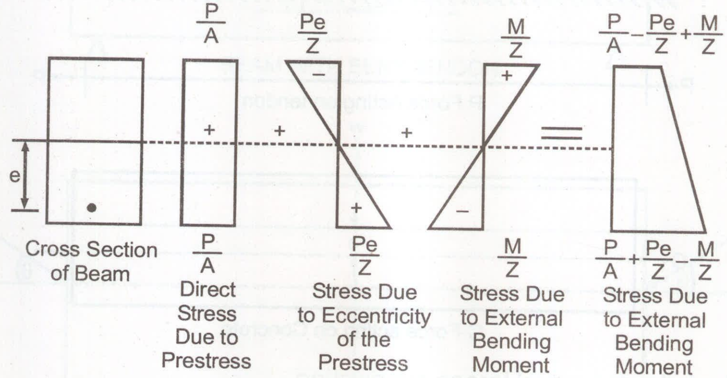
If due to dead load & external loads, the bending moment at the section is M, then the extreme stresses at the section due to bending moment alone is $f_b = \pm\frac{M}{Z}$.

Hence final stress at the extreme top edge $= \frac{P}{A} + \frac{M}{Z}$

and stress at the extreme bottom edge $= \frac{P}{A} - \frac{M}{Z}$

- **Case-(ii):** Beams with eccentrics tendon:





(i) Direct stresses due to prestressing force $= +\frac{P}{A}$

(ii) Extreme stresses due to eccentricity of the prestressing force $= \mp\frac{P.e}{Z}$

(iii) Extreme stresses due to bending moment $= \pm\frac{M}{Z}$.

- **Final stresses**

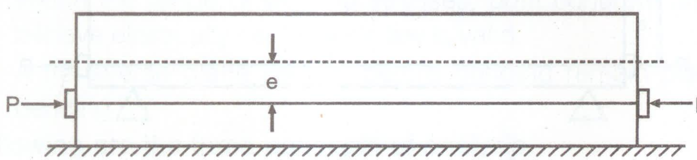
  Stress at top fibre $= \frac{P}{A} - \frac{P.e}{Z} + \frac{M}{Z}$

  Stress at bottom fibre $= \frac{P}{A} + \frac{P.e}{Z} - \frac{M}{Z}$

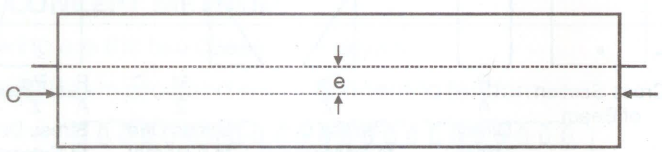
  By providing an eccentricity to the tendon, a hogging moment (P.e.) is developed which will produce stresses, which will counteract the stresses due to external bending moment.

STRENGTH CONCEPT METHOD

Consider a beam of length l provided with a tendon at an eccentricity e. Suppose the beam is lying on the ground i.e. the beam is not subjected to any external load. Hence there is no external bending moment on the beam.



P Force Acting on tendon



C-Force acting on Concrete

The following equal forces are existing.

(i) The P-force which is the tension in the tendon.

(ii) The C-force which is the compressive force acting on the concrete. Stresses in concrete are produced entirely due to C-force.

In the absence of any external bending moment the C-force and P-force act at the same level. Line of action of P-force is called the *P-line*. The *P-line* is nothing but the tendon line itself. The line of action of the C-force is called the *C-line* or *Pressure line*. Hence in the absence of any external bending moment the *P-line* and the *C-line* coincide.

Suppose the beam is subjected to a bending moment M, then the C-line will be shifted from the P-line by a distance 'a' called lever arm.

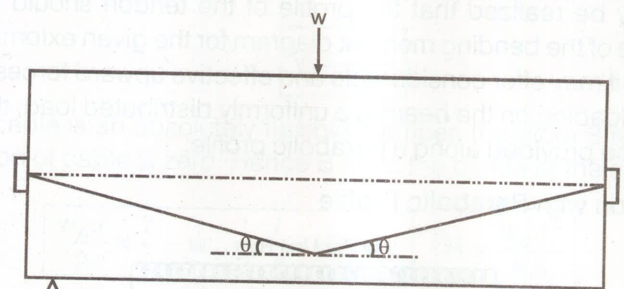
$$a = \frac{M}{P} = \frac{M}{C}$$

Extreme stresses in concrete are given by

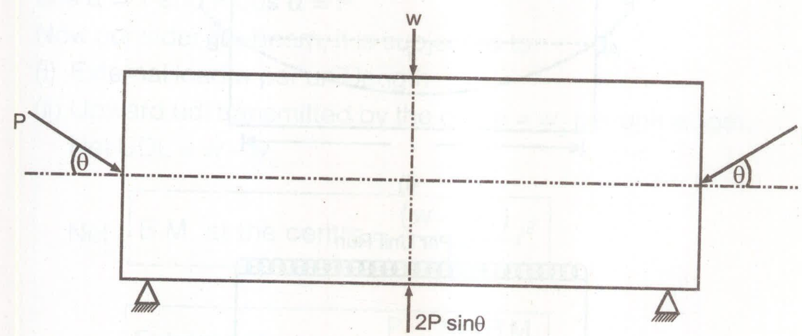
$$= \frac{C}{A} \pm \frac{C \times \text{eccentricity of } C}{Z}$$

LOAD BALANCING CONCEPT

- **Prestressed Beam with Bent Tendon**



BEAM WITH BENT TENDON



(b)

By providing bent tendons, the tendons will exert an upward pressure on the concrete beam and will therefore counter act a part of the external downward loading.

Considering the concrete as a free body. We find an upward force 2P sin θ.

The net downward load at the centre will be (W-2P sinθ).

The axial longitudinal force provided by the tendon = P cosθ = P {since θ is small}

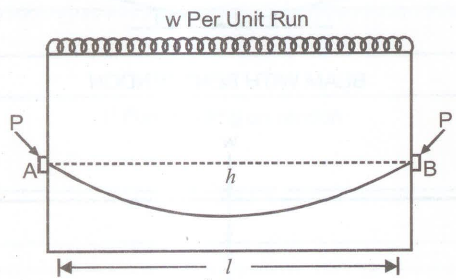
Direct stress on the section = $\frac{P\cos\theta}{A} = \frac{P}{A}$

Net B.M, $M = \frac{(W - 2P\sin\theta)\,l}{4} + \frac{wl^2}{8}$

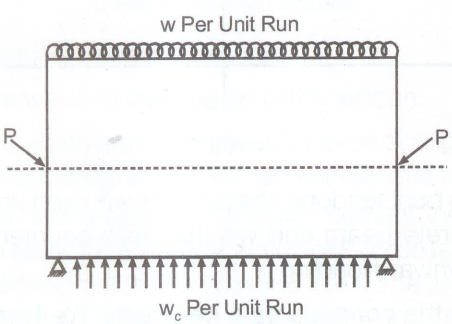
Where, w = dead load per unit length of the beam. Extreme fibre stress = $\frac{P}{A} \pm \frac{M}{Z}$

It may be realized that the profile of the tendon should follow the shape of the bending moment diagram for the given external loads in order it may offer considerable and effective upward forces. For e.g., if the loading on the beam is a uniformly distributed load, the tendon may be provided along a parabolic profile.

- **Tendon with Parabolic Profile**

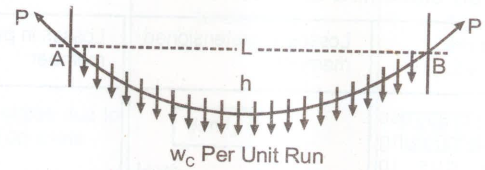


(i)



Loads Transmitted to Beam

(ii)



Loads Transmitted to Tendon

(iii)

Let l be the span of the beam and h be the dip of the cable.

The cable will exert an upward udl = w_c/m on the beam, but the cable will be subjected to downward udl of w_c per unit run.

Let V and H are vertical and horizontal components of P.

$$V = \frac{w_c . l}{2}$$

The cable is an absolutely flexible member, therefore B.M. at every section of cable is zero. Hence B.M. at the centre of the cable is

$$\frac{w_c . l}{2} \times \frac{l}{2} - w_c . \frac{l}{2} . \frac{l}{4} - H.h = o \qquad H = \frac{w_c . l^2}{8.h}$$

Since dip of the cable is very small, we can make approximation cos α = 1 and P cos α = P

Now consider the beam, it is subjected to

(i) External load w per unit length

(ii) Upward udl transmitted by the cable = w_c per unit length.

Net UDL = $w - w_c$

$$\text{Net B.M. at the centre} = \frac{(w - w_c)\, l^2}{8}$$

$$\text{Extreme stresses} = \frac{P}{A} \pm \frac{\text{Net B.M.}}{Z}$$

LOSSES OF PRESTRESS

The steel wires of a prestressed concrete member do not retain all the preliminary prestress. A certain amount of loss of prestress always takes place.

Losses may be classified as follows:

	Losses in pretensioned member	Losses in posttensioned member
1. Loss of prestress during tensioning process due to friction.		
(a) Loss due to length effect.	No Loss	$P_o kx$
(b) Loss due to curvature effect.	No Loss	$P_o \mu\alpha$
(c) Loss due to both length and curvature effect.	No Loss Here P_o = Prestressing force at the jacking end. K = Wobble friction factor 15×10^{-4} per meter $< K < 50 \times 10^{-4}$ per meter. α = Cumulative angle in radians through which tangent to the cable profile has turned between any two points under consid-eration. μ = Coefficient of friction in curves = 0.25 to 0.55.	$P_o (kx + \mu\alpha)$
2. Loss of prestress at the anchoring stage.	No Loss	$\frac{\Delta l}{l} . E_s$ Here Δl = effective slip of the wire. l = Length of the tendon E_s = Young's modulus for tendon wires.
3. Loss of prestress occuring Sub-sequently.		
(a) Loss of stress due to shrinkage of concrete.	$(3 \times 10^{-4})E_s$ Here E_s = Youngs modulus for tendon wire.	$\frac{2 \times 10^{-4}}{\log_{10}(T+2)} . E_s$ Here T = Age of concrete at the time of transfer of stress (in days).
(b) Loss of stress due to creep to concrete	$\phi.m.f_c$ Here m = Modular ratio = E_s/E_c f_c = Original prestress in concrete at the level of steel.	$\phi.m.f_c$
(c) Loss of stress due to elastic shortening of concrete.	$m.f_c$ Here f_c = Initial stress in concrete at the level of steel.	zero → it all the bars are tensioned at same time $\frac{\Delta l}{l} . E_s$ → for subsequent tensioning
(d) Loss of stress due to creep of steel or loss due to stress relaxations.	1 to 5% of initial prestress	1 to 5% of initial prestress

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