

6. Coordinates

Questions Pg-134

1 A. Question

Find the following:

The y coordinate of any point on the x – axis.

Answer

[To solve such questions, following are the key points:

1. X coordinate of any point on Y axis is 0.
2. Y coordinate of any point on X axis is 0.
3. If a line is parallel to X axis, then its X coordinate varies, but Y coordinate remains constant.
4. If a line is parallel to Y axis, then its Y coordinate varies, but X coordinate remains constant.]

Since, the equation of X – axis is $y = 0$, y coordinate of any point on X – axis is 0(zero).

1 B. Question

Find the following:

The x coordinate of any point on the y – axis.

Answer

Since, the equation of Y axis is $x = 0$, coordinate of any point on Y axis is 0(zero).

1 C. Question

Find the following:

The coordinates of the origin.

Answer

Origin is the intersection of X axis and Y axis, the i.e. intersection of lines $y = 0$ and $x = 0$.

Therefore,

Coordinates of origin is (0,0).

1 D. Question

Find the following:

The y – coordinate of any point on the line through (0, 1), parallel to the x – axis.

Answer

Since the line is parallel to x – axis, its equation will be of the form $y = c$, where c is a constant.

Point (0,1) is on the line, therefore, it will satisfy the equation of the line.

Putting $y = 1$ in the equation of a line,

We get,

$$1 = c.$$

Therefore, the equation of line will be $y = 1$.

Hence, y coordinate of any point on the line through (0,1),parallel to the x axis is 1.

1 E. Question

Find the following:

The y coordinate of any point on the line through (1, 0), parallel to the y - axis

Answer

Since the line is parallel to y - axis , its equation will be of the form $x = c$, where c is a constant.

Point (1,0) is on the line, therefore, it will satisfy the equation of the line.

Putting $x = 1$ in the equation of a line,

We get,

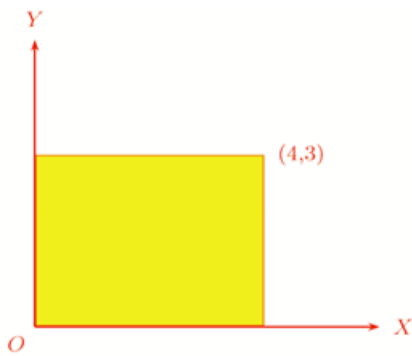
$$1 = c.$$

Therefore, the equation of line will be $x = 1$.

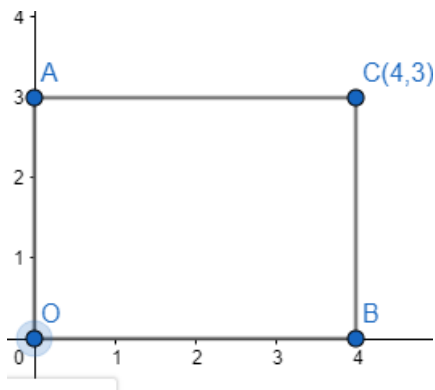
Hence, y coordinate of any point on the line through (1,0), parallel to the y axis is 1.

2. Question

Find the coordinates of the other three vertices of the rectangle below:



Answer



Vertex O is the origin.

Therefore, its coordinates are (0,0).

Vertex A is on the Y - axis.

Therefore its X coordinate = 0.

Line joining vertex A and (4,3) is parallel to X - axis, therefore its y coordinate will remain constant. Since point A is on the line, its Y coordinate = 3

Therefore coordinates of vertex A are (0,3)

Vertex B is on X - axis.

Therefore its Y coordinate = 0.

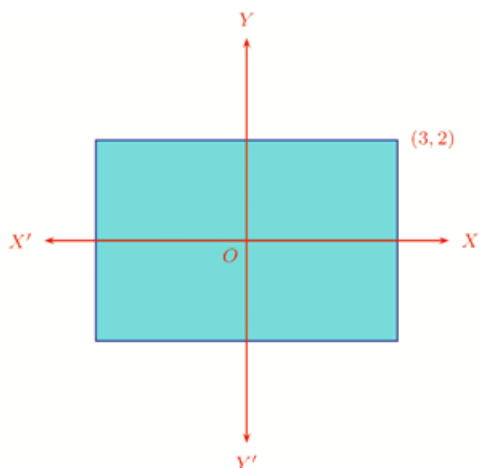
Line joining vertex B and (4,3) is parallel to Y - axis, therefore its x coordinate will remain constant. Since point A is on the line, its x coordinate = 4

Therefore coordinates of vertex A are (4,0)

Hence, the coordinate of the other three vertices of the given rectangle is $(0,0)$, $(0,3)$ and $(4,0)$.

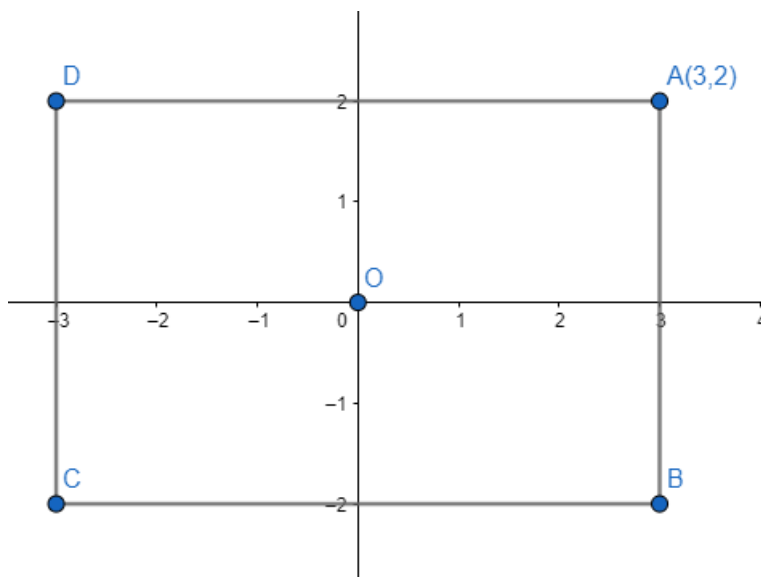
3. Question

In the rectangle shown below, the sides are parallel to the axes and origin is the midpoint:



What are the coordinates of the other three vertices?

Answer



Since, O is the midpoint of the rectangle and sides of the rectangle are parallel to coordinate axis,
The rectangle is symmetrical about both X and Y axis.

Line AB is parallel to Y – axis,

$\therefore$ X coordinate will remain same

$\therefore$ X coordinate of B = 3

A is 2 units above X – axis , therefore by symmetry B will be 2 units below X – axis.

$\therefore$ Y coordinate of B = - 2

$\therefore$ coordinates of B = (3, - 2)

BC is parallel to the X axis

$\therefore$ Y coordinate will remain constant

$\therefore$ Y coordinate of C = - 2

B is 3 units right to the Y axis,

$\therefore$ by symmetry C will be 3 units to left of Y – axis.

$\therefore$ X coordinate of C = - 3

$\therefore$ coordinates of C = (- 3, - 2)

CD is parallel to Y - axis,

$\therefore$ X coordinate will remain constant

$\therefore$ X coordinate of D = - 3

The AD is parallel to X - axis,

$\therefore$ Y coordinate will remain constant

$\therefore$ Y coordinate of D = 2

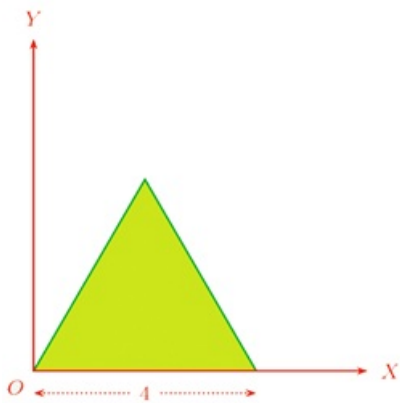
$\therefore$ coordinates of D = (- 3,2)

Hence, the coordinates of other three vertices are (3, - 2),(- 3, - 2)

And (- 3,2)

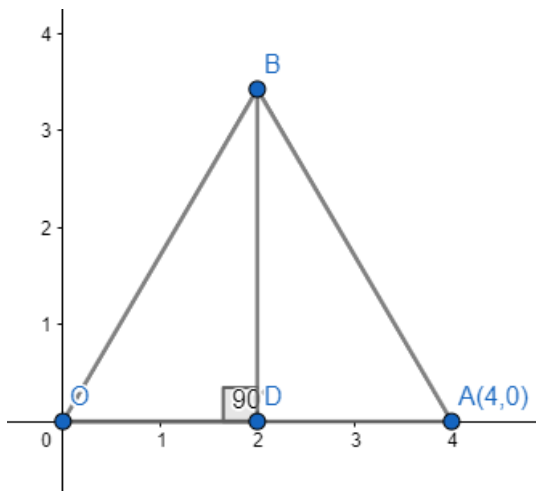
4. Question

The triangle shown below is equilateral:



Find the coordinates of its vertices.

Answer



Vertex O is the origin.

Therefore, its coordinates (0,0).

Vertex A is on X axis, therefore, its y coordinate = 0.

Vertex A is 4 units away from y - axis in the direction of the positive X - axis.

Therefore, the x coordinate of A = 4

∴ coordinates of A = (4,0).

Consider triangle OBD,

OB = 4 units(All sides of an equilateral triangle are equal)

∠ BOD = 60°

$$\sin \angle BOD = \frac{BD}{OB}$$

Let coordinates of B be (q,p)

Then,

BD = p

OD = q

$$\sin \angle BOD = \frac{p}{4}$$

$$\sin 60^\circ = \frac{p}{4}$$

$$\frac{\sqrt{3}}{2} = \frac{p}{4}$$

$$2\sqrt{3} = p$$

Therefore, the y coordinate of B = $2\sqrt{3}$

$$\cos \angle BOD = \frac{OD}{OB}$$

$$\cos 60^\circ = \frac{q}{4}$$

$$\frac{1}{2} = \frac{q}{4}$$

$$2 = q$$

Therefore, the x coordinate of B = 2

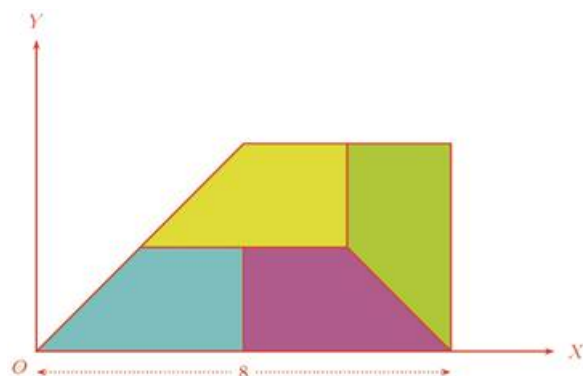
∴ coordinates of B = (2,2)

Hence, the coordinates of the vertices of the triangle are (0,0),(4,0) and (2,2)

5. Question

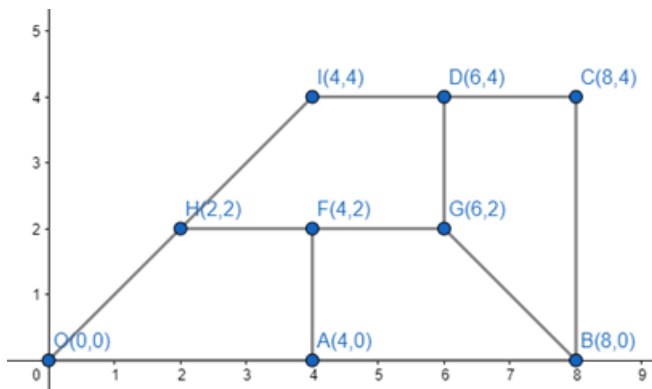
A large trapezium made up of four equal trapeziums:

Find the coordinates of the vertices of all these trapeziums.



Draw this picture in GeoGebra.

Answer



Since, the four trapeziums are equal

$$OA = AB$$

$\therefore$ A bisects OB

$$OB = 8$$

$$\therefore OA = 4$$

$\therefore$ coordinates of A = (4,0)[A is on X axis]

And coordinates of B = (8,0)[B is on X axis]

BC is parallel to Y - axis,

$\therefore$ X coordinate will remain constant

$\therefore$ X coordinate of C = 8

Since all four trapeziums are equal,

$$BC = OA = 4 = \text{Y coordinate of C}$$

$\therefore$ Y coordinate of C = 4

$\therefore$ coordinates of C = (8,4)

$$AF = GD[\text{All trapeziums are equal}]$$

$$AF + GD = BC = 4$$

$$\therefore AF = GD = 2$$

$$AF = \text{Y coordinate of F} = 2$$

Line AF is parallel to Y - axis,

$\therefore$ X coordinate will remain constant.

$\therefore$ X coordinate of F = 4

$\therefore$ coordinates of F = (4,2)

Line FG is parallel to X - axis,

$\therefore$ Y coordinate will remain constant

$\therefore$ Y coordinate of G = 2

$$FG = DC[\text{All trapeziums are equal}]$$

$$FG + DC = AB = 4$$

$$\therefore FG = DC = 2$$

$$\text{X coordinate of G} = \text{X coordinate of F} + FG$$

$$= 4 + 2 = 6$$

∴ **coordinates of G = (6,2)**

CD is parallel to X – axis,

∴ Y coordinate will remain constant

∴ Y coordinate of D = 4

GD is parallel to Y – axis,

∴ X coordinate will remain constant

∴ X coordinate of D = 6

∴ **coordinates of D = (6,4)**

CI is parallel to X axis

∴ Y coordinate will remain constant

∴ Y coordinate of I = 4

AI is parallel to Y – axis,

∴ X coordinate will remain constant

∴ X coordinate of I = 4

∴ **coordinates of I = (4,4)**

OH and HI will be equal in length and will be at equal angles with X axis[All trapeziums are equal]

∴ H will be the midpoint of OI

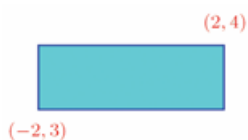
∴ H will be the mid point of (0,0) and (4,4)

∴ **coordinates of H = (2,2)**

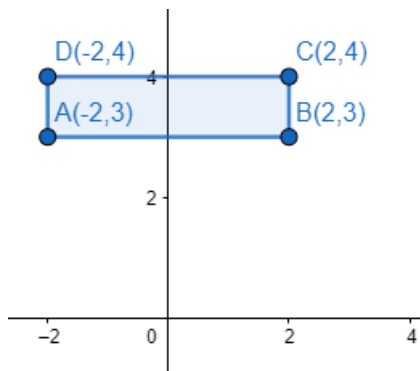
Questions Pg-139

1 A. Question

All rectangles below have sides parallel to the axes. Find the coordinates of the remaining vertices of each.



Answer



Coordinates of A = $(-2, 3)$

Coordinates of C = $(2, 4)$

Line AB is parallel to X – axis ,

∴ Y coordinate will remain constant.

$\therefore$ Y - coordinate of B = 3

Line BC is parallel to Y - axis ,

$\therefore$ X coordinate will remain constant.

$\therefore$ X coordinate of B = 2

$\therefore$ coordinates of B = (2,3)

Line AD is parallel to Y - axis ,

$\therefore$ X coordinate will remain constant.

$\therefore$ X coordinate of D = - 2

Line CD is parallel to X - axis ,

$\therefore$ Y coordinate will remain constant.

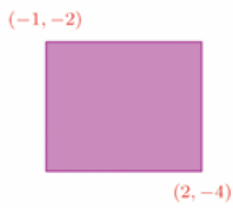
$\therefore$ Y coordinate of D = 4

$\therefore$ coordinates of D = (- 2,4)

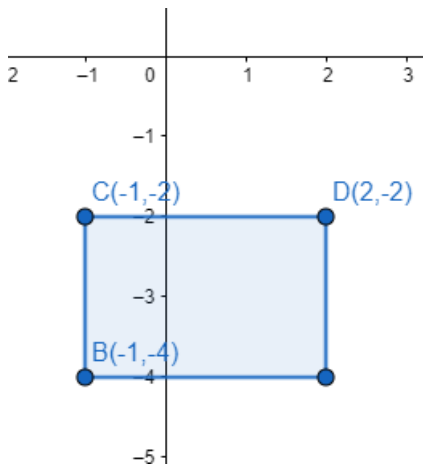
Hence, the remaining vertices of the rectangle are (2,3),(- 2,4).

1 B. Question

All rectangles below have sides parallel to the axes. Find the coordinates of the remaining vertices of each.



Answer



Coordinates of A = (2, - 4)

Coordinates of C = (- 1, - 2)

Line AB is parallel to X - axis ,

$\therefore$ Y coordinate will remain constant.

$\therefore$ Y - coordinate of B = - 4

Line BC is parallel to Y - axis ,

$\therefore$ X coordinate will remain constant.

$\therefore$ X coordinate of B = - 1

$\therefore$ coordinates of B = (- 1, - 4)

Line AD is parallel to Y – axis ,

$\therefore$ X coordinate will remain constant.

$\therefore$ X coordinate of D = 2

Line CD is parallel to X – axis ,

$\therefore$ Y coordinate will remain constant.

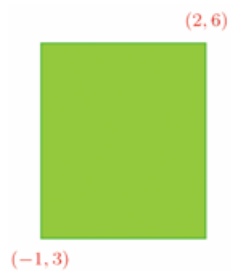
$\therefore$ Y coordinate of D = - 2

$\therefore$ coordinates of D = (2, - 2)

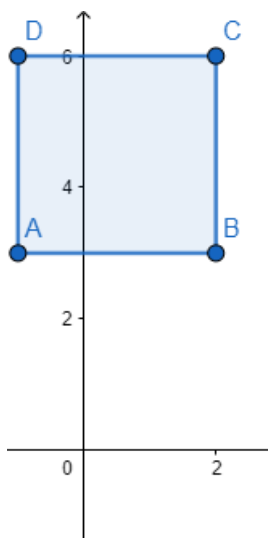
Hence, the remaining vertices of the rectangle are (- 1, - 4) and (2, - 2).

1 C. Question

All rectangles below have sides parallel to the axes. Find the coordinates of the remaining vertices of each.



Answer



Coordinates of A = (- 1,3)

Coordinates of C = (2,6)

Line AB is parallel to X – axis ,

$\therefore$ Y coordinate will remain constant.

$\therefore$ Y – coordinate of B = 3

Line BC is parallel to Y – axis ,

$\therefore$ X coordinate will remain constant.

$\therefore$ X coordinate of B = 2

$\therefore$ coordinates of B = (2,3)

Line AD is parallel to Y – axis ,

$\therefore$ X coordinate will remain constant.

$\therefore$ X coordinate of D = - 1

Line CD is parallel to X - axis ,

$\therefore$ Y coordinate will remain constant.

$\therefore$ Y coordinate of D = 6

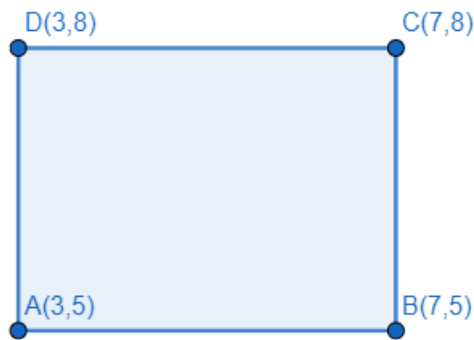
$\therefore$ **coordinates of D = (- 1,6)**

Hence, the remaining vertices of the rectangle are (2,3) and (- 1,6).

2 A. Question

Without drawing coordinates axes, mark each pair of points below with left - right, top - bottom position correct. Find the other coordinates of the rectangles drawn with these as opposite vertices and sides parallel to the axes.

(3, 5), (7, 8)



Answer

Steps for marking the given points:

- 1) Mark Point (3,5) anywhere.
- 2) Move $(7 - 3) = 4$ units right.
- 3) Move $(8 - 5) = 3$ units up.
- 4) Mark the point at the current position.

Since sides of the rectangle are parallel to coordinate axes,

Their equations will be

$$x = 3$$

$$x = 7$$

$$y = 5$$

$$y = 8$$

Point of intersection of $x = 3$ and $y = 5$ is (3,5) [Given point]

Point of intersection of $x = 3$ and $y = 8$ is (3,8)

Point of intersection of $x = 7$ and $y = 5$ is (7,5)

Point of intersection of $x = 7$ and $y = 8$ is (7,8) [Given point]

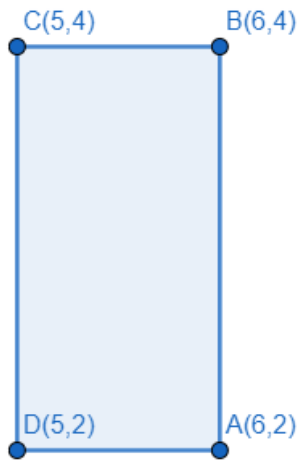
Hence, the coordinates of the other vertices of the rectangle are (3,8) and (7,5)

2 B. Question

Without drawing coordinates axes, mark each pair of points below with left - right, top - bottom position correct. Find the other coordinates of the rectangles drawn with these as opposite vertices and sides parallel to the axes.

(6, 2), (5, 4)

Answer



Steps for marking the given points:

- 1) Mark Point (6,2) anywhere.
- 2) Move $(5 - 6) = -1$ unit right, i.e. 1 unit left.
- 3) Move $(4 - 2) = 2$ units up.
- 4) Mark the point at the current position.

Since sides of the rectangle are parallel to coordinate axes,

Their equations will be

$$x = 6$$

$$x = 5$$

$$y = 2$$

$$y = 4$$

Point of intersection of $x = 6$ and $y = 2$ is (6,2) [Given point]

Point of intersection of $x = 5$ and $y = 2$ is (5,2)

Point of intersection of $x = 6$ and $y = 4$ is (6,4)

Point of intersection of $x = 5$ and $y = 4$ is (5,4) [Given point]

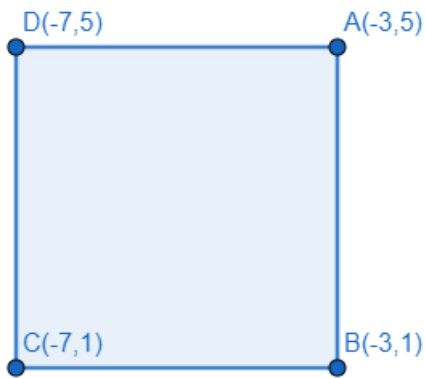
Hence, the coordinates of the other vertices of the rectangle are (5,2) and (6,4)

2 C. Question

Without drawing coordinates axes, mark each pair of points below with left – right, top – bottom position correct. Find the other coordinates of the rectangles drawn with these as opposite vertices and sides parallel to the axes.

(- 3, 5), (- 7, 1)

Answer



Steps for marking the given points:

- 1) Mark Point $(-3, 5)$ anywhere.
- 2) Move $(-7 - (-3)) = -4$ units right i.e. 4 units left.
- 3) Move $(1 - 5) = -4$ units up i.e. 4 units down.
- 4) Mark the point at the current position.

Since sides of the rectangle are parallel to coordinate axes,

Their equations will be

$$x = -3$$

$$x = -7$$

$$y = 5$$

$$y = 1$$

Point of intersection of $x = -3$ and $y = 5$ is $(-3, 5)$ [Given point]

Point of intersection of $x = -7$ and $y = 5$ is $(-7, 5)$

Point of intersection of $x = -3$ and $y = 1$ is $(-3, 1)$

Point of intersection of $x = -7$ and $y = 1$ is $(-7, 1)$ [Given point]

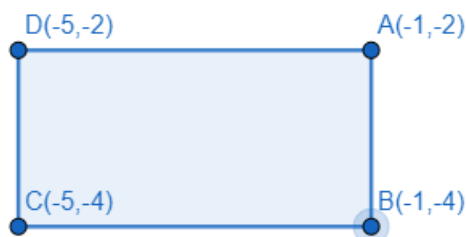
Hence, the coordinates of the other vertices of the rectangle are $(-3, 1)$ and $(-7, 5)$

2 D. Question

Without drawing coordinates axes, mark each pair of points below with left - right, top - bottom position correct. Find the other coordinates of the rectangles drawn with these as opposite vertices and sides parallel to the axes.

$(-1, -2)$, $(-5, -4)$

Answer



Steps for marking the given points:

- 1) Mark Point $(-1, -2)$ anywhere.
- 2) Move $(-5 - (-1)) = -4$ units right i.e. 4 units left.

3) Move $(-4 - (-2)) = -2$ units up i.e. 2 units down.

4) Mark the point at the current position.

Since sides of the rectangle are parallel to coordinate axes,

Their equations will be

$$x = -1$$

$$x = -5$$

$$y = -2$$

$$y = -4$$

Point of intersection of $x = -1$ and $y = -2$ is $(-1, -2)$ [Given point]

Point of intersection of $x = -1$ and $y = -4$ is $(-1, -4)$

Point of intersection of $x = -5$ and $y = -2$ is $(-5, -2)$

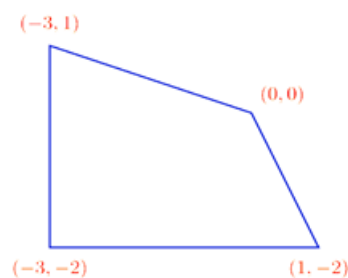
Point of intersection of $x = -5$ and $y = -4$ is $(-5, -4)$ [Given point]

Hence, the coordinates of the other vertices of the rectangle are $(-1, -4)$ and $(-5, -2)$

Questions Pg-146

1. Question

Calculate the length of the sides and diagonals of the quadrilateral below:



Answer

Let $A = (0, 0)$

$B = (1, -2)$

$C = (-3, -2)$

$D = (-3, 1)$

Here, AB, BC, CD, DA are the sides of the quadrilateral and AC and BD are the diagonals of the quadrilateral.

Length of Side

$$AB = \text{distance between point A and B} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here $x_2 = 1, y_2 = -2, x_1 = 0, y_1 = 0$

$$\therefore AB = \sqrt{(1 - 0)^2 + (-2 - 0)^2}$$

$$\therefore AB = \sqrt{1^2 + (-2)^2}$$

$$\therefore AB = \sqrt{1 + 4}$$

$$\therefore \mathbf{AB = \sqrt{5} \text{ unit}}$$

$$BC = \text{distance between point B and C} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here $x_2 = -3, y_2 = -2, x_1 = 1, y_1 = -2$

$$\therefore BC = \sqrt{(-3 - (-1))^2 + (-2 - (-2))^2}$$

$$\therefore BC = \sqrt{(-2)^2 + (0)^2}$$

$$\therefore BC = \sqrt{4 + 0}$$

$$\therefore \mathbf{BC = 2 \text{ units}}$$

$$CD = \text{distance between point C and D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here $x_2 = -3, y_2 = 1, x_1 = -3, y_1 = -2$

$$\therefore CD = \sqrt{(-3 - (-3))^2 + (1 - (-2))^2}$$

$$\therefore CD = \sqrt{(0)^2 + (3)^2}$$

$$\therefore CD = \sqrt{0 + 9}$$

$$\therefore \mathbf{CD = 3 \text{ units}}$$

$$DA = \text{distance between point D and A} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here $x_2 = 0, y_2 = 0, x_1 = -3, y_1 = 1$

$$\therefore DA = \sqrt{(0 - (-3))^2 + (0 - 1)^2}$$

$$\therefore DA = \sqrt{3^2 + 1^2}$$

$$\therefore DA = \sqrt{9 + 1}$$

$$\therefore \mathbf{DA = \sqrt{10} \text{ units}}$$

Length of diagonal

$$AC = \text{distance between point A and C} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here $x_2 = -3, y_2 = -2, x_1 = 0, y_1 = 0$

$$\therefore AC = \sqrt{(-3 - 0)^2 + (-2 - 0)^2}$$

$$\therefore AC = \sqrt{3^2 + 2^2}$$

$$\therefore AC = \sqrt{9 + 4}$$

$$\therefore \mathbf{AC = \sqrt{13} \text{ units}}$$

Length of diagonal

$$BD = \text{distance between point B and D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here $x_2 = -3, y_2 = 1, x_1 = 1, y_1 = -2$

$$\therefore BD = \sqrt{(-3 - 1)^2 + (1 - (-2))^2}$$

$$\therefore BD = \sqrt{(-4)^2 + 3^2}$$

$$\therefore BD = \sqrt{16 + 9}$$

$$\therefore \mathbf{BD = 5 \text{ units}}$$

Hence, length of the sides of the quadrilateral is $\sqrt{5}, 2, 3, \sqrt{10}$ units.

Length of the diagonals of the quadrilateral are $\sqrt{13}$ and 5 units.

2. Question

Prove that by joining the point (2, 1), (3, 4), (-3, 6) we get a right triangle.

Answer

Let A = (2,1)

B = (3,4)

C = (-3,6)

Length of Side

AB = distance between point A and B = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here $x_2 = 3, y_2 = 4, x_1 = 2, y_1 = 1$

$$\therefore AB = \sqrt{(3-2)^2 + (4-1)^2}$$

$$\therefore AB = \sqrt{1^2 + 3^2}$$

$$\therefore AB = \sqrt{1 + 9}$$

$$\therefore AB = \sqrt{10} \text{ units}$$

BC = distance between point B and C = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here $x_2 = -3, y_2 = 6, x_1 = 3, y_1 = 4$

$$\therefore BC = \sqrt{(-3-3)^2 + (6-4)^2}$$

$$\therefore BC = \sqrt{(-6)^2 + 2^2}$$

$$\therefore BC = \sqrt{36 + 4}$$

$$\therefore BC = \sqrt{40} \text{ units}$$

CA = distance between point C and A = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here $x_2 = 2, y_2 = 1, x_1 = -3, y_1 = 6$

$$\therefore CA = \sqrt{(2-(-3))^2 + (1-6)^2}$$

$$\therefore CA = \sqrt{5^2 + (-5)^2}$$

$$\therefore CA = \sqrt{25 + 25}$$

$$\therefore CA = \sqrt{50} \text{ units}$$

Here CA is the largest side.

For $\triangle ABC$ to be right angled triangle

$$CA^2 = AB^2 + BC^2$$

Here,

$$CA^2 = (\sqrt{50})^2 = 50$$

And

$$AB^2 + BC^2 = (\sqrt{10})^2 + (\sqrt{40})^2 = 10 + 40 = 50$$

$$50 = 50$$

$$\Rightarrow CA^2 = AB^2 + BC^2$$

∴ the given triangle is a right – angled triangle.

Hence Proved.

3. Question

A circle of radius 10 is drawn with the origin as the centre.

- i) Check whether each of the points with coordinates (6, 9), (5, 9), (6, 8) is inside, outside or on the circle.
- ii) Write the coordinates of 8 points on this circle.

Answer

(i) [If the distance between the centre and the point is greater than Radius, then the point is outside the circle.

If the distance between the centre and the point is smaller than Radius, then the point is inside the circle.

If the distance between the centre and the point is equal to Radius, then the point is on the circle.]

Centre of the circle = (0,0)(given)

Radius = 10

The distance between origin and (6,9) = $\sqrt{(6-0)^2 + (9-0)^2} = \sqrt{117} > 10$

⇒ **Point (6,9) is outside the circle.**

The distance between origin and (5,9) = $\sqrt{(5-0)^2 + (9-0)^2} = \sqrt{106} > 10$

⇒ **Point (5,9) is outside the circle.**

The distance between origin and (6,8) = $\sqrt{(6-0)^2 + (8-0)^2} = \sqrt{100} = 10$

⇒ **Point (6,8) is outside the circle.**

(ii) Let the coordinates of the point on the circle be (x,y)

Then, distance of it from the origin(center) = 10

$$\Rightarrow \sqrt{(x-0)^2 + (y-0)^2} = 10$$

$$\Rightarrow \sqrt{x^2 + y^2} = 10$$

$$\Rightarrow x^2 + y^2 = 100$$

All the possible solutions of the above equation will be on the circle.

Such 8 points are, **(6,8),**

(√10,√90),

(√20,√80),

(√30,√70),

(√40,√60),

(√50,√50),

(√60,√40),

(√70,√30).

4. Question

Find the coordinates of the points where a circle of radius $\sqrt{2}$, centered on the point with coordinates (1, 1) cut the axes.

Answer

Let the coordinates of the required point is (x, y).

Since the point is on the circle,

Its distance from the centre(1,1) = Radius = $\sqrt{2}$

$$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2} = \sqrt{2}$$

$$\Rightarrow (x-1)^2 + (y-1)^2 = 2$$

If the point is on X axis,

$$y = 0$$

$$\Rightarrow (x-1)^2 + (0-1)^2 = 2$$

$$\Rightarrow (x-1)^2 + 1 = 2$$

$$\Rightarrow (x-1)^2 = 1$$

$$\Rightarrow x-1 = 1 \text{ or } x-1 = -1$$

$$\Rightarrow x = 2 \text{ or } x = 0$$

Hence, coordinates of the points where a circle of radius $\sqrt{2}$, centred on the point with coordinates (1, 1) cut the axis are (2,0) and (0,0)

If the point is on Y axis,

$$x = 0$$

$$\Rightarrow (0-1)^2 + (y-1)^2 = 2$$

$$\Rightarrow (-1)^2 + (y-1)^2 = 2$$

$$\Rightarrow (y-1)^2 = 1$$

$$\Rightarrow y-1 = 1 \text{ or } y-1 = -1$$

$$\Rightarrow y = 2 \text{ or } y = 0$$

Hence, coordinates of the points where a circle of radius $\sqrt{2}$, centred on the point with coordinates (1, 1) cut the axis are (0,2) and (0,0)

Hence, the coordinates of the points where a circle of radius $\sqrt{2}$, centred on the point with coordinates (1, 1) cut the axes are (0,0),(2,0) and (0,2).

5. Question

The coordinates of the vertices of a triangle are (1, 2), (2, 3), (3, 1). Find the coordinates of the centre of its circumcircle and the circumradius.

Answer

Let the centre of the circumcircle be (x,y)

Since, centre of circumcircle is a point which is equidistant from all the points,

$$(x-1)^2 + (y-2)^2 = (x-2)^2 + (y-3)^2 = (x-3)^2 + (y-1)^2$$

$$\Rightarrow x^2 + y^2 - 2x - 4y + 5 = x^2 + y^2 - 4x - 6y + 13 = x^2 + y^2 - 6x - 2y + 10$$

$$\Rightarrow -2x - 4y + 5 = -4x - 6y + 13 = -6x - 2y + 10$$

$$-2x - 4y + 5 = -4x - 6y + 13$$

$$\Rightarrow 2x + 2y = 8$$

$$\Rightarrow x + y = 4 \dots\dots(1)$$

$$-4x - 6y + 13 = -6x - 2y + 10$$

$$\Rightarrow 2x - 4y = -3 \dots (2)$$

Substituting $x = 4 - y$ from eq(1)

$$2(4 - y) - 4y = -3$$

$$\Rightarrow 8 - 2y - 4y = -3$$

$$\Rightarrow 8 - 6y = -3$$

$$\Rightarrow -6y = -11$$

$$\Rightarrow y = \frac{11}{6}$$

And

$$x = 4 - y$$

$$\Rightarrow x = \frac{13}{6}$$

Hence coordinates of the centre of the circumcircle of the triangle = $(\frac{13}{6}, \frac{11}{6})$

And its radius =

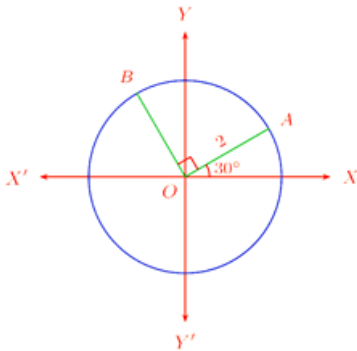
$$\sqrt{(\frac{13}{6} - 1)^2 + (\frac{11}{6} - 2)^2}$$

$$= \sqrt{\frac{50}{36}}$$

$$= \frac{5\sqrt{2}}{6} \text{ units.}$$

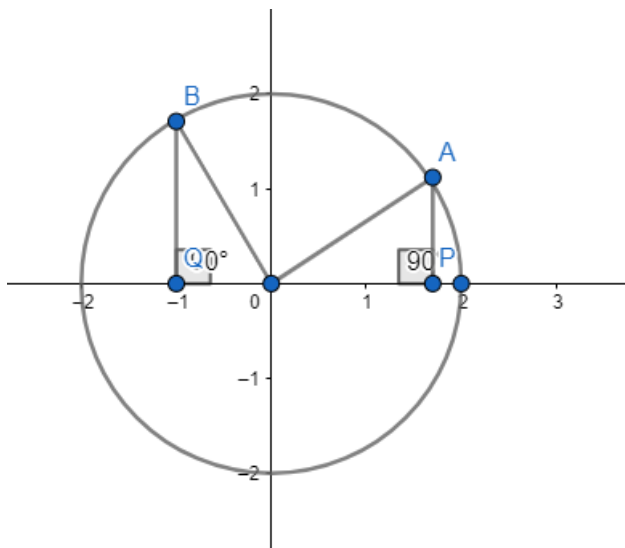
6. Question

The centre of the circle below is the origin and A, B are points on it.



Calculate the length of the chord AB.

Answer



Let coordinates of A be (x_1, y_1) and coordinates of B be (x_2, y_2) .

Drop a perpendicular from A on X – axis intersecting X axis at P.

Now,

In ΔOPA

$$\sin 30^\circ = \frac{AP}{OA}$$

$$\Rightarrow \frac{1}{2} = \frac{y_1}{2}$$

$$\Rightarrow y_1 = 1$$

And

$$\cos 30^\circ = \frac{OP}{OA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{x_1}{2}$$

$$\Rightarrow x_1 = \sqrt{3}$$

$\therefore$ coordinates of A = $(\sqrt{3}, 1)$

Drop a perpendicular from B on X – axis intersecting X axis at Q.

Now,

In ΔOQB

$$\cos 60^\circ = \frac{OQ}{OB}$$

$$\Rightarrow \frac{1}{2} = -\frac{x_2}{2}$$

$$\Rightarrow x_2 = -1$$

And

$$\sin 60^\circ = \frac{BQ}{OB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{y_2}{2}$$

$$\Rightarrow y_2 = \sqrt{3}$$

$\therefore$ coordinates of B = $(-1, \sqrt{3})$

Hence, length of the chord AB =

The distance between point A and B =

$$\sqrt{(-1 - \sqrt{3})^2 + (\sqrt{3} - 1)^2}$$

$$= \sqrt{8} = \sqrt{2}$$