Chapter 9. Factorization

Formulae

1. **Factor Theorem:** If f(x) is a polynomial and a is a real number, then (x – a) is a factor of f(x) if f(a) = 0.
2. **Remainder Theorem:** If a polynomial f(x) is divided by (x – a), then remainder =f(x).

Determine the Following

Question 1. Use remainder theorem and find the remainder when the polynomial $g(x) = x^3 + x^2 - 2x + 1$ is divided by x – 3.

Solution : By the remainder theorem, required remainder

$$g(3) = (3)^3 + (3)^2 - 2 \times 3 + 1 = 27 + 9 - 6 + 1 = 31. \quad \text{Ans.}$$

Question 2. (i) When $x^3 + 3x^2 - kx + 4$ is divided by (x – 2), the remainder is k. Find the value of k.

(ii) Find the value of p if the division of $px^3 + 9x^2 + 4x - 10$ by $(x + 3)$ leaves the remainder 5.

Solution : (i) Here, $P(2) = k$

$\Rightarrow \quad 2^3 + 3(2)^2 - k(2) + 4 = k$

$\Rightarrow \quad 8 + 12 - 2k + 4 = k$

$\Rightarrow \quad 3k = 24$

$\Rightarrow \quad k = 8$ Ans.

(ii) Here, $P(-3) = 5$

$\Rightarrow \quad p(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$

$\Rightarrow \quad -27p + 81 - 12 - 10 = 5$

$\Rightarrow \quad -27p = -54$

$\Rightarrow \quad p = 2$ Ans.

Question 3. Use the factor theorem to determine that $x - 1$ is a factor of $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$.

Solution : Let $f(x) = x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$ to check whether $x - 1$ is a factor of $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$ we find $f(1)$.

Put $x = 1$ in equation (i) we get

$$f(1) = (1)^6 - (1)^5 + (1)^4 - (1)^3 + (1)^2 - (1) + 1$$
$$= 1 - 1 + 1 - 1 + 1 - 1 + 1$$
$$= 4 - 3 = 1.$$

Since, $f(1) \neq 0$, So by factor theorem $(x - 1)$ is not a factor of $f(x)$. Ans.

Question 4. Use the factor theoem to factorise completely $x^3 + x^2 - 4x - 4$.

Solution : $x^3 + x^2 - 4x - 4$

Let $x + 1 = 0$

$\therefore x = -1$

On substituting value of x in the experssion

$\because \quad f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = 0$

Clearly $x + 1$ is a factor of

$f(x) = x^3 + x^2 - 4x - 4$

$\therefore f(x) = (x+1)(x^2-4)$ (By actual division)

$= (x+1)(x-2)(x+2)$ Ans.

Question 5. Find the remainder when the polynomial $f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$ is divided by $x + 2$.

Solution : If $x + 2 = 0$

$x = -2$

$f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2,$

[By remainder theorem]

$$\begin{aligned} f(-2) &= 2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2 \\ &= 2(16) - 6(-8) + 2(4) + 2 + 2 \\ &= 32 + 48 + 8 + 2 + 2 = 92 \end{aligned}$$

Hence, required remainder = 92. Ans.

Question 6. Find the value of a and b so that the polynomial $x^3 - ax^2 - 13x + b$ has $(x-1)(x+3)$ as factor.

Solution : Let $p(x) = x^3 - ax^2 - 13x + b$ be the given polynomial.

If $(x-1)$ and $(x+3)$ are the factors of $p(x)$ then

$p(1) = 0$

and $p(-3) = 0$

$$\begin{aligned} p(1) &= (1)^3 - a(1)^2 - 13(1) + b = 0 \\ &= 1 - a - 13 + b = 0 \\ a - b &= -12 \quad \ldots (1) \\ p(-3) &= (-3)^3 - a(-3)^2 - 13(-3) + b = 0 \\ &= -27 - 9a + 39 + b = 0 \\ 9a - b &= 12 \quad \ldots (2) \end{aligned}$$

Solving (1) and (2) we get

$a = 3$

and $b = 15.$ Ans.

Question 7. If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x - 3)$, find the value of a.

Solution : Let $p(x) = ax^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + a$ be the given polynomials.

When $p(x)$ and $q(x)$ are divided by $(x - 3)$ the remainder are $p(3)$ and $q(3)$ respectively.

$$p(3) = q(3) \text{ given}$$

$$a(3)^3 + 4(3)^2 + 3 \times 3 - 4 = 3^3 - 4 \times 3 + a$$

$$\Rightarrow \quad 27a + 36 + 9 - 4 = 27 - 12 + a$$

$$\Rightarrow \quad 26a = 15 - 41$$

$$\Rightarrow \quad 26a = -26$$

$$\therefore \quad a = -\frac{26}{26} = -1. \qquad \text{Ans.}$$

Question 8. Find the value of a, if $(x - a)$ is a factor of $x^3 - a^2x + x + 2$.

Solution : Let $f(x) = x^3 - a^2x + x + 2$

Put $x - a = 0$

$$\therefore \quad x = a$$

$$f(a) = a^3 - a^2 \cdot a + a + 2$$

$$0 = a^3 - a^3 + a + 2$$

$$\text{or} \quad a + 2 = 0$$

$$\therefore \quad a = -2$$

Question 9. If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b.

Solution : Let $f(x) = 2x^3 + ax^2 + bx - 14 \quad \ldots(1)$

as $(x - 2)$ is factor of (1)

Put $x - 2 = 0$

$$\Rightarrow \quad x = 2 \text{ in (1)}$$

$$f(2) = 2(2)^3 + a(2)^2 + b(2) - 14$$

$$0 = 16 + 4a + 2b - 14$$

$$\text{or} \quad 4a + 2b = -2$$

$$\text{or} \quad 2a + b = -1 \qquad \ldots(2)$$

Again when $f(x)$ is divided by $(x - 3)$, it leaves remainder 52

Put $x - 3 = 0$

$$\Rightarrow \quad x = 3$$

$$f(3) = 2(3)^3 + a(3)^2 + b(3) - 14$$

$$52 = 54 + 9a + 3b - 14$$

$$52 = 9a + 3b + 40$$

$$52 - 40 = 9a + 3b$$

$$\Rightarrow \quad 12 = 9a + 3b$$

$$\text{or} \quad 4 = 3a + b \qquad \ldots(3)$$

Solving (2) and (3)

$$3a + b = 4$$

$$2a + b = -1$$

$$\text{Sub} \quad - \quad - \quad +$$

$$a = 5$$

Substitute $a = 5$ in $3a + b = 4$

$$\Rightarrow \quad 3 \times 5 + b = 4$$
$$15 + b = 4$$
$$\Rightarrow \quad b = 4 - 15$$
$$b = -11$$

Question 10. Show that $(x - 1)$ is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise the above expression.

Solution : If $(x - 1)$ is a factor of $x^3 - 7x^2 + 14x - 8$ then on putting $x - 1 = 0$

$$x = 1$$
$$f(1) = 0$$
$$= 1^3 - 7(1)^2 + 14(1) - 8$$
$$= 1 - 7 + 14 - 8 = 0$$

Hence $x - 1$ is one factor.

To find other factors

$$= x^3 - 7x^2 + 14x - 8$$
$$= x^2(x - 1) - 6x(x - 1) + 8(x - 1)$$
$$= (x - 1)(x^2 - 6x + 8)$$
$$= (x - 1)(x^2 - 4x - 2x + 8)$$
$$= (x - 1)\{x(x - 4) - 2(x - 4)\}$$
$$= (x - 1)(x - 2)(x - 4).$$

Question 11. In the following two polynomials. Find the value of 'a' if x + a is a factor of each of the two:

(i) $x^3 + ax^2 - 2x + a + 4$

(ii) $x^4 - a^2x^2 + 3x - a$.

Solution : (i) Let

$$p(x) = x^3 + ax^2 - 2x + a + 4 \quad \ldots(i)$$

Since, $(x + a)$ is a factor of $p(x)$, so $p(-a) = 0$

Put $x = -a$ in equation (i), we get

$$p(-a) = (-a)^3 + a(-a)^2 - 2(-a) + a + 4$$
$$= -a^3 + a(a^2) + 2a + a + 4$$
$$= -a^3 + a^3 + 3a + 4$$
$$= 3a + 4$$

But $\quad p(-a) = 0$

$$\Rightarrow \quad 3a + 4 = 0$$
$$\Rightarrow \quad 3a = -4$$
$$\Rightarrow \quad a = -\frac{4}{3}. \qquad \text{Ans.}$$

(ii) Let $p(x) = x^4 - a^2x^2 + 3x - a \quad \ldots(i)$

Put $x = -a$ in equation (i) we get

$$p(-a) = (-a)^4 - a^2(-a)^2 + 3(-a) - a$$
$$= a^4 - a^2 \times a^2 - 3a - a = -4a$$

But $\quad p(-a) = 0$

$$\Rightarrow \quad -4a = 0$$
$$\Rightarrow \quad a = \frac{0}{-4}$$
$$\Rightarrow \quad a = 0. \qquad \text{Ans.}$$

Question 12. In the following two polynomials, find the value of 'a' if x – a is a factor of each of the two:

(i) $x^6 - ax^5 + x^4 - ax^3 + 3a + 2$

(ii) $x^5 - a^2x^3 + 2x + a + 1$.

Solution : (i) Let

$$p(x) = x^6 - ax^5 + x^4 - ax^3 + 3a + 2 \quad \ldots(i)$$

Put $x = a$ in equation (i) we get

$$p(a) = (a)^6 - a(a)^5 + (a)^4 - a(a)^3 + 3(a) + 2$$
$$= a^6 - a^6 + a^4 - a^4 + 3a + 2 = 0$$
$$\therefore \quad 3a = -2$$
$$\therefore \quad a = \frac{-2}{3}. \qquad \text{Ans.}$$

(ii) Let $p(x) = x^5 - a^2x^3 + 2x + a + 1$

Since$(x - a)$ in a factor of $p(x)$, so $p(a) = 0$.

Put $x = a$ in equation (i) we get

$$p(a) \equiv (a)^5 - a^2(a)^3 + 2a + a + 1 = 0$$
$$= a^5 - a^2 \times a^3 + 3a + 1 = 0$$
$$= a^5 - a^5 + 3a + 1 = 0$$
$$= 3a + 1 = 0$$
$$\Rightarrow \quad 3a + 1 = 0$$
$$\Rightarrow \quad 3a = -1$$
$$\Rightarrow \quad a = -\frac{1}{3}.$$

Question 13. Given that $x + 2$ and $x + 3$ are factors of $2x^3 + ax^2 + 7x - b$. Determine the values of a and b.

Solution : If $x + 2$ is a factor is $2x^3 + ax^2 + 7x - b$ then $x + 2 = 0$, $x = -2$ in equation

$$2(-2)^3 + a(-2)^2 + 7(-2) - b = 0$$
$$2(-8) + a(4) + 7(-2) - b = 0$$
$$-16 + 4a - 14 - b = 0$$
$$4a - b = 30 \quad \ldots(1)$$

Also given that $x + 3$ is a factor of $2x^3 + ax^2 + 7x - b$, then $x + 3 = 0$

$x = -3$ in equation

$$2(-3)^3 + a(-3)^2 + 7(-3) - b = 0$$
$$2(-27) + a(9) + 7(-3) - b = 0$$
$$-54 + 9a - 21 - b = 0$$
$$9a - b = 75 \quad \ldots(2)$$

Solving (1) and (2) we get

$$a = 9, \; b = 6 \qquad \text{Ans.}$$

Question 14. If $p(x) = 4x^3 - 3x^2 + 2x - 4$ find the remainder when $p(x)$ is divided by :

(i) $x - 4$ (ii) $x + 2$ (iii) $x + \frac{1}{2}$.

Solution : (i)

$$p(x) = 4x^3 - 3x^2 + 2x - 4 \quad \text{...(i)}$$

Put $x = 4$ in equation (i), we get

$$\begin{aligned} p(4) &= 4(4)^3 - 3(4)^2 + 2(4) - 4 \\ &= 4 \times 64 - 3 \times 16 + 8 - 4 \\ &= 256 - 48 + 8 - 4 \\ &= 264 - 52 = 212 \end{aligned}$$

Hence, the remainder is 212. Ans.

(ii) $p(x) = 4x^3 - 3x^2 + 2x - 4$ ...(i)

By the remainder theorem the required remainder $= p(-2)$.

Put $x = -2$ in equation (i), we get

$$\begin{aligned} p(-2) &= 4(-2)^3 - 3(-2)^2 + 2(-2) - 4 \\ &= 4 \times (-8) - 3 \times 4 - 4 - 4 \\ &= -32 - 12 - 4 - 4 = -52 \end{aligned}$$

Hence, the remainder is -52. Ans.

(iii) $p(x) = 4x^3 - 3x^2 + 2x - 4$ ...(i)

By the remainder theorem the required remainder

$$= p\left(-\frac{1}{2}\right).$$

Put $x = \left(-\frac{1}{2}\right)$ in equation (i) we get

$$\begin{aligned} p\left(-\frac{1}{2}\right) &= 4\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) - 4 \\ &= 4 \times \left(-\frac{1}{8}\right) - 3 \times \frac{1}{4} + 2 \times \left(-\frac{1}{2}\right) - 4 \\ &= -\frac{1}{2} - \frac{3}{4} - 1 - 4 \\ &= \frac{-2 - 3 - 4 - 16}{4} = -\frac{25}{4} \end{aligned}$$

Hence, the remainder is $-\frac{25}{4}$. Ans.

Question 15. In the following problems use the factor theorem to find if g(x) is a factor of p(x):

(i) $p(x) = x^3 - 3x^2 + 4x - 4$ and $g(x) = x - 2$

(ii) $p(x) = 2x^3 + 4x + 6$ and $g(x) = x + 1$

(iii) $p(x) = x^3 + x^2 + 3x + 175$ and $g(x) = x + 5$.

Solution : (i)

$$p(x) = x^3 - 3x^2 + 4x - 4$$

and $g(x) = x - 2$

To check whether $x - 2$ is a foctor of $p(x)$ now put $x = 2$ in equation (i), we get

$$\begin{aligned} p(2) &= (2)^3 - 3(2)^2 + 4(2) - 4 \\ &= 8 - 3 \times 4 + 8 - 4 \\ &= 8 - 12 + 8 - 4 \\ &= 16 - 16 = 0 \end{aligned}$$

Since, $p(2) = 0$, so by factor theorem $(x - 2)$ is a factor of $p(x)$. Ans.

(ii) $p(x) = 2x^3 + 4x + 6$

and $g(x) = x + 1$

Now put $x = -1$ in equation (i), we get

$$\begin{aligned} p(-1) &= 2(-1)^3 + 4(-1) + 6 \\ &= 2 \times -1 - 4 + 6 \\ &= -2 - 4 + 6 \\ &= -6 + 6 = 0 \end{aligned}$$

Since, $p(-1) = 0$, so by factor theorem $(x + 1)$ is a factor of $p(x)$.

Ans.

(iii) $p(x) = x^3 + x^2 + 3x + 175$ ...(i)

and $g(x) = x + 5$

To check whether $(x + 5)$ is a factor of $p(x)$, we have to find $p(-5)$, put $x = -5$ in equation (i), we get

$$\begin{aligned} p(-5) &= (-5)^3 + (-5)^2 + 3(-5) + 175 \\ &= -125 + 25 - 15 + 175 \\ &= -140 + 200 = 60 \end{aligned}$$

Since, $p(-5) \neq 0$, so by factor theorem $(x + 5)$ is not a factor of $p(x)$. Ans.

Question 16. If x – 2 is a factor of each of the following three polynomials. Find the value of ‘a’ in each case:

(i) $x^2 - 3x + 5a$
(ii) $x^3 + 2ax^2 + ax - 1$
(iii) $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$.

Solution : (i) Let

$$p(x) = x^2 - 3x + 5a \qquad \ldots(i)$$

Since, $(x - 2)$ is a factor of $p(x)$, so $p(2) = 0$
(by factor theorem)

Put $x = 2$ in equation (i), we get

$$\begin{aligned} p(2) &= (2)^2 - 3 \times 2 + 5a \\ &= 4 - 6 + 5a \\ &= 5a - 2 \end{aligned}$$

But $p(2) = 0$

$\Rightarrow \quad 5a - 2 = 0$

$\Rightarrow \quad 5a = 2$

$\Rightarrow \quad a = \frac{2}{5}$. Ans.

(ii) Let $p(x) = x^3 + 2ax^2 + ax - 1 \qquad \ldots(i)$

Since, $(x - 2)$ is a factor of $p(x)$, so $p(2) = 0$

Put $x = 2$ in equation (i), we get

$$\begin{aligned} p(2) &= (2)^3 - 2a(2)^2 + a(2) - 1 \\ &= 8 - 2a \times 4 + 2a - 1 \\ &= 8 - 8a + 2a - 1 \\ &= 7 - 6a \end{aligned}$$

But $p(2) = 0$

$7 - 6a = 0$

$\Rightarrow \quad -6a = -7$

$\Rightarrow \quad a = \frac{+7}{+6}$

$\Rightarrow \quad a = \frac{7}{6}$. Ans.

(iii) Let $p(x) = x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4 \qquad \ldots(i)$

Since, $(x - 2)$ is factor of $p(x)$, so $p(2) = 0$

Put $x = 2$ in equation (i) we get

$$\begin{aligned} p(2) &= (2)^5 - 3(2)^4 - a(2)^3 + 3a\,(2)^2 + 2a\,(2) + 4 \\ &= 32 - 3 \times 16 - a \times 8 + 3a \times 4 + 4a + 4 \\ &= 32 - 48 - 8a + 12a + 4a + 4 \\ &= 8a - 12 \end{aligned}$$

But $p(2) = 0$

$\Rightarrow \quad 8a - 12 = 0$

$\Rightarrow \quad 8a = 12$

$\Rightarrow \quad a = \frac{12}{8}$

$\Rightarrow \quad a = \frac{3}{2}$. Ans.

Question 17. Find the value of the constant a and b, if (x – 2) and (x + 3) are both factors of expression $x^3 + ax^2 + bx - 12$.

Solution : Expression $x^3 + ax^2 + bx - 12$

$(x-2)$ is a factor *i.e.*, at $x = 2$

the remainder will be zero

$\Rightarrow \quad (2)^3 + a(2)^2 + b(2) - 12 = 0$

$\Rightarrow \quad 8 + 4a + 2b - 12 = 0$

$\Rightarrow \quad 4a + 2b = 4$

$\Rightarrow \quad 2a + b = 2 \quad$ ...(i)

when $x + 3$ is a factor *i.e.*, at $x = -3$ the remainder will be zero.

$\Rightarrow (-3)^3 + a(-3)^2 + b(-3) - 12 = 0$

$\Rightarrow \quad -27 + 9a - 3b - 12 = 0$

$\Rightarrow \quad 9a - 3b = 39$

$\Rightarrow \quad 3a - b = 13 \quad$ (ii)

Solving (i) and (ii) simultaneously

$2a + b = 2$

By adding $\quad 3a - b = 13$

$5a = 15$

$a = 3$

Substituting the value of a in the equation (i)

$\Rightarrow \quad 2 \times 3 + b = 2$

$\Rightarrow \quad 6 + b = 2$

$\Rightarrow \quad b = 2 - 6 = -4$

$\Rightarrow \quad a = 3, b = -4 \quad$ Ans.

Question 18. The expression $2x^3 + ax^2 + bx - 2$ leaves the remainder 7 and 0 when divided by $(2x - 3)$ and $(x + 2)$ respectively calculate the value of a and b. With these value of a and b factorise the expression completely.

Solution : Let $P(x) = 2x^3 + ax^2 + bx - 2$

when $P(x)$ is divided by $2x - 3$

$$P\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 + b\left(\frac{3}{2}\right) - 2 = 7$$

$$= \frac{27}{4} + \frac{9}{4}a + \frac{3}{2}b - 2 = 7$$

$$= \frac{27 + 9a + 6b - 8}{4} = 7$$

$$= 9a + 6b = 28 + 8 - 27$$

$$= 9a + 6b = 9$$

$$\Rightarrow 3a + 2b = 3 \quad \text{...(1)}$$

Similarly when $P(x)$ is divided by $x + 2$

$x = -2$

$2(-2)^3 + a(-2)^2 + b(-2) - 2 = 0$

$-16 + 4a - 2b - 2 = 0$

$\Rightarrow \quad 4a - 2b = 18 \quad$ ...(2)

On Solving equation (1) and (2)

$$3a + 2b = 3$$
$$4a - 2b = 18$$
$$7a = 21$$
$$a = 3$$

On substituting value of a in equation (1)

$$3 \times 3 + 2b = 3$$
$$2b = 3 - 9$$
$$b = \frac{-6}{2} = -3$$
$$b = -3$$

$a = 3, b = -3$

On substituting value of a and b

$2x^3 + 3a^2 - 3x - 2$

When $x + 2$ is a factor

$$x + 2 \overline{) 2x^3 + 3x^2 - 3x - 2} (2x^2 - x - 1$$
$$2x^3 + 4x^2$$
$$- \quad -$$
$$- x^2 - 3x$$
$$- x^2 - 2x$$
$$+ \quad +$$
$$- x - 2$$
$$- x + 2$$
$$+ \quad +$$
$$\times$$

$$2x^2 - x - 1 = 2x^2 - 2x + x - 1$$
$$= 2x(x - 1) + 1(x - 1)$$
$$(x - 1)(2x + 1)$$

Hence required factors are

$$(x - 1)(x + 2)(2x + 1)$$

Question 19. If x – 2 is a factor of

$$2x^3 - x^2 - px - 2.$$

(i) find the value of p

(ii) with the value of p, factorize the above expression completely.

Solution : Given expression is $2x^3 - x^2 - px - 2$ and $x - 2$ is the factor.

(i) $x - 2 = 0, x = 2$ in expression

$$2(2)^3 - (2)^2 - p(2) - 2 = 0$$
$$16 - 4 - 2p - 2 = 0$$
$$10 - 2p = 0$$
$$p = 5$$

(ii) Putting the value of P

$$(2x^2 + 3x + 1)$$
$$x - 2 \overline{) 2x^3 - x^2 - 5x - 2} ($$
$$2x^3 - 4x^2$$
$$- \quad +$$
$$3x^2 - 5x$$
$$3x^2 - 6x$$
$$- \quad +$$
$$x - 2$$
$$x - 2$$
$$- \quad +$$
$$\times$$

$\therefore 2x^3 - x^2 - 5x - 2 = (x - 2)(2x^2 + 3x + 1)$

The expression can be the written as

$(2x^2 + 3x + 1)(x - 2)$ or $(2x + 1)(x + 1)(x - 2)$.

Prove the Following

Question 1. Show that $x^2 - 9$ is factor of

$$x^3 + 5x^2 - 9x - 45.$$

Solution : We know that

$$x^2 - 9 = (x+3)(x-3)$$

$x^2 - 9$ will be a factor of

$$f(x) = x^3 + 5x^2 - 9x - 45$$

Only when both $x + 3$ and $x - 3$ are factors of this polynomial.

Now, $f(-3) = (-3)^3 + 5(-3)^2 - 9(-3) - 45$

$= -27 + 45 + 27 - 45 = 0$

And $f(3) = (3)^3 + 5(3)^2 - 9(3) - 45$

$= 27 + 45 - 27 - 45 = 0$

So, both $x + 3$ and $x - 3$ are factors of

$$x^3 + 5x^2 - 9x - 45.$$

Hence, $x^2 - 9$ is a factor of the given polynomial. Ans.

Question 2. Show that $2x + 7$ is a factor of $2x^3 + 5x^2 - 11x - 14$. Hence factorise the given expression completely, using the factor theorem.

Solution : If $2x + 7$ in factor of $2x^3 + 5x^2 - 11x - 14$

then on putting $2x + 7 = 0$

$$x = -7/2$$

$$f(-7/2) = 0$$

$$= 2\left(-\frac{7}{2}\right)^3 + 5\left(-\frac{7}{2}\right)^2 - 11\left(-\frac{7}{2}\right) - 14$$

$$= \frac{-343}{4} + \frac{245}{4} + \frac{77}{2} - 14$$

$$= \frac{-399}{4} + \frac{245 + 154}{4}$$

$$= \frac{-399 + 399}{4} = 0$$

Hence $2x + 7$ is one factor.

Now $2x^3 + 5x^2 - 11x - 14$

$= x^2(2x+7) - x(2x+7) - 2(2x+7)$

$= (2x+7)(x^2 - x - 2)$

$= (2x+7)(x^2 + x - 2x - 2)$

$= (2x+7)[x(x+1) - 2(x+1)]$

$= (2x+7)(x-2)(x+1)$ Ans.

Question 3. Using factor theorem, show that $(x - 3)$ is a factor of $x^3 - 7x^2 + 15x - 9$. Hence, factorise the given expression completely.

Solution : Let $p(x) = x^3 - 7x^2 + 15x - 9$

For checking that $(x - 3)$ is a factor of $p(x)$, we find : $p(3)$.

$$\begin{aligned} p(3) &= (3)^3 - 7(3)^2 + 15(3) - 9 \\ &= 27 - 63 + 45 - 9 \\ &= 72 - 72 \\ &= 0. \end{aligned}$$

Hence, $(x - 3)$ is a factor of $p(x)$.

By division of $p(x)$ by $x - 3$, we get the quotient

$$= x^2 - 4x + 3.$$

$$\begin{aligned} \therefore\ x^3 - 7x^2 + 15x - 9 &= (x - 3)(x^2 - 4x + 3) \\ &= (x - 3)(x - 3)(x - 1) \\ &= (x - 3)^2 (x - 1). \end{aligned}$$ Ans.