33. Linear Programming

Exercise 33A

1. Question

Graph the solution sets of the following inequations:

x + y $\geq$ 4

Answer

Given x + y ≥ 4

⇒ y ≥ 4 - x

Consider the equation y = 4 - x.

Finding points on the coordinate axes:

If x = 0, the y value is 4 i.e, y = 4

⇒ the point on the Y axis is A(0,4)

If y = 0, 0 = 4 - x

⇒x = 4

The point on the X axis is B(4,0)

Plotting the points on the graph: fig. 1a

Now consider the inequality y ≥ 4 - x

Here we need the y value greater than or equal to 4 - x

⇒ the required region is above point A.

Therefore the graph of the inequation x + y ≥ 4 is fig. 1b

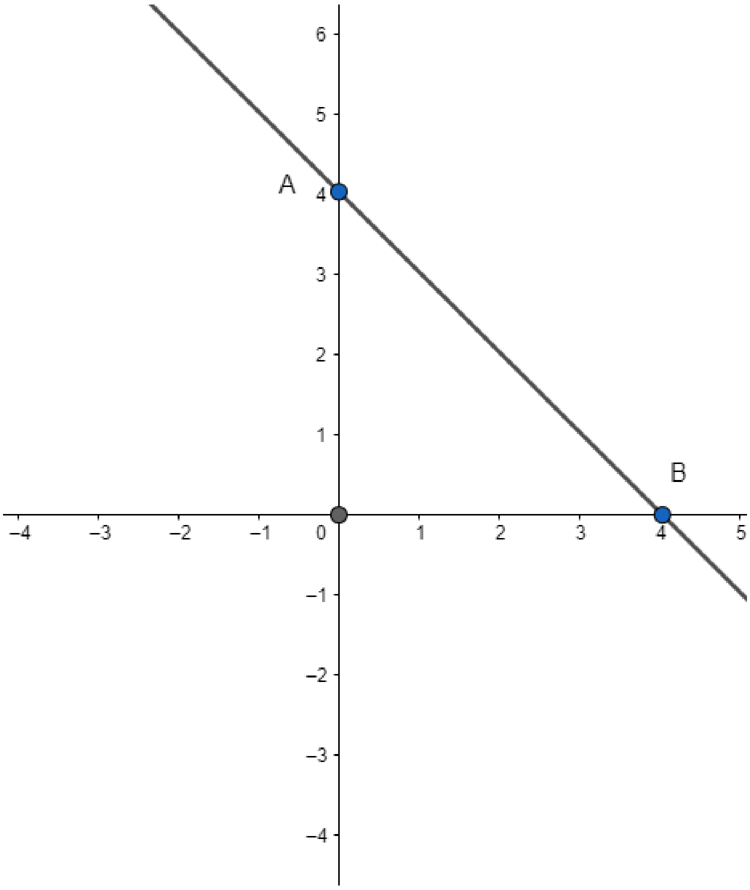


Fig 1a

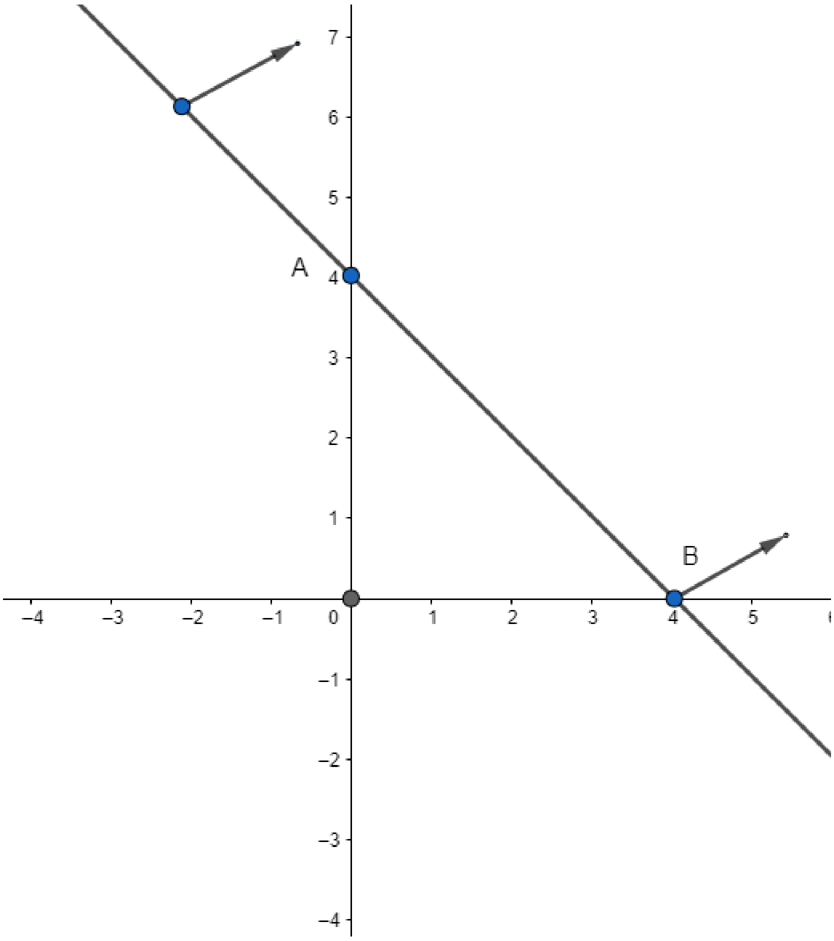


Fig 1b

2. Question

Graph the solution sets of the following inequations:

$x - y \leq 3$

Answer

Given $x - y \leq 3$

$\Rightarrow - y \leq 3 - x$

Multiplying by minus on both the sides, we'll get

$y \geq - 3 + x$

$y \geq x - 3$

Consider the equation $y = x - 3$.

Finding points on the coordinate axes:

If $x = 0$, the y value is - 3 i.e, $y = - 3$

$\Rightarrow$ the point on the Y axis is A(0, - 3)

If $y = 0$, $0 = x - 3$

$\Rightarrow x = 3$

The point on the X axis is B(3,0)

Plotting the points on the graph: fig. 2a

Now consider the inequality $y \geq x - 3$

Here we need the y value greater than or equal to $x - 3$

⇒ the required region is above point A.

Therefore the graph of the inequation x + y ≥ 4 is fig. 2b

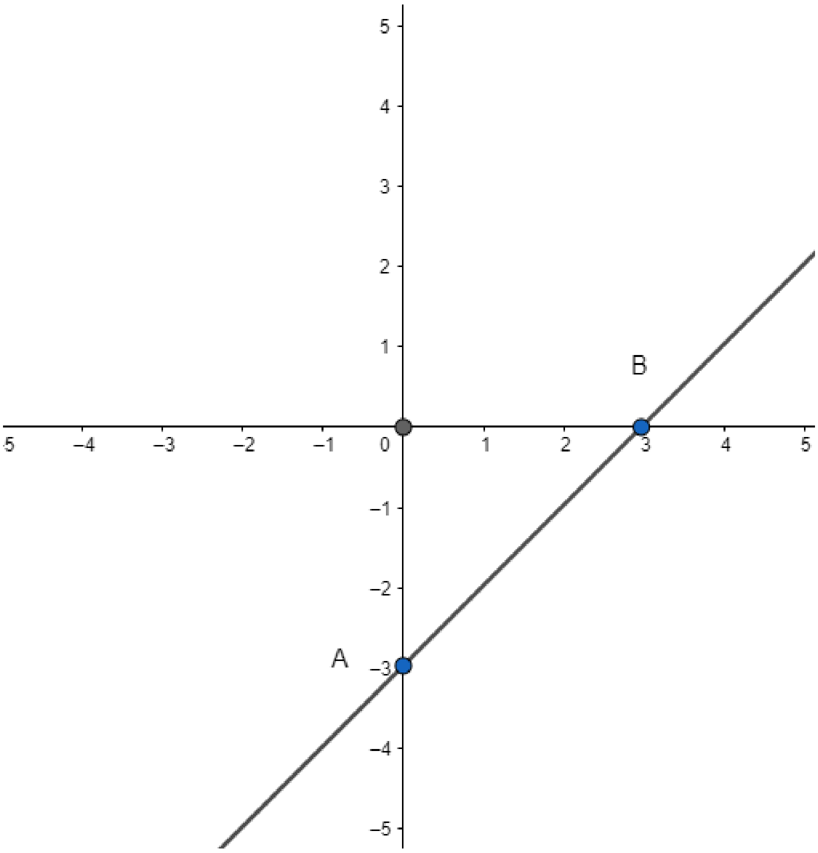


Fig 2a

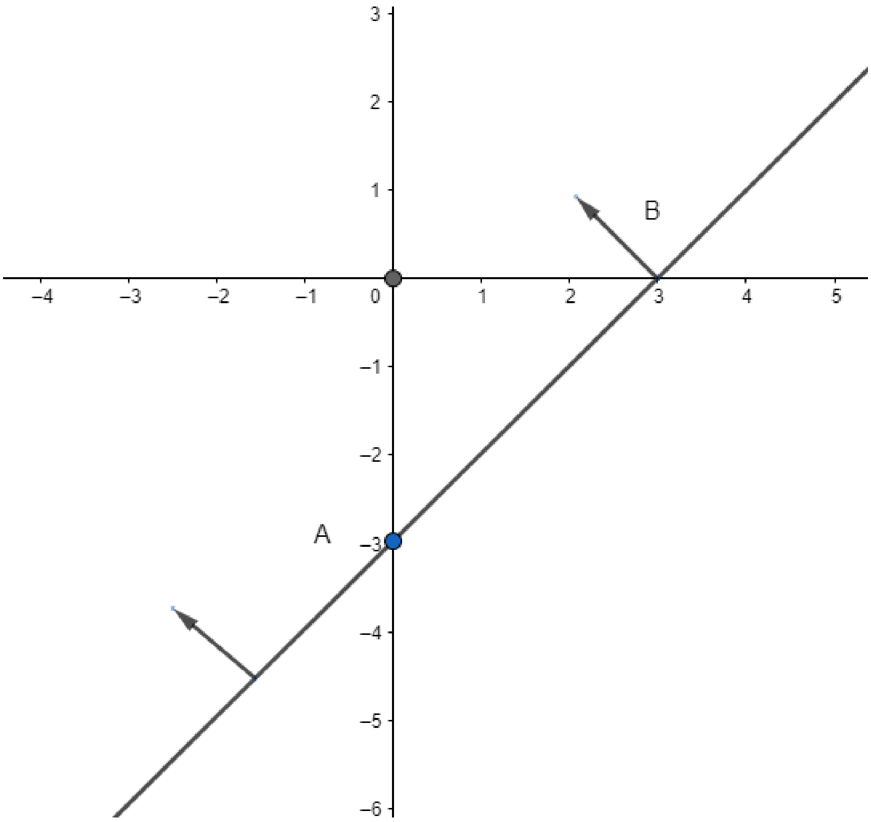


Fig 2b

3. Question

Graph the solution sets of the following inequations:

x + 2y>1

Answer

Given x + 2y>1

⇒ 2y>1 - x

⇒$y > \frac{1}{2} - \frac{x}{2}$

Consider the equation $y = \frac{1}{2} - \frac{x}{2}$

Finding points on the coordinate axes:

If x = 0, the y value is $\frac{1}{2}$ i.e., y = 4

⇒ the point on the Y axis is A(0, $\frac{1}{2}$)

If y = 0, x = 1

The point on the X axis is B(1,0)

Plotting the points on the graph: fig. 3a

Now consider the inequality $y > \frac{1}{2} - \frac{x}{2}$

Here we need the y value greater than $\frac{1}{2} - \frac{x}{2}$

⇒ the required region is above point A.

Also , the line AB is represented in dotted line. This is s done because $y \neq \frac{1}{2} - \frac{x}{2}$

Therefore the graph of the inequation $y > \frac{1}{2} - \frac{x}{2}$is fig. 3b

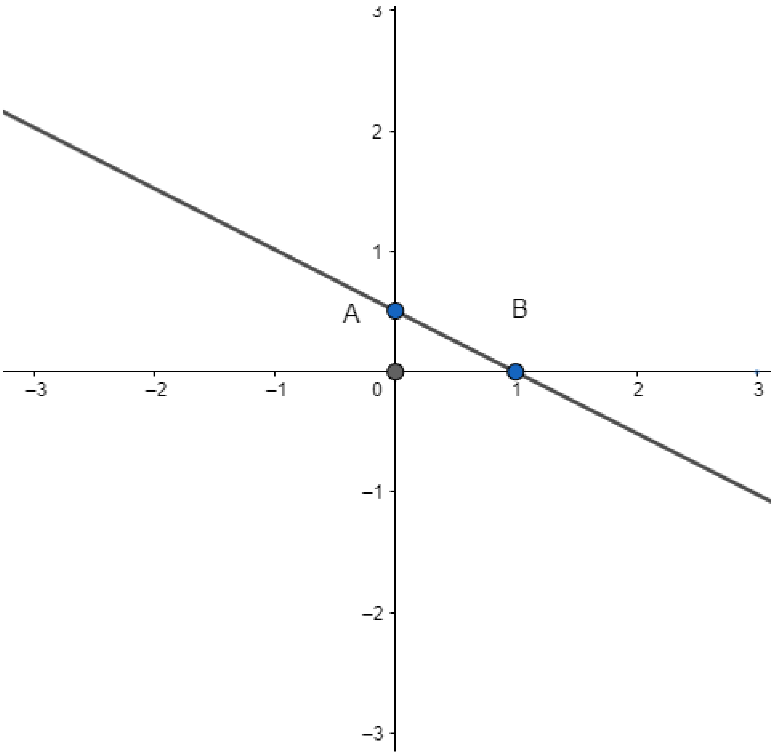


Fig 3a

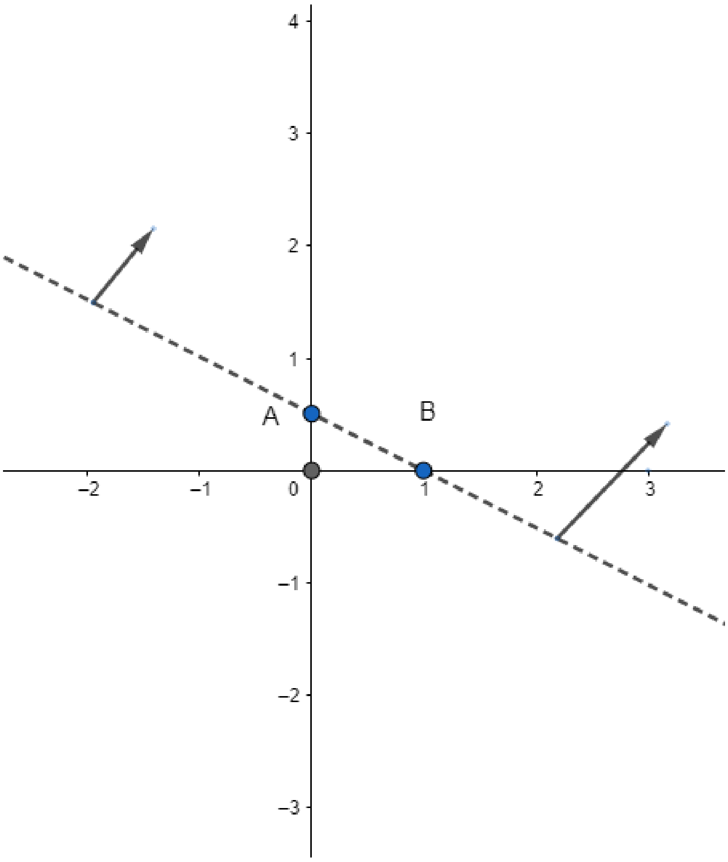


Fig 3b

4. Question

Graph the solution sets of the following inequations:

2x - 3y<4

Answer

Given 2x - 3y<4

⇒ 2x - 4<3y

⇒3y>2x - 4

⇒$y > \frac{2}{3}x - \frac{4}{3}$

Consider the equation $y = \frac{2}{3}x - \frac{4}{3}$

Finding points on the coordinate axes:

If x = 0, the y value is $\frac{1}{2}$ i.e., y = $-\frac{4}{3}$

⇒ the point on the Y axis is A(0, $-\frac{4}{3}$)

If y = 0, x = 2

The point on the X axis is B(2,0)

Plotting the points on the graph: fig. 4a

Now consider the inequality $y > \frac{2}{3}x - \frac{4}{3}$

Here we need the y value greater than $\frac{2}{3}x - \frac{4}{3}$

⇒ the required region is above point A.

Also , the line AB is represented in dotted line. This is s done because $y \neq \frac{2}{3}x - \frac{4}{3}$

Therefore the graph of the inequation $y > \frac{2}{3}x - \frac{4}{3}$is fig. 4b

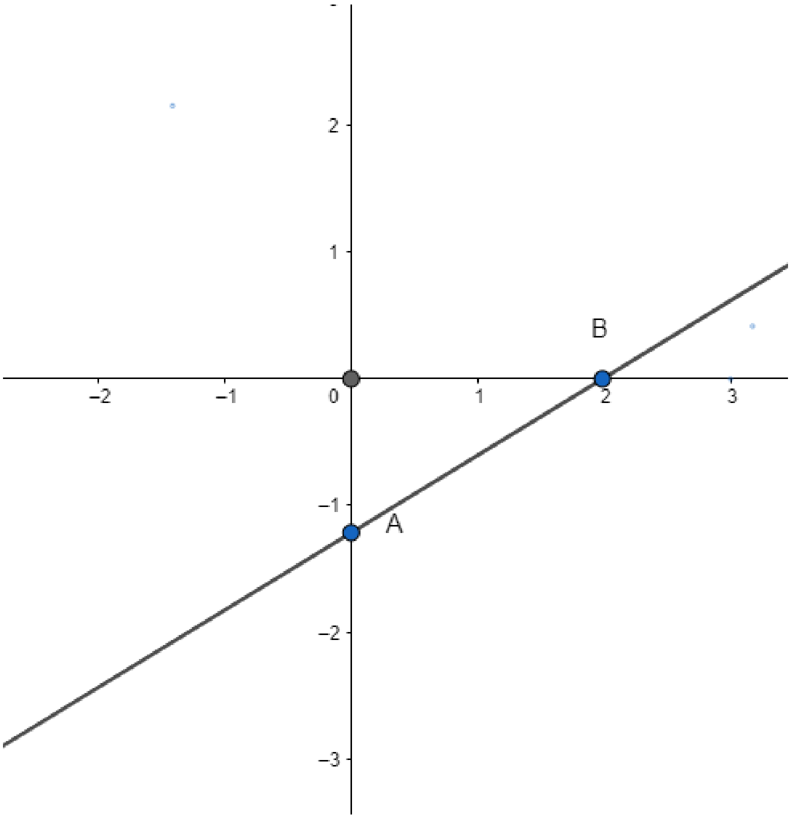


Fig 4a

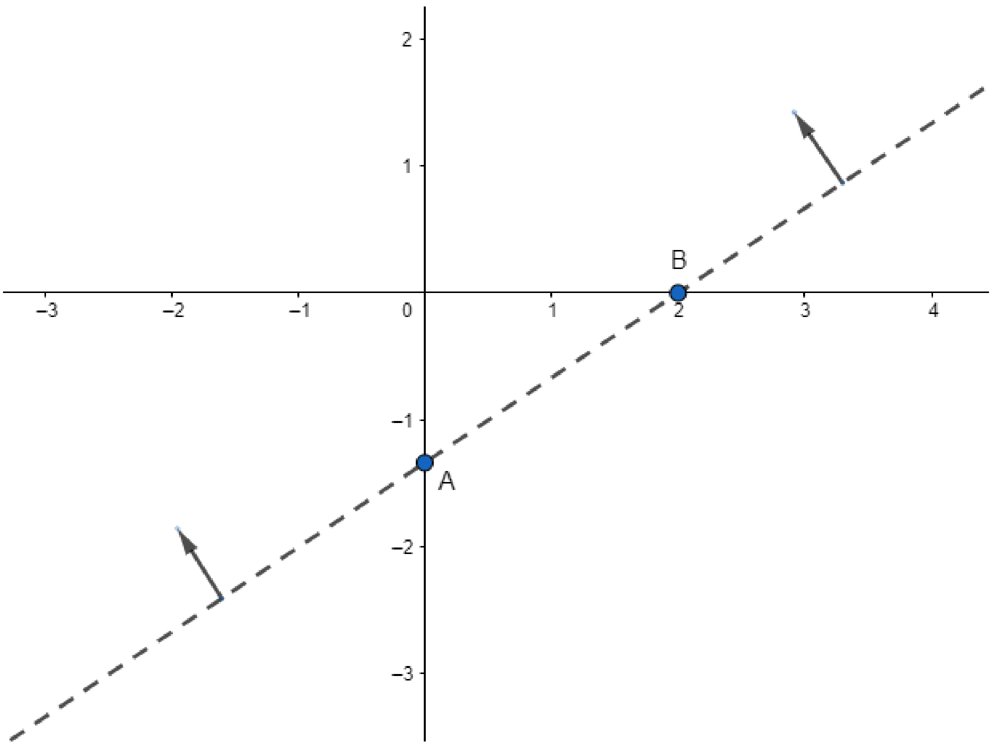


Fig 4b

5. Question

Graph the solution sets of the following inequations:

x $\geq$ y - 2

Answer

Given x ≥ y - 2

⇒ y≤x + 2

Consider the equation y = x + 2

Finding points on the coordinate axes:

If x = 0, the y value is 2 i.e, y = 2

⇒ the point on the Y axis is A(0,2)

If y = 0, 0 = x + 2

⇒x = - 2

The point on the X axis is B(- 2,0)

Plotting the points on the graph: fig. 5a

Now consider the inequality y≤x + 2

Here we need the y value less than or equal to x + 2

⇒ the required region is below point A.

Therefore the graph of the inequation x ≥ y - 2 is fig. 5b

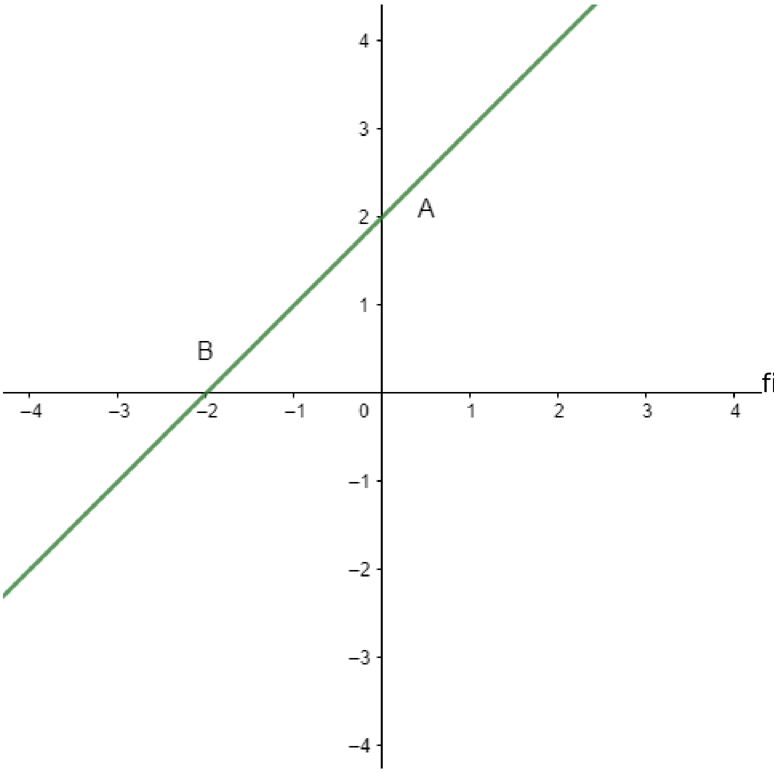


fig 5a

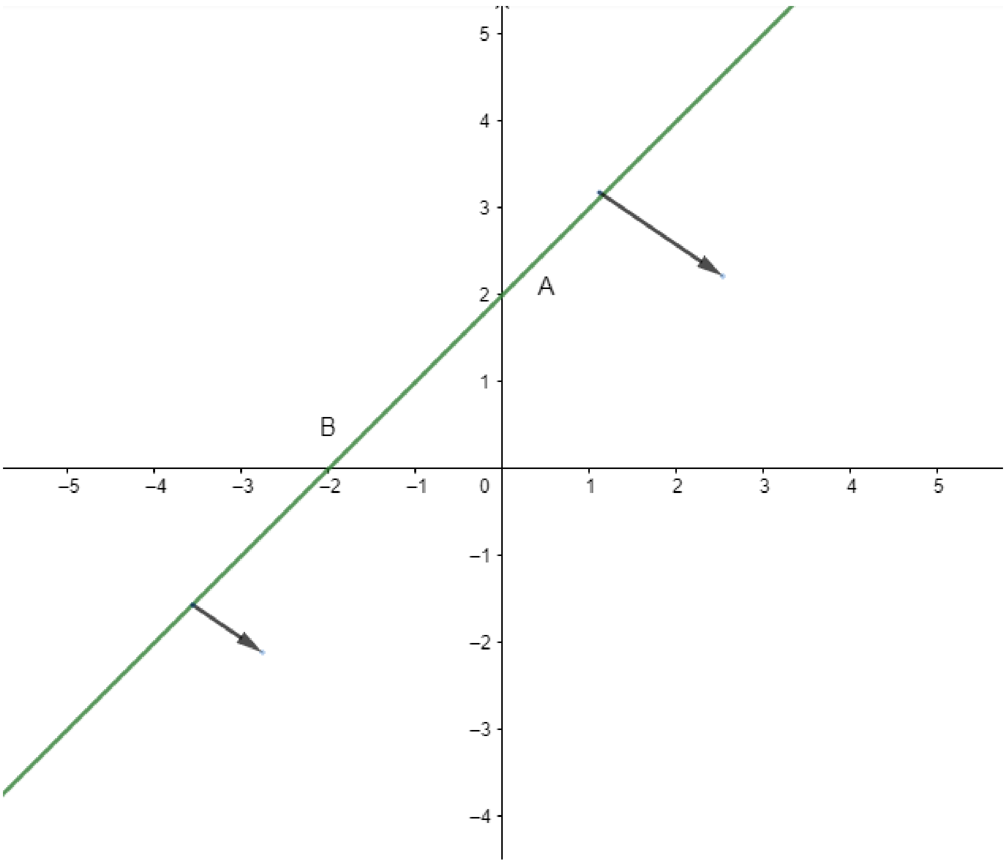


Fig 5b

6. Question

Graph the solution sets of the following inequations:

y - 2$\leq$3x

Answer

Given y - 2≤3x

⇒ y≤3x + 2

Consider the equation y = 3x + 2

Finding points on the coordinate axes:

If x = 0, the y value is 2 i.e, y = 2

⇒ the point on Y axis is A(0,2)

If y = 0, 0 = 3x + 2

⇒x = $-\frac{3}{2}$

The point on the X axis is B(- $-\frac{3}{2}$,0)

Plotting the points on the graph: fig. 6a

Now consider the inequality y≤3x + 2

Here we need the y value less than or equal to 3x + 2

⇒ the required region is below point A.

Therefore the graph of the inequation y≤3x + 2 is fig. 5b

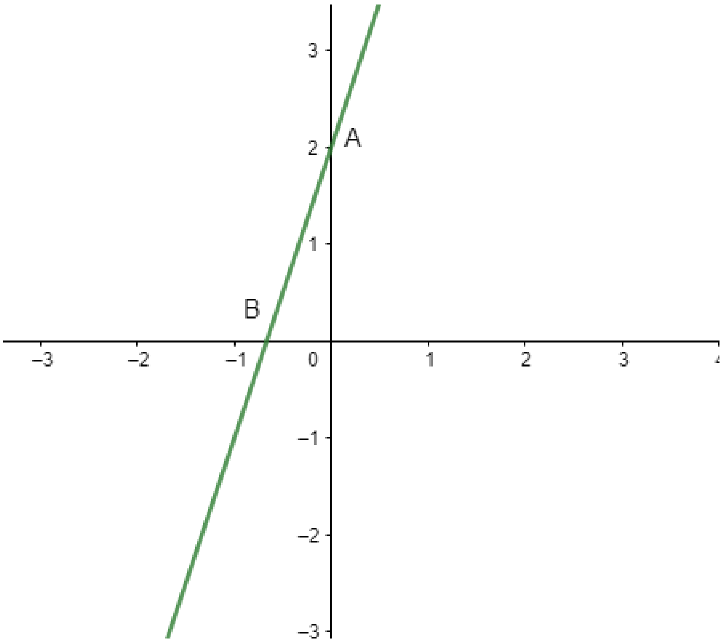


Fig 6a

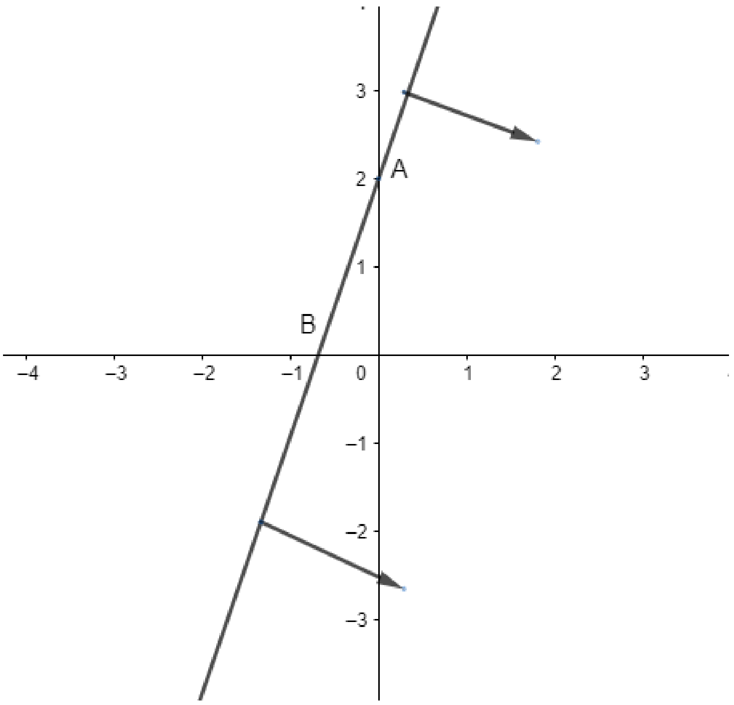


Fig 6b

7. Question

Solve each of the following systems of simultaneous inequations:

2x + y>1 and 2x - y $\geq$ - 3

Answer

Consider the inequation 2x + y>1:

⇒ y>1 - 2x

Consider the equation y = 1 - 2x

Finding points on the coordinate axes:

If x = 0, the y value is 1 i.e, y = 1

⇒ the point on Y axis is A(0,1)

If y = 0, 0 = x + 2

⇒x = $\frac{1}{2}$

The point on the X axis is B($\frac{1}{2}$,0)

Plotting the points on the graph: fig. 7a

Now consider the inequality y>1 - 2x

Here we need the y value greater than x + 2

⇒ the required region is below point A.

Therefore the graph of the inequation y>1 - 2x is fig. 7b

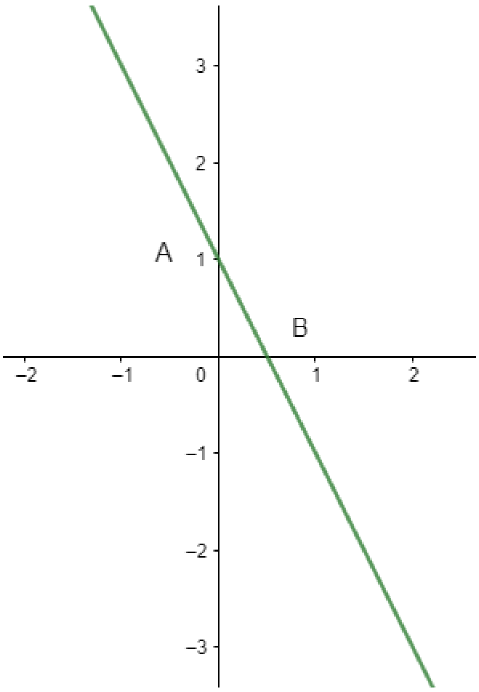


Fig 7a

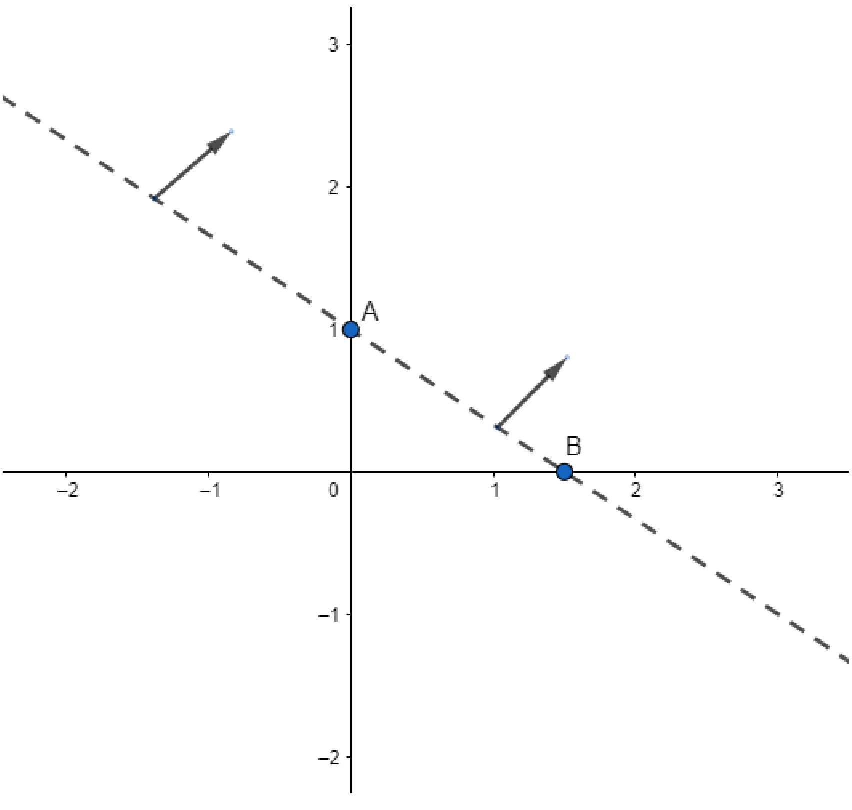


Fig 7b

Consider the inequation 2x - y ≥ 3

⇒ y≤2x - 3

Consider the equation y = 2x - 3

Finding points on the coordinate axes:

If x = 0, the y value is - 3 i.e, y = - 3

⇒ the point on the Y axis is C(0, - 3)

If y = 0, 0 = 2x + 3

⇒x = $\frac{3}{2}$

The point on the X axis is D($\frac{3}{2}$,0)

Plotting the points on the graph: fig. 7c

Now consider the inequality y≤2x - 3

Here we need the y value less than or equal to 2x - 3

⇒ the required region is below point C.

Therefore the graph of the inequation y≤2x - 3 is fig. 7d

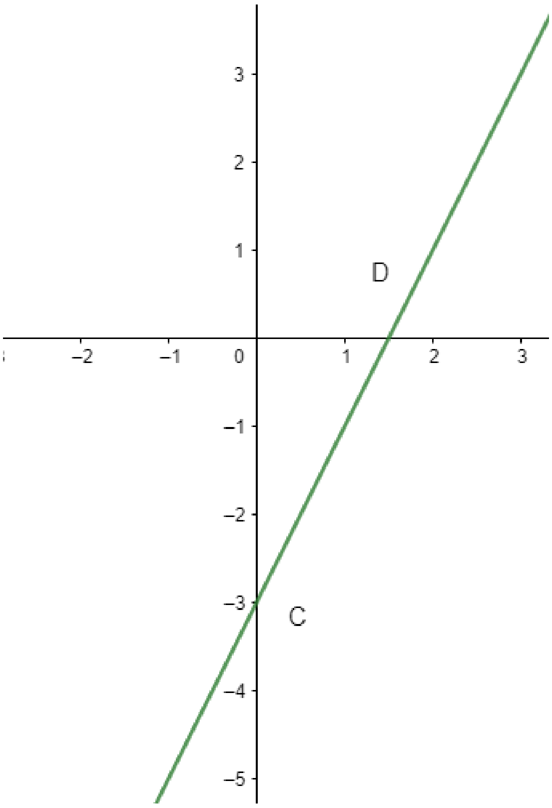


Fig 7c

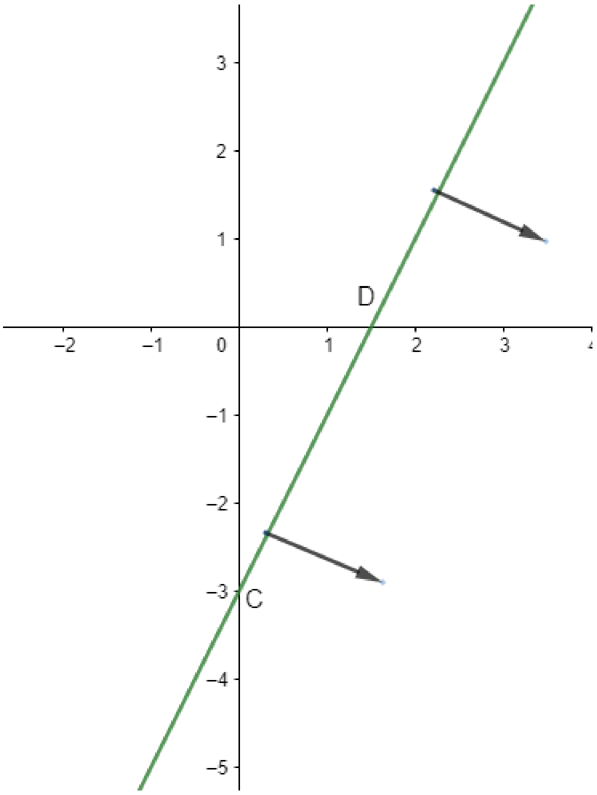
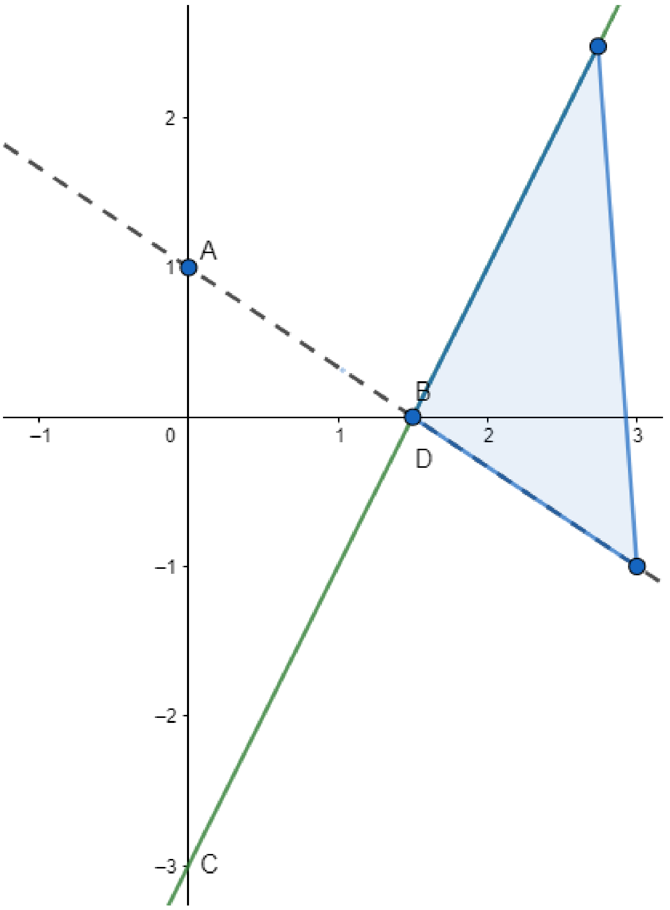


Fig 7d

Combining the graphs 7c and 7d, we'll get,



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

8. Question

Solve each of the following systems of simultaneous inequations:

$x - 2y \geq 0,\ 2x - y \leq -2$

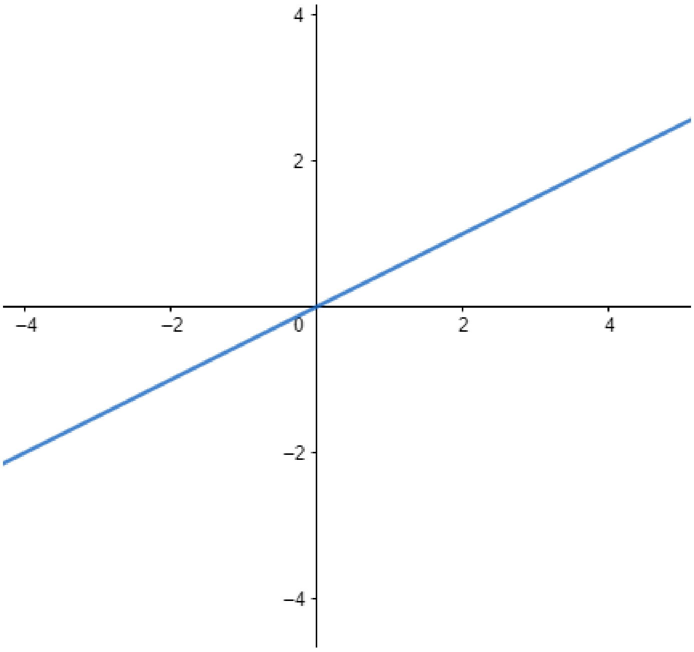
Answer

Consider the inequation x - 2y ≥ 0:

⇒x ≥ 2y

⇒y≤$\frac{x}{2}$

consider the equation y = $\frac{x}{2}$. This equation's graph is a straight line passing through origin.



Now consider the inequality y≤$\frac{x}{2}$

Here we need the y value less than or equal to $\frac{x}{2}$

⇒ the required region is below the origin.

Therefore the graph of the inequation y≤$\frac{x}{2}$is fig.8a

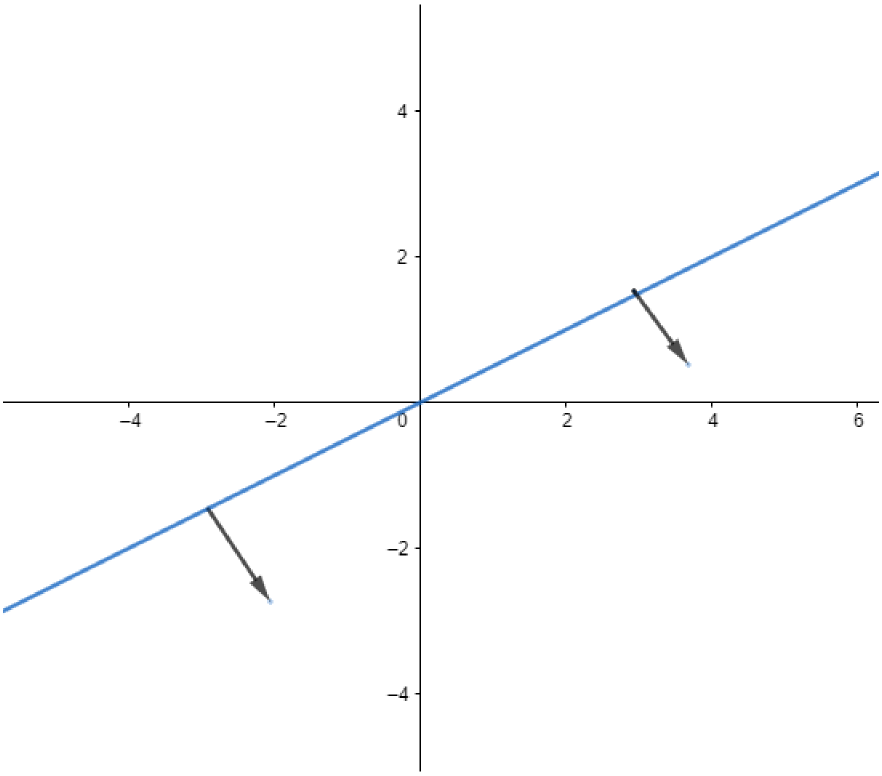


Fig 8a

Consider the inequation $2x - y \leq -2$:

$\Rightarrow y \geq 2x + 2$

Consider the equation $y = 2x + 2$

Finding points on the coordinate axes:

If $x = 0$, the y value is 2 i.e, $y = 2$

$\Rightarrow$ the point on the Y axis is A(0,2)

If $y = 0$, $0 = 2x + 2$

$\Rightarrow x = -1$

The point on the X axis is B(- 1,0)

Plotting the points on the graph: fig. 8b.

Now consider the inequality $y \geq 2x + 2$

Here we need the y value greater than or equal to $2x + 2$

$\Rightarrow$ the required region is above point A.

Therefore the graph of the inequation $y \geq 2x + 2$ is fig. 8c

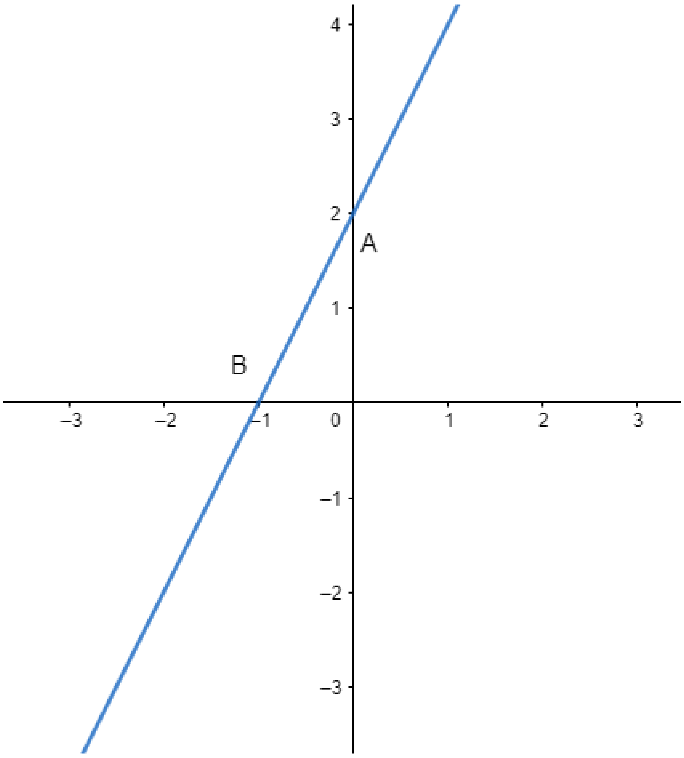


Fig 8b

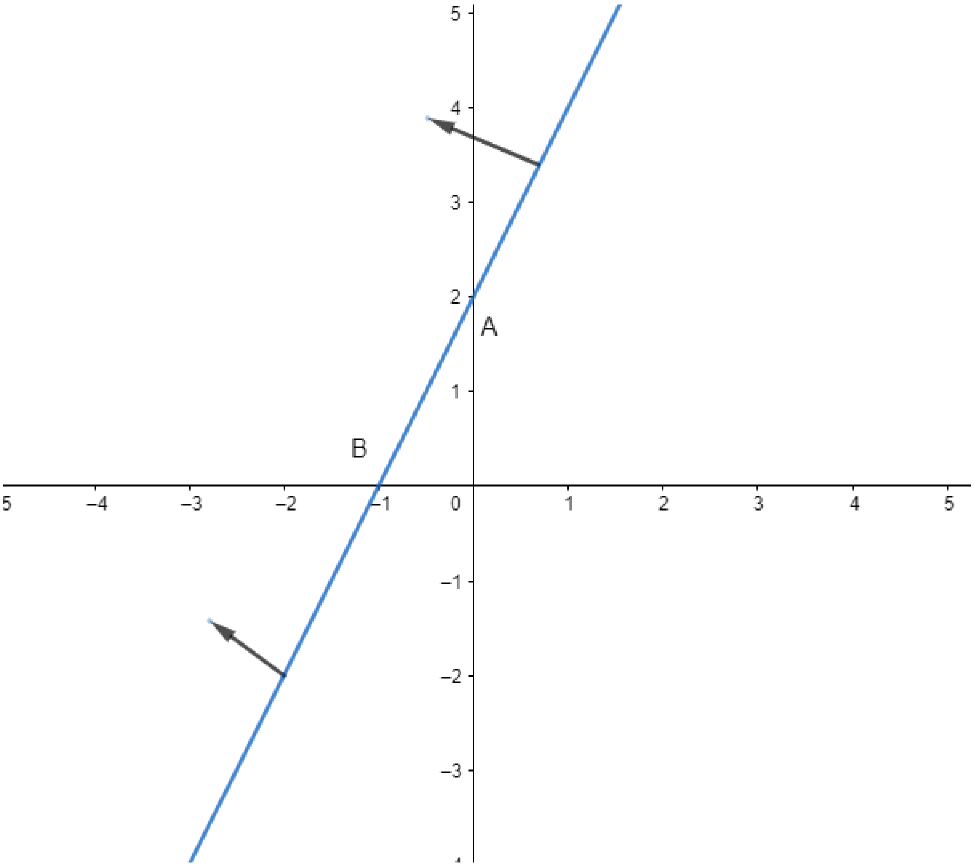
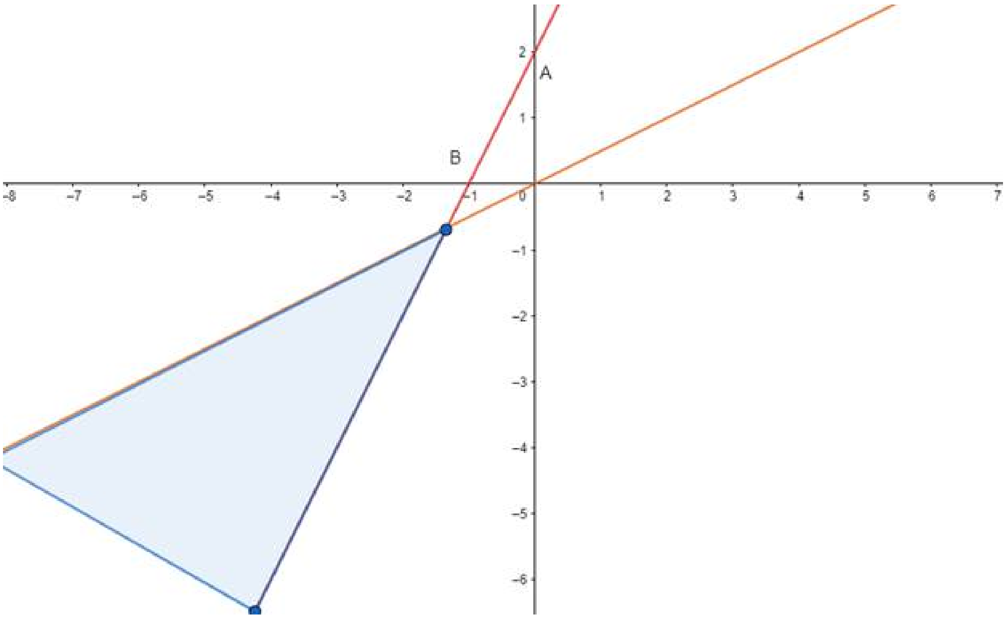


Fig 8c

Combining the graphs of 8a and 8c, we'll get



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

9. Question

Solve each of the following systems of simultaneous inequations:

$3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$ and $4x + 7y \leq 28$

Answer

Consider the inequation $3x + 4y \geq 12$:

$\Rightarrow 4y \geq 12 - 3x$

$\Rightarrow y \geq 3 - \frac{3}{4}x$

Consider the equation $y = 3 - \frac{3}{4}x$

Finding points on the coordinate axes:

If x = 0, the y value is 3 i.e, y = 3

⇒ the point on the Y axis is A(0,3)

If y = 0, $0 = 3 - \frac{3}{4}x$

$\Rightarrow x = 4$

The point on the X axis is B(4,0)

Now consider the inequality $y \geq 3 - \frac{3}{4}x$

Here we need the y value greater than or equal to $y \geq 3 - \frac{3}{4}x$

⇒ the required region is above point A.

Therefore the graph of the inequation $y \geq 3 - \frac{3}{4}x$ is fig. 9a

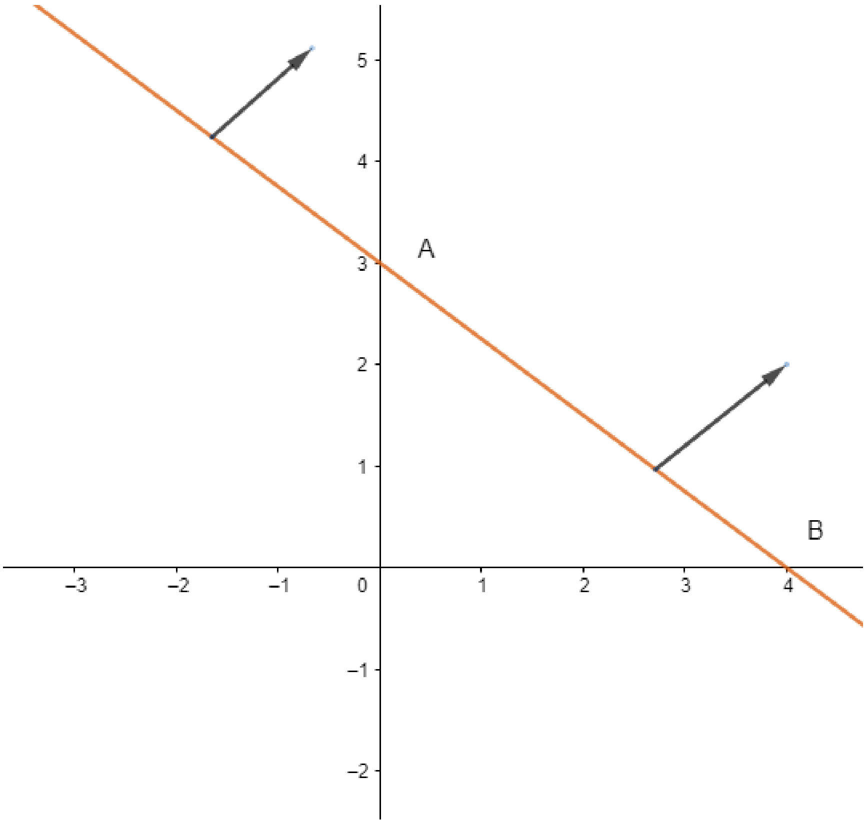


Fig 9a

Consider the inequation $4x + 7y \leq 28$

$\Rightarrow 7y \leq 28 - 4x$

$\Rightarrow y \leq 4 - \frac{4}{7}x$

Consider the equation $y = 4 - \frac{4}{7}x$

Finding points on the coordinate axes:

If x = 0, the y value is 4 i.e, y = 4

⇒ the point on the Y axis is C(0,4)

If y = 0, 0 = 4 - $\frac{4}{7}x$

⇒x = 7

The point on the X axis is D(7,0)

Now consider the inequality y≤4 - $\frac{4}{7}x$

Here we need the y value less than or equal to 4 - $\frac{4}{7}x$

⇒ the required region is below point C.

Therefore the graph of the inequation y≤4 - $\frac{4}{7}x$ is fig. 9b

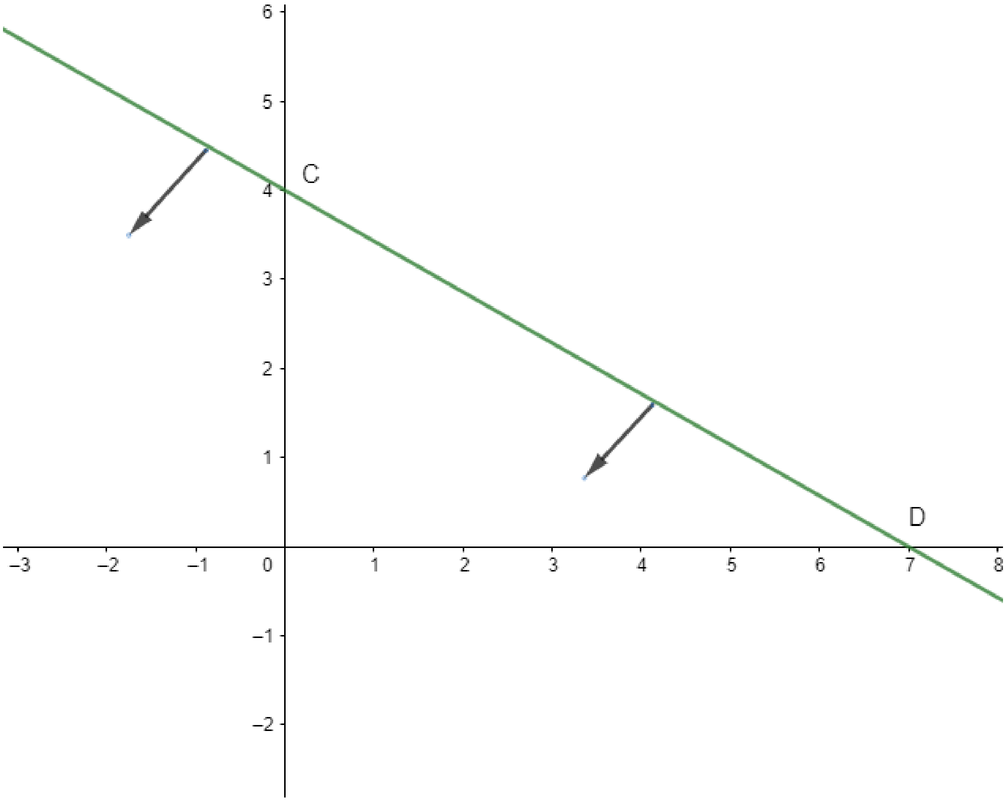
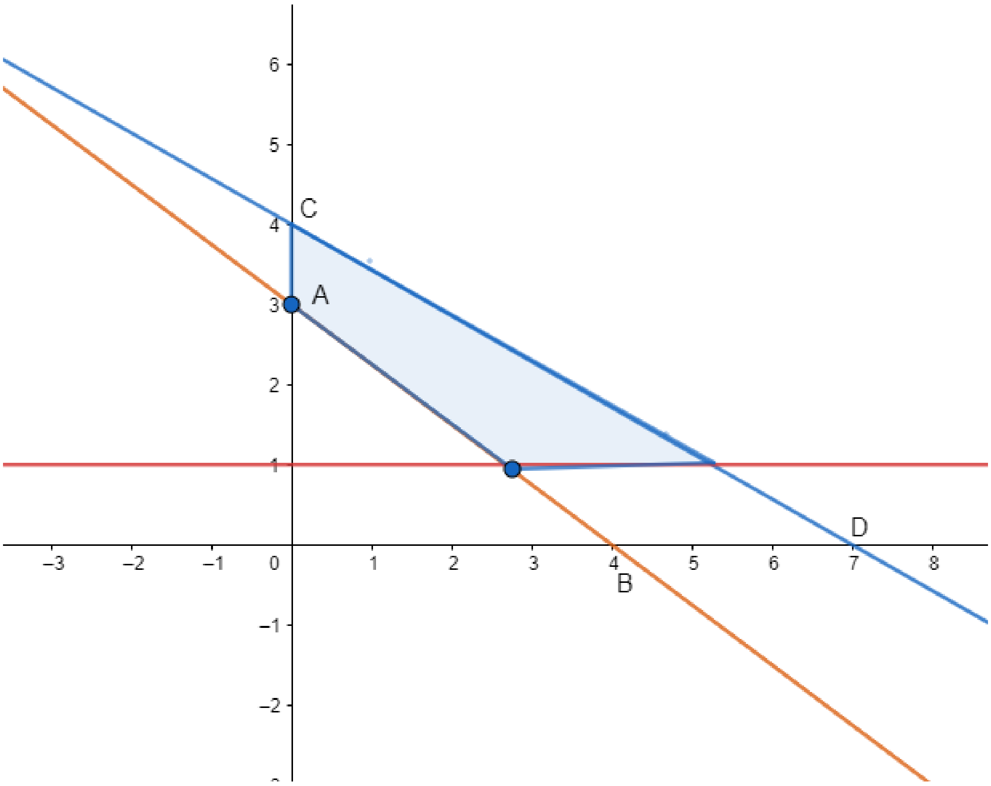


Fig 9b

x ≥ 0 is the region right side of Y - axis.

y ≥ 1 is the region above the line y = 1

Combining all the above results in a single graph , we’ll get



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

10. Question

Show that the solution set of the following linear constraints is empty:

x - 2y $\geq$ 0, 2x - y$\leq$ - 2, x $\geq$ 0 and y $\geq$ 0

Answer

Consider the inequation x - 2y ≥ 0:

⇒x ≥ 2y

⇒y≤$\frac{x}{2}$

consider the equation y = $\frac{x}{2}$. This equation’s graph is a straight line passing through origin.

Now consider the inequality y≤$\frac{x}{2}$

Here we need the y value less than or equal to $\frac{x}{2}$

⇒ the required region is below origin.

Therefore the graph of the inequation y≤$\frac{x}{2}$ is fig.10a

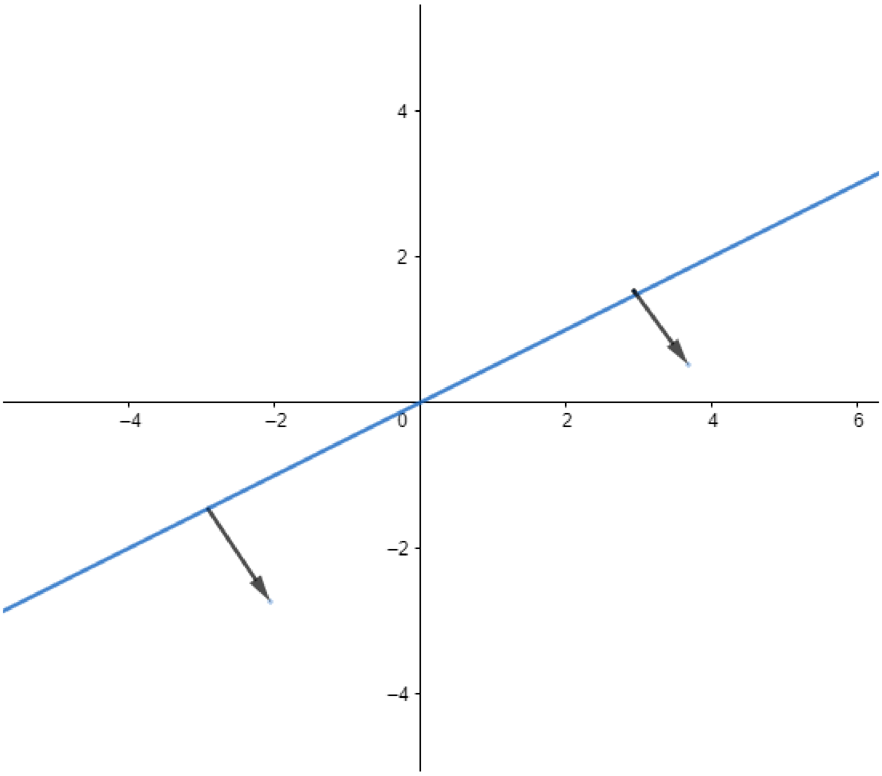


Fig 10a

Consider the inequation 2x - y≤ - 2 :

⇒y ≥ 2x + 2

Consider the equation y = 2x + 2

Finding points on the coordinate axes:

If x = 0, the y value is 2 i.e, y = 2

⇒ the point on Y axis is A(0,2)

If y = 0, 0 = 2x + 2

⇒x = - 1

The point on X axis is B(- 1,0)

Now consider the inequality y ≥ 2x + 2

Here we need the y value greater than or equal to 2x + 2

⇒ the required region is above point A.

Therefore the graph of the inequation y ≥ 2x + 2 is fig. 10b

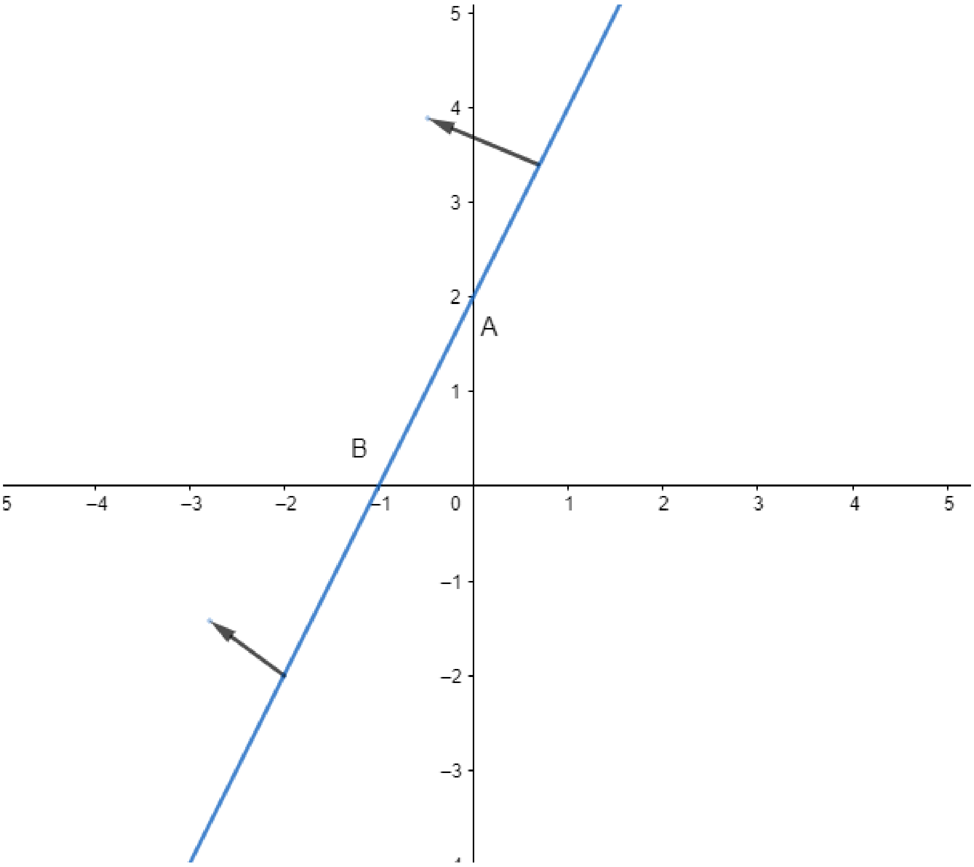
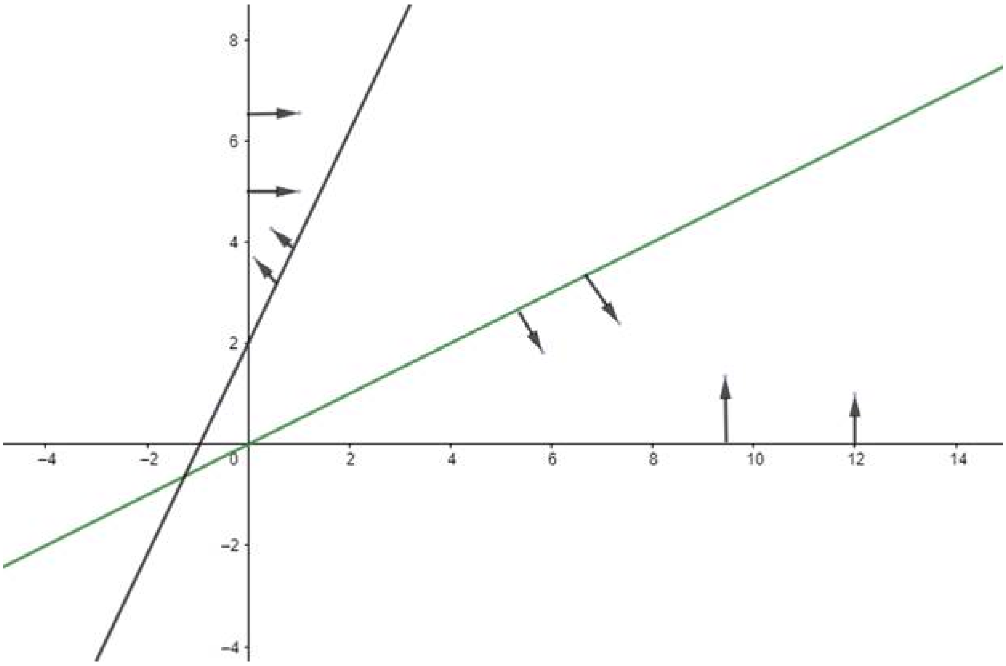


Fig 10b

y ≥ 0 is the region above X - axis

x ≥ 0 is the region right side of Y - axis

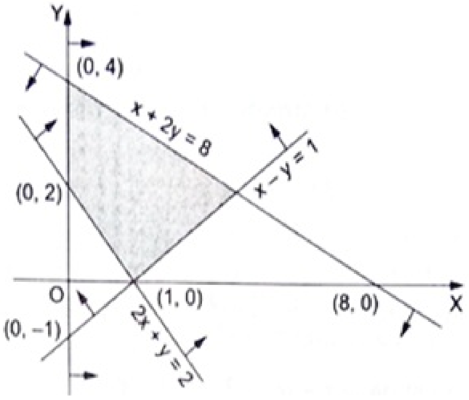
Combining the above results, we'll get



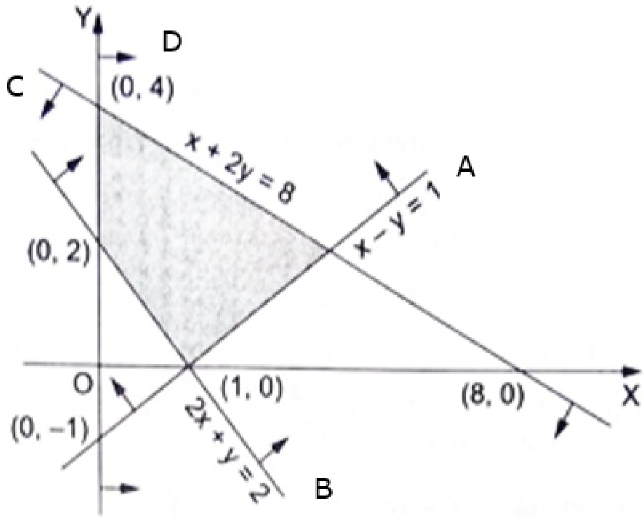
As they is no common area of intersection , there is no solution for the given set of simultaneous inequations.

11. Question

Find the linear constraints for which the shaded area in the figure given is the solution set.



Answer



Consider A:

Given line x - y = 1

⇒y = x - 1

As the region given in the figure is above the y - intercept's coordinates (0, - 1),

⇒y ≥ x - 1

⇒x - y≤1

Consider B:

Given line 2x + y = 2

⇒y = 2 - 2x

As the region given in the figure is above the y - intercept's coordinates (0,2),

⇒y ≥ 2 - 2x

⇒2x + y ≥ 2

Consider C:

Given line x + 2y = 8

⇒2y = 8 - x

⇒y = 4 - $\frac{x}{2}$

As the region given in the figure is below the y - intercept's coordinates (0,4),

⇒y≤4 - $\frac{x}{2}$

⇒$2y \le 8 - x$

⇒$x + 2y \le 8$

Consider D:

It is the region right side of the Y - axis.

It is $x \ge 0$.

All the results derived:

$x - y \le 1$

$2x + y \ge 2$

$x + 2y \le 8$

$x \ge 0$

Exercise 33B

1. Question

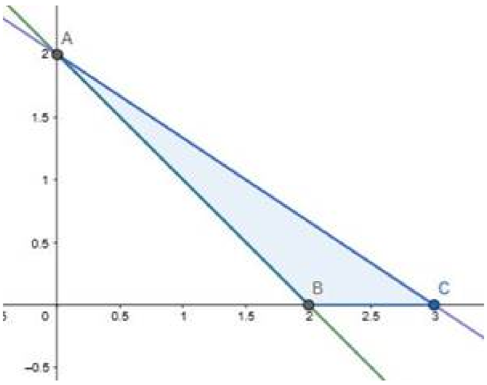
Find the maximum value of $Z = 7X + 7Y$, subject to the constraints.

$x \ge 0$, $y \ge 0$, $x + y \ge 2$ and $2x + 3y \le 6$.

Answer

The feasible region determined by the constraints $x \ge 0$, $y \ge 0$,

$x + y \ge 2$, $2x + 3y \le 6$ is given by



The corner points of the feasible region is A(0,2),B(2,0),C(3,0).

The values of Z at the following points is

Corner point	Z = 7x + 7y	
A(0,2)	14	
B(2,0)	14	
C(3,0)	21	Maximum

The maximum value of Z is 21 at point C(3,0) .

1. Question

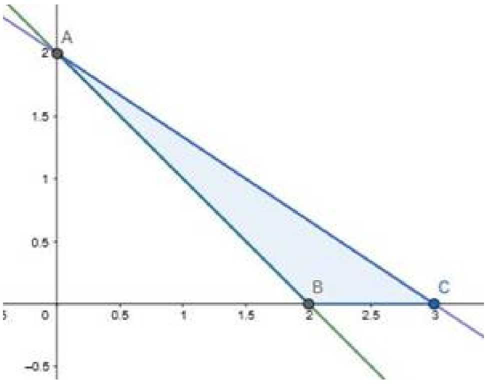
Find the maximum value of $Z = 7X + 7Y$, subject to the constraints.

$x \geq 0$, $y \geq 0$, $x + y \geq 2$ and $2x + 3y \leq 6$.

Answer

The feasible region determined by the constraints $x \geq 0$, $y \geq 0$,

$x + y \geq 2$, $2x + 3y \leq 6$ is given by



The corner points of the feasible region is A(0,2),B(2,0),C(3,0).

The values of Z at the following points is

Corner point	Z = 7x + 7y	
A(0,2)	14	
B(2,0)	14	
C(3,0)	21	Maximum

The maximum value of Z is 21 at point C(3,0) .

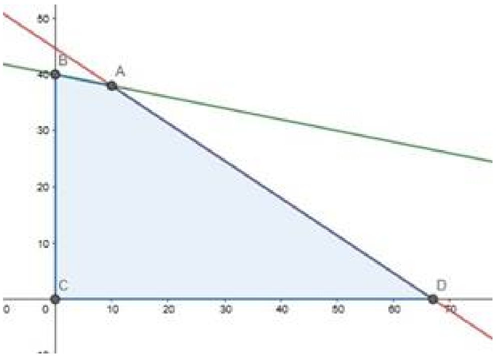
2. Question

Maximize $Z = 4x + 9y$, subject to the constraints

$x\geq0$, $y\geq0$, $x + 5y \leq 200$, $2x + 3y \leq 134$.

Answer

The feasible region determined by the constraints $x\geq0$, $y\geq0$, $x + 5y \leq 200$, $2x + 3y \leq 134$ is given by



The corner points of feasible region are A(10,38) ,B(0,40) ,C(0,0), D(67,0) . The values of Z at the following points is

Corner Point	Z = 4x + 9y	
A(10,38)	382	Maximum
B(0,40)	360	
C(0,0)	0	
D(67,0)	268	

The maximum value of Z is 382 at point A(10,38) .

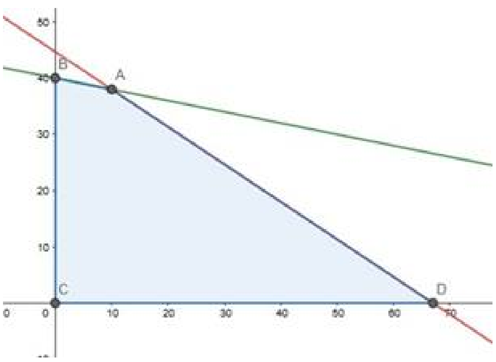
2. Question

Maximize Z = 4x + 9y, subject to the constraints

$x \geq 0$, $y \geq 0$, $x + 5y \leq 200$, $2x + 3y \leq 134$.

Answer

The feasible region determined by the constraints $x \geq 0$, $y \geq 0$, $x + 5y \leq 200$, $2x + 3y \leq 134$ is given by



The corner points of feasible region are A(10,38) ,B(0,40) ,C(0,0), D(67,0) . The values of Z at the following points is

Corner Point	Z = 4x + 9y	
A(10,38)	382	Maximum
B(0,40)	360	
C(0,0)	0	
D(67,0)	268	

The maximum value of Z is 382 at point A(10,38) .

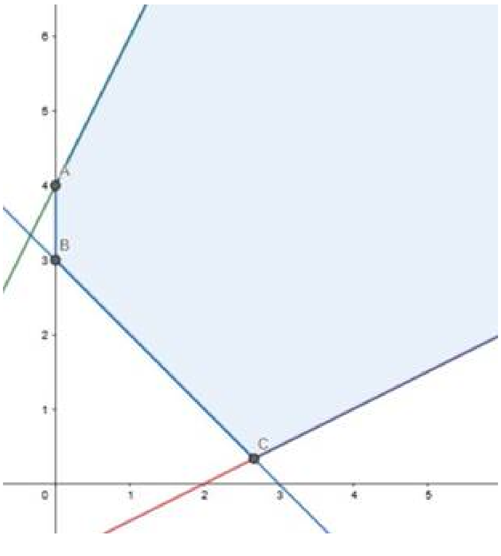
3. Question

Find the minimum value of Z = 3x + 5y, subject to the constraints

$-2x + y \leq 4$, $x + y \geq 3$, $x - 2y \leq 2$, $x \geq 0$ and $y \geq 0$

Answer

The feasible region determined by the $-2x + y \leq 4$, $x + y \geq 3$, $x - 2y \leq 2$, $x \geq 0$ and $y \geq 0$ is given by



Here the feasible region is unbounded. The vertices of the region are A(0,4) ,B(0,3) ,C$\left(\frac{8}{3},\frac{1}{3}\right)$. The values of Z at the following points is

Corner Point	Z = 3x + 5y	
A(0,4)	20	
B(0,3)	15	
C$(\frac{8}{3},\frac{1}{3})$	$\frac{29}{3}$	Minimum

The minimum value of Z is $\frac{29}{3}$ at point C$(\frac{8}{3},\frac{1}{3})$.

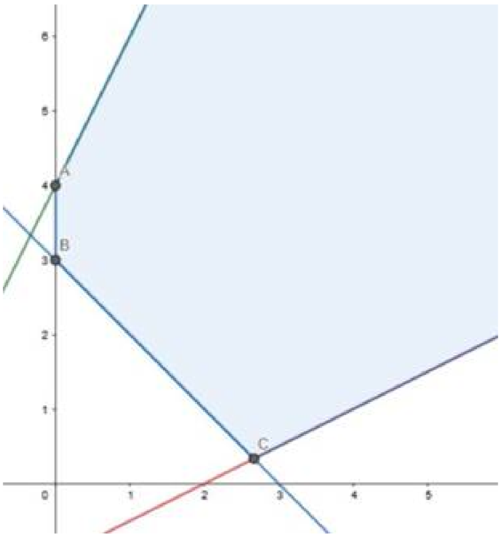
3. Question

Find the minimum value of Z = 3x + 5y, subject to the constraints

$-2x + y \leq 4$, $x + y \geq 3$, $x - 2y \leq 2$, $x \geq 0$ and $y \geq 0$

Answer

The feasible region determined by the $-2x + y \leq 4$, $x + y \geq 3$, $x - 2y \leq 2$, $x \geq 0$ and $y \geq 0$ is given by



Here the feasible region is unbounded. The vertices of the region are A(0,4) ,B(0,3) ,C$(\frac{8}{3},\frac{1}{3})$. The values of Z at the following points is

Corner Point	Z = 3x + 5y	
A(0,4)	20	
B(0,3)	15	
$C(\frac{8}{3},\frac{1}{3})$	$\frac{29}{3}$	Minimum

The minimum value of Z is $\frac{29}{3}$ at point $C(\frac{8}{3},\frac{1}{3})$.

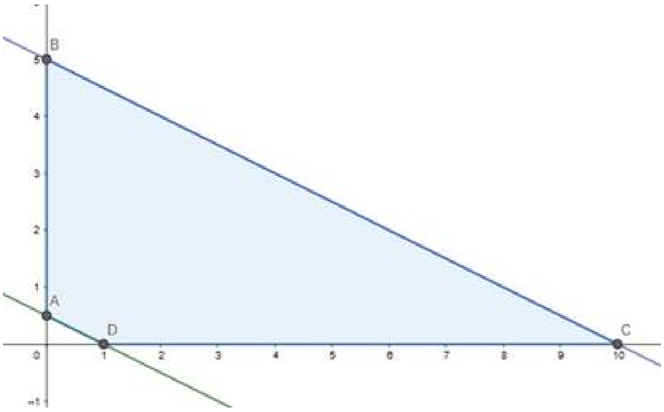
4. Question

Minimize Z = 2x + 3y, subject to the constraints

$x \geq 0$, $y \geq 0$, $x + 2y \geq 1$ and $x + 2y \leq 10$.

Answer

The feasible region determined by the $x \geq 0$, $y \geq 0$, $x + 2y \geq 1$ and $x + 2y \leq 10$ is given by



The corner points of the feasible region is $A(0,\frac{1}{2})$, B(0,5), C(10,0), D(1,0).The value of Z at corner points are

Corner Points	Z = 2x + 3y	
A(0,$\frac{1}{2}$)	$\frac{3}{2}$	Minimum
B(0,5)	15	
C(10,0)	20	
D(1,0)	2	

The minimum value of Z is $\frac{3}{2}$ at point A(0,$\frac{1}{2}$).

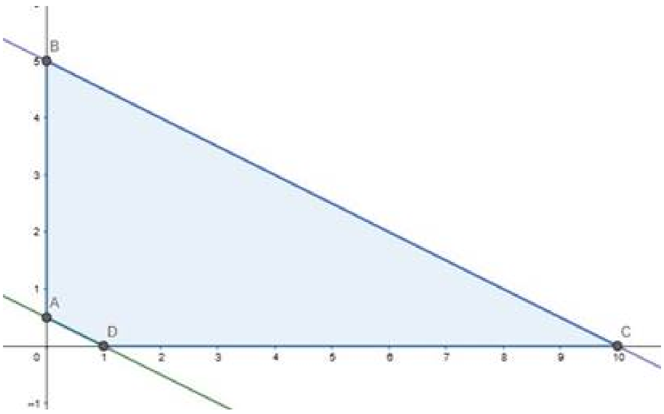
4. Question

Minimize Z = 2x + 3y, subject to the constraints

x $\geq$ 0, y $\geq$ 0, x + 2y $\geq$ 1 and x + 2y $\leq$ 10.

Answer

The feasible region determined by the x $\geq$ 0, y $\geq$ 0, x + 2y $\geq$ 1 and x + 2y $\leq$ 10 is given by



The corner points of the feasible region is A(0,$\frac{1}{2}$), B(0,5), C(10,0), D(1,0).The value of Z at corner points are

Corner Points	Z = 2x + 3y	
A(0, $\frac{1}{2}$)	$\frac{3}{2}$	Minimum
B(0,5)	15	
C(10,0)	20	
D(1,0)	2	

The minimum value of Z is $\frac{3}{2}$ at point A(0,$\frac{1}{2}$).

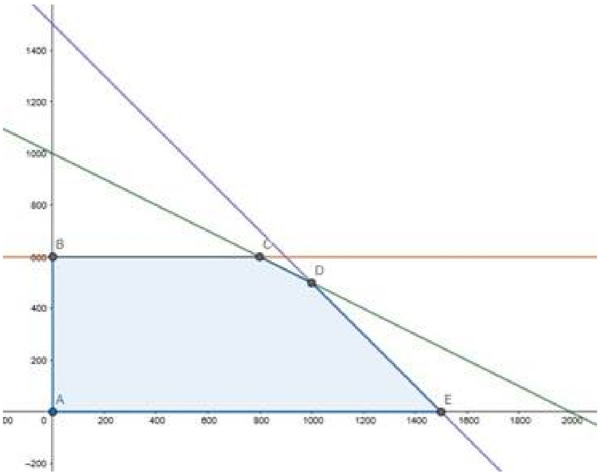
5. Question

Maximize Z = 3x + 5y, subject to the constraints

X + 2y $\leq$ 2000, x + y $\leq$ 1500, y $\leq$ 00, y $\leq$ 600, x $\geq$ 0 and y $\geq$ 0.

Answer

The feasible region determined by the X + 2y $\leq$ 2000, x + y $\leq$ 1500, y $\leq$ 600, x $\geq$ 0 and y $\geq$ 0 is given by



The corner points of the feasible region are A(0,0), B(0,600), C(800,600), D(1000,500), E(1500,0).The value of Z at the corner points are

Corner Point	Z = 3x + 5y	
A(0,0)	0	
B(0,600)	3000	
C(800,600)	5400	
D(1000,500)	5500	Maximum
E(1500,0)	4500	

The maximum value of Z is 5500 at point D(1000,500).

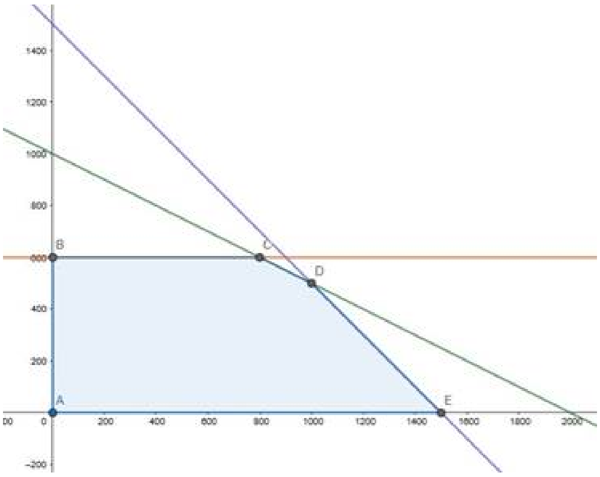
5. Question

Maximize Z = 3x + 5y, subject to the constraints

X + 2y $\leq$ 2000, x + y $\leq$ 1500, y $\leq$ 00, y $\leq$ 600, x $\geq$ 0 and y $\geq$ 0.

Answer

The feasible region determined by the X + 2y $\leq$ 2000, x + y $\leq$ 1500, y $\leq$ 600, x $\geq$ 0 and y $\geq$ 0 is given by



The corner points of the feasible region are A(0,0), B(0,600), C(800,600), D(1000,500), E(1500,0).The value of Z at the corner points are

Corner Point	Z = 3x + 5y	
A(0,0)	0	
B(0,600)	3000	
C(800,600)	5400	
D(1000,500)	5500	Maximum
E(1500,0)	4500	

The maximum value of Z is 5500 at point D(1000,500).

6. Question

Find the maximum and minimum values of Z = 2x + y, subject to the constraints

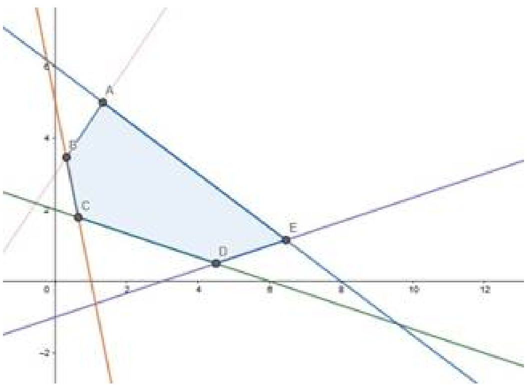
X + 3y $\geq$ 6, x - 3y $\leq$ 3, 3x + 4y $\leq$ 24,

- 3x + 2y $\leq$ 6, 5x + y $\geq$ 5, x $\geq$ 0 and y $\geq$ 0.

Answer

The feasible region determined by X + 3y $\geq$ 6, x - 3y $\leq$ 3, 3x + 4y $\leq$ 24,

- 3x + 2y $\leq$ 6, 5x + y $\geq$ 5, x $\geq$ 0 and y $\geq$ 0 is given by



The corner points of the feasible region are A(4/3,5) , B(4/13,45/13), C(9/14,25/14) , D(9/2,1/2) , E(84/13,15/13).The value of Z at corner points are

Corner Point	Z = 2x + y	
A(4/3,5)	23/3	
B(4/13,45/13)	53/13	
C(9/14,25/14)	43/14	Minimum
D(9/2,1/2)	19/2	
E(84/13,15/13)	183/13	Maximum

The maximum and minimum value of Z is 183/13 and 43/14 at points E(84/13,15/13) and C(9/14,25/14).

6. Question

Find the maximum and minimum values of Z = 2x + y, subject to the constraints

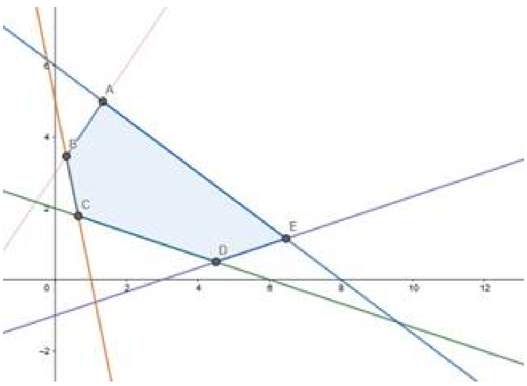
X + 3y $\geq$ 6, x - 3y $\leq$ 3, 3x + 4y $\leq$ 24,

- 3x + 2y $\leq$ 6, 5x + y $\geq$ 5, x $\geq$ 0 and y $\geq$ 0.

Answer

The feasible region determined by X + 3y $\geq$ 6, x - 3y $\leq$ 3, 3x + 4y $\leq$ 24,

- 3x + 2y $\leq$ 6, 5x + y $\geq$ 5, x $\geq$ 0 and y $\geq$ 0 is given by



The corner points of the feasible region are A(4/3,5) , B(4/13,45/13), C(9/14,25/14) , D(9/2,1/2) , E(84/13,15/13).The value of Z at corner points are

Corner Point	Z = 2x + y	
A(4/3,5)	23/3	
B(4/13,45/13)	53/13	
C(9/14,25/14)	43/14	Minimum
D(9/2,1/2)	19/2	
E(84/13,15/13)	183/13	Maximum

The maximum and minimum value of Z is 183/13 and 43/14 at points E(84/13,15/13) and C(9/14,25/14).

7. Question

Mr.Dass wants to invest ₹12000 in public provident fund (PPF) and in national bonds. He has to invest at least ₹1000 in PPF and at least ₹2000 in bonds. If the rate of interest on PPF is 12% per annum and that on bonds is 15% per annum, how should he invest the money to earn maximum annual income? Also find the maximum annual income.

Answer

Let the invested money in PPF be x and in national bonds be y.

∴According to the question,

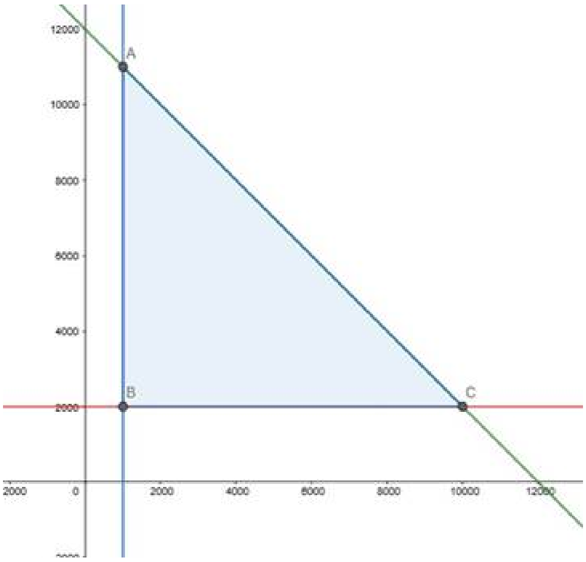
X + y ≤ 12000

x ≥ 1000 , y ≥ 2000

Maximize Z = 0.12x + 0.15y

The feasible region determined by X + y ≤ 12000 , x ≥ 1000 ,

y ≥ 2000 is given by



The corner points of the feasible region are A(1000,11000) , B(1000,2000) and C(10000,2000) . The value of Z at the corner point are

Corner Point	Z = 0.12x + 0.15y	
A(1000,11000)	1770	Maximum
B(1000,2000)	420	
C(10000,2000)	1500	

The maximum value of Z is 1770 at point A(1000,11000).

So, he must invest Rs.1000 in PPF and Rs.11000 in national bonds.

The maximum annual income is Rs.1770 .

7. Question

Mr.Dass wants to invest ₹12000 in public provident fund (PPF) and in national bonds. He has to invest at least ₹1000 in PPF and at least ₹2000 in bonds. If the rate of interest on PPF is 12% per annum and that on bonds is 15% per annum, how should he invest the money to earn maximum annual income? Also find the maximum annual income.

Answer

Let the invested money in PPF be x and in national bonds be y.

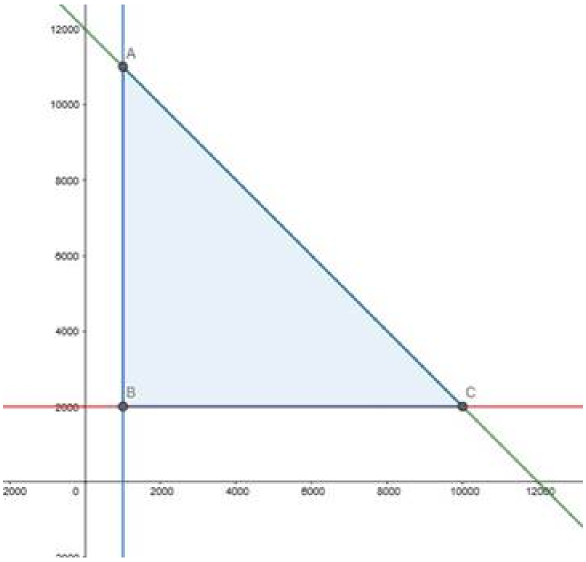
∴According to the question,

$X + y \leq 12000$

$x \geq 1000$, $y \geq 2000$

Maximize Z = 0.12x + 0.15y

The feasible region determined by $X + y \leq 12000$, $x \geq 1000$, y ≥ 2000 is given by



The corner points of the feasible region are A(1000,11000) , B(1000,2000) and C(10000,2000) . The value of Z at the corner point are

Corner Point	Z = 0.12x + 0.15y	
A(1000,11000)	1770	Maximum
B(1000,2000)	420	
C(10000,2000)	1500	

The maximum value of Z is 1770 at point A(1000,11000).

So, he must invest Rs.1000 in PPF and Rs.11000 in national bonds.

The maximum annual income is Rs.1770 .

8. Question

A small firm manufactures necklace and bracelets. The total number of necklace and bracelet that it can handle per day is at most 24. It takes 1 hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹100 and that on a bracelet is ₹300, how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced.

Answer

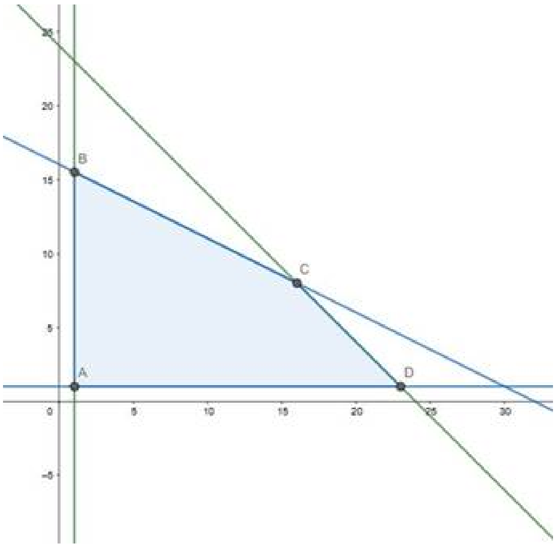
Let the firm manufacture x number of necklaces and y number of bracelets a day.

∴According to the question,

X + y ≤ 24 , 0.5x + y ≤ 16 x $\geq 1, y \geq 1$

Maximize Z = 100x + 300y

The feasible region determined by X + y ≤ 24 , 0.5x + y ≤ 16 , x $\geq 1, y \geq 1$ is given by



The corner points of the feasible region are A(1,1) , B(1,15.5) , C(16,8) , D(23,1).The number of bracelets should be whole number. Therefore, considering point (2,15). The value of Z at corner point is

Corner Point	Z = 100x + 300y	
A(1,1)	400	
(2,15)	4700	Maximum
C(16,8)	4000	
D(23,1)	2600	

The maximum value of Z is 4700 at point B(2,15).

∴ The firm should make 2 necklaces and 15 bracelets.

8. Question

A small firm manufactures necklace and bracelets. The total number of necklace and bracelet that it can handle per day is at most 24. It takes 1 hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹100 and that on a bracelet is ₹300, how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced.

Answer

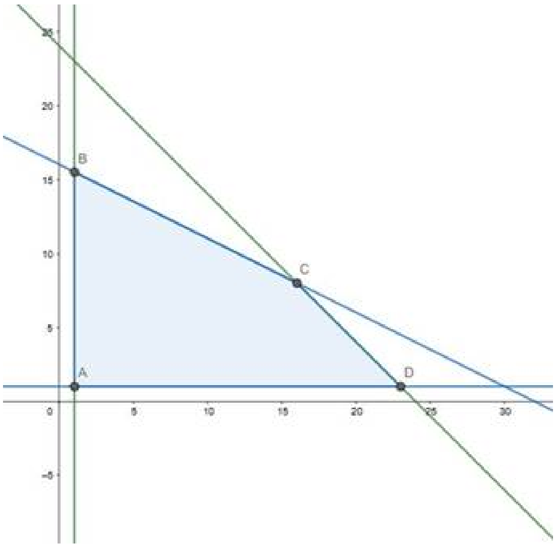
Let the firm manufacture x number of necklaces and y number of bracelets a day.

∴According to the question,

X + y ≤ 24 , 0.5x + y ≤ 16 x $\geq 1, y \geq 1$

Maximize Z = 100x + 300y

The feasible region determined by X + y ≤ 24 , 0.5x + y ≤ 16, x $\geq 1, y \geq 1$ is given by



The corner points of the feasible region are A(1,1) , B(1,15.5) , C(16,8) , D(23,1).The number of bracelets should be whole number. Therefore, considering point (2,15). The value of Z at corner point is

Corner Point	Z = 100x + 300y	
A(1,1)	400	
(2,15)	4700	Maximum
C(16,8)	4000	
D(23,1)	2600	

The maximum value of Z is 4700 at point B(2,15).

∴ The firm should make 2 necklaces and 15 bracelets.

9. Question

A man has ₹1500 to purchase rice and wheat. A bag of rice and a bag of wheat cost ₹180 and 120 respectively. He has storage capacity of 10 bags only. He earns a profit of ₹11 and ₹8 per bag of rice and wheat respectively. How many bags of each must he buy to make maximum profit?

Answer

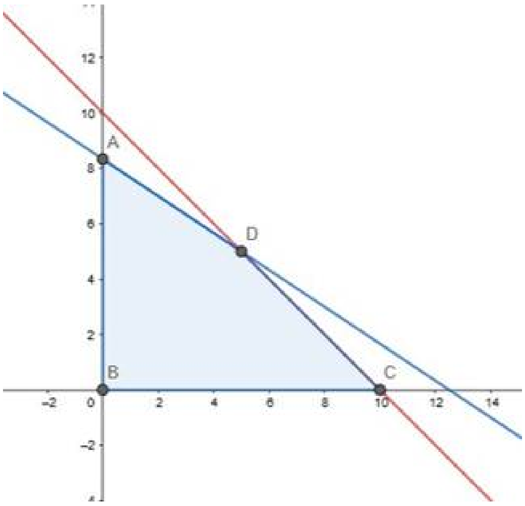
Let the number of wheat and rice bags be x and y.

∴According to the question,

120x + 180y ≤ 1500, x + y ≤ 10, x $\geq 0, y \geq 0$

Maximize Z = 8x + 11y

The feasible region determined by 120x + 180y ≤ 1500, x + y ≤ 10, x $\geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,8), B(0,0), C(10,0), D(5,5) .

The value of Z at corner point is

Corner Point	Z = 8x + 11y	
A(0,8)	88	
B(0,0)	0	
C(10,0)	80	
D(5,5)	95	Maximum

The maximum value of Z is 95 at point (5,5).

Hence, the man should 5 bags each of wheat and rice to earn maximum profit.

9. Question

A man has ₹1500 to purchase rice and wheat. A bag of rice and a bag of wheat cost ₹180 and 120 respectively. He has storage capacity of 10 bags only. He earns a profit of ₹11 and ₹8 per bag of rice and wheat respectively. How many bags of each must he buy to make maximum profit?

Answer

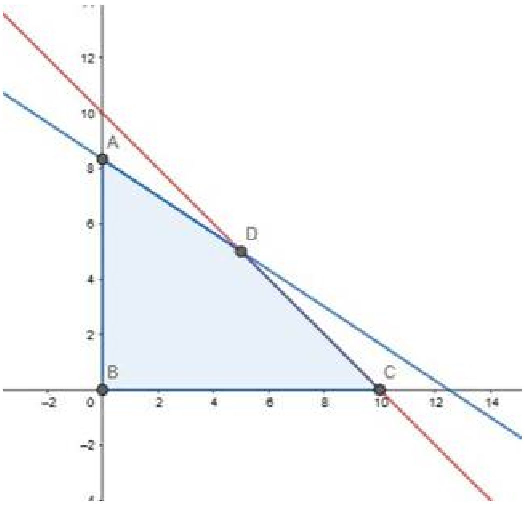
Let the number of wheat and rice bags be x and y.

∴According to the question,

120x + 180y ≤ 1500, x + y ≤ 10, x $\geq 0, y \geq 0$

Maximize Z = 8x + 11y

The feasible region determined by 120x + 180y ≤ 1500, x + y ≤ 10, x $\geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,8), B(0,0), C(10,0), D(5,5) .

The value of Z at corner point is

Corner Point	Z = 8x + 11y	
A(0,8)	88	
B(0,0)	0	
C(10,0)	80	
D(5,5)	95	Maximum

The maximum value of Z is 95 at point (5,5).

Hence, the man should 5 bags each of wheat and rice to earn maximum profit.

10. Question

A manufacture produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine B to produces a packet of nuts while it takes 3 hours on machine A and 1 hours on machine B to produce a packet of bolts. He earns a profit ₹17.50 per packet on nuts and ₹7 per packet on bolts. How many packets of each should be produced each day so as to maximize his profit if he operates each machine for at the most 12 hours a day? Also find the maximum profit.

Answer

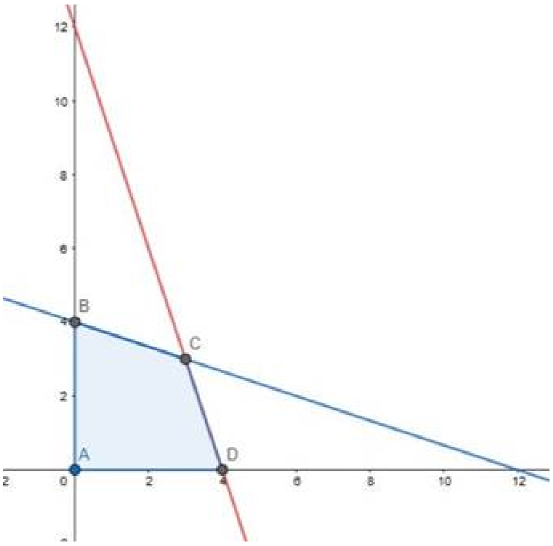
Let the number of packets of nuts and bolts be x and y respectively.

∴According to the question,

X + 3y $\leq\ 12\,, 3x\ +\ y\ \leq\ 12, x\ \geq\ 0, y\ \geq\ 0$

Maximize Z = 17.50x + 7y

The feasible region determined by X + 3y $\leq\ 12\,, 3x\ +\ y\ \leq\ 12, x\ \geq\ 0, y\ \geq\ 0$ is given by



The corner points of the feasible region are A(0,0), B(0,4), C(3,3)), D(4,0).The value of Z at the corner point is

Corner Point	Z = 17.50x + 7y	
A(0,0)	0	
B(0,4)	28	
C(3,3)	73.50	Maximum
D(4,0)	70	

The maximum value of Z is 73.50 at (3,3).

The manufacturer should make 3 packets each of nuts and bolts to make maximum profit of Rs.73.50.

10. Question

A manufacture produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine B to produces a packet of nuts while it takes 3 hours on machine A and 1 hours on machine B to produce a packet of bolts. He earns a profit ₹17.50 per packet on nuts and ₹7 per packet on bolts. How many packets of each should be produced each day so as to maximize his profit if he operates each machine for at the most 12 hours a day? Also find the maximum profit.

Answer

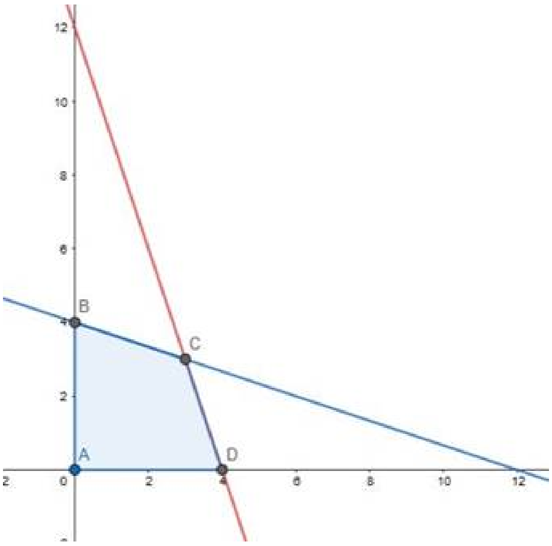
Let the number of packets of nuts and bolts be x and y respectively.

∴According to the question,

X + 3y $\leq 12, 3x + y \leq 12, x \geq 0, y \geq 0$

Maximize Z = 17.50x + 7y

The feasible region determined by X + 3y $\leq 12, 3x + y \leq 12, x \geq 0, y \geq 0$ is given by



The corner points of the feasible region are A(0,0), B(0,4), C(3,3)), D(4,0).The value of Z at the corner point is

Corner Point	Z = 17.50x + 7y	
A(0,0)	0	
B(0,4)	28	
C(3,3)	73.50	Maximum
D(4,0)	70	

The maximum value of Z is 73.50 at (3,3).

The manufacturer should make 3 packets each of nuts and bolts to make maximum profit of Rs.73.50.

11. Question

Two tailors, A and B, earn ₹300 and ₹400 per day respectively. A can stitch 6 shirts and 4 pair of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. How many days should each of them work if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labor cost?

Answer

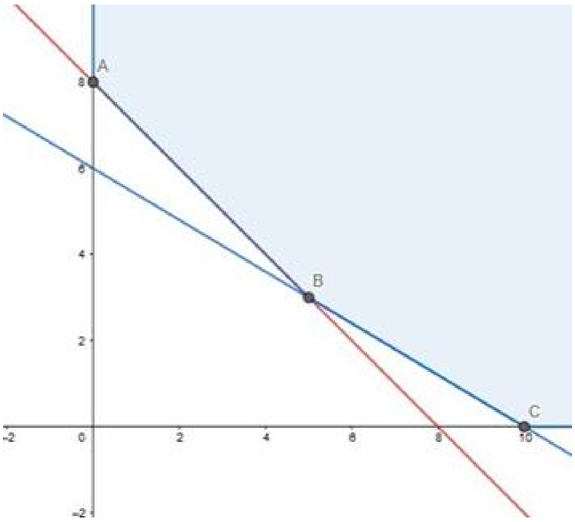
Let the total number of days tailor A work be x and tailor B be y.

∴According to the question,

6x + 10 y ≥ 60, 4x + 4y $\geq 32, x \geq 0, y \geq 0$

Minimize Z = 300x + 400y

The feasible region determined by 6x + 10 y ≥ 60, 4x + 4y $\geq 32, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,8),B(5,3),C(10,0) .The value of Z at corner point is

Corner Point	Z = 300x + 400y	
A(0,8)	3200	
B(5,3)	2700	Minimum
C(10,0)	3000	

The minimum value of Z is 2700 at point (5,3).

∴Tailor A must work for 5 days and tailor B must work for 3 days for minimum expenses.

11. Question

Two tailors, A and B, earn ₹300 and ₹400 per day respectively. A can stitch 6 shirts and 4 pair of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. How many days should each of them work if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labor cost?

Answer

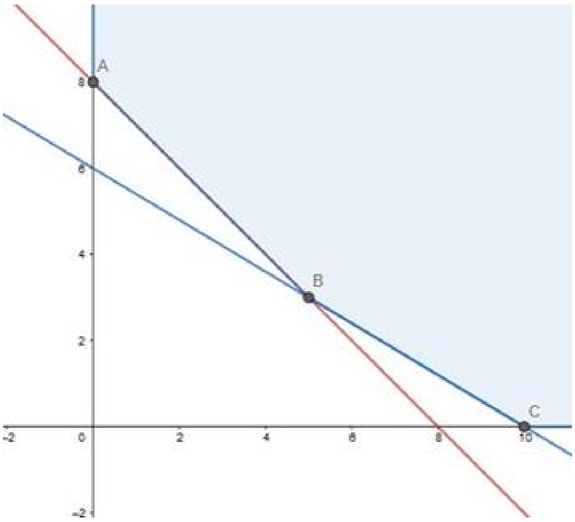
Let the total number of days tailor A work be x and tailor B be y.

∴According to the question,

$6x + 10y \geq 60$, $4x + 4y \geq 32, x \geq 0, y \geq 0$

Minimize Z = 300x + 400y

The feasible region determined by $6x + 10y \geq 60$, $4x + 4y \geq 32, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,8),B(5,3),C(10,0) .The value of Z at corner point is

Corner Point	Z = 300x + 400y	
A(0,8)	3200	
B(5,3)	2700	Minimum
C(10,0)	3000	

The minimum value of Z is 2700 at point (5,3).

∴Tailor A must work for 5 days and tailor B must work for 3 days for minimum expenses.

12. Question

A dealer wishes to purchase a number of fans and sewing machines. He has only ₹5760 to invest and space for at most 20 items. A fan costs him ₹360 and a sewing machine, ₹240. He expects to gain ₹22 on a fan and ₹18 on a sewing machine. Assuming that he can sell all the items he can buy, how should he invest the money in order to maximize the profit?

Answer

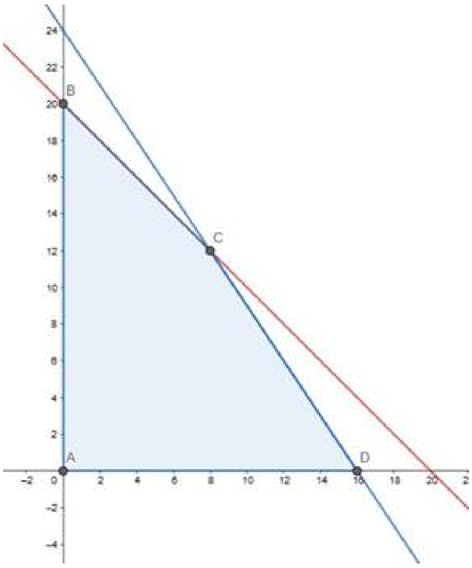
Let the number of fans bought be x and sewing machines bought be y.

∴According to the question,

$360x + 240y \leq 5760, x + y \leq 20, x \geq 0, y \geq 0$

Maximize Z = 22x + 18y

The feasible region determined by $360x + 240y \leq 5760, x + y \leq 20, x \geq 0, y \geq 0$ is given by



The corner points of the feasible region are A(0,0) , B(0,20),C(8,12) , D(16,0).The value of Z at corner points is

Corner Point	Z = 22x + 18y	
A(0,0)	0	
B(0,20)	360	
C(8,12)	392	Maximum
D(16,0)	352	

The maximum value of Z is 392 at point (8,12).

The dealer must buy 8 fans and 12 sewing machines to make the maximum profit.

12. Question

A dealer wishes to purchase a number of fans and sewing machines. He has only ₹5760 to invest and space for at most 20 items. A fan costs him ₹360 and a sewing machine, ₹240. He expects to gain ₹22 on a fan and ₹18 on a sewing machine. Assuming that he can sell all the items he can buy, how should he invest the money in order to maximize the profit?

Answer

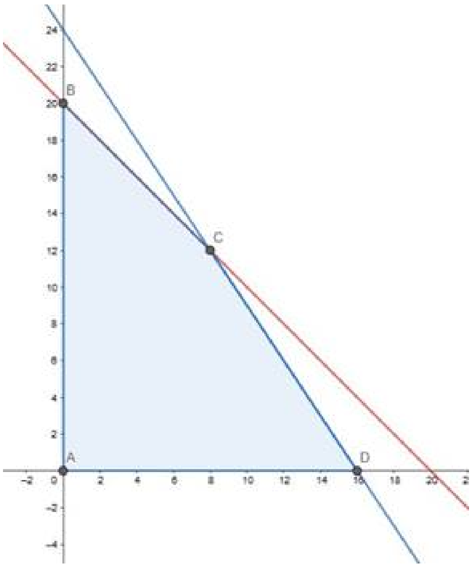
Let the number of fans bought be x and sewing machines bought be y.

∴According to the question,

360x + 240y ≤ 5760,x + y $\leq 20, x \geq 0, y \geq 0$

Maximize Z = 22x + 18y

The feasible region determined by 360x + 240y ≤ 5760,x + y $\leq 20, x \geq 0, y \geq 0$ is given by



The corner points of the feasible region are A(0,0) , B(0,20),C(8,12) , D(16,0).The value of Z at corner points is

Corner Point	Z = 22x + 18y	
A(0,0)	0	
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C(8,12)	392	Maximum
D(16,0)	352	

The maximum value of Z is 392 at point (8,12).

The dealer must buy 8 fans and 12 sewing machines to make the maximum profit.

13. Question

A firm manufactures two types of products, A and B, and sells them at a profit of ₹2 on type A and ₹2 on type B. Each product is processed on two machines, M_1 and M_2. Type A requires one minute of processing time on M_1 and two minutes on M_2. Type B requires one minute on M_1 and one minute on M_2 is available for not more than 6 hours 40 minutes while M_2 is available for at most 10 hours a day.

Find how many products of each type the firm should produce each day in order to get maximum profit.

Answer

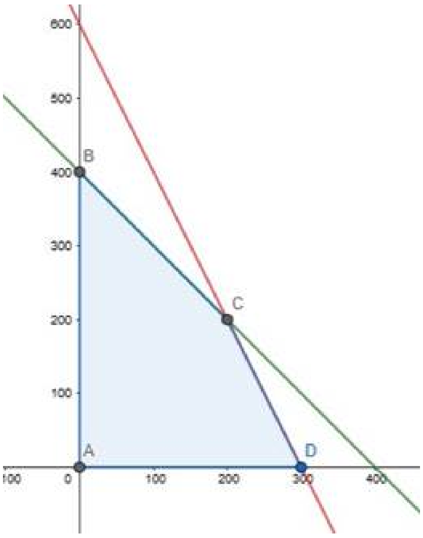
Let the firm manufacture x number of Aand y number of B products.

∴According to the question,

X + y $\leq 400, 2x + y \leq 600, x \geq 0, y \geq 0$

Maximize Z = 2x + 2y

The feasible region determined by X + y$\leq 400, 2x + y \leq 600, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,400) , C(200,200) , D(300,0).The value of Z at corner point is

Corner Point	Z = 2x + 2y	
A(0,0)	0	
B(0,400)	800	Maximum
C(200,200)	800	Maximum
D(300,0)	600	

The maximum value of Z is 800 and occurs at two points. Hence the line BC is a feasible solution.

The firm should produce 200 number of Aproducts and 200 number of B products.

13. Question

A firm manufactures two types of products, A and B, and sells them at a profit of₹2 on type A and ₹2 on type B. Each product is processed on two machines, M_1 and M_2. Type A requires one minute of processing time on M_1 and two minutes on M_2. Type B requires one minute on M_1 and one minute on M_2 is available for not more than 6 hours 40 minutes while M_2 is available for at most 10 hours a day.

Find how many products of each type the firm should produce each day in order to get maximum profit.

Answer

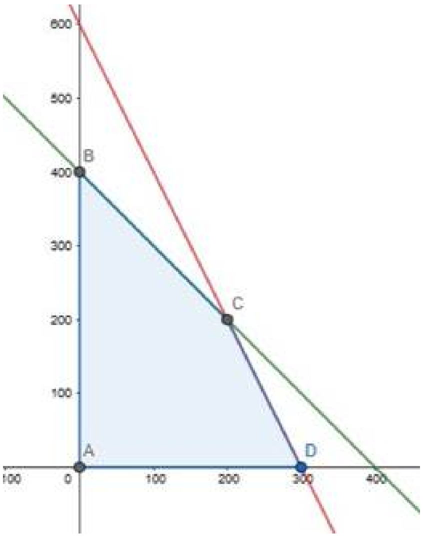
Let the firm manufacture x number of Aand y number of B products.

∴According to the question,

X + y$\leq 400, 2x + y \leq 600, x \geq 0, y \geq 0$

Maximize Z = 2x + 2y

The feasible region determined by X + y ≤ 400 , $2x + y \leq 600$, $x \geq 0$, $y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,400) , C(200,200) , D(300,0).The value of Z at corner point is

Corner Point	Z = 2x + 2y	
A(0,0)	0	
B(0,400)	800	Maximum
C(200,200)	800	Maximum
D(300,0)	600	

The maximum value of Z is 800 and occurs at two points. Hence the line BC is a feasible solution.

The firm should produce 200 number of Aproducts and 200 number of B products.

14. Question

A manufactures produces two types of soap bars using two machines, A and B. A is operated for 2 minutes and B for 3 minutes to manufacture the first type, while it takes 3 minutes on machine A and 5 minutes on machine B to manufacture the second type. Each machine can be operated at the most for 8 hours per day. The two types of soap bars are sold at a profit of ₹0.25 and ₹0.50 each. Assuming that the manufacture can sell all the soap bars he can manufacture, how many bars of soap of each type should be manufactured per day so as to maximize his profit?

Answer

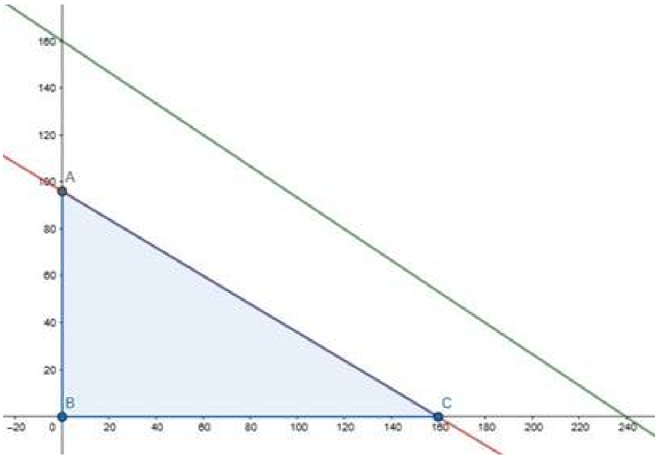
Let x and y be number of soaps be manufactured of 1^{st} and 2^{nd} type.

∴According to the question,

2x + 3y $\leq$ 480 , 3x + 5y $\leq$ 480, x $\geq$ 0 , y $\geq$ 0

Maximize Z = 0.25x + 0.50y

The feasible region determined by 2x + 3y $\leq$ 480 , 3x + 5y $\leq$ 480, x $\geq$ 0 , y $\geq$ 0 is given by



The corner points of feasible region are A(0,96) , B(0,0) , C(160,0).

The value of Z at corner points are

Corner Point	Z = 0.25x + 0.50y	
A(0,96)	48	Maximum
B(0,0)	0	
C(160,0)	40	

The maximum value of Z is 48 at point (0,96).

Hence, the manufacturer should make 96 soaps of the 2nd type to make maximum profit.

14. Question

A manufactures produces two types of soap bars using two machines, A and B. A is operated for 2 minutes and B for 3 minutes to manufacture the first type, while it takes 3 minutes on machine A and 5 minutes on machine B to manufacture the second type. Each machine can be operated at the most for 8 hours per day. The two types of soap bars are sold at a profit of ₹0.25 and ₹0.50 each. Assuming that the manufacture can sell all the soap bars he can manufacture, how many bars of soap of each type should be manufactured per day so as to maximize his profit?

Answer

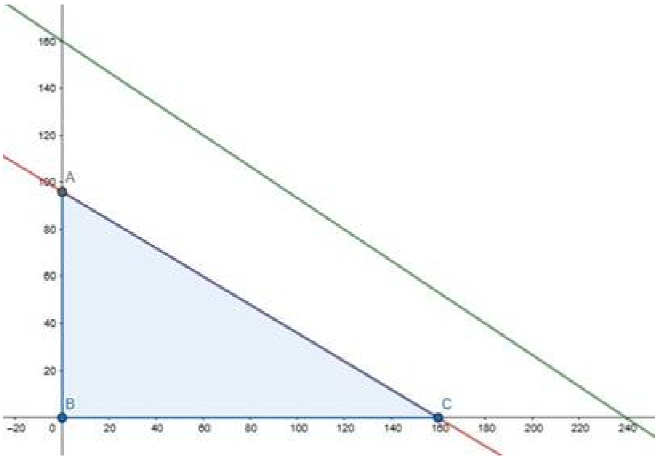
Let x and y be number of soaps be manufactured of 1st and 2nd type.

∴According to the question,

2x + 3y $\leq$ 480 , 3x + 5y $\leq$ 480, x $\geq$ 0 , y $\geq$ 0

Maximize Z = 0.25x + 0.50y

The feasible region determined by 2x + 3y $\leq$ 480 , 3x + 5y $\leq$ 480, x $\geq$ 0 , y $\geq$ 0 is given by



The corner points of feasible region are A(0,96) , B(0,0) , C(160,0).

The value of Z at corner points are

Corner Point	Z = 0.25x + 0.50y	
A(0,96)	48	Maximum
B(0,0)	0	
C(160,0)	40	

The maximum value of Z is 48 at point (0,96).

Hence, the manufacturer should make 96 soaps of the 2^{nd} type to make maximum profit.

15. Question

A manufacture of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20000 bottles of A and 40000 bottles of B but there are only 45000 bottles into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough material to fill 1000 bottles of A and it takes 1 hour to prepare enough material to fill 1000 bottles of B, and there are 66 hours available for this operation. The profit is ₹8 per bottle for A and ₹7 per bottle for B.

How should the manufacture schedule the production in order to maximize his profit? Also, find the maximum profit.

Answer

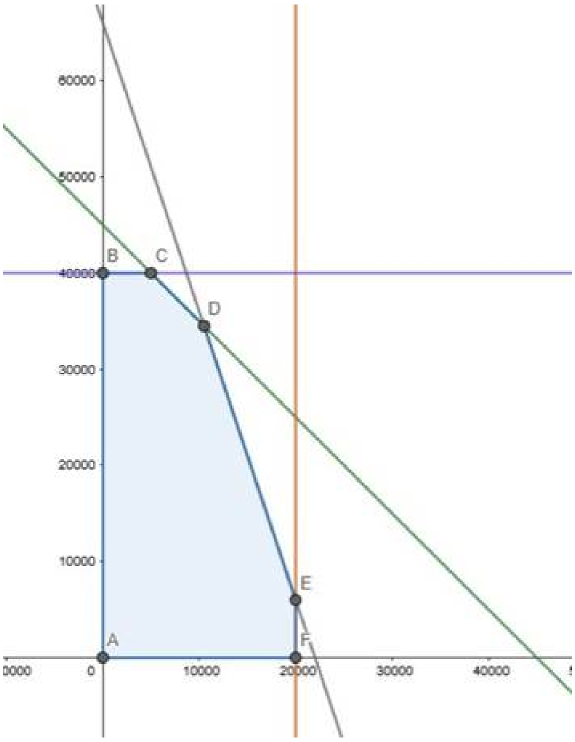
Let x and y be number of bottles of medicines A and B be prepared.

∴According to the question,

x + y $\leq$ 45000 , 3x + y $\leq$ 66000, x $\leq$ 20000 , y $\leq$ 40000, $x \geq 0, y \geq 0$

Maximize Z = 8x + 7y

The feasible region determined by x + y $\leq$ 45000 , 3x + y $\leq$ 66000, x $\leq$ 20000 , y $\leq$ 40000, $x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,40000) , C(5000,40000),D(10500,34500),E(20000,6000),F(20000,0).

The value of Z at corner points are

Corner Point	Z = 8x + 7y	
A(0,0)	0	
B(0,40000)	280000	
C(5000,40000)	320000	
D(10500,34500)	325500	Maximum
E(20000,6000)	202000	
F(20000,0)	160000	

The maximum value of Z is 325500 at point (10500,34500).

Hence, the manufacturer should produce 10500 bottles of medicine A and 34500 bottles of medicine B.

15. Question

A manufacture of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20000 bottles of A and 40000 bottles of B but there are only 45000 bottles into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough material to fill 1000 bottles of A and it takes 1 hour to prepare enough material to fill 1000 bottles of B, and there are 66 hours available for this operation. The profit is ₹8 per bottle for A and ₹7 per bottle for B.

How should the manufacture schedule the production in order to maximize his profit? Also, find the maximum profit.

Answer

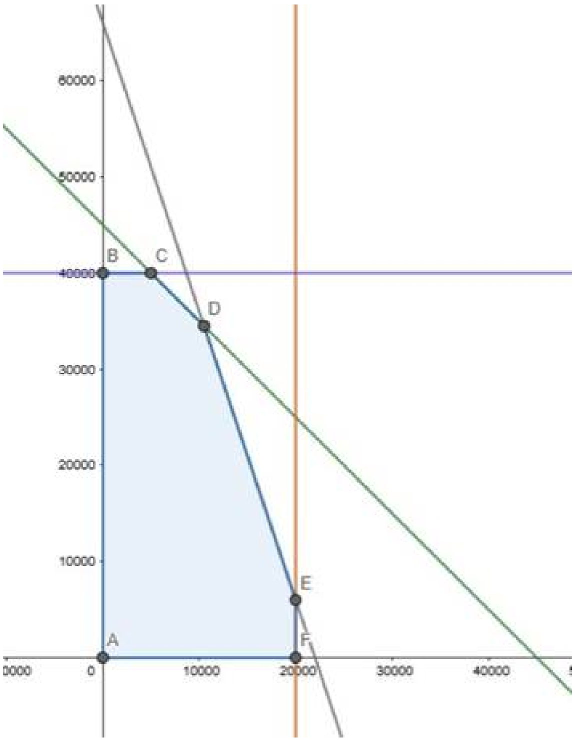
Let x and y be number of bottles of medicines A and B be prepared.

∴According to the question,

x + y $\leq$ 45000 , 3x + y $\leq$ 66000, x $\leq\ 20000$, y $\leq\ 40000, x\ \geq\ 0, y\ \geq\ 0$

Maximize Z = 8x + 7y

The feasible region determined by x + y $\leq$ 45000 , 3x + y $\leq$ 66000, x $\leq\ 20000$, y $\leq\ 40000, x\ \geq\ 0, y\ \geq\ 0$ is given by



The corner points of feasible region are A(0,0) , B(0,40000) , C(5000,40000),D(10500,34500),E(20000,6000),F(20000,0).

The value of Z at corner points are

Corner Point	Z = 8x + 7y	
A(0,0)	0	
B(0,40000)	280000	
C(5000,40000)	320000	
D(10500,34500)	325500	Maximum
E(20000,6000)	202000	
F(20000,0)	160000	

The maximum value of Z is 325500 at point (10500,34500).

Hence, the manufacturer should produce 10500 bottles of medicine A and 34500 bottles of medicine B.

16. Question

A toy company manufactures two types of dolls, A and B. Each doll of type B take twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 per day, if it produces only type A. the supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). Type B requires a fancy dress of which there are only 600 per day available. If the company makes profit of ₹3 and ₹5 per dolls respectively on dolls A and B, how many of each should be produced per day in order to maximize the profit? Also, find the maximum profit.

Answer

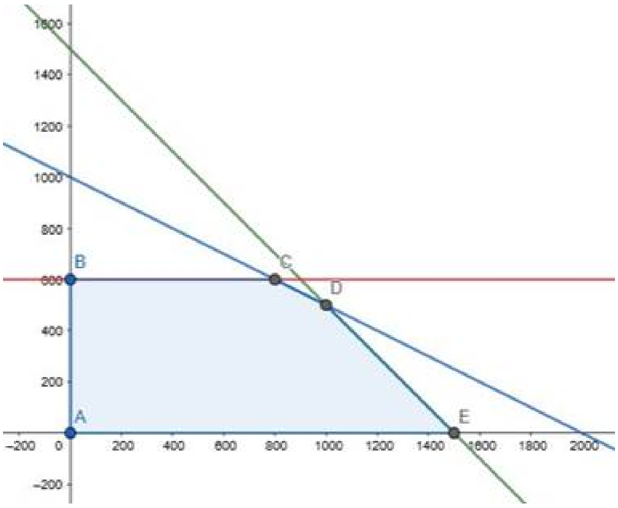
Let x and y be number of doll A manufactured and doll B manufactured.

∴According to the question,

x + y $\leq$ 1500, x + 2y ≤ 2000, y $\leq 600, x \geq 0, y \geq 0$

Maximize Z = 3x + 5y

The feasible region determined by x + y $\leq$ 1500, x + 2y ≤ 2000, y $\leq 600, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,600) , C(800,600),D(1000,500),E(1500,0).

The value of Z at corner points are

Corner Point	Z = 3x + 5y	
A(0,0)	0	
B(0,600)	3000	
C(800,600)	5400	
D(1000,500)	5500	Maximum
E(1500,0)	4500	

The maximum value of Z is 5500 at point (1000,500).

Hence, the manufacturer should produce 1000 types of doll A and 500 types of doll B to make maximum profit of Rs.5500.

16. Question

A toy company manufactures two types of dolls, A and B. Each doll of type B take twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 per day, if it produces only type A. the supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). Type B requires a fancy dress of which there are only 600 per day available. If the company makes profit of ₹3 and ₹5 per dolls respectively on dolls A and B, how many of each should be produced per day in order to maximize the profit? Also, find the maximum profit.

Answer

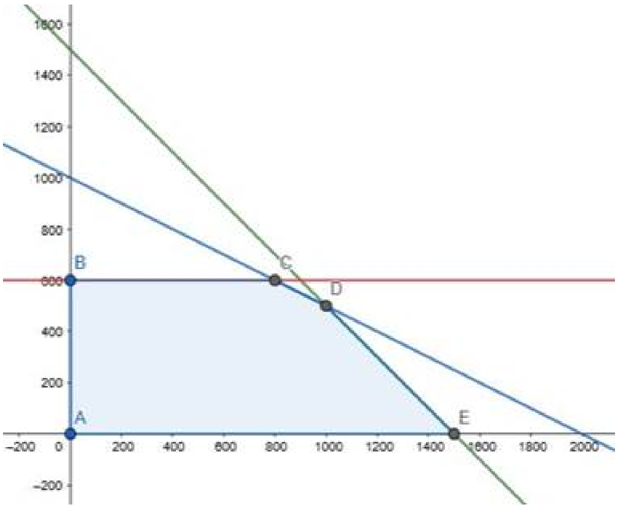
Let x and y be number of doll A manufactured and doll B manufactured.

∴According to the question,

x + y $\leq$ 1500, x + 2y ≤ 2000, y $\leq 600, x \geq 0, y \geq 0$

Maximize Z = 3x + 5y

The feasible region determined by x + y $\leq$ 1500, x + 2y ≤ 2000, y $\leq 600, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,600) , C(800,600),D(1000,500),E(1500,0).

The value of Z at corner points are

Corner Point	Z = 3x + 5y	
A(0,0)	0	
B(0,600)	3000	
C(800,600)	5400	
D(1000,500)	5500	Maximum
E(1500,0)	4500	

The maximum value of Z is 5500 at point (1000,500).

Hence, the manufacturer should produce 1000 types of doll A and 500 types of doll B to make maximum profit of Rs.5500.

17. Question

A small manufacture has employed 5 skilled men and 10 semiskilled men and makes an article in two qualities, a deluxe model and an ordinary model. The making of a deluxe model requires 2hours work by a skilled man and 2hours work by a semiskilled man. The ordinary model requires 1 hour by a skilled man and

3 hours by a semiskilled man. By union rules, no man can work more than 8 hours per day. The manufacture gains ₹15 on the deluxe model and ₹10 on the ordinary model. How many of each type should be made in order to maximize his total daily profit? Also, find the maximum daily profit.

Answer

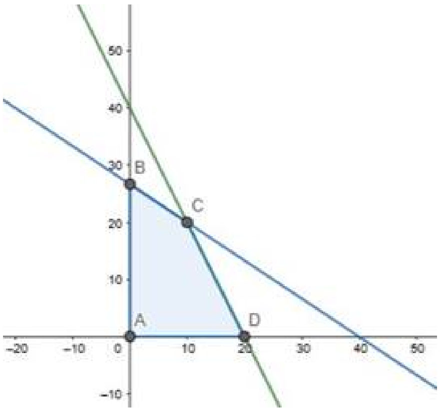
Let x and y be number of deluxe article manufactured and ordinary article manufactured.

∴According to the question,

2x + y ≤ 40, 2x + 3y $\leq 80, x \geq 0, y \geq 0$

Maximize Z = 15x + 10y

The feasible region determined by 2x + y ≤ 40, 2x + 3y $\leq 80, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,80/3) , C(10,20),D(20,0).

The value of Z at corner points are

Corner Point	Z = 15x + 10y	
A(0,0)	0	
B(0,80/3)	266.67	
C(10,20)	350	Maximum
D(20,0)	300	

The maximum value of Z is 350 at point (10,20).

Hence, the manufacturer should produce 10 types of deluxe article and 20 types of ordinary article to make maximum profit of Rs.350.

17. Question

A small manufacture has employed 5 skilled men and 10 semiskilled men and makes an article in two qualities, a deluxe model and an ordinary model. The making of a deluxe model requires 2hours work by a skilled man and 2hours work by a semiskilled man. The ordinary model requires 1 hour by a skilled man and

3 hours by a semiskilled man. By union rules, no man can work more than 8 hours per day. The manufacture gains ₹15 on the deluxe model and ₹10 on the ordinary model. How many of each type should be made in order to maximize his total daily profit? Also, find the maximum daily profit.

Answer

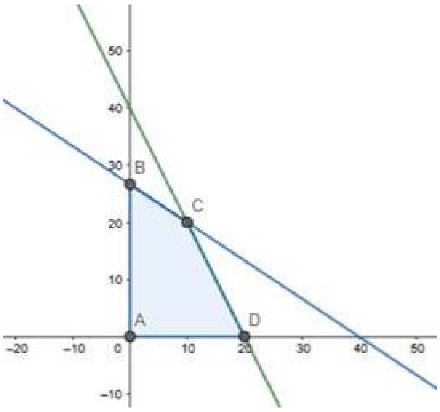
Let x and y be number of deluxe article manufactured and ordinary article manufactured.

∴According to the question,

2x + y ≤ 40, 2x + 3y $\leq 80, x \geq 0, y \geq 0$

Maximize Z = 15x + 10y

The feasible region determined by 2x + y ≤ 40, 2x + 3y $\leq 80, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,80/3) , C(10,20),D(20,0).

The value of Z at corner points are

Corner Point	Z = 15x + 10y	
A(0,0)	0	
B(0,80/3)	266.67	
C(10,20)	350	Maximum
D(20,0)	300	

The maximum value of Z is 350 at point (10,20).

Hence, the manufacturer should produce 10 types of deluxe article and 20 types of ordinary article to make maximum profit of Rs.350.

18. Question

A company producing soft drinks has a contrast which requires a minimum of 80 units of chemical A and 60 units of chemical B to go in each bottle of the drink. The chemical are available in a prepared mix from two different suppliers. Supplier X has a mix of 4 units of A and 2 units of B that costs ₹10, and the supplier Y has a mix of 1 unit of A and 1 unit of B that costs ₹4. How many mixes from X and Y should the company purchase to honor the contract requirement and yet minimize the cost?

Answer

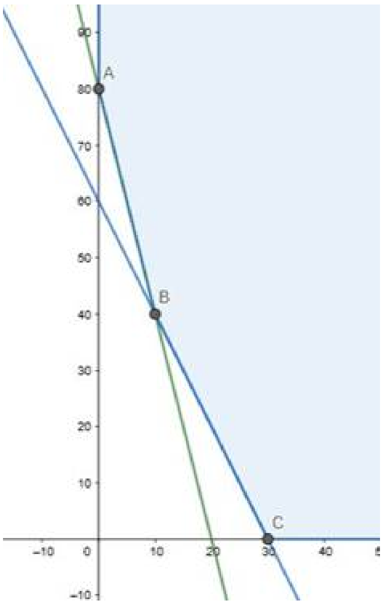
Let x and y be number of mixes from suppliers X and Y.

∴According to the question,

4x + y ≥ 80, 2x + y $\geq 60, x \geq 0, y \geq 0$

Minimize Z = 10x + 4y

The feasible region determined by 4x + y ≥ 80, 2x + y $\geq 60, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded .The corner points of feasible region are A(0,80) , B(10,40) , C(30,0).

The value of Z at corner points are

Corner Point	Z = 10x + 4y	
A(0,80)	320	
B(10,40)	260	Minimum
C(30,0)	300	

The minimum value of Z is 260 at point (10,40).

Hence, the company should buy 10 mixes from supplier X and 40 mixes from supplier Y to minimize the cost.

18. Question

A company producing soft drinks has a contrast which requires a minimum of 80 units of chemical A and 60 units of chemical B to go in each bottle of the drink. The chemical are available in a prepared mix from two different suppliers. Supplier X has a mix of 4 units of A and 2 units of B that costs ₹10, and the supplier Y has a mix of 1 unit of A and 1 unit of B that costs ₹4. How many mixes from X and Y should the company purchase to honor the contract requirement and yet minimize the cost?

Answer

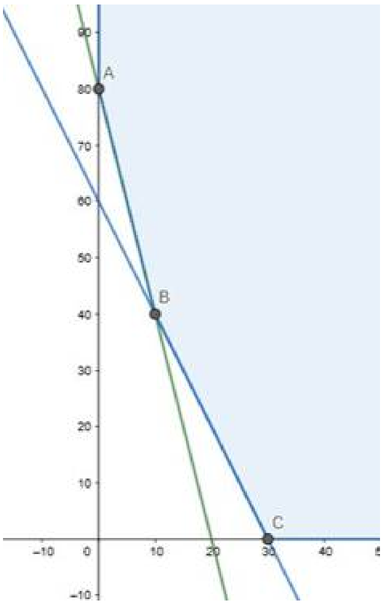
Let x and y be number of mixes from suppliers X and Y.

∴According to the question,

4x + y ≥ 80, 2x + y $\geq 60, x \geq 0, y \geq 0$

Minimize Z = 10x + 4y

The feasible region determined by 4x + y ≥ 80, 2x + y $\geq 60, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded .The corner points of feasible region are A(0,80) , B(10,40) , C(30,0).

The value of Z at corner points are

Corner Point	Z = 10x + 4y	
A(0,80)	320	
B(10,40)	260	Minimum
C(30,0)	300	

The minimum value of Z is 260 at point (10,40).

Hence, the company should buy 10 mixes from supplier X and 40 mixes from supplier Y to minimize the cost.

19. Question

A small firm manufactures gold rings and chains. The combined number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and half an hour for a chain. The maximum number of hour to available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, how many of each should be manufactured daily so as to maximize the profit?

Answer

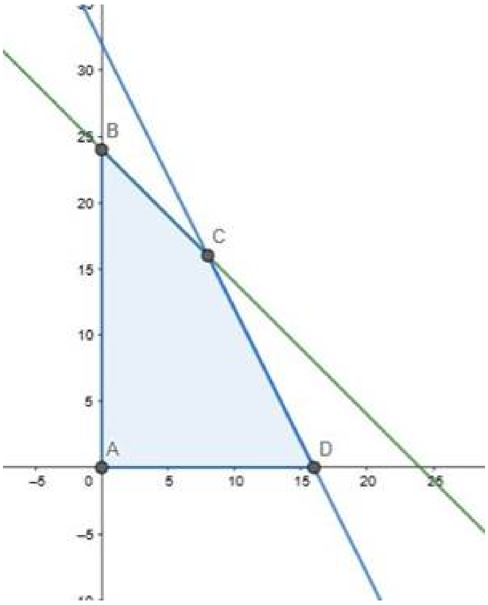
Let x and y be number of gold rings and chains.

∴According to the question,

x + y ≤ 24 , x + 0.5y $\leq 16, x \geq 0, y \geq 0$

Maximize Z = 300x + 190y

The feasible region determined by x + y ≤ 24 , x + 0.5y $\leq 16, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,24) , C(8,16), D(16,0).The value of Z at corner points are

Corner Point	Z = 300x + 190y	
A(0,0)	0	
B(0,24)	4560	
C(8,16)	5440	Maximum
D(16,0)	4800	

The maximum value of Z is 5440 at point (8,16).

Hence, the firm should manufacture 8 gold rings and 16 gold chains to maximize their profit.

19. Question

A small firm manufactures gold rings and chains. The combined number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and half an hour for a chain. The maximum number of hour to available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, how many of each should be manufactured daily so as to maximize the profit?

Answer

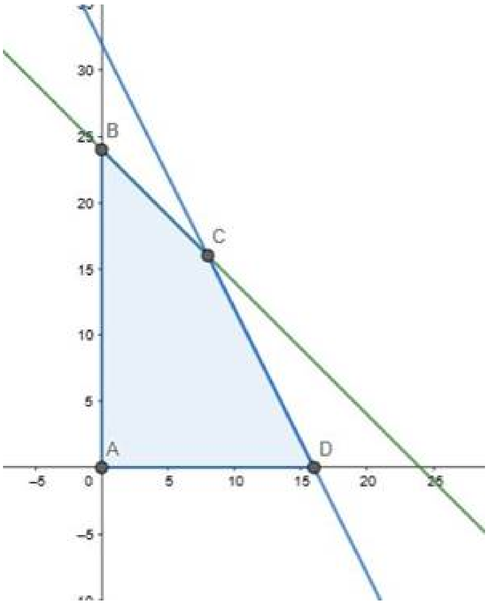
Let x and y be number of gold rings and chains.

∴According to the question,

x + y ≤ 24 , x + 0.5y $\le 16, x \ge 0, y \ge 0$

Maximize Z = 300x + 190y

The feasible region determined by x + y ≤ 24 , x + 0.5y $\le 16, x \ge 0, y \ge 0$ is given by



The corner points of feasible region are A(0,0) , B(0,24) , C(8,16), D(16,0).The value of Z at corner points are

Corner Point	Z = 300x + 190y	
A(0,0)	0	
B(0,24)	4560	
C(8,16)	5440	Maximum
D(16,0)	4800	

The maximum value of Z is 5440 at point (8,16).

Hence, the firm should manufacture 8 gold rings and 16 gold chains to maximize their profit.

20. Question

A manufacture makes two types, A and B, of teapots. Three machines are needed for the manufacture and the time required for each teapot on the machines is given below.

Each machine is available for a maximum of 6 hours per day. If the profit on each teapot of type A is 75 paise and that on each teapot of type B is 50 paise, show that 15 teapots of type A and 30 of type B should be manufactured in a day to get the maximum profit.

Machine	Time (in minutes)		
Type	I	II	III
A	12	18	6
B	6	0	9

Answer

Let x teapots of type A and y teapots of type B manufactured.

Then,

$x \geq 0, y \geq 0$

Also,

$12x + 6y \leq 6 \times 60$

$12x + 6y \leq 360$

$2x + y \leq 60$.....(1)

And,

$18x + 0y \leq 6 \times 60$

$X \leq 20$......(2)

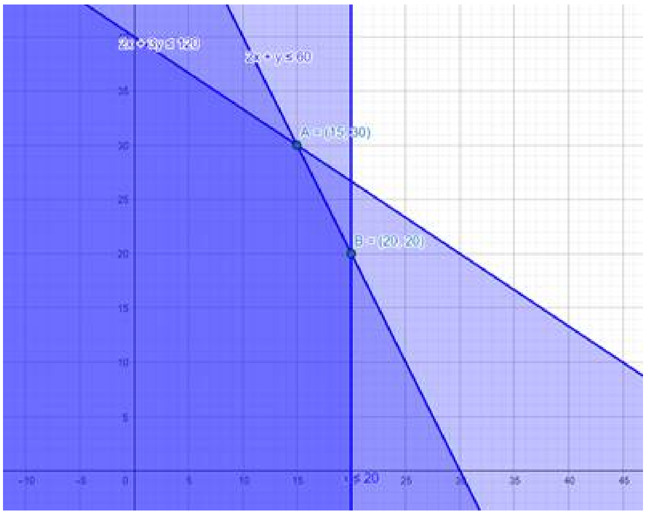
Also,

$6x + 9y \leq 6 \times 60$

$2x + 3y \leq 120$.....(3)

The profit will be given by: $Z = \frac{75}{100}x + \frac{50}{100}y \Rightarrow Z = \frac{3}{4}x + \frac{1}{2}y$

On plotting the constraints, we get,



Profit will be maximum when x = 30 and y = 15

Hence, Proved.

20. Question

A manufacture makes two types, A and B, of teapots. Three machines are needed for the manufacture and the time required for each teapot on the machines is given below.

Each machine is available for a maximum of 6 hours per day. If the profit on each teapot of type A is 75 paise and that on each teapot of type B is 50 paise, show that 15 teapots of type A and 30 of type B should be manufactured in a day to get the maximum profit.

Machine	Time (in minutes)		
Type	I	II	III
A	12	18	6
B	6	0	9

Answer

Let x teapots of type A and y teapots of type B manufactured.

Then,

$x \geq 0, y \geq 0$

Also,

$12x + 6y \leq 6 \times 60$

$12x + 6y \leq 360$

$2x + y \leq 60$.....(1)

And,

$18x + 0y \leq 6 \times 60$

X ≤ 20……(2)

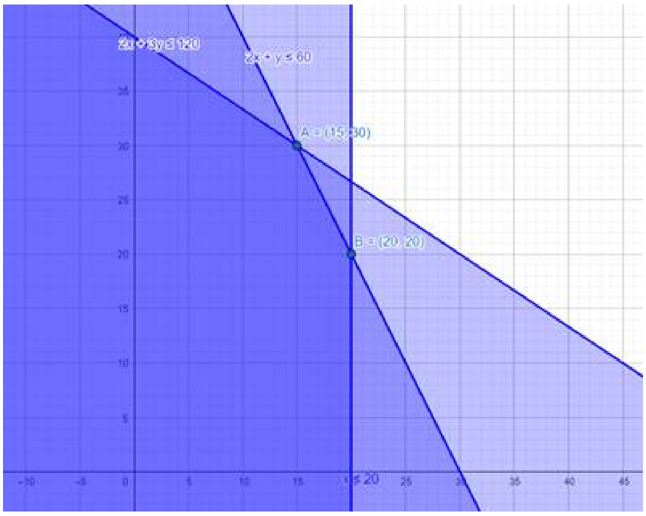
Also,

6x + 9y ≤ 6 × 60

2x + 3y ≤ 120…..(3)

The profit will be given by: Z $= \frac{75}{100}x + \frac{50}{100}y \Rightarrow Z = \frac{3}{4}x + \frac{1}{2}y$

On plotting the constraints, we get,



Profit will be maximum when x = 30 and y = 15

Hence, Proved.

21. Question

A manufacture makes two product, A and B. product A sells at ₹200 each and takes $\frac{1}{2}$ hour to make. Product B sells at ₹300 each and takes 1 hour to make. There is a permanent order for 14 of product A and 16 of product B. A working week consist of 40 hours of production and the weekly turnover must not be less than ₹10000. If the profit on each of the product A is ₹20 and on product B, it is ₹30 then how many of each should be produced so that the profit is maximum? Also, find the maximum profit.

Answer

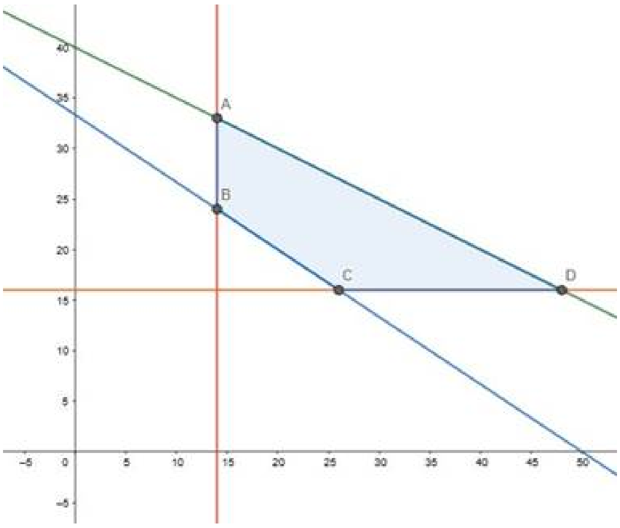
Let x and y be number of A and B products.

∴According to the question,

0.5x + y ≤ 40 , 200x + 300y $\geq 10000, x \geq 14, y \geq 16$

Maximize Z = 20x + 30y

The feasible region determined by 0.5x + y ≤ 40 , 200x + 300y $\geq 10000, x \geq 14, y \geq 16$ is given by



The corner points of feasible region are A(14,33) , B(14,24) , C(26,16), D(48,16).The value of Z at corner points are

Corner Point	Z = 20x + 30y	
A(14,33)	1270	
B(14,24)	1000	
C(26,16)	1000	
D(48,16)	1440	Maximum

The maximum value of Z is 1440 at point (48,16).

Hence, the manufacturer should manufacture 48 A products and 16 B products to maximize their profit of Rs.1440.

21. Question

A manufacture makes two product, A and B. product A sells at ₹200 each and takes $\frac{1}{2}$ hour to make. Product B sells at ₹300 each and takes 1 hour to make. There is a permanent order for 14 of product A and 16 of product B. A working week consist of 40 hours of production and the weekly turnover must not be less than ₹10000. If the profit on each of the product A is ₹20 and on product B, it is ₹30 then how many of each should be produced so that the profit is maximum? Also, find the maximum profit.

Answer

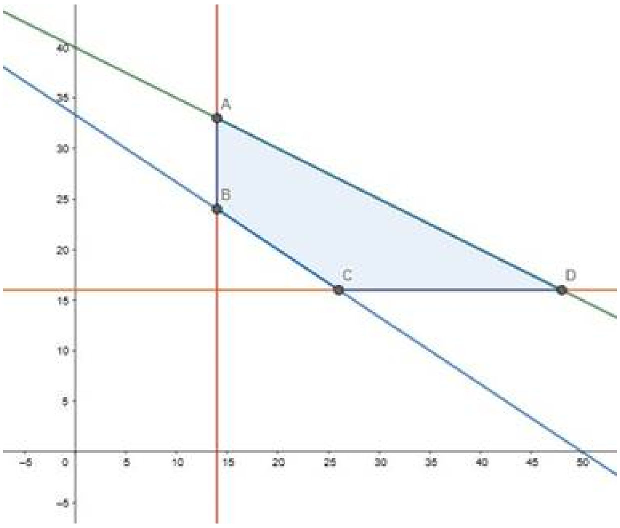
Let x and y be number of A and B products.

∴According to the question,

0.5x + y ≤ 40 , 200x + 300y $\geq 10000, x \geq 14, y \geq 16$

Maximize Z = 20x + 30y

The feasible region determined by 0.5x + y ≤ 40 , 200x + 300y $\geq 10000, x \geq 14, y \geq 16$ is given by



The corner points of feasible region are A(14,33) , B(14,24) , C(26,16), D(48,16).The value of Z at corner points are

Corner Point	Z = 20x + 30y	
A(14,33)	1270	
B(14,24)	1000	
C(26,16)	1000	
D(48,16)	1440	Maximum

The maximum value of Z is 1440 at point (48,16).

Hence, the manufacturer should manufacture 48 A products and 16 B products to maximize their profit of Rs.1440.

22. Question

A man owns a field area 1000 m^2. He wants to plant fruit trees in it. He has a sum of ₹1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 m^2 of ground per trees and costs ₹20 per tree, and type B requires 20 m^2 of ground per tree and costs ₹25 per tree. When full grown, a type - A tree produces an average of 20 kg of fruit which can be sold at a profit ₹2 per kg and type - B tree produces an average of 40 kg of fruit which can be sold at a profit of ₹1.50 per kg. How many of each type should be planted to achieve maximum profit when tree are full grown? What is the maximum profit?

Answer

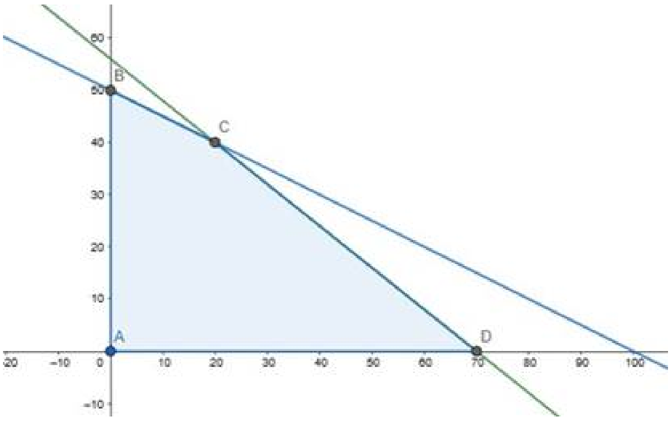
Let x and y be number of A and B trees.

∴According to the question,

20x + 25y ≤ 1400 , 10x + 20y $\leq 1000, x \geq 0, y \geq 0$

Maximize Z = 40x + 60y

The feasible region determined by 20x + 25y ≤ 1400 , 10x + 20y $\leq 1000, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,50) , C(20,40), D(70,0).The value of Z at corner points are

Corner Point	Z = 40x + 60y	
A(0,0)	0	
B(0,50)	3000	
C(20,40)	3200	Maximum
D(70,0)	2800	

The maximum value of Z is 3200 at point (20,40).

Hence, the man should plant 20 A trees and 40 B trees to make maximum profit of Rs.3200.

22. Question

A man owns a field area 1000 m^2. He wants to plant fruit trees in it. He has a sum of ₹1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 m^2 of ground per trees and costs ₹20 per tree, and type B requires 20 m^2 of ground per tree and costs ₹25 per tree. When full grown, a type - A tree produces an average of 20 kg of fruit which can be sold at a profit ₹2 per kg and type - B tree produces an average of 40 kg of fruit which can be sold at a profit of ₹1.50 per kg. How many of each type should be planted to achieve maximum profit when tree are full grown? What is the maximum profit?

Answer

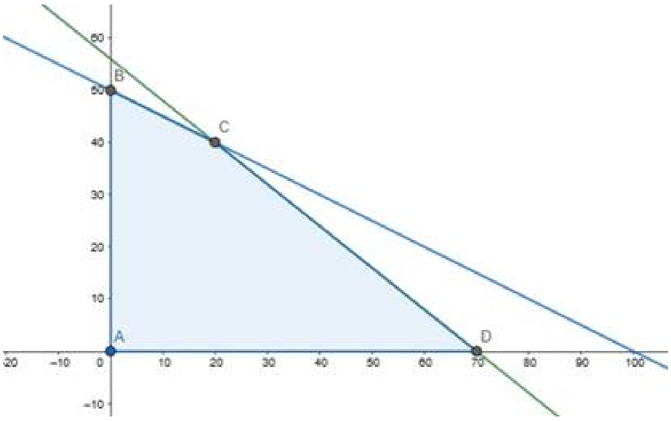
Let x and y be number of A and B trees.

∴According to the question,

20x + 25y ≤ 1400 , 10x + 20y $\leq 1000, x \geq 0, y \geq 0$

Maximize Z = 40x + 60y

The feasible region determined by 20x + 25y ≤ 1400 , 10x + 20y $\leq 1000, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,50) , C(20,40), D(70,0).The value of Z at corner points are

Corner Point	Z = 40x + 60y	
A(0,0)	0	
B(0,50)	3000	
C(20,40)	3200	Maximum
D(70,0)	2800	

The maximum value of Z is 3200 at point (20,40).

Hence, the man should plant 20 A trees and 40 B trees to make maximum profit of Rs.3200.

23. Question

A publisher sells a hardcover edition of a book for ₹72 and a paperback edition of the same for ₹40. Costs to the publisher are ₹56 and ₹28 respectively in addition to weekly costs of ₹9600. Both types require 5 minutes of printing time although the hardcover edition requires 10 minutes of binding time and the paperback edition requires only 2 minutes. Both the printing and binding operations have 4800 minutes available each week. How many of each type of books should be produced in order to maximize the profit? Also, find the maximum profit per week.

Answer

Let x and y be number of hardcover and paperback edition of the book.

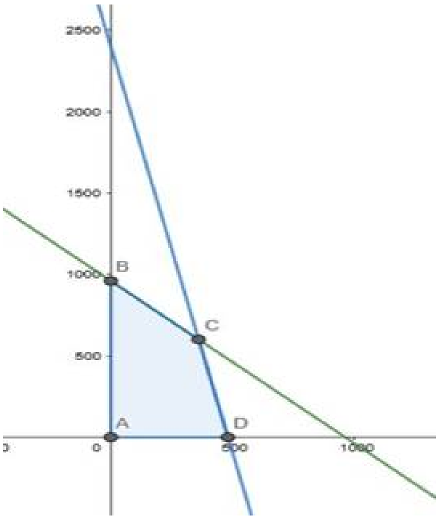
∴According to the question,

5x + 5y ≤ 4800 , 10x + 2y $\leq 4800, x \geq 0, y \geq 0$

Maximize Z = (72x + 40y) – (56x + 28y + 9600)

= 16x + 12y - 9600

The feasible region determined by 5x + 5y ≤ 4800 , 10x + 2y $\leq 4800, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,960) , C(360,600), D(480,0).The value of Z at corner points are

Corner Point	Z = 16x + 12y - 9600	
A(0,0)	0	
B(0,960)	1920	
C(360,600)	3360	Maximum
D(480,0)	- 1920	

The maximum value of Z is 3360 at point (360,600).

Hence, the publisher should publish 360 hardcover edition and 600 and paperback edition of the book to earn maximum profit of Rs.3360.

23. Question

A publisher sells a hardcover edition of a book for ₹72 and a paperback edition of the same for ₹40. Costs to the publisher are ₹56 and ₹28 respectively in addition to weekly costs of ₹9600. Both types require 5 minutes of printing time although the hardcover edition requires 10 minutes of binding time and the paperback edition requires only 2 minutes. Both the printing and binding operations have 4800 minutes available each week. How many of each type of books should be produced in order to maximize the profit? Also, find the maximum profit per week.

Answer

Let x and y be number of hardcover and paperback edition of the book.

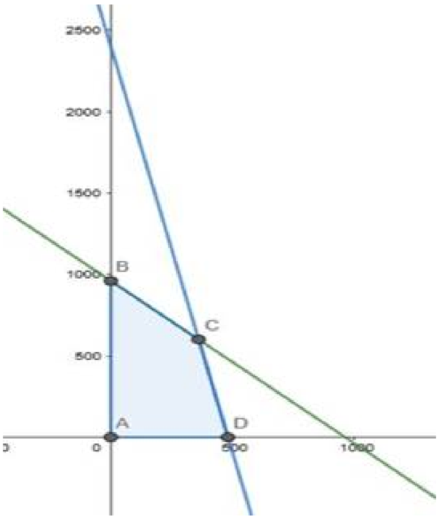
∴According to the question,

5x + 5y ≤ 4800 , 10x + 2y $\leq 4800, x \geq 0, y \geq 0$

Maximize Z = (72x + 40y) – (56x + 28y + 9600)

= 16x + 12y - 9600

The feasible region determined by 5x + 5y $\leq$ 4800 , 10x + 2y $\leq$ 4800, $x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,960) , C(360,600), D(480,0).The value of Z at corner points are

Corner Point	Z = 16x + 12y - 9600	
A(0,0)	0	
B(0,960)	1920	
C(360,600)	3360	Maximum
D(480,0)	- 1920	

The maximum value of Z is 3360 at point (360,600).

Hence, the publisher should publish 360 hardcover edition and 600 and paperback edition of the book to earn maximum profit of Rs.3360.

24. Question

A gardener has a supply of fertilizers of the type 1 which consist of 10% nitrogen and 6% phosphoric acid, and of the type II which consist of 5% nitrogen and 10% phosphoric acid. After testing the soil condition, he finds that he needs at least 14kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type - I fertilizer costs 60 paise per kg and the type - II fertilizer costs 40 paise per kg, determine how many kilograms of each type of fertilizer should be used so that the nutrient requirement are met at a minimum cost. What is the minimum cost?

Answer

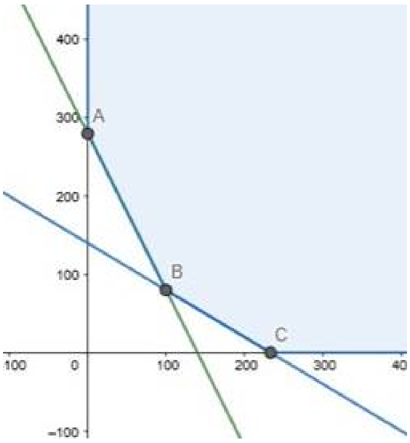
Let x and y be number of kilograms of fertilizer I and II

∴According to the question,

0.10x + 0.05y ≥ 14 , 0.06x + 0.10y $\geq 14, x \geq 0, y \geq 0$

Minimize Z = 0.60x + 0.40y

The feasible region determined by 0.10x + 0.05y ≥ 14 , 0.06x + 0.10y $\geq 14, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,280) , B(100,80) , C(700/3,0).The value of Z at corner points are

Corner Point	Z = 0.60x + 0.40y	
A(0,280)	112	
B(100,80)	92	Minimum
C(700/3,0)	140	

The minimum value of Z is 92 at point (100,80).

Hence, the gardener should by 100 kilograms o fertilizer I and 80 kg of fertilizer II to minimize the cost which is Rs.92.

24. Question

A gardener has a supply of fertilizers of the type 1 which consist of 10% nitrogen and 6% phosphoric acid, and of the type II which consist of 5% nitrogen and 10% phosphoric acid. After testing the soil condition, he finds that he needs at least 14kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type - I fertilizer costs 60 paise per kg and the type - II fertilizer costs 40 paise per kg, determine how many kilograms of each type of fertilizer should be used so that the nutrient requirement are met at a minimum cost. What is the minimum cost?

Answer

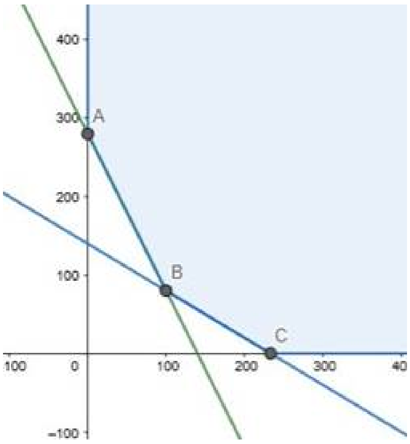
Let x and y be number of kilograms of fertilizer I and II

∴According to the question,

0.10x + 0.05y ≥ 14 , 0.06x + 0.10y $\geq 14, x \geq 0, y \geq 0$

Minimize Z = 0.60x + 0.40y

The feasible region determined by 0.10x + 0.05y ≥ 14 , 0.06x + 0.10y $\geq 14, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,280) , B(100,80) , C(700/3,0).The value of Z at corner points are

Corner Point	Z = 0.60x + 0.40y	
A(0,280)	112	
B(100,80)	92	Minimum
C(700/3,0)	140	

The minimum value of Z is 92 at point (100,80).

Hence, the gardener should by 100 kilograms o fertilizer I and 80 kg of fertilizer II to minimize the cost which is Rs.92.

25. Question

Two godowns, A and B, have a grain storage capacity of 100 quintals and 50 quintals respectively. Their supply goes to three ration shops, D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The costs of transportation per quintal from the godowns to the shops are given in the following table.

	Cost of transportation (in ₹ per quintal)	
From / To	*A*	*B*
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

How should the supplies be transported in order that the transportation cost is minimum?

Answer

Let x quintals of supplies be transported from A to D and y quintals be transported from A to E.

Therefore, 100 - (x + y) will be transported to F.

Also, (60 - x) quintals, (50 - y) quintals and (40 – (100 – (x + y))) quintals will be transported to D, E, F by godown B.

∴According to the question,

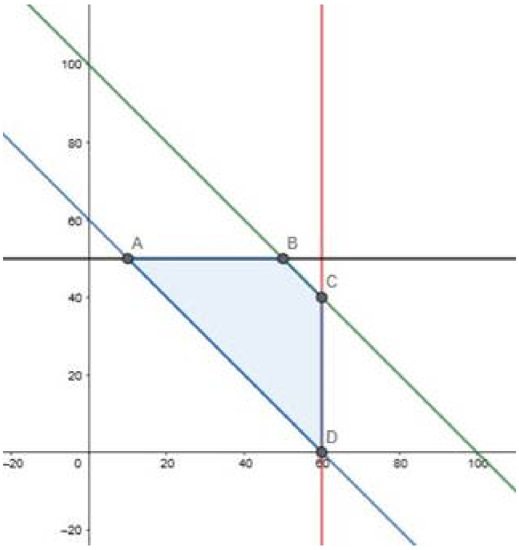
$x \geq 0, y \geq 0, x + y \leq 100, x \leq 60, y \leq 50, x + y \geq 60$

Minimize Z = 6x + 4(60 - x) + 3y + 2(50 – y) + 2.50(100 - (x + y)) + 3((x + y) – 60)

Z = 6x + 240 – 4x + 3y + 100 – 2y + 250 - 2.5x - 2.5y + 3x + 3y – 180

Z = 2.5x + 1.5y + 210

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 100, x \leq 60, y \leq 50, x + y \geq 60$ is given by



The corner points of feasible region are A(10,50) , B(50,50) , C(60,40) , D(60,0)

Corner Point	Z = 2.5x + 1.5y + 210	
A(10,50)	310	Minimum
B(50,50)	410	
C(60,40)	420	
D(60,0)	360	

The minimum value of Z is 310 at point (10,50).

Hence, 10, 50, 40 quintals of supplies should be transported from A to D, E, F and 50, 0, 0 quintals of supplies should be transported from B to D, E, F.

25. Question

Two godowns, A and B, have a grain storage capacity of 100 quintals and 50 quintals respectively. Their supply goes to three ration shops, D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The costs of transportation per quintal from the godowns to the shops are given in the following table.

From / To	Cost of transportation (in ₹ per quintal)	
	A	B
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

How should the supplies be transported in order that the transportation cost is minimum?

Answer

Let x quintals of supplies be transported from A to D and y quintals be transported from A to E.

Therefore, 100 - (x + y) will be transported to F.

Also, (60 - x) quintals, (50 - y) quintals and (40 – (100 – (x + y))) quintals will be transported to D, E, F by godown B.

∴According to the question,

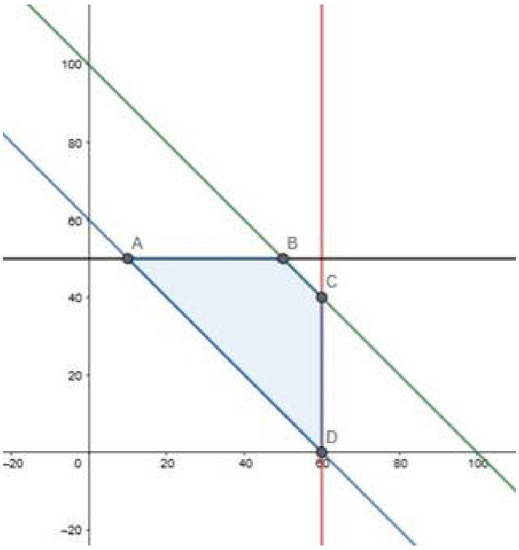
$x \geq 0, y \geq 0, x + y \leq 100, x \leq 60, y \leq 50, x + y \geq 60$

Minimize Z = 6x + 4(60 - x) + 3y + 2(50 – y) + 2.50(100 - (x + y)) + 3((x + y) – 60)

Z = 6x + 240 – 4x + 3y + 100 – 2y + 250 - 2.5x - 2.5y + 3x + 3y – 180

Z = 2.5x + 1.5y + 210

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 100, x \leq 60, y \leq 50, x + y \geq 60$ is given by



The corner points of feasible region are A(10,50) , B(50,50) , C(60,40) , D(60,0)

Corner Point	Z = 2.5x + 1.5y + 210	
A(10,50)	310	Minimum
B(50,50)	410	
C(60,40)	420	
D(60,0)	360	

The minimum value of Z is 310 at point (10,50).

Hence, 10, 50, 40 quintals of supplies should be transported from A to D, E, F and 50, 0, 0 quintals of supplies should be transported from B to D, E, F.

26. Question

A brick manufacture has two depots, P and Q, with stocks of 30000 and 20000 bricks respectively. He receives order from three building A, B, C, for 15000, 20000 and 15000 bricks respectively. The costs of transporting 1000 bricks to the building from the depots are given below.

	Cost of transportation (in ₹ per 1000 bricks)		
To / From	A	B	C
P	40	20	30
Q	20	60	40

How should the manufacture fulfill the orders so as to keep the cost of transportation minimum?

Answer

Let x bricks be transported from P to A and y bricks be transported from P to B.

Therefore, 30000 - (x + y) will be transported to C.

Also, (15000 - x) bricks, (20000 - y) bricks and (15000 - (30000 - (x + y))) bricks will be transported to A, B, C from Q.

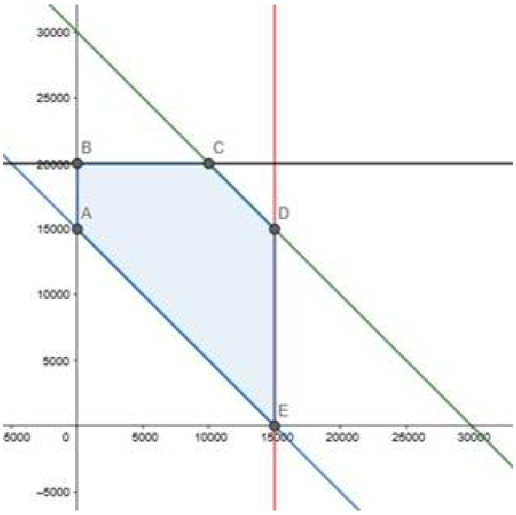
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 30000, x \leq 15000, y \leq 20000, x + y \geq 15000$

Minimize Z = 0.04x + 0.02(15000 - x) + 0.02y + 0.06(20000 - y) + 0.03(30000 - (x + y)) + 0.04((x + y) – 15000)

Z = 0.03x - 0.03y + 1800

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 30000, x \leq 15000, y \leq 20000, x + y \geq 15000$ is given by



The corner points of feasible region are A(0,15000) , B(0,20000) , C(10000,20000) , D(15000,15000), E(15000,0).

Corner Point	Z = 0.03x - 0.03y + 1800	
A(0,15000)	1350	
B(0,20000)	1200	Minimum
C(10000,20000)	1500	
D(15000,15000)	1800	
E(15000,0)	2250	

The minimum value of Z is 1200 at point (0,20000).

Hence, 0, 20000, 10000 bricks should be transported from P to A, B, C and 15000, 0, 5000 bricks should be transported from Q to A, B, C.

26. Question

A brick manufacture has two depots, P and Q, with stocks of 30000 and 20000 bricks respectively. He receives order from three building A, B, C, for 15000, 20000 and 15000 bricks respectively. The costs of transporting 1000 bricks to the building from the depots are given below.

	Cost of transportation (in ₹ per 1000 bricks)		
From \ To	A	B	C
P	40	20	30
Q	20	60	40

How should the manufacture fulfill the orders so as to keep the cost of transportation minimum?

Answer

Let x bricks be transported from P to A and y bricks be transported from P to B.

Therefore, 30000 - (x + y) will be transported to C.

Also, (15000 - x) bricks, (20000 - y) bricks and (15000 - (30000 - (x + y))) bricks will be transported to A, B, C from Q.

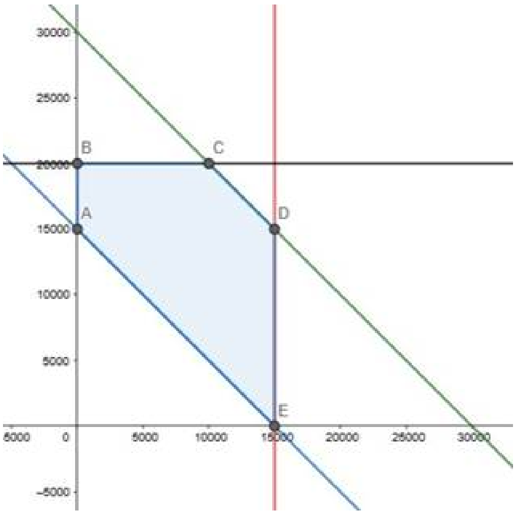
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 30000, x \leq 15000, y \leq 20000, x + y \geq 15000$

Minimize Z = 0.04x + 0.02(15000 - x) + 0.02y + 0.06(20000 - y) + 0.03(30000 - (x + y)) + 0.04((x + y) – 15000)

Z = 0.03x - 0.03y + 1800

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 30000, x \leq 15000, y \leq 20000, x + y \geq 15000$ is given by



The corner points of feasible region are A(0,15000) , B(0,20000) , C(10000,20000) , D(15000,15000), E(15000,0).

Corner Point	Z = 0.03x - 0.03y + 1800	
A(0,15000)	1350	
B(0,20000)	1200	Minimum
C(10000,20000)	1500	
D(15000,15000)	1800	
E(15000,0)	2250	

The minimum value of Z is 1200 at point (0,20000).

Hence, 0, 20000, 10000 bricks should be transported from P to A, B, C and 15000, 0, 5000 bricks should be transported from Q to A, B, C.

27. Question

A medicine company has factories at two places, X and Y. From these places, supply is made to each of its three agencies situated at P, Q and R. the monthly requirement of the agencies are respectively 40 packets, 40 packets and 50 packets of medicine, while the production capacity of the factories at X and Y are 60 packets and 70 packets respectively. The transportation costs per packet from the factories to the agencies are given as follows.

	Transportation cost per packet (in ₹)	
From / To	X	Y
P	5	4
Q	4	2
R	3	5

How many packets from each factory should be transported to each agency so that the cost of transportation is minimum? Also, find the minimum cost.

Answer

Let x packets of medicines be transported from X to P and y packets of medicines be transported from X to Q.

Therefore, 60 - (x + y) will be transported to R.

Also, (40 - x) packets, (40 - y) packets and (50 - (60 - (x + y))) packets will be transported to P, Q, R from Y.

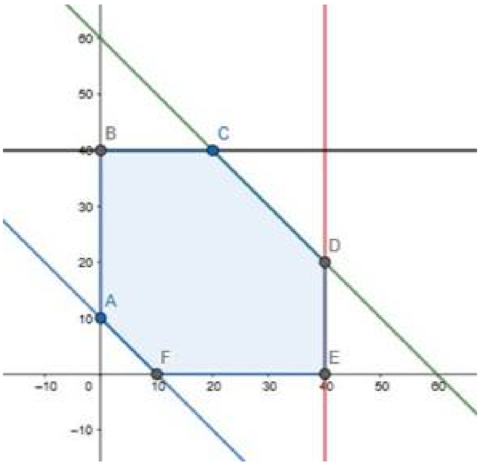
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 60, x \leq 40, y \leq 40, x + y \geq 10$

Minimize Z = 5x + 4(40 - x) + 4y + 2(40 - y) + 3(60 - (x + y)) + 5((x + y) – 10)

Z = 3x + 4y + 370

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 60, x \leq 40, y \leq 40, x + y \geq 10$ is given by



The corner points of feasible region are A(0,10) , B(0,40) , C(20,40) , D(40,20), E(10,0).

Corner Point	Z = 3x + 4y + 370	
A(0,10)	410	
B(0,40)	530	
C(20,40)	590	
D(40,20)	570	
E(10,0)	400	Minimum

The minimum value of Z is 40 at point (10,0).

Hence, 10, 0, 50 packets of medicines should be transported from X to P, Q, R and 30, 40, 0 packets of medicines should be transported from Y to P, Q, R.

27. Question

A medicine company has factories at two places, X and Y. From these places, supply is made to each of its three agencies situated at P, Q and R. the monthly requirement of the agencies are respectively 40 packets, 40 packets and 50 packets of medicine, while the production capacity of the factories at X and Y are 60 packets and 70 packets respectively. The transportation costs per packet from the factories to the agencies are given as follows.

	Transportation cost per packet (in ₹)	
From / To	X	Y
P	5	4
Q	4	2
R	3	5

How many packets from each factory should be transported to each agency so that the cost of transportation is minimum? Also, find the minimum cost.

Answer

Let x packets of medicines be transported from X to P and y packets of medicines be transported from X to Q.

Therefore, 60 - (x + y) will be transported to R.

Also, (40 - x) packets, (40 - y) packets and (50 - (60 - (x + y))) packets will be transported to P, Q, R from Y.

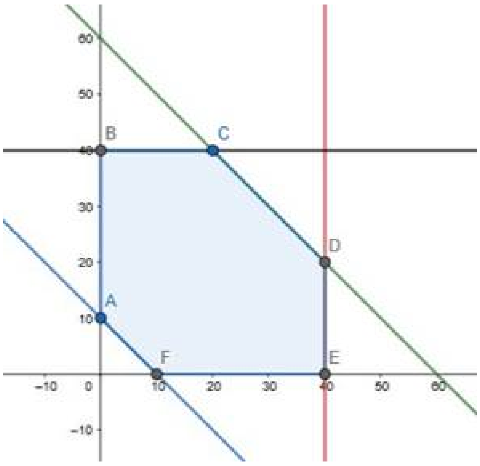
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 60, x \leq 40, y \leq 40, x + y \geq 10$

Minimize Z = 5x + 4(40 - x) + 4y + 2(40 - y) + 3(60 - (x + y)) + 5((x + y) – 10)

Z = 3x + 4y + 370

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 60, x \leq 40, y \leq 40, x + y \geq 10$ is given by



The corner points of feasible region are A(0,10) , B(0,40) , C(20,40) , D(40,20), E(10,0).

Corner Point	Z = 3x + 4y + 370	
A(0,10)	410	
B(0,40)	530	
C(20,40)	590	
D(40,20)	570	
E(10,0)	400	Minimum

The minimum value of Z is 40 at point (10,0).

Hence, 10, 0, 50 packets of medicines should be transported from X to P, Q, R and 30, 40, 0 packets of medicines should be transported from Y to P, Q, R.

28. Question

An oil company has two depots, A and B, with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three pumps, D, E, F, whose requirements are 4500 L, 3000 L, and 3500 L respectively. The distances (in km) between the depots and the petrol pumps are given in the following table:

	Distance (in km)	
From / To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost per km is re 1 per litre, how should the delivery be scheduled in order that the transportation cost is minimum?

Answer

Let x liters of petrol be transported from A to D and y liters of petrol be transported from A to E.

Therefore, 7000 - (x + y) will be transported to F.

Also, (4500 - x) liters of petrol, (3000 - y) liters of petrol and (3500 – (7000 – (x + y))) liters of petrol will be transported to D, E, F by B.

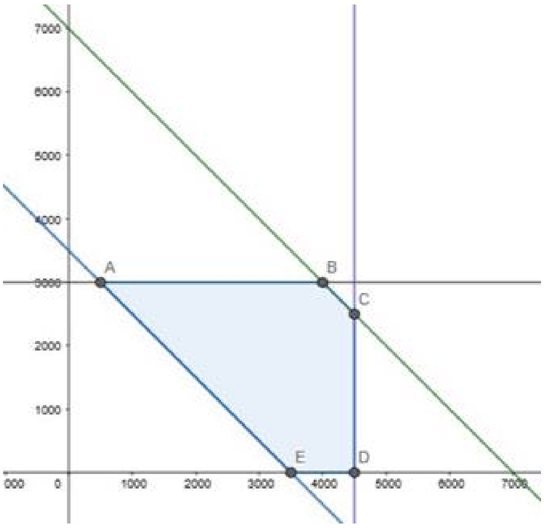
∴According to the question,

$x \geq 0, y \geq 0, x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$

Minimize Z = 7x + 3(4500 – x) + 6y + 4(3000 – y) + 3(7000 –(x + y)) + 2 ((x + y) – 3500)

Z = 3x + y + 39500

The feasible region represented by $x \geq 0, y \geq 0, x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$ is given by



The corner points of feasible region are A(500,3000) , B(4000,3000) , C(4500,2500) , D(4500,0) , E(3500,0)

Corner Point	Z = 3x + y + 39500	
A(500,3000)	44000	Minimum
B(4000,3000)	54500	
C(4500,2500)	55500	
D(4500,0)	53000	
E(3500,0)	50000	

The minimum value of Z is 44000 at point (500,3000).

Hence, 500,3000,3500 liters of petrol should be transported from A to D, E, F and 4000, 0, 0 liters of petrol should be transported from B to D, E, F.

28. Question

An oil company has two depots, A and B, with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three pumps, D, E, F, whose requirements are 4500 L, 3000 L, and 3500 L respectively. The distances (in km) between the depots and the petrol pumps are given in the following table:

To \ From	Distance (in km) A	Distance (in km) B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost per km is re 1 per litre, how should the delivery be scheduled in order that the transportation cost is minimum?

Answer

Let x liters of petrol be transported from A to D and y liters of petrol be transported from A to E.

Therefore, 7000 - (x + y) will be transported to F.

Also, (4500 - x) liters of petrol, (3000 - y) liters of petrol and (3500 – (7000 – (x + y))) liters of petrol will be transported to D, E, F by B.

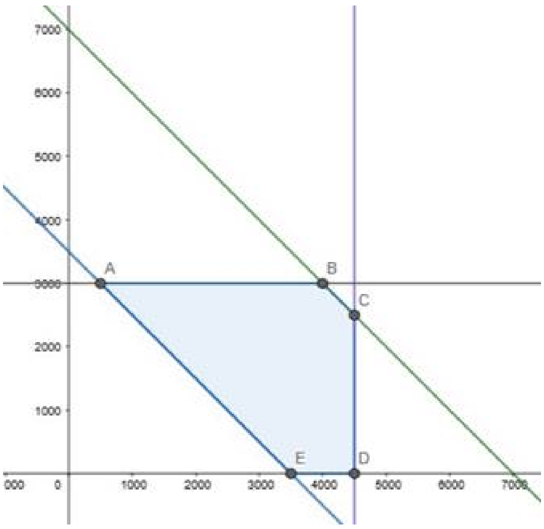
∴According to the question,

x $\geq 0, y \geq 0, x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$

Minimize Z = 7x + 3(4500 – x) + 6y + 4(3000 – y) + 3(7000 –(x + y)) + 2 ((x + y) – 3500)

Z = 3x + y + 39500

The feasible region represented by x $\geq 0, y \geq 0, x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$ is given by



The corner points of feasible region are A(500,3000) , B(4000,3000) , C(4500,2500) , D(4500,0) , E(3500,0)

Corner Point	Z = 3x + y + 39500	
A(500,3000)	44000	Minimum
B(4000,3000)	54500	
C(4500,2500)	55500	
D(4500,0)	53000	
E(3500,0)	50000	

The minimum value of Z is 44000 at point (500,3000).

Hence, 500,3000,3500 liters of petrol should be transported from A to D, E, F and 4000, 0, 0 liters of petrol should be transported from B to D, E, F.

29. Question

A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. They need certain nutrients, named as X,Y,Z. the pigs are fed on two products, A and B. One unit of product A contain 36 unit of X, 3 units of Y and 20 units of Z, while one unit of product B contain 6 units of X, 12 units of Y and 10 units of Z. the minimum requirement of X, Y, Z are 108 units, 36 units and 100 units respectively. Product A costs ₹20 per unit and product B costs ₹40 per unit. How many units of each product must be taken to minimize the cost? Also, find the minimum cost.

Answer

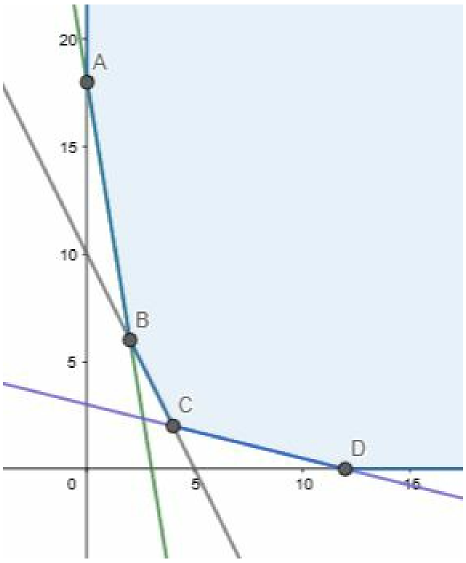
Let x and y be number of units of products of A and B.

∴According to the question,

36x + 6y ≥ 108 , 3x + 12y $\geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$

Minimize Z = 20x + 40y

The feasible region determined 36x + 6y ≥ 108 , 3x + 12y $\geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,18) , B(2,6) , C(4,2) , D(12,0).The value of Z at corner points are

Corner Point	Z = 20x + 40y	
A(0,18)	720	
B(2,6)	280	
C(4,2)	160	Minimum
D(12,0)	240	

The minimum value of Z is 160 at point (4,2).

Hence, the firm should buy 4 units of fertilizer A and 2 units of fertilizer B to achieve minimum expense of Rs.160.

29. Question

A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. They need certain nutrients, named as X,Y,Z. the pigs are fed on two products, A and B. One unit of product A contain 36 unit of X, 3 units of Y and 20 units of Z, while one unit of product B contain 6 units of X, 12 units of Y and 10 units of Z. the minimum requirement of X, Y, Z are 108 units, 36 units and 100 units respectively. Product A costs ₹20 per unit and product B costs ₹40 per unit. How many units of each product must be taken to minimize the cost? Also, find the minimum cost.

Answer

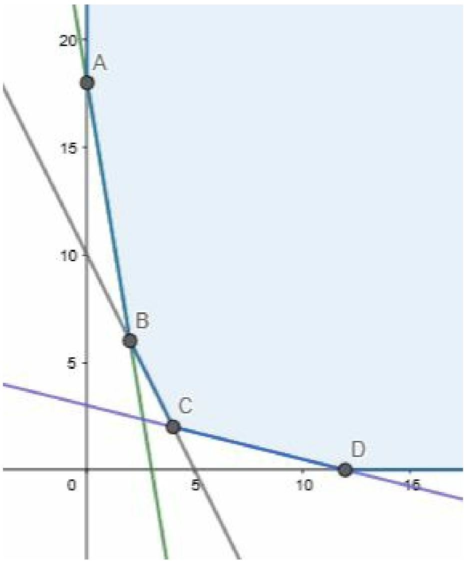
Let x and y be number of units of products of A and B.

∴According to the question,

$36x + 6y \geq 108$, $3x + 12y \geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$

Minimize Z = 20x + 40y

The feasible region determined 36x + 6y ≥ 108 , 3x + 12y $\geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,18) , B(2,6) , C(4,2) , D(12,0).The value of Z at corner points are

Corner Point	Z = 20x + 40y	
A(0,18)	720	
B(2,6)	280	
C(4,2)	160	Minimum
D(12,0)	240	

The minimum value of Z is 160 at point (4,2).

Hence, the firm should buy 4 units of fertilizer A and 2 units of fertilizer B to achieve minimum expense of Rs.160.

30. Question

A dietician wishes to mix two types of food, X and Y, in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food X contains 2 units/kg of vitamin A and 1 unit /kg of vitamin C, while food Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5 per kg to purchase the food X and ₹7 per kg to purchase the food Y. Determine the minimum cost of such a mixture.

Answer

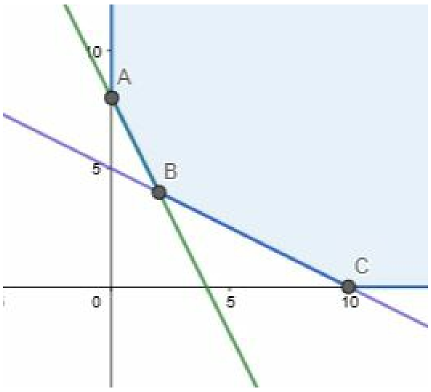
Let x and y be number of units of X and Y.

∴According to the question,

2x + y ≥ 8 , x + 2y $\geq 10, x \geq 0, y \geq 0$

Minimize Z = 5x + 7y

The feasible region determined 2x + y ≥ 8 , x + 2y $\geq 10, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,8) , B(2,4) , C(10,0).The value of Z at corner points are

Corner Point	Z = 5x + 7y	
A(0,8)	56	
B(2,4)	38	Minimum
C(10,0)	50	

The minimum value of Z is 160 at point (4,2).

Hence, the dietician should mix 2 units of X and 4 units of Y to meet the requirements at minimum cost of Rs.38.

30. Question

A dietician wishes to mix two types of food, X and Y, in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food X contains 2 units/kg of vitamin A and 1 unit /kg of vitamin C, while food Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5 per kg to purchase the food X and ₹7 per kg to purchase the food Y. Determine the minimum cost of such a mixture.

Answer

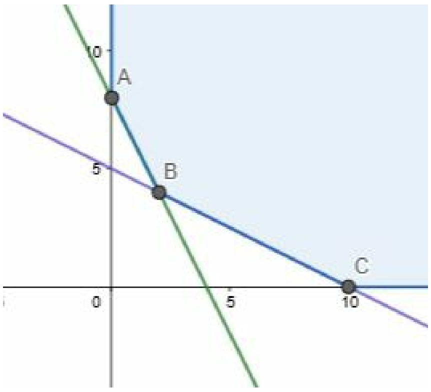
Let x and y be number of units of X and Y.

∴According to the question,

2x + y ≥ 8 , x + 2y $\geq 10, x \geq 0, y \geq 0$

Minimize Z = 5x + 7y

The feasible region determined 2x + y ≥ 8 , x + 2y $\geq 10, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,8) , B(2,4) , C(10,0).The value of Z at corner points are

Corner Point	Z = 5x + 7y	
A(0,8)	56	
B(2,4)	38	Minimum
C(10,0)	50	

The minimum value of Z is 160 at point (4,2).

Hence, the dietician should mix 2 units of X and 4 units of Y to meet the requirements at minimum cost of Rs.38.

31. Question

A diet for a sick person must contain at least 4000 units of vitamins, 50 units of mineral and 1400 calories. Two food, A and B, are available at a cost of ₹4 and ₹3 per unit respectively. If one unit of A contains 200 units of vitamins, 1 unit of mineral and 40 calories, and 1 unit of B contains 100 units of vitamins, 2 units of mineral and 40 calories, find what combination of foods should be used to have the least cost.

Answer

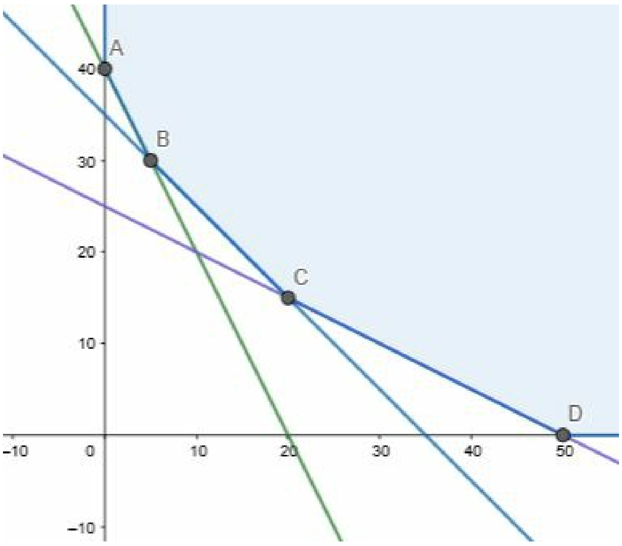
Let x and y be number of units of food A and B.

∴According to the question,

200x + 100y ≥ 4000 , x + 2y $\geq 50, 40x + 40y \geq 1400, x \geq 0, y \geq 0$

Minimize Z = 4x + 3y

The feasible region determined 200x + 100y ≥ 4000 , x + 2y $\geq 50, 40x + 40y \geq 1400, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,40) , B(5,30) , C(20,15) , D(50,0).The value of Z at corner points are

Corner Point	Z = 4x + 3y	
A(0,40)	120	
B(5,30)	110	Minimum
C(20,15)	125	
D(50,0)	200	

The minimum value of Z is 110 at point (5,30).

Hence, the diet should contain 5 units of food A and 30 units of food B for the least cost.

31. Question

A diet for a sick person must contain at least 4000 units of vitamins, 50 units of mineral and 1400 calories. Two food, A and B, are available at a cost of ₹4 and ₹3 per unit respectively. If one unit of A contains 200 units of vitamins, 1 unit of mineral and 40 calories, and 1 unit of B contains 100 units of vitamins, 2 units of mineral and 40 calories, find what combination of foods should be used to have the least cost.

Answer

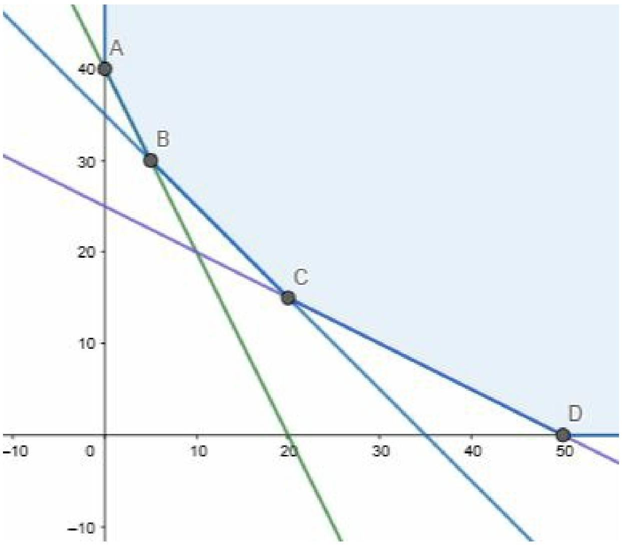
Let x and y be number of units of food A and B.

∴According to the question,

200x + 100y ≥ 4000 , x + 2y$\geq 50, 40x + 40y \geq 1400, x \geq 0, y \geq 0$

Minimize Z = 4x + 3y

The feasible region determined 200x + 100y ≥ 4000 , x + 2y$\geq 50, 40x + 40y \geq 1400, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,40) , B(5,30) , C(20,15) , D(50,0).The value of Z at corner points are

Corner Point	Z = 4x + 3y	
A(0,40)	120	
B(5,30)	110	Minimum
C(20,15)	125	
D(50,0)	200	

The minimum value of Z is 110 at point (5,30).

Hence, the diet should contain 5 units of food A and 30 units of food B for the least cost.

32. Question

A housewife wishes to mix together two kinds of food, X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of 1 kg of each food are given below.

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

If 1 kg of food X cost ₹6 and 1 kg of food Y costs ₹10, find the minimum cost of the mixture which will produce the diet.

Answer

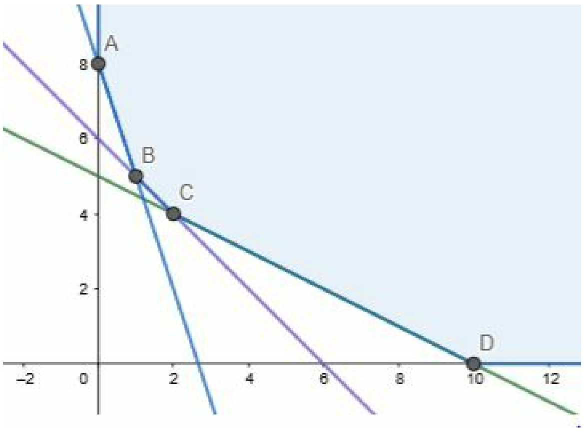
Let x and y be number of kilograms of food X and Y.

∴According to the question,

x + 2y ≥ 10 , 2x + 2y $\geq 12, 3x + y \geq 8, x \geq 0, y \geq 0$

Minimize Z = 6x + 10y

The feasible region determined x + 2y ≥ 10 , 2x + 2y $\geq 12, 3x + y \geq 8, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,8) , B(1,5) , C(2,4) , D(10,0).The value of Z at corner points are

Corner Point	Z = 6x + 10y	
A(0,8)	80	
B(1,5)	56	
C(2,4)	52	Minimum
D(10,0)	60	

The minimum value of Z is 52 at point (2,4).

Hence, the diet should contain 2 kgs of food X and 4 kgs of food Y for the least cost of Rs. 52.

32. Question

A housewife wishes to mix together two kinds of food, X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of 1 kg of each food are given below.

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

If 1 kg of food X cost ₹6 and 1 kg of food Y costs ₹10, find the minimum cost of the mixture which will produce the diet.

Answer

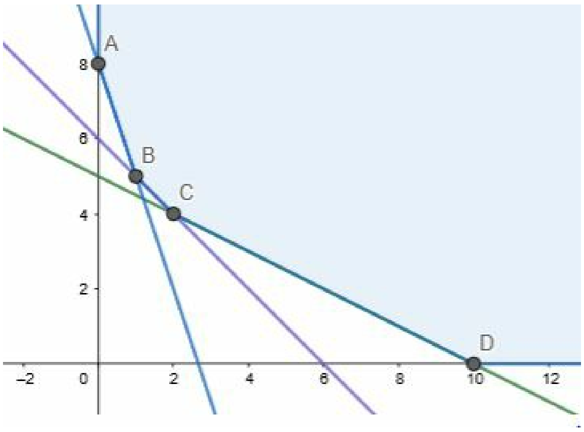
Let x and y be number of kilograms of food X and Y.

∴According to the question,

x + 2y ≥ 10 , 2x + 2y $\geq 12, 3x + y \geq 8, x \geq 0, y \geq 0$

Minimize Z = 6x + 10y

The feasible region determined x + 2y ≥ 10 , 2x + 2y $\geq 12, 3x + y \geq 8, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,8) , B(1,5) , C(2,4) , D(10,0).The value of Z at corner points are

Corner Point	Z = 6x + 10y	
A(0,8)	80	
B(1,5)	56	
C(2,4)	52	Minimum
D(10,0)	60	

The minimum value of Z is 52 at point (2,4).

Hence, the diet should contain 2 kgs of food X and 4 kgs of food Y for the least cost of Rs. 52.

33. Question

A firm manufactures two types of product, A and B, and sells them at a profit of ₹5 per unit of type A and ₹3 per unit of type B. Each product is processed on two machines, M_1 and M_2 . one unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2; whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2. Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product the firm should produce a day in order to maximize the profit. Solve the problem graphically.

Answer

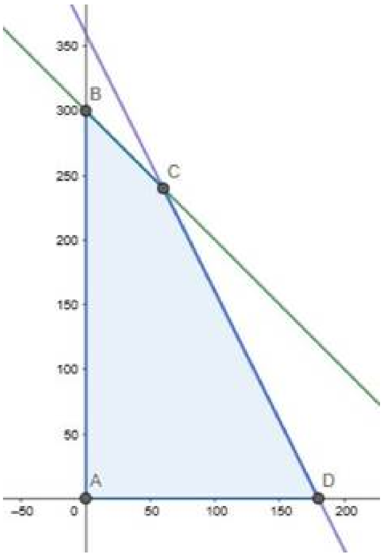
Let the firm manufacture x number of Aand y number of B products.

∴According to the question,

X + y$\leq 300, 2x + y \leq 360, x \geq 0, y \geq 0$

Maximize Z = 5x + 3y

The feasible region determined X + y$\leq 300, 2x + y \leq 360, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,300) , C(60,240) , D(180,0).The value of Z at corner point is

Corner Point	Z = 5x + 3y	
A(0,0)	0	
B(0,300)	900	
C(60,240)	1020	Maximum
D(180,0)	900	

The maximum value of Z is 1020 and occurs at point (60,240).

The firm should produce 60 Aproducts and 240 B products to earn maximum profit of Rs.1020.

33. Question

A firm manufactures two types of product, A and B, and sells them at a profit of ₹5 per unit of type A and ₹3 per unit of type B. Each product is processed on two machines, M_1 and M_2 . one unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2; whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2. Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product the firm should produce a day in order to maximize the profit. Solve the problem graphically.

Answer

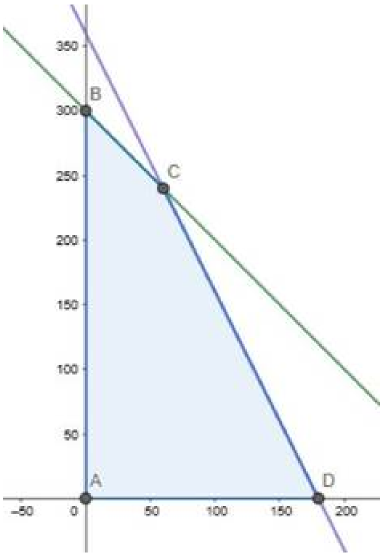
Let the firm manufacture x number of Aand y number of B products.

∴According to the question,

X + y$\leq 300, 2x + y \leq 360, x \geq 0, y \geq 0$

Maximize Z = 5x + 3y

The feasible region determined X + y$\leq 300, 2x + y \leq 360, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,300) , C(60,240) , D(180,0).The value of Z at corner point is

Corner Point	Z = 5x + 3y	
A(0,0)	0	
B(0,300)	900	
C(60,240)	1020	Maximum
D(180,0)	900	

The maximum value of Z is 1020 and occurs at point (60,240).

The firm should produce 60 Aproducts and 240 B products to earn maximum profit of Rs.1020.

34. Question

A small firm manufactures items A and B. The total number of items that it can manufacture in a day is at most 24. Item A takes one hour to make while item B take only half an hour. The maximum time available per day is 16 hours. If the profit on one unit item A be ₹300 and that on one unit of item B be ₹160, how many of each type of item should be produced to maximize the profit? Solve the problem graphically.

Ans. Z is maximum at (8, 16) and its maximum value is ₹4960

Answer

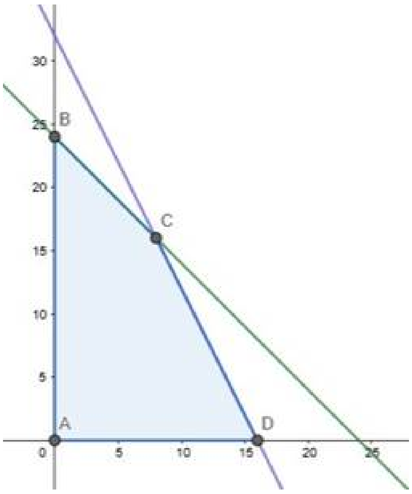
Let the firm manufacture x number of A and y number of B products.

∴According to the question,

X + y$\leq 24, x + 0.5y \leq 16, x \geq 0, y \geq 0$

Maximize Z = 300x + 160y

The feasible region determined X + y$\leq 24, x + 0.5y \leq 16, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,24) , C(8,16) , D(16,0).The value of Z at corner point is

Corner Point	Z = 300x + 160y	
A(0,0)	0	
B(0,24)	3840	
C(8,16)	4960	Maximum
D(16,0)	4800	

The maximum value of Z is 4960 and occurs at point (8,16).

The firm should produce 8 Aproducts and 16 B products to earn maximum profit of Rs.4960.

34. Question

A small firm manufactures items A and B. The total number of items that it can manufacture in a day is at most 24. Item A takes one hour to make while item B take only half an hour. The maximum time available per day is 16 hours. If the profit on one unit item A be ₹300 and that on one unit of item B be ₹160, how many of each type of item should be produced to maximize the profit? Solve the problem graphically.

Ans. Z is maximum at (8, 16) and its maximum value is ₹4960

Answer

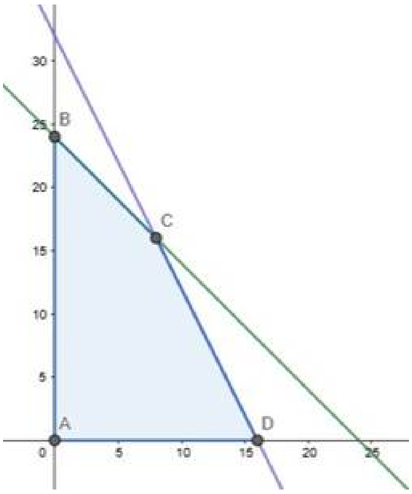
Let the firm manufacture x number of A and y number of B products.

∴According to the question,

X + y$\leq 24, x + 0.5y \leq 16, x \geq 0, y \geq 0$

Maximize Z = 300x + 160y

The feasible region determined X + y$\leq 24, x + 0.5y \leq 16, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,24) , C(8,16) , D(16,0).The value of Z at corner point is

Corner Point	Z = 300x + 160y	
A(0,0)	0	
B(0,24)	3840	
C(8,16)	4960	Maximum
D(16,0)	4800	

The maximum value of Z is 4960 and occurs at point (8,16).

The firm should produce 8 Aproducts and 16 B products to earn maximum profit of Rs.4960.

35. Question

A manufacture produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type required 3 hours on machine A and 2 hours on Machine A and 2 hours on machine B. Machine A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹30 and ₹25 per trunk of the first type and second type respectively. How may trunks of each type must he make each day to make the maximum profit?

Answer

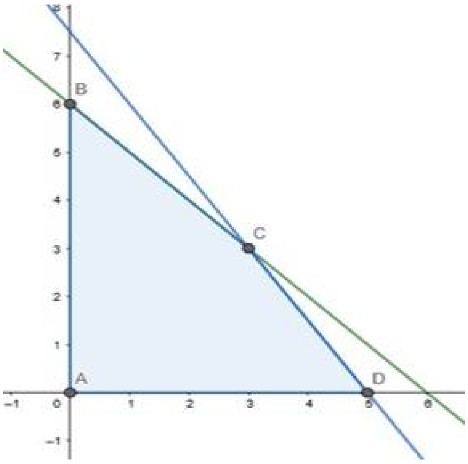
Let the manufacturer manufacture x and y numbers of type 1 and type 2trunks.

∴According to the question,

3X + 3y $\leq 18, 3x + 2y \leq 15, x \geq 0, y \geq 0$

Maximize Z = 30x + 25y

The feasible region determined 3X + 3y $\leq 18, 3x + 2y \leq 15, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,6) , C(3,3) , D(5,0).The value of Z at corner point is

Corner Point	Z = 30x + 25y	
A(0,0)	0	
B(0,6)	150	
C(3,3)	165	Maximum
D(5,0)	150	

The maximum value of Z is165 and occurs at point (3,3).

The manufacturer should manufacture 3 trunks of each type to earn maximum profit of Rs.165.

35. Question

A manufacture produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type required 3 hours on machine A and 2 hours on Machine A and 2 hours on machine B. Machine A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹30 and ₹25 per trunk of the first type and second type respectively. How may trunks of each type must he make each day to make the maximum profit?

Answer

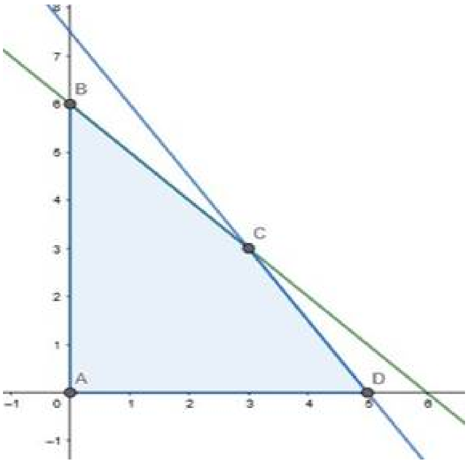
Let the manufacturer manufacture x and y numbers of type 1 and type 2trunks.

∴According to the question,

3X + 3y $\leq 18, 3x + 2y \leq 15, x \geq 0, y \geq 0$

Maximize Z = 30x + 25y

The feasible region determined 3X + 3y $\leq 18, 3x + 2y \leq 15, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,6) , C(3,3) , D(5,0).The value of Z at corner point is

Corner Point	Z = 30x + 25y	
A(0,0)	0	
B(0,6)	150	
C(3,3)	165	Maximum
D(5,0)	150	

The maximum value of Z is165 and occurs at point (3,3).

The manufacturer should manufacture 3 trunks of each type to earn maximum profit of Rs.165.

36. Question

A company manufactures two types of toys, A and B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B required 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is ₹50 each on type A and ₹60 each on type B. how many toys of each types should the company manufactures in a day to maximize the profit?

Answer

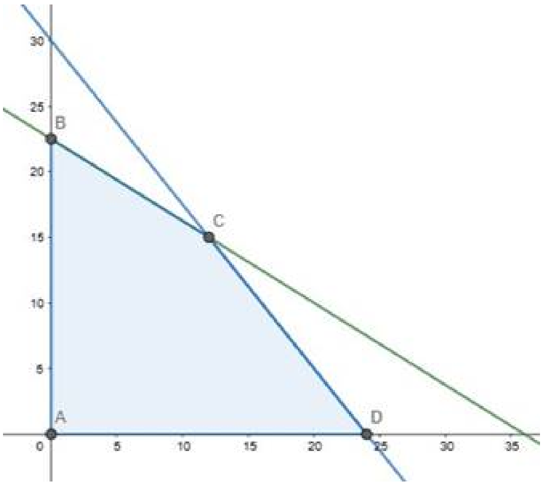
Let the company manufacture x and y numbers of toys A and B.

∴According to the question,

5X + 8y $\leq 180, 10x + 8y \leq 240, x \geq 0, y \geq 0$

Maximize Z = 50x + 60y

The feasible region determined 5X + 8y $\leq 180, 10x + 8y \leq 240, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,22.5) , C(12,15) , D(24,0).The value of Z at corner point is

Corner Point	Z = 50x + 60y	
A(0,0)	0	
B(0,22.5)	1350	
C(12,15)	1500	Maximum
D(24,0)	1200	

The maximum value of Z is1500 and occurs at point (12,15).

The company should manufacture 12 A toys and 15 B toys to earn profit of rupees 1500.

36. Question

A company manufactures two types of toys, A and B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B required 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is ₹50 each on type A and ₹60 each on type B. how many toys of each types should the company manufactures in a day to maximize the profit?

Answer

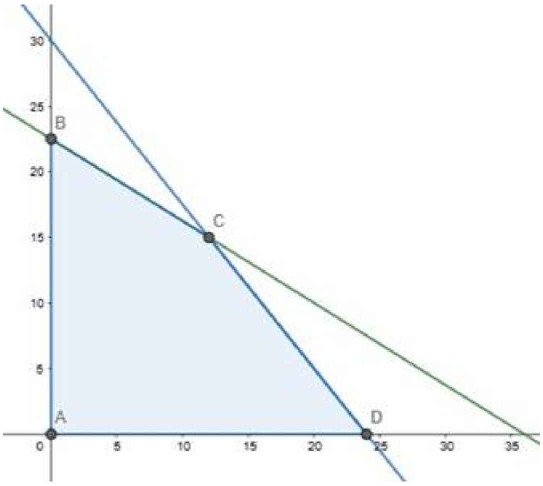
Let the company manufacture x and y numbers of toys A and B.

∴According to the question,

5X + 8y $\leq 180, 10x + 8y \leq 240, x \geq 0, y \geq 0$

Maximize Z = 50x + 60y

The feasible region determined 5X + 8y $\leq 180, 10x + 8y \leq 240, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,22.5) , C(12,15) , D(24,0).The value of Z at corner point is

Corner Point	Z = 50x + 60y	
A(0,0)	0	
B(0,22.5)	1350	
C(12,15)	1500	Maximum
D(24,0)	1200	

The maximum value of Z is1500 and occurs at point (12,15).

The company should manufacture 12 A toys and 15 B toys to earn profit of rupees 1500.

37. Question

Kellogg is a new cereal formed of a mixture of bran and rice, that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilograms, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost producing this new cereal if bran costs ₹5 per kilogram and rice costs ₹4 per kilogram.

Answer

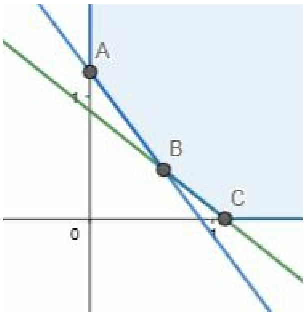
Let x and y be number of kilograms of bran and rice.

∴According to the question,

80x + 100y ≥ 88 , 40x + 30y $\geq 36, x \geq 0, y \geq 0$

Minimize Z = 5x + 4y

The feasible region determined 80x + 100y ≥ 88 , 40x + 30y $\geq 36, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,1.2) , B(0.6,0.4) , C(1.1,0).The value of Z at corner points are

Corner Point	Z = 5x + 4y	
A(0,1.2)	4.8	
B(0.6,0.4)	4.6	Minimum
C(1.1,0)	5.5	

The minimum value of Z is 4.6 at point (0.6,0.4).

Hence, the diet should contain 0.6 kgs of bran and 0.4 kgs of rice for achieving minimum cost of Rs.4.6.

37. Question

Kellogg is a new cereal formed of a mixture of bran and rice, that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilograms, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost producing this new cereal if bran costs ₹5 per kilogram and rice costs ₹4 per kilogram.

Answer

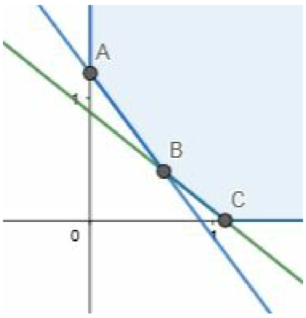
Let x and y be number of kilograms of bran and rice.

∴According to the question,

80x + 100y ≥ 88 , 40x + 30y $\geq 36, x \geq 0, y \geq 0$

Minimize Z = 5x + 4y

The feasible region determined 80x + 100y ≥ 88 , 40x + 30y $\geq 36, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,1.2) , B(0.6,0.4) , C(1.1,0).The value of Z at corner points are

Corner Point	Z = 5x + 4y	
A(0,1.2)	4.8	
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The minimum value of Z is 4.6 at point (0.6,0.4).

Hence, the diet should contain 0.6 kgs of bran and 0.4 kgs of rice for achieving minimum cost of Rs.4.6.

38. Question

A dealer wishes to purchase a number of fans and sewing machines. He has only ₹5760 to invest and has space for at most 20 items. A fan costs him ₹360 and a sewing machine ₹240. He expects to sell a fan at a profit of ₹22 and a sewing machine at a profit of ₹18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? Solve the graphically and find the maximum profit.

Answer

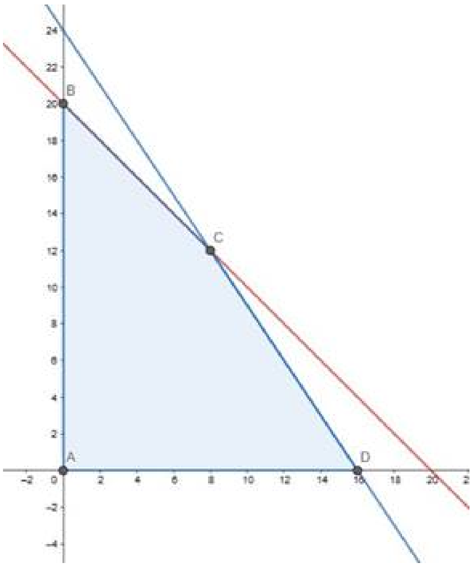
Let the number of fans bought be x and sewing machines bought be y.

∴According to the question,

360x + 240y ≤ 5760,x + y $\leq 20, x \geq 0, y \geq 0$

Maximize Z = 22x + 18y

The feasible region determined by 360x + 240y ≤ 5760,x + y $\leq 20, x \geq 0, y \geq 0$ is given by



The corner points of the feasible region are A(0,0) , B(0,20),C(8,12) , D(16,0).The value of Z at corner points is

Corner Point	Z = 22x + 18y	
A(0,0)	0	
B(0,20)	360	
C(8,12)	392	Maximum
D(16,0)	352	

The maximum value of Z is 392 at point (8,12).

The dealer must buy 8 fans and 12 sewing machines to make the maximum profit.

38. Question

A dealer wishes to purchase a number of fans and sewing machines. He has only ₹5760 to invest and has space for at most 20 items. A fan costs him ₹360 and a sewing machine ₹240. He expects to sell a fan at a profit of ₹22 and a sewing machine at a profit of ₹18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? Solve the graphically and find the maximum profit.

Answer

Let the number of fans bought be x and sewing machines bought be y.

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360x + 240y ≤ 5760,x + y$\leq 20, x \geq 0, y \geq 0$

Maximize Z = 22x + 18y

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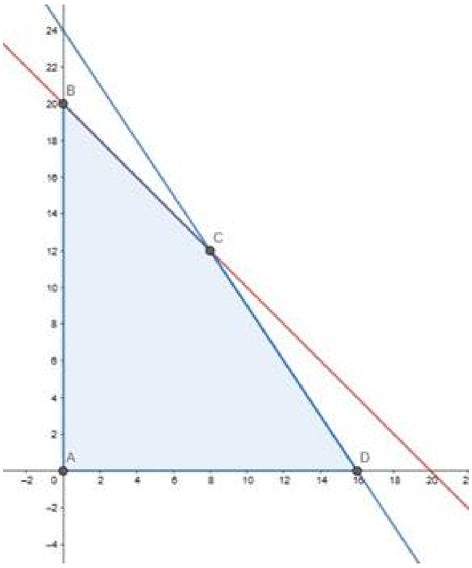

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B(0,20)	360	
C(8,12)	392	Maximum
D(16,0)	352	

The maximum value of Z is 392 at point (8,12).

The dealer must buy 8 fans and 12 sewing machines to make the maximum profit.

39. Question

Anil wants to invest at the most ₹12000 in bonds A and B. According to rules, he has to invest at least ₹2000 in bond A and at least ₹4000 in bond B. if the rate of interest of bond A is 8% per annum and on bond B, it is 10% per annum, how should he invest his money for maximum interest? Solve the problem graphically.

Answer

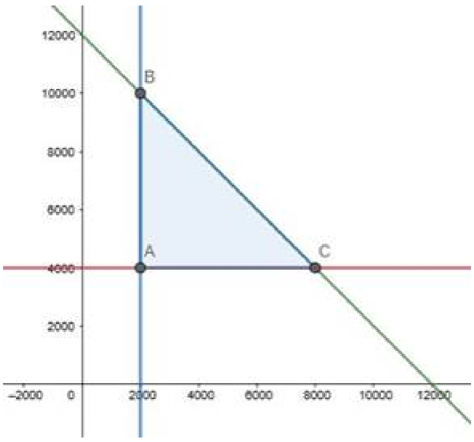
Let the invested money in bond A be x and in bond B be y.

∴According to the question,

X + y $\leq$ 12000 , x $\geq$ 2000 , y $\geq$ 4000

Maximize Z = 0.08x + 0.10y

The feasible region determined by X + y $\leq$ 12000 , x $\geq$ 2000 , y $\geq$ 4000 is given by



The corner points of the feasible region are A(2000,4000) , B(2000,10000) and C(8000,4000) . The value of Z at the corner point are

Corner Point	Z = 0.08x + 0.10y	
A(2000,4000)	560	
B(2000,10000)	1160	Maximum
C(8000,4000)	1040	

The maximum value of Z is 116770 at point (2000,10000)

So, he must invest Rs.2000 in bond A and Rs.10000 in bond B.

The maximum annual income is Rs.1160 .

39. Question

Anil wants to invest at the most ₹12000 in bonds A and B. According to rules, he has to invest at least ₹2000 in bond A and at least ₹4000 in bond B. if the rate of interest of bond A is 8% per annum and on bond B, it is 10% per annum, how should he invest his money for maximum interest? Solve the problem graphically.

Answer

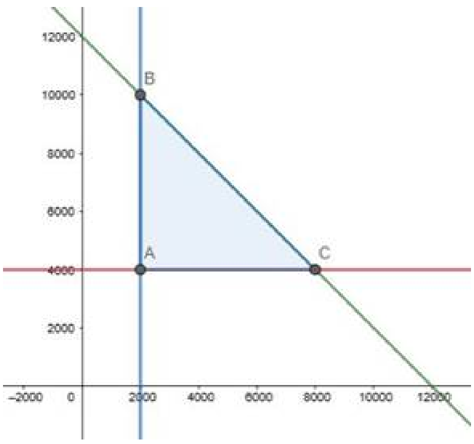
Let the invested money in bond A be x and in bond B be y.

∴According to the question,

X + y $\leq$ 12000 , x $\geq$ 2000 , y $\geq$ 4000

Maximize Z = 0.08x + 0.10y

The feasible region determined by X + y $\leq$ 12000 , x $\geq$ 2000 , y $\geq$ 4000 is given by



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Corner Point	Z = 0.08x + 0.10y	
A(2000,4000)	560	
B(2000,10000)	1160	Maximum
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The maximum value of Z is 116770 at point (2000,10000)

So, he must invest Rs.2000 in bond A and Rs.10000 in bond B.

The maximum annual income is Rs.1160 .

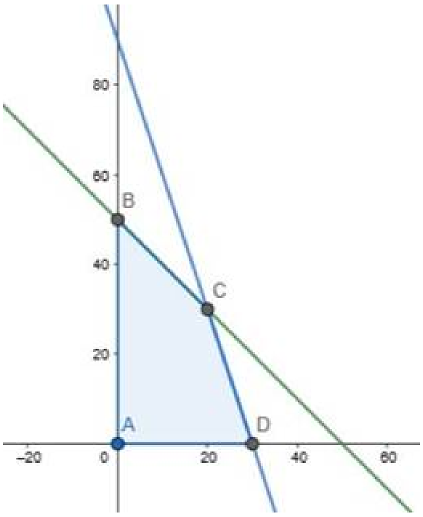
40. Question

Maximize = 60x + 15y, subject to the constraints

x + y $\leq$ 50, 3x + $\leq$ y90, x, y $\geq$ 0.

Answer

The feasible region determined by the constraints x + y $\leq$ 50, 3x + y $\leq$ 90, x, y $\geq$ 0. is given by



The corner points of feasible region are A(0,0) ,B(0,50) ,C(20,30), D(30,0) . The values of Z at the following points is

Corner Point	Z = 60x + 15y	
A(0,0)	0	
B(0,50)	750	
C(20,30)	1650	
D(30,0)	1800	Maximum

The maximum value of Z is 1800 at point A(30,0) .

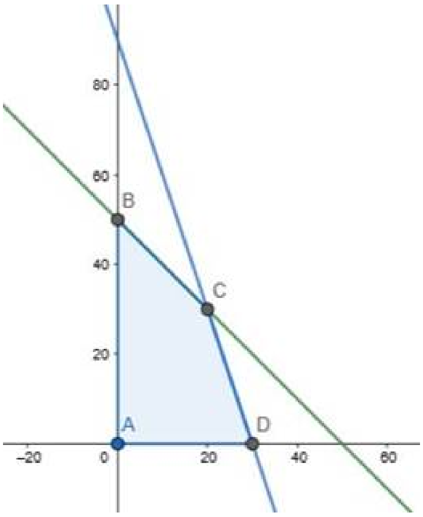
40. Question

Maximize = 60x + 15y, subject to the constraints

x + y $\leq$ 50, 3x + $\leq$ y90, x, y $\geq$ 0.

Answer

The feasible region determined by the constraints x + y $\leq$ 50, 3x + y $\leq$ 90, x, y $\geq$ 0. is given by



The corner points of feasible region are A(0,0) ,B(0,50) ,C(20,30), D(30,0) . The values of Z at the following points is

Corner Point	Z = 60x + 15y	
A(0,0)	0	
B(0,50)	750	
C(20,30)	1650	
D(30,0)	1800	Maximum

The maximum value of Z is 1800 at point A(30,0) .

41. Question

A company manufacture two types of toys A and B. type A requires 5 minutes each for cutting and 10 minutes for each assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. He earns a profit of ₹50 each on type A and ₹60 each on type B. How many toys of each type should the company manufacture in a day to maximize the profit?

Answer

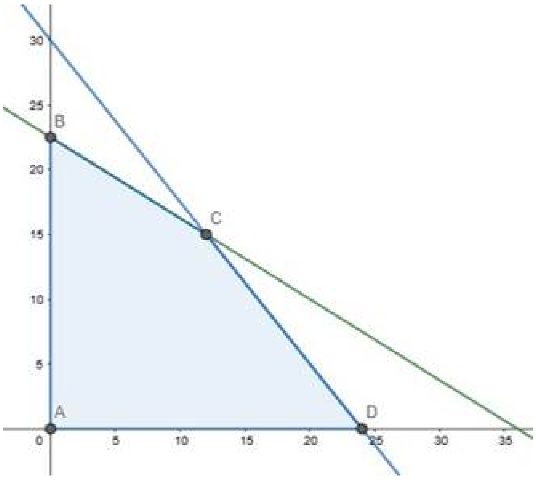
Let the company manufacture x and y numbers of toys A and B.

∴According to the question,

5X + 8y $\leq 180, 10x + 8y \leq 240, x \geq 0, y \geq 0$

Maximize Z = 50x + 60y

The feasible region determined 5X + 8y $\leq 180, 10x + 8y \leq 240, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,22.5) , C(12,15) , D(24,0).The value of Z at corner point is

Corner Point	Z = 50x + 60y	
A(0,0)	0	
B(0,22.5)	1350	
C(12,15)	1500	Maximum
D(24,0)	1200	

The maximum value of Z is1500 and occurs at point (12,15).

The company should manufacture 12 A toys and 15 B toys to earn profit of rupees 1500.

41. Question

A company manufacture two types of toys A and B. type A requires 5 minutes each for cutting and 10 minutes for each assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. He earns a profit of ₹50 each on type A and ₹60 each on type B. How many toys of each type should the company manufacture in a day to maximize the profit?

Answer

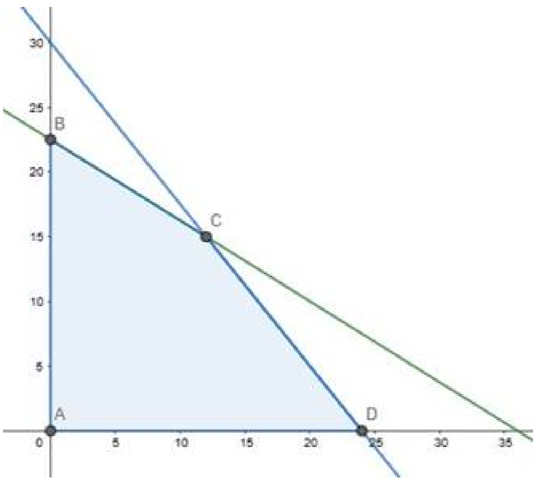
Let the company manufacture x and y numbers of toys A and B.

∴According to the question,

5X + 8y $\leq 180, 10x + 8y \leq 240, x \geq 0, y \geq 0$

Maximize Z = 50x + 60y

The feasible region determined 5X + 8y $\leq 180, 10x + 8y \leq 240, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,22.5) , C(12,15) , D(24,0).The value of Z at corner point is

Corner Point	Z = 50x + 60y	
A(0,0)	0	
B(0,22.5)	1350	
C(12,15)	1500	Maximum
D(24,0)	1200	

The maximum value of Z is1500 and occurs at point (12,15).

The company should manufacture 12 A toys and 15 B toys to earn profit of rupees 1500.

42. Question

One kind of cake requires 200 g of flour and 25 g of fat, another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it an LPP and solve it graphically.

Answer

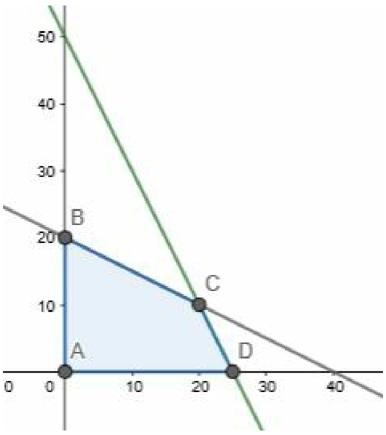
Let the company make x no of 1^{st} kind and y no of 2^{nd} cakes.

∴According to the question,

200x + 100y $\leq 5000, 25x + 50y \leq 1000, x \geq 0, y \geq 0$

Maximize Z = x + y

The feasible region determined by 200x + 100y $\leq 5000, 25x + 50y \leq 1000, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,20) , C(20,10) , D(25,0).The value of Z at corner point is

Corner Point	Z = x + y	
A(0,0)	0	
B(0,20)	20	
C(20,10)	30	Maximum
D(25,0)	25	

The maximum value of Z is 30 and occurs at point (20,10).

The company should make 20 of 1^{st} type and 10 of 2^{nd} type.

42. Question

One kind of cake requires 200 g of flour and 25 g of fat, another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it an LPP and solve it graphically.

Answer

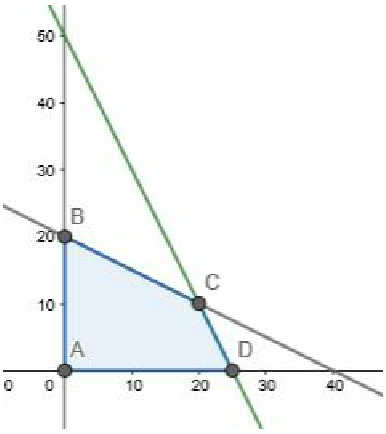
Let the company make x no of 1^{st} kind and y no of 2^{nd} cakes.

∴According to the question,

$200x + 100y \leq 5000, 25x + 50y \leq 1000, x \geq 0, y \geq 0$

Maximize Z = x + y

The feasible region determined by $200x + 100y \leq 5000, 25x + 50y \leq 1000, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,20) , C(20,10) , D(25,0).The value of Z at corner point is

Corner Point	Z = x + y	
A(0,0)	0	
B(0,20)	20	
C(20,10)	30	Maximum
D(25,0)	25	

The maximum value of Z is 30 and occurs at point (20,10).

The company should make 20 of 1st type and 10 of 2nd type.

43. Question

A manufacturing company makes two types of teaching aids A and B of mathematics for class XII. Each type of A requires 9 labor hours of fabricating and 1 labor hour for finishing. Each type of B requires 12 labors hour for fabricating and 3 labor hour for finishing. For fabricating and finishing, the maximum labor hours available per week are 180 and 30 respectively. The company makes a profit of ₹80 on each piece of type A and ₹120 on each piece of type B. how many pieces of type A and type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LLP and solve graphically. What is the maximum profit per week?

Answer

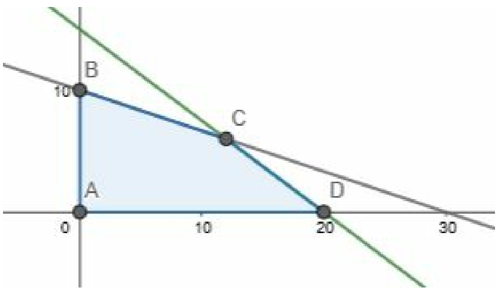
Let the company make x no of 1st type of teaching aid and y no of 2nd type of teaching aid.

∴According to the question,

9x + 12y $\leq 180, x + 3y \leq 30, x \geq 0, y \geq 0$

Maximize Z = 80x + 120y

The feasible region determined by 9x + 12y $\leq 180, x + 3y \leq 30, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,10) , C(12,6) , D(20,0).The value of Z at corner point is

Corner Point	Z = 80x + 120y	
A(0,0)	0	
B(0,10)	1200	
C(12,6)	1680	Maximum
D(20,0)	1600	

The maximum value of Z is 1680 and occurs at point (12,6).

The company should make 12 of 1st type and 6 of 2nd type of teaching aid. Maximum profit is Rs.1680.

43. Question

A manufacturing company makes two types of teaching aids A and B of mathematics for class XII. Each type of A requires 9 labor hours of fabricating and 1 labor hour for finishing. Each type of B requires 12 labors hour for fabricating and 3 labor hour for finishing. For fabricating and finishing, the maximum labor hours available per week are 180 and 30 respectively. The company makes a profit of ₹80 on each piece of type A and ₹120 on each piece of type B. how many pieces of type A and type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LLP and solve graphically. What is the maximum profit per week?

Answer

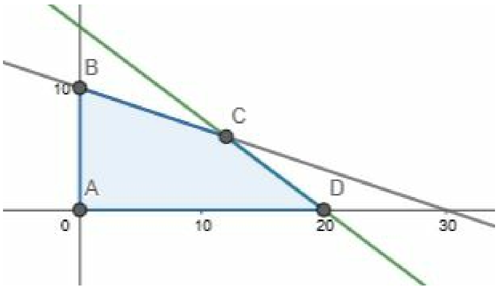
Let the company make x no of 1st type of teaching aid and y no of 2nd type of teaching aid.

∴According to the question,

9x + 12y $\leq 180, x + 3y \leq 30, x \geq 0, y \geq 0$

Maximize Z = 80x + 120y

The feasible region determined by 9x + 12y $\leq 180, x + 3y \leq 30, x \geq 0, y \geq 0$ is given by



The corner points of feasible region are A(0,0) , B(0,10) , C(12,6) , D(20,0).The value of Z at corner point is

Corner Point	Z = 80x + 120y	
A(0,0)	0	
B(0,10)	1200	
C(12,6)	1680	Maximum
D(20,0)	1600	

The maximum value of Z is 1680 and occurs at point (12,6).

The company should make 12 of 1^{st} type and 6 of 2^{nd} type of teaching aid. Maximum profit is Rs.1680.