Quadratic Equations

A quadratic equation in the variable x is an equation of the form $ax^2 + bx + c = 0$, where a , b, c are real numbers, $a \neq 0$.

Roots of a Quadratic Equation:

- A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$ if
  $a\alpha^2 + b\alpha + c = 0$.
- $\boldsymbol{x} = \alpha$ is a solution of the quadratic equation, or α satisfies the quadratic equation.
- The zeroes of the quadratic polynomial $\boldsymbol{ax^2 + bx + c}$ and the roots of the quadratic. Equation $\boldsymbol{ax^2 + bx + c = 0}$ are the same.

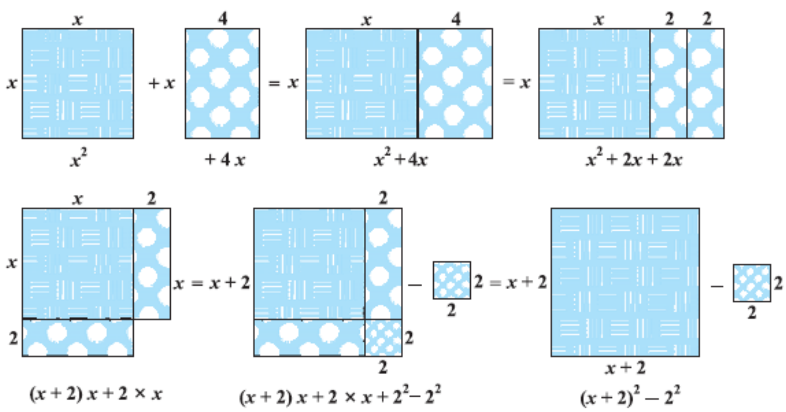
Solution of Quadratic Equation by Factorisation:

- To factorise quadratic polynomials the middle term is split.
- By factorizing the equation into linear factors and equating each factor to zero the roots are determined.

Quadratic Equations - Method of Squares

Solution of Quadratic Equation by method of Squares

- We can convert any quadratic equation to the form
  $(x + a)^2 - b^2 = 0$
  $x^2 + 4x$ is being converted to
  $(x + 2)^2 - 4. = (x + 2)^2 - 2^2$



The process is as follows:

$x^2 + 4x = \left(x^2 + \frac{4}{2}x\right) + \frac{4}{2}x$

$= x^2 + 2x + 2x$

$= (x+2)\ x + 2\ \times x$

$= (x+2)\ x + 2\ \times x + 2\ \times 2 - 2\ \times 2$

$= (x+2)\ x + (x + 2\ ) \times 2 - 2\ \times 2$

$= (x+2)\ (x + 2\ ) - 2^2$

$= (x+2)^2 - 4$

So, $x^2 + 4x - 5\ (x+2)^2 - 4 - 5 = (x+2)^2 - 9$

So, $x^2 + 4x - 5 = 0$ can be written as $(x+2)^2 - 9 = 0$ by this process of completing the square. This is known as the **method of completing the square.**

Solution of Quadratic Equation by using Formula.

The formula is as follows:

The roots of $ax^2 + bx + c = 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$

If $b^2 - 4ac\ \geq 0$.

Thus, if $b^2 - 4ac\ \geq 0$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

This formula for finding the roots of a quadratic equation is known as the Quadratic formula.

Nature of Roots

We know that roots of the equation $ax^2 + \mathrm{bx} + \mathrm{c} = 0, \mathrm{a} \neq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}\ and\ x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Where $b^2 - 4\mathrm{ac} = \Delta$ is known as discriminant.

Nature of roots based on the discriminant value

1. If $\Delta = 0$, then the roots are real and equal.
2. If $\Delta > 0$, then the roots are real and distinct (unequal)
3. If $\Delta < 0$, then the roots are imaginary (not real)