Ratio Proportion and Unitary Method
Ex 10A

Ratio and Proportion

- A ratio is a comparison of two values expressed as a quotient
  - Example: A class has 12 girls and 18 boys. The ratio of girls to boys is $\frac{12}{18}$
  - This ratio can also be expressed as an equivalent fraction $\frac{2}{3}$
- A proportion is an equation stating that two ratios are equal.
  - Example: $\frac{12}{18} = \frac{2}{3}$

1. **Ratio:**

   The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

   In the ratio $a : b$, we call a as the first term or **antecedent** and b, the second term or **consequent**.

   Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

   Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

   Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. **Proportion:**

   The equality of two ratios is called proportion.

   If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

   Here a and d are called **extremes**, while b and c are called **mean terms**.

   Product of means = Product of extremes.

   Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. **Fourth Proportional:**

   If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

   Third Proportional:

   $a : b = c : d$, then c is called the third proportion to a and b.

   Mean Proportional:

   Mean proportional between a and b is $\sqrt{ab}$.

4. **Comparison of Ratios:**

   We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

   Compounded Ratio:

   The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

5. **Duplicate Ratios:**

   Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

   Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

   Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

   Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

6. **Variations:**

   We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

   We say that x is inversely proportional to y, if $xy = k$ for some constant k *and*

   we write, $x \propto \frac{1}{y}$.

Properties of proportions:

Convertendo: If a : b :: c : d ,then a : (a - b) :: c : (c – d) .

Invertendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{b}{a}=\frac{d}{c}$.

Alternendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a}{c}=\frac{b}{d}$.

Componendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{b}=\frac{c+d}{d}$.

Dividendo: $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a-b}{b}=\frac{c-d}{d}$

Componendo and Dividendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

Q1

Answer :

(i) 24:56 = $\frac{24}{56}=\frac{24\div8}{56\div8}=\frac{3}{7}$

As the H.C.F. of 3 and 7 is 1, the simplest form of 24:56 is 3:7.

(ii) 84 paise to Rs 3 = Rs 0.84 to R. 3 = $\frac{0.84}{3}=\frac{0.84\div3}{3\div3}=\frac{0.28}{1}=\frac{28}{100}=\frac{28\div4}{100\div4}=\frac{7}{25}$

As the H.C.F. of 7 and 25 is 1, the simplest form of 0.84:3 is 7:25.

(iii) 4 kg:750 g = 4000 g:750 g = $\frac{4000\div250}{750\div250}=\frac{16}{3}$

As the H.C.F. of 16 and 3 is 1, the simplest form of 4000:750 is 16:3.

(iv) 1.8 kg:6 kg = $\frac{1.8}{6}=\frac{18}{60}=\frac{18\div6}{60\div6}=\frac{3}{10}$

As the H.C.F. of 3 and 10 is 1, the simplest form of 1.8:6 is 3:1.

(v) 48 minutes to 1 hour = 48 minutes to 60 minutes = 48:60 = $\frac{48\div12}{60\div12}=\frac{4}{5}$

As the H.C.F. of 4 and 5 is 1, the simplest form of 48:60 is 4:5.

(vi) 2.4 km to 900 m = 2400m:900m = $\frac{2400}{900}=\frac{24}{9}=\frac{24\div3}{9\div3}=\frac{8}{3}$

As the H.C.F. of 8 and 3 is 1, the simplest form of 2400:900 is 8:3.

Q2

Answer :

(i) 36:90 = $\frac{36}{90}=\frac{36\div18}{90\div18}=\frac{2}{5}$ (As the H.C.F. of 36 and 90 is 18.)

Since the H.C.F. of 2 and 5 is 1, the simplest form of 36:90 is 2:5.

(ii) 324:144 = $\frac{324}{144}=\frac{324\div36}{144\div36}=\frac{9}{4}$ (As the H.C.F. of 324 and 144 is 36.)

Since the H.C.F. of 9 and 4 is 1, the simplest form of 324:144 is 9:4.

(iii) 85:561 = $\frac{85}{561}=\frac{85\div17}{561\div17}=\frac{5}{33}$ (As the H.C.F. of 85 and 561 is 17.)

Since the H.C.F. of 5 and 33 is 1, the simplest form of 85:561 is 5:33.

(iv) 480:384 = $\frac{480}{384}=\frac{480\div96}{384\div96}=\frac{5}{4}$ (As the H.C.F. of 480 and 384 is 96.)

Since the H.C.F. of 5 and 4 is 1, the simplest form of 480:384 is 5:4.

(v) 186:403 = $\frac{186}{403}=\frac{186\div31}{403\div31}=\frac{6}{13}$ (As the H.C.F. of 186 and 403 is 31.)

Since the H.C.F. of 6 and 13 is 1, the simplest form of 186:403 is 6:13.

(vi) 777:1147 = $\frac{777\div37}{1147\div37}=\frac{21}{31}$ (As the H.C.F. of 777 and 1147 is 37.)

Since the H.C.F. of 21 and 31 is 1, the simplest form of 777:1147 is 21:31.

Q3

Answer :

(i) Rs 6.30:Rs 16.80

$\frac{6.30}{16.80} = \frac{63}{168} = \frac{63 \div 21}{168 \div 21} = \frac{3}{8}$ (H.C.F. of 63 and 168 is 21.)

Ratio = 3 : 8

(ii)3 weeks:30 days = 21days:30 days (1 week = 7 days)

$\frac{21}{30} = \frac{21 \div 3}{30 \div 3} = \frac{7}{10}$ (H.C.F. of 21 and 30 is 3.)

Ratio = 7 : 10

(iii) 3 m 5 cm:35 cm = 305 cm:35 cm (1 m = 100 cm)

$\frac{305}{35} = \frac{305 \div 5}{35 \div 5} = \frac{61}{7}$ (H.C.F. of 305 and 35 is 5.)

Ratio = 61:7

(iv) 48 min:2 hours 40 min = 48 min:160 min (1 hour = 60 mins)

$\frac{48}{160} = \frac{48 \div 16}{160 \div 16} = \frac{3}{10}$ (H.C.F. of 48 and 160 is 16.)

Ratio = 3:10

(v) 1 L 35 mL:270 mL = 1035 mL:270 mL (1 L = 1000 mL)

$\frac{1035}{270} = \frac{1035 \div 45}{270 \div 45} = \frac{23}{6}$ (H.C.F. of 1035 and 270 is 45.)

Ratio = 23:6

(vi) 4 kg:2 kg 500 g = 4000 g:2500 g (1 kg= 1000 g)

$\frac{4000}{2500} = \frac{40}{25} = \frac{40 \div 5}{25 \div 5} = \frac{8}{5}$ (H.C.F. of 40 and 25 is 5.)

Ratio = 8:5

Q4

Answer :

Mr Sahai's earning = Rs 16800

Mrs Sahai's earning = Rs 10500

(i) Ratio = 16800:10500 = 168:105 = $\frac{168 \div 21}{105 \div 21} = \frac{8}{5}$ (H.C.F. of 168 and 105 is 21.)

Mr Sahai's income:Mrs Sahai's income = 8:5

(ii)Ratio = 10500:16800 = 105:168 = $\frac{105 \div 21}{168 \div 21} = \frac{5}{8}$ (H.C.F. of 168 and 105 is 21.)

Mrs Sahai's income:Mr Sahai's income = 5:8

(iii) Total income = 16800 + 10500 = Rs 27300

Ratio = 16800:27300 = 168:273 = $\frac{168}{273} = \frac{168 \div 21}{273 \div 21} = \frac{8}{13}$ (H.C.F. of 168 and 273 is 21.)

Mrs Sahai's income:Total income = 8:13

Q5

Answer :

Rohit's income = Rs 15300
Rohit's savings = Rs 1224
(i) Income:Savings = 15300:1224 = $\frac{15300 \div 612}{1224 \div 612} = \frac{25}{2}$ (H.C.F. of 15300 and 1224 is 612.)
Income:Savings = 25:2
(ii) Monthly expenditure = Rs (15300 − 1224) = Rs 14076
Income:Expenditure = 15300:14076 = $\frac{15300 \div 612}{14076 \div 612} = \frac{25}{23}$ (H.C.F. of 15300 and 14076 is 612.)
Income:Expenditure = 25:23
(iii) Expenditure : Savings = 14076:1224 = $\frac{14076 \div 612}{1224 \div 612} = \frac{23}{2}$ (H.C.F. of 14076 and 1224 is 612.)
Expenditure:Savings = 23:2

Q6

Answer :

Number of male:Number of female = 5:3
Let the number be x.
Number of male = $5x$
Number of female = $3x$
Number of male workers = 115
Now, $5x = 115$
$\Rightarrow x = \frac{115}{5} = 23$
Number of female workers in the mill = $3x = 3 \times 23 = 69$

Q7

Answer :

Boys:Girls = 9:5
Let the number of boys = $9x$
Let the number of girls = $5x$
Total strength of the school = 448
According to given condition, we have:

$$9x + 5x = 448$$
$$\Rightarrow 14x = 448$$
$$\Rightarrow x = \frac{448}{14} = 32$$

Number of boys = $9x = 9 \times 32 = 288$
Number of girls = $5x = 5 \times 32 = 160$

Q8

Answer :

Kamal:Madhu = 7:2
Sum of the ratio terms = 7 + 2 = 9
Kamal's share = $\frac{7}{9} \times 1575 = \frac{11025}{9}$ = Rs 1225
Madhu's share = $\frac{2}{9} \times 1575 = \frac{3150}{9}$ = Rs 350

Q9

Answer :

A:B:C = 3:5:7
Sum of the ratio terms = 3 + 5 +7 = 15
A's share = $\frac{3}{15} \times 3450 = \frac{10350}{15}$ = Rs 690

B's share = $\frac{5}{15} \times 3450 = \frac{17250}{15}$ = Rs 1150

C's share = $\frac{7}{15} \times 3450 = \frac{24150}{15}$ = Rs 1610

Q10

Answer :

Two number are in the ratio 11:12.
Let the numbers be $11x$ and $12x$.
Given: $11x + 12x = 460$
$\Rightarrow 23x = 460$
$\Rightarrow x = \frac{460}{23} = 20$
First number = $11x = 11 \times 20 = 220$
Second number = $12x = 12 \times 20 = 240$
Hence, the numbers are 220 and 240.

Q11

Answer :

Ratio of the two parts of line segment = 4:3
Sum of the ratio terms = 4 + 3 = 7
First part = $\frac{4}{7} \times 35$ cm = 4 × 5 cm = 20 cm
Second part = $\frac{3}{7} \times 35$ cm = 3 × 5 cm = 15 cm

Q12

Answer :

Number of bulbs produced each day = 630
Out of 10 bulbs, 1 is defective.
Number of defective bulbs = $\frac{630}{10} = 63$

$\therefore$ Number of defective bulbs produced each day = 63

Q13

Answer :

Price of pencil = Rs 96 per score

Price of ball pen = Rs 50.40 per dozen

Price per unit of pencil = $\frac{96}{20} = 4.8$

Price per unit of ball pen = $\frac{50.40}{12} = 4.2$

Ratio = $\frac{4.8}{4.2} = \frac{48}{42} = \frac{48 \div 6}{42 \div 6} = \frac{8}{7}$

Price of a pencil:Price of a ball pen = 8:7

Q14

Answer :

Length:Width = 5:3

Let the length and the width of the field be $5x$ m and $3x$ m, respectively.

Width = 42 m

$3x = 42$

$x = \frac{42}{3} = 14$

$\therefore$ Length = $5x = 5 \times 14 = 70$ metres

Q15

Answer :

Income:Savings = 11:2

Let the income and the saving be Rs $11x$ and Rs $2x$, respectively.

Saving = Rs 1520

$2x = 1520$

$x = \frac{1520}{2} = 760$

$\therefore$ Income = Rs $11x$ =Rs (11×760) = Rs 8360

Expenditure = Income — Saving

= Rs (8360 — 1520)

= Rs 6840

Q16

Answer :

Income:Expenditure = 7:6

Let the income and the expenditure be Rs $7x$ and Rs $6x$, respectively.

Income = Rs 14000

$7x = 14000$

$x = \frac{14000}{7} = 2000$

Expenditure = Rs $6x$ = Rs 6×2000 = Rs 12000

$\therefore$ Saving = Income — Expenditure

= Rs (14000 — 12000)

= Rs 2000

Q17

Answer :

Let the weight of zinc be x kg.

Ratio of zinc and copper = 7:9

Weight of copper in the alloy = 11.7 kg

$\frac{7}{9} = \frac{x}{11.7}$

$\Rightarrow x = \frac{11.7 \times 7}{9} = \frac{81.9}{9} = 9.1$

Weight of zinc = 9.1 kg

Q18

Answer :

A bus covers 128 km in 2 hours.

Speed of the bus = $\frac{\text{Distance}}{\text{Time}} = \frac{128 \text{ km}}{2 \text{ hr}}$ = 64 km/ hr

A train covers 240 km in 3 hours.

Speed of the train = $\frac{\text{Distance}}{\text{Time}} = \frac{240}{3}$ = 80 km /hr

Ratio of their speeds = 64:80 = $\frac{64}{80} = \frac{64 \div 16}{80 \div 16} = \frac{4}{5}$

∴ Ratio of the speeds of the bus and the train = 4:5

Q19

Answer :

(i) (3:4) or (9:16)

Making the denominator equal:

$\frac{3 \times 4}{4 \times 4} = \frac{12}{16}$ and $\frac{12}{16} > \frac{9}{16}$

∴ (3:4) > (9:16)

(ii) (5:12) or (17:30)

Making the denominator equal:

$\frac{5 \times 5}{12 \times 5} = \frac{25}{60}$ and $\frac{17 \times 2}{30 \times 2} = \frac{34}{60}$

$\Rightarrow \frac{25}{60} < \frac{34}{60}$

∴ (5:12) < (17:30)

(iii) (3:7) or (4:9)

Making the denominator equal:

$\frac{3 \times 9}{7 \times 9} = \frac{27}{63}$ and $\frac{4 \times 7}{9 \times 7} = \frac{28}{63}$

$\Rightarrow \frac{27}{63} < \frac{28}{63}$

(3:7) < (4:9)

(iv) (1:2) or (13:27)

Making the denominator equal:

$\frac{1 \times 27}{2 \times 27} = \frac{27}{54}$ and $\frac{13 \times 2}{27 \times 2} = \frac{26}{54}$

$\Rightarrow \frac{27}{54} > \frac{26}{54}$

(1:2) > (13:27)

Q20 **Answer :**

(i) $\frac{24}{40} = \frac{24 \div 8}{40 \div 8} = \frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$

(ii) $\frac{36}{63} = \frac{36 \div 9}{63 \div 9} = \frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$

(iii) $\frac{5}{7} = \frac{5 \times 4}{7 \times 4} = \frac{20}{28} = \frac{5 \times 7}{7 \times 7} = \frac{35}{49}$

Ratio Proportion and Unitary Method
Ex 10B

Q1

Answer :

(i) 4, 6, 8, 12

$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$; $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

Hence, 4:9::8:12 are in proportion.

(ii) 7, 42, 13, 78

$\frac{7}{42} = \frac{7 \div 7}{42 \div 7} = \frac{1}{6}$; $\frac{13}{78} = \frac{13 \div 13}{78 \div 13} = \frac{1}{6}$

Hence, 7:42::13:78 are in proportion.

(iii) 33, 121, 9, 96

$\frac{33}{121} = \frac{33 \div 11}{121 \div 11} = \frac{3}{11}$; $\frac{9}{96} = \frac{9 \div 3}{96 \div 3} = \frac{3}{32}$

Hence, 33:121::9:96 are not in proportion.

(iv) 22, 33, 42, 63

$\frac{22}{33} = \frac{22 : 11}{33 : 11} = \frac{2}{3}$ and $\frac{42}{63} = \frac{42 : 21}{63 : 21} = \frac{2}{3}$

Hence, 22:33 :: 42 : 63 are not in proportion.

(v) 32, 48, 70, 210

$\frac{32}{48} = \frac{32 \div 6}{48 \div 6} = \frac{7}{8}$; $\frac{70}{210} = \frac{70 \div 70}{210 \div 70} = \frac{1}{3}$

Hence, 32:48::70:210 are not in proportion.

(vi) 150, 200, 250, 300

$\frac{150}{200} = \frac{150 \div 50}{200 \div 50} = \frac{3}{4}$; $\frac{250}{300} = \frac{250 \div 50}{300 \div 50} = \frac{5}{6}$

Hence, 150:200::250:300 are not in proportion.

Q2

Answer :

(i) 60:105::84:147

$\frac{60}{105} = \frac{60 \div 15}{105 \div 15} = \frac{4}{7}$ (H.C.F. of 60 and 105 is 15.)

$\frac{84}{147} = \frac{84 \div 21}{147 \div 21} = \frac{4}{7}$ (H.C.F. of 84 and 147 is 21.)

Hence, 60:105::84:147 are in proportion.

(ii) 91:104::119:136

$\frac{91}{104} = \frac{91 \div 13}{104 \div 13} = \frac{7}{8}$ (H.C.F. of 91 and 104 is 13.)

$\frac{119}{136} = \frac{119 \div 17}{136 \div 17} = \frac{7}{8}$ (H.C.F. of 11 and 136 is 17.)

Hence, 91:104::119:136 are in proportion.

(iii) 108:72::129:86

$\frac{108}{72} = \frac{108 \div 36}{72 \div 36} = \frac{3}{2}$ (H.C.F. of 108 and 72 is 36.)

$\frac{129}{86} = \frac{129 \div 43}{86 \div 43} = \frac{3}{2}$ (H.C.F. of 129 and 86 is 43.)

Hence, 108:72::129:86 are in proportion.

(iv) 39:65::141:235

$\frac{39}{65} = \frac{39 \div 13}{65 \div 13} = \frac{3}{5}$ (H.C.F. of 39 and 65 is 13.)

$\frac{141}{235} = \frac{141 \div 47}{235 \div 47} = \frac{3}{5}$ (H.C.F. of 141 and 235 is 47.)

Hence, 39:65::141:235 are in proportion.

Q3

Answer :

(i) 55:11::x:6

Product of extremes = Product of means

$55 \times 6 = 11 \times x$

$\Rightarrow 11x = 330$

$\Rightarrow x = \frac{330}{11} = 30$

(ii) 27:x::63:84

Product of extremes = Product of means

$27 \times 84 = x \times 63$

$\Rightarrow 63x = 2268$

$\Rightarrow x = \frac{2268}{63} = 36$

(iii) 51:85::57:x

Product of extremes = Product of means

$51 \times x = 85 \times 57$

$\Rightarrow 51x = 4845$

$\Rightarrow x = \frac{4845}{51} = 95$

(iv) x:92::87:116

Product of extremes = Product of means

$x \times 116 = 92 \times 87$

$\Rightarrow 116x = 8004$

$\Rightarrow x = \frac{8004}{116} = 69$

Q4

Answer :

(i) 51:68::85:102

Product of means = $68 \times 85 = 5780$

Product of extremes = $51 \times 102 = 5202$

Product of means ≠ Product of extremes

Hence, (F).

(ii) 36:45::80:100

Product of means = $45 \times 80 = 3600$

Product of extremes = $36 \times 100 = 3600$

Product of means = Product of extremes

Hence, (T).

(iii) 30 bags:18 bags::Rs 450:Rs 270

or 30:18::450:270

Product of means = $18 \times 450 = 8100$

Product of extremes = $30 \times 270 = 8100$

Product of means = Product of extremes

Hence, (T).

(iv) 81 kg:45 kg::18 men:10 men

or 81:45::18:10

Product of means = $45 \times 18 = 810$

Product of extremes = $81 \times 10 = 810$

Product of means = Product of extremes

Hence, (T).

(v) 45 km:60 km::12 h:15 h

or,45:60::12:15

Product of means = $60 \times 12 = 720$

Product of extremes = $45 \times 15 = 675$

Product of means ≠ Product of extremes

Hence, (F).

(vi) 32 kg:Rs 36::8 kg:Rs 9

Product of means = $36 \times 8 = 288$

Product of extremes = $32 \times 9 = 288$

Product of means = Product of extremes

Hence, (T).

Q5

Answer :

(i) 25 cm:1 m and Rs 40:Rs 160 (or) 25 cm:100 cm and Rs 40:Rs 160

$\frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$ and $\frac{40}{160} = \frac{40 \div 40}{160 \div 40} = \frac{1}{4}$

Hence, they are in proportion.

(ii) 39 litres:65 litres and 6 bottles:10 bottles

$\frac{39}{65} = \frac{39 \div 13}{65 \div 13} = \frac{3}{5}$ and $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

Hence they are in proportion.

(iii) 200 mL:2.5 L and Rs 4:Rs 50 (or) 200 mL:2500 mL and Rs 4:Rs 50

$\frac{200}{2500} = \frac{2}{25}$ and $\frac{4}{50} = \frac{4 \div 2}{50 \div 2} = \frac{2}{25}$

Hence, they are in proportion.

(iv) 2 kg:80 kg and 25 g:625 kg (or) 2 kg:80 kg and 25 g:625000 g

$\frac{2}{80} = \frac{2 \div 2}{80 \div 2} = \frac{1}{40}$ and $\frac{25}{625000} = \frac{25 \div 25}{625000 \div 25} = \frac{1}{25000}$

Hence, they are not in proportion.

Q6

Answer :

Let the 3rd term be x.

Thus, 51:68::x:108

We know:

Product of extremes = Product of means

$51 \times 108 = 68 \times x$

$\Rightarrow 5508 = 68x$

$\Rightarrow x = \frac{5508}{68} = 81$

Hence, the third term is 81.

Q7

Answer :

Let the second term be x.

Then. 12:x::8:14

We know:

Product of extremes = Product of means

$12 \times 14 = 8x$

$\Rightarrow 168 = 8x$

$\Rightarrow x = \frac{168}{8} = 21$

Hence, the second term is 21.

Q8

Answer :

(i) 48:60, 60:75

Product of means = 60 × 60 = 3600

Product of extremes = 48 × 75 = 3600

Product of means = Product of extremes

Hence, 48:60::60:75 are in continued proportion.

(ii) 36:90, 90:225

Product of means = 90 × 90 = 8100

Product of extremes = 36 × 225 = 8100

Product of means = Product of extremes

Hence, 36:90::90:225 are in continued proportion.

(iii) 16:84, 84:441

Product of means = 84 × 84 = 7056

Product of extremes = 16 × 441 = 7056

Product of means = Product of extremes

Hence, 16:84::84:441 are in continued proportion.

Q9

Answer :

Given: $9:x::x:49$

We know:

Product of means = Product of extremes

$$x \times x = 9 \times 49$$

$$\Rightarrow \quad x^2 = 441$$

$$\Rightarrow \quad x^2 = (21)^2$$

$$\Rightarrow \quad x = 21$$

Q10

Answer :

Let the height of the pole = x m

Then, we have:

$x:20::6:8$

Now, we know:

Product of extremes = Product of means

$$8x = 20 \times 6$$

$$x = \frac{120}{8} = 15$$

Hence, the height of the pole is 15 m.

Q11

Answer :

$5:3::x:6$

We know:

Product of means = Product of extremes

$$3x = 5 \times 6$$

$$\Rightarrow x = \frac{30}{3} = 10$$

$\therefore x = 10$

Ratio Proportion and Unitary Method
Ex 10C

Q1

Answer :

Cost of 14 m of cloth = Rs 1890

Cost of 1 m of cloth = $\frac{1890}{14}$ = Rs 135

Cost of 6 m of cloth = 6 × 135 = Rs 810

Q2

Answer :

Cost of dozen soaps = Rs 285.60

Cost of 1 soap = $\frac{285.60}{12}$

Cost of 15 soaps = 15 × $\frac{285.60}{12}$ = $\frac{4284}{12}$ = Rs 357

Q3

Answer :

Cost of 9 kg of rice = Rs 327.60

Cost of 1 kg of rice = $\frac{327.60}{9}$

Cost of 50 kg of rice = 50 × $\frac{327.60}{9}$ = $\frac{16380}{9}$ = Rs 1820

Hence, the cost of 50 kg of rice is Rs 1820.

Q4

Answer :

Weight of 22.5 m of uniform iron rod = 85.5 kg

Weight of 1 m of uniform iron rod = $\frac{85.5}{22.5}$ kg

Weight of 5 m of uniform iron rod = $5 \times \frac{85.5}{22.5} = \frac{427.5}{22.5}$ = 19 kg

Thus, the weight of 5 m of iron rod is 19 kg.

Q5

Answer :

Oil contained by 15 tins = 234 kg

Oil contained by 1 tin = $\frac{234}{15}$ kg

Oil contained by 10 tins = $10 \times \frac{234}{15} = \frac{2340}{15}$ = 156 kg

Q6

Answer :

Distance covered by a car in 12 L diesel = 222 km

Distance covered by it in 1 L diesel = $\frac{222}{12}$ km

Distance covered by it in 22 L diesel = $22 \times \frac{222}{12} = \frac{4884}{12}$ = 407 km

Q7

Answer :

Cost of transporting 25 tonnes of weight = Rs 540

Cost of transporting 1 tone of weight = $\frac{540}{25}$

Cost of transporting 35 tonnes of weight = $35 \times \frac{540}{25} = \frac{18900}{25}$ = Rs 756

Q8

Answer :

Let the weight of copper be x g.

Then, 4.5:3.5::18.9:x

Product of extremes = Product of means

$4.5 \times x = 3.5 \times 18.9$

$\Rightarrow x = \frac{66.15}{4.5} = 14.7$

So, the weight of copper is 14.7 g.

Q9

Answer :

Number of inland letters whose total cost is Rs 87.50 = 35

Number of inland letters of whose cost is Re 1 = $\frac{35}{87.50}$

Number of inland letters whose cost is Rs 315 = $315 \times \frac{35}{87.50} = \frac{11025}{87.50}$ = 126

Hence, we can buy 126 inland letters for Rs 315.

Q10

Answer :

Number of bananas that can be purchased for Rs 104 = 48 (4 dozen)

Number of bananas that can be purchased for Re 1 = $\frac{48}{104}$

Number of bananas that can be purchased for Rs 6.50 = $6.50 \times \frac{48}{104} = \frac{312}{104}$ = 3

Hence, 3 bananas can be purchased for Rs 6.50.

Q11

Answer :

Number of chairs that can be bought for Rs 22770 = 18

Number of chairs that can be bought for Re 1 = $\frac{18}{22770}$

Number of chairs that can be bought for Rs 10120 = $10120 \times \frac{18}{22770} = \frac{182160}{22770} = 8$

Q12

Answer :

(i) Time taken by the car to travel 195 km = 3 hours

Time taken by it to travel 1 km = $\frac{3}{195}$ hours

Time taken by it to travel 520 km = $520 \times \frac{3}{195} = \frac{1560}{195} = 8$ hours

(ii) Distance covered by the car in 3 hours = 195 km

Distance covered by it in 1 hour = $\frac{195}{3} = 65$ km

Distance covered by it in 7 hours = $7 \times 65 = 455$ km

Q13

Answer :

(i) Earning of a labourer in 12 days = Rs 1980

Earning of the labourer in 1 day = $\frac{1980}{12}$ = Rs 165

Earning of the labourer in 7 days = 7×165 = Rs 1155

(ii) Number of days taken by the labourer to earn Rs 1980 = 12 days

Number of days taken by him to earn Re 1 = $\frac{12}{1980}$ days

Number of days taken by him to earn Rs 2640 = $2640 \times \frac{12}{1980} = \frac{31680}{1980} = 16$ days

Q14

Answer :

Weight of 65 books = 13 kg

(i) Weight of 1 book = $\frac{13}{65}$ kg

Weight of 80 books = $80 \times \frac{13}{65} = \frac{1040}{65} = 16$ kg

(ii) Number of books weighing 13 kg = 65

Number of books weighing 1 kg = $\frac{65}{13} = 5$

Number of books weighing 6.4 kg = $6.4 \times 5 = 32$

Q15

Answer :

Number of boxes containing 6000 pens = 48

Number of boxes containing 1 pen = $\frac{48}{6000}$

Number of boxes containing 1875 pens = $1875 \times \frac{48}{6000} = \frac{90000}{6000} = 15$

15 boxes are needed for 1875 pens.

Q16

Answer :

Number of days taken by 24 workers to build a wall = 15 days

Number of days taken by 1 worker to build the wall = $15 \times 24 = 360$ days (less worker means more days)

Number of days taken by 9 workers to build the wall = $\frac{360}{9} = 40$ days

Q17

Answer :

Number of men required to complete the work in 26 days = 40
Number of men required to complete the work in 1 day = 40 × 26 = 1040 men (less men more days)
Number of men required to complete the work in 16 days = $\frac{1040}{16}$ = 65

Q18

Answer :

Number of days the provisions will last for 550 men = 28 days
Number of days the provisions will last for 1 man = 28 × 550 = 15400 days (less men means more days)
Number of days the provisions will last for 700 men = $\frac{15400}{700}$ = 22 days

The provision will last for 22 days.

Q19

Answer :

Number of days for which the given quantity of rice is sufficient for 60 persons = 3 days
Number of days for which it is sufficient for 1 person = 3 × 60 = 180 days (less men means more days)
Number of days for which it is sufficient for 18 persons = $\frac{180}{18}$ = 10 days

Ratio Proportion and Unitary Method
Ex 10D

Q1

Answer :

(d) 4 : 5

$92:115 = \frac{92 \div 23}{115 \div 23} = \frac{4}{5}$ (As H.C.F. of 92 and 115 is 23.)

Q2

Answer :

(a) 95

$57:x::51:85$

$\frac{57}{x} = \frac{51}{85}$

$\Rightarrow x = \frac{57 \times 85}{51}$

$\Rightarrow x = \frac{4845}{51} = 95$

Q3

Answer :

(a) 63

$25:35::45:x$

$\frac{25}{35} = \frac{45}{x}$

$\Rightarrow x = \frac{35 \times 45}{25} = \frac{1575}{25} = 63$

Q4

Answer :

(c) 28

$4:5::x:35$

$\Rightarrow \frac{4}{5} = \frac{x}{35}$

$\Rightarrow x = \frac{4 \times 35}{5} = 4 \times 7 = 28$

Q5

Answer :

(b) $ad = bc$

Given:

a, b, c, d are in proportion.

$a:b::c:d$

$\frac{a}{b} = \frac{c}{d}$

$\Rightarrow ad = bc$

Q6

Answer :

(b) $b^2 = ac$

Given:

a, b, c are in proportion.

$a:b::b:c$

Product of means = Product of extremes

$\Rightarrow b^2 = ac$

Q7

Answer :

(b) (5 : 8) < (3 : 4)

We can write

$\left(5:8\right) = \frac{5}{8}$ and $\left(3:4\right) = \frac{3}{4}$

Making the denominator equal:

$\frac{5}{8}$ and $\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

As 6 > 5, $\frac{5}{8} < \frac{3}{4}$

Q8

Answer :

(a) Rs 440

A:B = 8:11

Sum of ratio terms = 8 + 11 = 19

B's share = $\frac{11}{19} \times 760 = \frac{8360}{19}$ = Rs 440

Q9

Answer :

(d) 147

Ratio = 5:7

Let x be any number such that we have:

$5x + 7x = 252$

$\Rightarrow 12x = 252$

$\Rightarrow x = \frac{252}{12} = 21$

Now, $5x = 5 \times 21 = 105$

$7x = 7 \times 21 = 147$

The largest number is 147.

Q10

Answer :

(b) 50 cm
The sides of the triangle are in the ratio 1:3:5.
Let x be any number such that the sides are $1x$ cm, $3x$ cm and $5x$ cm.

$1x + 3x + 5x = 90$

$\Rightarrow 9x = 90$

$\Rightarrow x = \frac{90}{9} = 10$

First side = $1x = 1 \times 10 = 10$ cm
Second side = $3x = 3 \times 10 = 30$ cm
Third side = $5x = 5 \times 10 = 50$ cm
The length of the largest side is 50 cm.

Q11

Answer :

(c) 2856
Ratio of boys and girls = 12:5
Let x be any number such that the number of boys and girls are $12x$ and $5x$, respectively.
Number of girls = 840

$5x = 840$

$\Rightarrow x = \frac{840}{5} = 168$

Number of boys = $12x = 12 \times 168 = 2016$
Number of girls = 840
Total strength of the school = 2016 + 840 = 2856

Q12

Answer :

(b) Rs 161
Cost of 12 pens = Rs 138
Cost of 1 pen = Rs $\frac{138}{12}$
Cost of 14 pens = Rs $\frac{138}{12} \times 14$ = Rs $\frac{1932}{12}$ = Rs 161

Q13

Answer :

(b) 45 days
Time taken by 24 workers to build a wall = 15 days
Time taken by 1 worker to build a wall = $24 \times 15 = 360$ days (clearly less workers will take more time to build a wall)
Time taken by 8 workers to build a wall = $\frac{360}{8}$ = 45 days

Q14

Answer :

(a) 52
Number of men required to finish the work in 26 days = 40
Number of men required to finish it in 1 day = $40 \times 26 = 1040$ men (More men means less days)
Number of men required to finish it in 20 days = $\frac{1040}{20}$ = 52

Q15

Answer :

(b) 185 km
Distance covered in 6 L of petrol = 111 km
Distance covered in 1 L of petrol = $\frac{111}{6}$ km
Distance covered in 10 L of petrol = $\frac{111}{6} \times 10 = \frac{1110}{6}$ = 185 km

Q16

Answer :

(a) 22 days
Number of days for which 550 men had provisions = 28 days
Number of days for which 1 man had provisions = 28 × 550 = 15400 days (more men means less days)
Number of days for which 700 men had provisions = $\frac{15400}{700}$ = 22 days

Q17

Answer :

(c) 90°
Ratio of the angles of a triangle is 3:1: 2
Let x be any number such that the three angles are $(3x)^\circ$, $(1x)^\circ$ and $(2x)^\circ$.
We know, the sum of the angles of a triangle is 180°.

$3x + 1x + 2x = 180$
$\Rightarrow 6x = 180$
$\Rightarrow x = \frac{180}{6} = 30$

$\therefore (3x)^\circ = (3 \times 30)^\circ = 90^o$
$(1x)^\circ = (1 \times 30)^\circ = 30^o$
$(2x)^\circ = (2 \times 30)^\circ = 60^o$
The measure of the largest angle is 90^o.

Q18

Answer :

(b) 45 m
Length:Breadth = 5:4
Let x be any number such that the length and the breadth are $5x$ and $4x$, respectively.
Now , $4x = 36$

$x = \frac{36}{4} = 9$

Length = $5x = 5 \times 9 = 45$ m

Q19

Answer :

(a) 13 : 15

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

Speed of the bus = $\frac{195 \text{ km}}{3 \text{ hr}}$ = 65 km/hr

Speed of the train = $\frac{300 \text{ km}}{4 \text{ hr}}$ = 75 km/hr

Ratio = $\frac{65}{75} = \frac{65 \div 5}{75 \div 5} = \frac{13}{15}$ = 13:15

Q20

Answer :

(c) Rs 198
Cost of 5 bars of soap = Rs 82.50
Cost of 1 bar of soap = $\frac{82.50}{5}$ = Rs 16.5
Cost of 12 (1 dozen) bars of soap = 16.5 × 12 = Rs 198

Q21

Answer :

(b) Rs 750
Cost of 30 packets of 8 pencils each = Rs 600
Cost of 1 packet of 8 pencils = $\frac{600}{30}$ = Rs 20
Cost of 1 pencil = Rs $\frac{20}{8}$
Cost of 1 packet of 12 pencils = $12 \times \frac{20}{8} = \frac{240}{8}$ = Rs 30
Cost of 25 packets of 12 pencils each = 25 × 30 = Rs 750

Q22

Answer :

(a) Rs 344
Cost of rail journey of 75 km = Rs 215
Cost of rail journey of 1 km = Rs $\frac{215}{75}$
Cost of rail journey of 120 km = $120 \times \frac{215}{75} = \frac{25800}{75}$ = Rs 344

Q23

Answer :

(d) 8
Let the third term be x.
Then, we have:
12:21::x:14
We know:
Product of means = Product of extremes
$21x = 12 \times 14$
$\Rightarrow 21x = 168$
$\Rightarrow x = \frac{168}{21} = 8$
The third term is 8

Q24

Answer :

(b) 15 h
Time taken by 10 boys to dig a pitch = 12 hours
Time taken by 1 boy to dig a pitch = 12 × 10 = 120 hours (less boys means more time)
Time taken by 8 boys to dig a pitch = $\frac{120}{8}$ = 15 hours