# Sample Paper 14

## Class- X Exam - 2022-23

## Mathematics - Basic

**Time Allowed: 3 Hours** **Maximum Marks : 80**

**General Instructions :**

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

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## SECTION - A **20 marks**

*(Section - A consists of 20 questions of 1 mark each.)*

**1.** Two positive integers $p$ and $q$ are expressible as $p = a^3b$ and $q = ab^2$. The HCF ($p$, $q$) and LCM ($p$, $q$) is:

(a) $ab^2$ (b) $a^3b^2$

(c) $a^2b^3$ (d) $a^3b$ 1

**2.** If given A.P. is 11, 8, 5, 2, ........, then the sum of $10^{th}$ term is:

(a) –23 (b) 28

(c) 24 (d) –25 1

**3.** The ideal times of year to have chilled shakes and ice creams are during the summer. During lockdown, Saumya was eager to try the watermelon sharbat she had recently learned to make from her online culinary classes. She cut a watermelon slice with a semi-circular cross section. If the perimeter of a semi- circular portion is 36cm, then its radius is:

(a) 12 cm (b) 15 cm

(c) 7 cm (d) 14 cm 1

**4.** The solution of the pair of equations: $2x + 3y = 9$; $3x + 4y = 5$ is:

(a) 21, –17 (b) 20, 14

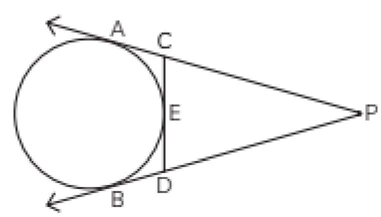
(c) –21, 17 (d) 20, 3 1

**5.** Two vertices of a triangle are (4, –5) and (–5, –2). If the centroid of the triangle is the origin, the third vertex of the triangle is:

(a) (1, 5) (b) (2, 4)

(c) (1, 4) (d) (1, 7) 1

**6.** In the adjoining figure, if PA = 10 cm, then the perimeter of $\Delta$PCD is:




1

(a) 16 cm (b) 21 cm

(c) 18 cm (d) 20 cm 1

**7.** What is mid-point of the line segment AB where A (– 5, 0) and B (0, 5) ?

(a) $\left(-\frac{5}{2}, \frac{5}{2}\right)$ (b) (3, 5)

(c) $\left(\frac{5}{2}, \frac{3}{2}\right)$ (d) (2, 4) 1

**8.** If $x \sec 45° = 2$, then what is the value of $x$ ?

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$ 1

**9.** If $\tan\theta + \cot\theta = 4$, then the value of $\tan^4\theta + \cot^4\theta$ is:

(a) 196 (b) 200

(c) 194 (d) 198 1

**10.** In an A.P., if $a = 3.5$, $d = 0$, $n = 101$, then the value of $a_n$ is:

(a) 2.5 (b) 3

(c) 4 (d) 3.5 1

**11.** If A = 900, $\Sigma f_i d_i = -400$ and $\Sigma f_i = 100$, then what is the value of $\bar{x}$?

(a) 890 (b) 986

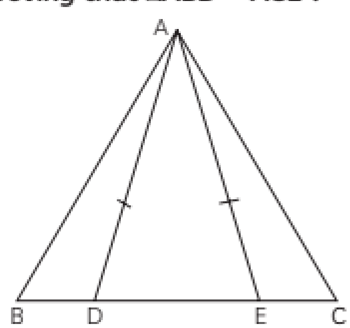
(c) 465 (d) 896 1

**12.** A 6 faced cube has letters A, B, C, D, A and C on its six faces. This cube is rolled once. What is the probability of getting B or C ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{4}$ 1

**13.** Which criterion of similarity will be used in proving that $\Delta ABD \sim ACE$ ?




(a) SAS (b) AA

(c) RHS (d) SSS 1

**14.** A letter is chosen from the letters of the word MAINTENANCE. The probability that it is N is:

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$

(c) $\frac{3}{11}$ (d) $\frac{4}{11}$ 1

**15.** If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then the value of '$k$' is:

(a) $k > 4$ (b) $k \geq 4$

(c) $k < 4$ (d) $k = 4$ 1

**16.** Which term of the A.P.: –2, –7, –12, ... will be –77?

(a) 16th (b) 10th

(c) 15th (d) 12th 1

**17.** What type of lines are represented by the pair of equations $10x + 6y = 9$ and $5x + 3y + 4 = 0$ ?

(a) Straight lines (b) Intersecting lines

(c) Parallel lines (d) None of these 1

**18.** If an event is sure to occur, then what is its probability of occurrence ?

(a) 0 (b) 1

(c) 2 (d) $\frac{1}{2}$ 1

**Direction for questions 19 and 20:** In question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct option:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**19.** Assertion (A) : The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. The radius of the circle is 17 cm

Reason (R) : Tangent is perpendicular to radius at the point of contact. 1

**20.** Assertion (A) : If circumference of two circles are equal, then their areas are also equal.

Reason (R) : Two circles are congruent if their radii are equal. 1

## SECTION - B

**10 marks**

*(Section - B consists of 5 questions of 2 mark each.)*

**21.** Assuming that $\sqrt{2}$ is irrational, show that $5\sqrt{2}$ is an irrational number. 2

**22.** Find the greatest number that divides 45 and 210 completely. 2

**23.** If $x = a\cos^3\theta$ and $y = b\sin^3\theta$, then prove that:

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$$

**OR**

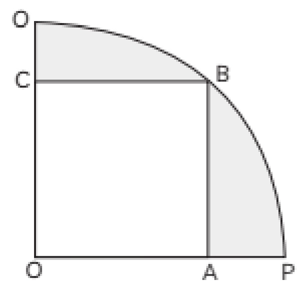
Prove that $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$. 2

**24.** The largest possible sphere is carved out of wooden solid cube of side 7 cm. What is the radius of this sphere ?

**OR**

A line intersect the $y$-axis and $x$-axis at the points P and Q respectively. If (2, – 5) is the mid point of PQ, then find the coordinates of the points P and Q. 2

**25.** In figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region.

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## SECTION - C

**18 marks**

*(Section - C consists of 6 questions of 3 mark each.)*

**26.** Solve for $x$ and $y$:

$$x + \frac{y}{4} = 11; \quad \frac{5x}{6} - \frac{y}{3} = 7$$

**OR**

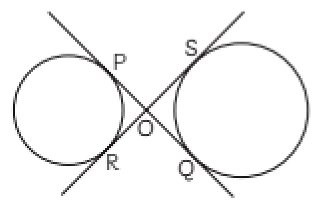
A 2-digit number is such that the product of the digits is 20. If 9 is subtracted from the number, the digits interchange their places. Find the number. 3

**27.** Show that $\triangle ABC$ with vertices A (– 2, 0), B (2, 0) and C(0, 2) is similar to $\triangle DEF$ with vertices D (– 4, 0), E (4, 0) and F (0, 4). 3

**28.** Prove that the lengths of tangents drawn from an external point to a circle are equal.

**OR**

In the figure, PQ and RS are the common tangents to two circles intersecting at O.




Prove that: PQ = RS 3

**29.** A number $x$ is selected from the numbers 1, 2, 3 and then a second number $y$ is selected randomly from the numbers 1, 4, 9. What is the probability that the product $xy$ of the two numbers will be less than 9? 3

**30.** Find the value of:

$$\frac{5\sin^2 30° + \cos^2 45° - 4\tan^2 30°}{2\sin 30°.\cos 30° + \tan 45°} + \cos 0°.$$

3

**31.** The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there in the A.P. ? 3

## SECTION - D **20 marks**

*(Section - D consists of 4 questions of 5 mark each)*

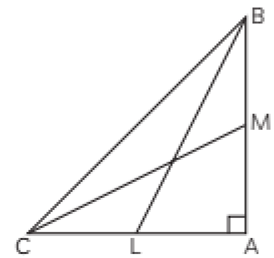
**32.** From the top of a building 60 m high, the angle of depression of the top and bottom of a vertical lamp-post are observed to be 30° and 60° respectively. Find the height of the lamp-post, and the distance between the top of building and the top of lamp-post. 5

**33.** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

**OR**

BL and CM are medians of $\triangle ABC$, right-angled at A.

Prove that: $4(BL^2 + CM^2) = 5\ BC^2$

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**34.** Find the median marks for the following frequency distribution :

| Marks | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
|---|---|---|---|---|---|
| Number of Students | 7 | 12 | 23 | 18 | 10 |

5

**35.** Between Mysore and Bangalore, 132 km apart, an express train travels in 1 hour less time than a passenger train (without taking into consideration the time they stop at intermediate stations). Find the average speed of the two trains if the express train's average speed is 11 km/h higher than the passenger train's.

**OR**

Form a pair of linear equations for the following problems and find their solution by substitution method.

(A) The cost of a taxi in a city consists of a fixed charge and a charge for the distance travelled. The cost for a 10 km travel is ₹ 105, while for a 15 km journey, the cost is ₹ 155. What are the fixed charges and the km charged? How much will it cost someoneto drive 25 Km?

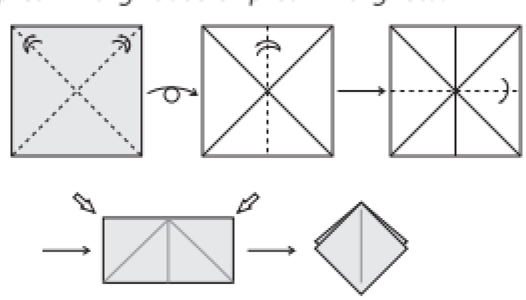
(B) For ₹ 3800, the cricket team's coach purchases 6 balls and 7 bats. he then spends 1750 for 3 bats and 5 balls. Find out how much each ball and bat costs. 5

## SECTION - E **12 marks**

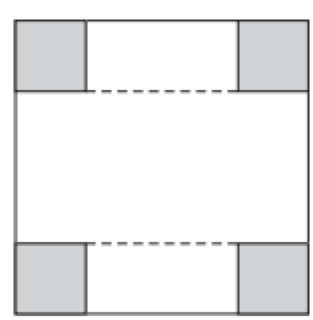
### (Case Study Based Questions)

*(Section - E consists of 3 questions. All are compulsory.)*

**36.** *'Origami' is the art of paper folding, which is often associated with Japanese culture. Gurmeet is trying to learn Origami using paper cutting and folding technique. A square base is sometimes referred to as a "preliminary" base or preliminary fold.*



*Here is a 20 cm × 20 cm square. Gurmeet wants to first cut the squares of integral length from the corners and by folding the flaps along the sides.*



On the basis of the above information, answer the following questions:

(A) How many different sizes of boxes Gurmeet can make? 1

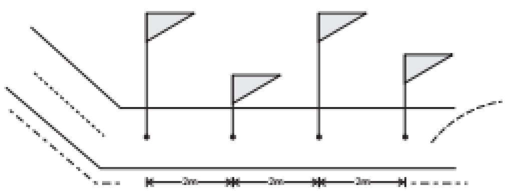
(B) How many different sizes of boxes Gurmeet can make if sides of the squares are not integral length? 1

(C) Find the equation relating the size of the square cut out and volume of the box.

**OR**

Find the dimension of the box with maximum volume and minimum volume. 2

**37.** *The students of a school decided to beautify the school on the Annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag.*



*Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flag were stored. She could carry only one flag at a time.*

**On the basis of the above information, answer the following questions:**

(A) What is the position of the middle flag? 1

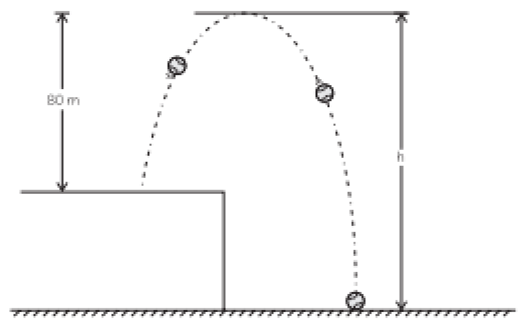
(B) Find total distance travelled for placing all the flags. 1

(C) Find total distance travelled for placing 13 flags on left.

**OR**

Find the maximum distance she travelled carrying a flag. 2

**38.** *Soumya throws a ball upwards, from a rooftop, 80 m above. It will reach a maximum height and then fall back to the ground. The height of the ball from the ground at time 't' is 'h', which is given by $h = -16t^2 + 64t + 80$.*




**On the basis of the above information, answer the following questions:**

(A) What is the height reached by the ball after 1 second? 1

(B) How long will the ball take to hit the ground? 1

(C) What are the two possible times to reach the ball at the same height of 128 m?

**OR**

What is the maximum height reached by the ball? 2

# SOLUTION

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## SECTION - A

**1.** (b) $a^3b^2$

**Explanation:** HCF $(p, q) = ab$;

$$\text{LCM } (p, q) = a^3b^2$$

**2.** (d) $-25$

**Explanation:** Given AP is 11, 8, 5, 2, ...

$$a = 11$$

$$d = 8 - 11 = -3$$

$$n = 10$$

$$S_{10} = \frac{10}{2}\,(2 \times 11 + (10 - 1) \times -3)$$

$$= 5\,(22 - 27)$$

$$= 5 \times -5$$

$$= -25$$

### ⚠ Caution

➥ *Use the appropriate formula for finding the sum of first 'n' terms, as per the values mentioned in the question.*

**3.** *(c) 7 cm*

**Explanation:** Perimeter of semicircular protractor = 36 cm

$$\pi r + 2r = 36$$

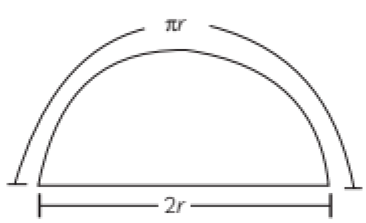
$$\Rightarrow \quad (\pi + 2)r = 36$$

$$\Rightarrow \quad \left(\frac{22}{7}+2\right) r = 36$$

$$\Rightarrow \quad \frac{36}{7}r = 36$$

$$\Rightarrow \quad r = \frac{36\times7}{36}$$

$$\Rightarrow \quad r = 7$$




**4.** *(c) –21, 17*

**Explanation:**

$$2x + 3y = 9 \quad \text{...(i)}$$

$$3x + 4y = 5 \quad \text{...(ii)}$$

Multiplying eq. (i) and (ii) by 3 and 2, respectively and then subtracting them, we get

$$6x + 9y = 27$$

$$6x + 8y = 10$$

$$- \quad - \quad -$$

$$y = 17$$

From eq. (i),

$$2x + 51 = 9$$

$$\Rightarrow \quad 2x = -42$$

$$\Rightarrow \quad x = -21$$

Thus, $x = -21$, $y = 17$ is the required solution.

### ⚠ Caution

➥ *Derive the value of either x or y, but do which is more convenient and don't mess up the process.*

**5.** *(d) (1, 7)*

**Explanation:** Let the third vertex be $(x, y)$. Then,

$$\left(\frac{4 - 5 + x}{3}, \frac{-5 - 2 + y}{3}\right) = (0, 0)$$

$$\Rightarrow \quad \frac{x - 1}{3} = 0; \quad \frac{y - 7}{3} = 0$$

$$\Rightarrow \quad x = 1, \quad y = 7$$

Thus, the third vertex is (1, 7).

**6.** *(d) 20 cm*

**Explanation:** We know lengths of tangents drawn from an external point to a circle are equal.

∴ From the figure, we have

PA = PB, CA = CE and DE = DB.

Now,

Perimeter of ΔPCD = PC + CE + ED + DP

= (PC + CE) + (ED + DP)

= (PC + CA) + (BD + DP)

= PA + PB

= 2 PA

= 2 × 10 cm = 20 cm

**7.** *(a)* $\left(\frac{-5}{2}, \frac{5}{2}\right)$

**Explanation:** The mid-point of AB is

$$\left(\frac{-5 + 0}{2}, \frac{0 + 5}{2}\right), \text{ i.e. } \left(-\frac{5}{2}, \frac{5}{2}\right).$$

**8.** *(c)* $\sqrt{2}$

**Explanation:** Given,

$$x \sec 45° = 2$$

$$\Rightarrow \quad x(\sqrt{2}) = 2$$

$$\Rightarrow \quad x = \sqrt{2}$$

### ⚠ Caution

➥ *Learn the table of trigonometric ratios for specific angles properly for solving such types of problems.*

**9.** *(c) 194*

**Explanation:** Given, $\tan\theta + \cot\theta = 4$ Squaring on both sides,

We get,

$\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 16$

$\tan^2\theta + \cot^2\theta + 2 = 16$

$\tan^2\theta + \cot^2\theta = 14$

Again, squaring on both sides, we get

$\tan^4\theta + \cot^4\theta + 2\tan^2\theta\cot^2\theta = 196$

$\Rightarrow \quad \tan^4\theta + \cot^4\theta + 2 = 196$

$\Rightarrow \quad \tan^4 + \cot^4\theta = 194$

**10.** *(d) 3.5*

**Explanation:** In the given A.P., $d = 0$

So, all its terms are same as $a = 3.5$,

$\therefore \quad a_{101} = 3.5$

**11.** *(d) 896*

**Explanation:** We know that,

$$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 900 + \frac{(-400)}{100}$$

$$= 900 - 4 = 896$$

**12.** *(a)* $\frac{1}{2}$

**Explanation:** Total number of outcomes = 6

$\therefore$ Number of favourable outcomes = 3

$$P\text{ (B or C)} = \frac{3}{6} \text{ i.e. } \frac{1}{2}$$

**13.** *(b) AA*

**Explanation:** In $\Delta$s ABD and ACE, we have

AD = AE (given)

So, $\angle D = \angle E$ (Angles opposite to equal sides are equal)

$\angle A = \angle A$ (Common)

So, by AA similarly criterion, $\Delta ABD \sim \Delta ACE$.

Hence, AA similarity criteria will be used.

**14.** *(c)* $\frac{3}{11}$

**Explanation:** In the given word, there are in all eleven letters, of which three are N.

So, the required probability is $\frac{3}{11}$.

**15.** *(c) $k < 4$*

**Explanation:** As the given equation has real and distinct roots,

$\therefore$ Discriminant $= (4)^2 - 4(1)(k) > 0$

*i.e.* $\quad 16 - 4k > 0$

or $\quad k < 4$

**16.** *(a) $16^{th}$*

**Explanation:** Let, the $n^{th}$ term of the A.P. be $-77$.

Then, $\quad a + (n - 1)d = -77$

For the given A.P., $a = -2$ and $d = -5$.

So, $\quad a + (n - 1)d = (-2) + (n - 1)(-5) = -77$

$-5(n - 1) = -75$

or $\quad n - 1 = 15 \quad$ or $\quad n = 16$

So, the $16^{th}$ term of the A.P. is $(-77)$.

**17.** *(c) Parallel lines*

**Explanation:** The pair of equations satisfy the relation

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \uparrow \frac{c_1}{c_2}$$

as $$\frac{10}{5} = \frac{6}{3} \neq \frac{9}{-4}$$

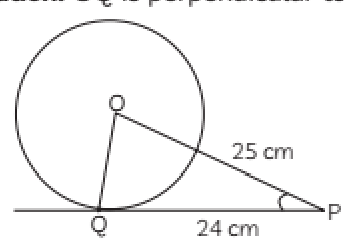
Hence, these equations represent parallel lines.

**18.** *(b) 1*

**Explanation:** The probability of occurrence of a sure event is 1.

**19.** *(d) Assertion (A) is false but reason (R) is true.*

**Explanation:** OQ is perpendicular to PQ.




$\therefore \quad$ In $\Delta POQ$,

$PQ^2 + OQ^2 = OP^2$

$25^2 = OQ^2 + 24^2$

$OQ^2 = 625 - 576$

$= 49$

$OQ = 7$ cm

**20.** *(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)*

**Explanation:** Let's consider 2 circles of radii $r_1$ and $r_2$.

Then, $2\pi r_1 = 2\pi r_2$

$r_1 = r_2 = r$

Then, $A_1 = \pi {r^2}_2 = \pi r^2$

$A_1 = A_2$

## SECTION - B

**21.** Let $5\sqrt{2}$ be rational. Then,

$5\sqrt{2} = \frac{p}{q}$, where $p$ and $q$ are co-prime and $q \neq 0$.

$\Rightarrow \quad \sqrt{2} = \frac{p}{5q}$

Here, $\frac{p}{5q}$ is rational, which implies $\sqrt{2}$ is rational, which is a contradiction, as it is given than $\sqrt{2}$ is irrational.

$\Rightarrow 5\sqrt{2}$ is irrational.

**22.** The greatest number that divides 45 and 240

completely is the HCF of 45 and 210.

Now, $45 = 3 \times 3 \times 5$, or $3^2 \times 5^1$

$210 = 2 \times 3 \times 5 \times 7$,

So, HCF (45, 210) = $3^1 \times 5^1$, *i.e.* 15

Hence, the required number is 15.

**Caution**

➥ *While calculating prime factors, start with the lowest prime number.*

**23.** $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = \left(\frac{a\cos^3\theta}{a}\right)^{\frac{2}{3}} + \left(\frac{b\sin^3\theta}{b}\right)^{\frac{2}{3}}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

**OR**

$\sqrt{\sec^2\theta + \text{cosec}^2\theta}$

$= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{(\tan\theta + \cot\theta)^2}$

$[\because 2 = 2\tan\theta\cot\theta]$

$= \tan\theta + \cot\theta$

**24.** The largest possible sphere that can be carved out of a sphere, is equal to diameter of the sphere.

$\therefore$ Diameter of sphere = 7 cm.

So, the radius of the largest possible sphere is 3.5 cm.

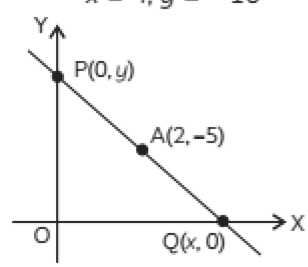
**OR**

Let the coordinates of P and Q be $(0, y)$ and $(x, 0)$ respectively.

Here, the mid-point of PQ is (2, – 5)

So, $\left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (2, -5)$

$\Rightarrow \quad x = 4, y = -10$




Thus, the coordinates of P and Q are (0, – 10) and (4, 0) respectively.

**25.** $OB = \sqrt{OA^2 + AB^2} = \sqrt{OA^2 + OA^2}$

$= \sqrt{2}\,OA = \sqrt{2} \times 20 = 20\sqrt{2}$ cm

Area of shaded region = Area of quadrant OPBQ – Area of square OABC

$= \frac{90°}{360°} \times 3.14 (20\sqrt{2})^2 - 20 \times 20$

$= \frac{1}{4} \times 3.14 \times 800 - 400$

$= 200 \times 3.14 - 400$

$= 228$ cm$^2$

**Caution**

➥ *Remember that total angle at the centre of a circle is 360° and equal sectors means each sector subtends equal angle.*

## SECTION - C

**26.** The given equations are rewritten as:

$4x + y - 44 = 0$ ;

$5x - 2y - 42 = 0$

From $4x + y - 44 = 0$, we have

$y = 44 - 4x$ .....(i)

Substituting this value of $y$ in $5x - 2y - 42 = 0$, we have:

$$5x - 2(44 - 4x) - 42 = 0$$

$\Rightarrow$ $13x - 88 - 42 = 0$

$\Rightarrow$ $x = 10$

From (i) $y = 44 - 40 = 4$

Thus, $x = 10$, $y = 4$ is the required solution.

**OR**

Let ten's digit and one's digit of the two-digit number be $a$ and $b$ respectively. Then, the number is $10a + b$.

Here, $ab = 20$ ...(i)

and $(10a + b) - 9 = 10b + a$

i.e. $9a - 9b = 9$

or $a - b = 1$ ...(ii)

Solving (i) and (ii) simultaneously, we get

$$a = 5, \quad b = 4$$

Thus, the number is 54.

**27.** Let's say that $\Delta ABC \sim \Delta DEF$.

If $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$

Now, using distance formula,

$AB = 4$, $BC = \sqrt{8} = 2\sqrt{2}$

$CA = \sqrt{8}$, $2\sqrt{2}$

$DE = 8$, $EF = \sqrt{32} = 4\sqrt{2}$

$FD = \sqrt{32}$, i.e. $4\sqrt{2}$

Here, $\dfrac{AB}{DE} = \dfrac{BC}{DE} = \dfrac{CA}{FD} = \dfrac{1}{2}$

So, $\Delta ABC \sim \Delta DEF$.

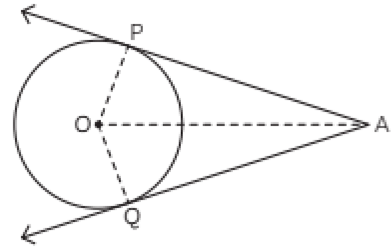
**28.** Let AP and AQ be the two tangents drawn to the circle from an external point A.

We need to show that AP = AQ.

Join OA, OP and OQ.

We know that tangent is perpendicular to radius at the point of contact.

$\therefore$ $OP \perp AP$ and $OQ \perp QA$.




Consider $\Delta OPA$ and $\Delta OQA$.

Here, $OQ = OP$ (radii of the circle)

$OA = OA$ (common)

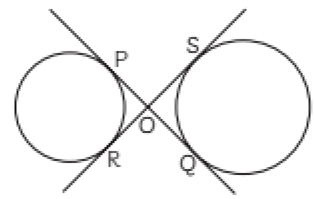
$\angle OPA = \angle OQA$

So, $\Delta OPA \cong \Delta OQA$

$\Rightarrow$ $PA = QA$ or $AP = AQ$

**OR**

We know that lengths of tangent drawn from an external point to a circle are equal.




So, from the figure, OP = OR and OS = OQ

Now, $PQ = PO + OQ$

$= OR + OS$

$= RS$

**29.** Total number of possible outcomes of product $xy = \{1, 4, 9, 2, 8, 18, 3, 12, 27\}$.

Of these, 5 are less than 9.

So, $P(xy < 9) = \dfrac{5}{9}$

**30.** $$\frac{5\sin^2 30° + \cos^2 45° - 4\tan^2 30°}{2\sin 30° . \cos 30° + \tan 45°} + \cos 0°$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2}{2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + 1} + 1$$

$$= \frac{\frac{5}{4} + \frac{1}{2} - \frac{4}{3}}{\frac{\sqrt{3}}{2} + 1} + 1 = \frac{15 + 6 - 16}{6(\sqrt{3} + 2)} + 1$$

$$= \frac{5 + 6(\sqrt{3} + 2)}{6(\sqrt{3} + 2)} = \frac{17 + 6\sqrt{3}}{6(\sqrt{3} + 2)}$$

**31.** Let the A.P. contains '$n$' terms. Then,

$$a_n = l = 350$$

$\Rightarrow \quad a + (n-1)d = 350$

$\Rightarrow \quad 17 + (n-1)(9) = 350$

$\Rightarrow \quad 9(n-1) = 333$

$\Rightarrow \quad n - 1 = 37$

$\Rightarrow \quad n = 38$

Thus, A.P. contains 38 terms.

## SECTION - D

**32.** Let AB be the building and XY be the lamp post.

$\therefore$ AB = 60 m, $\angle$BYM = 30° and $\angle$BXA = 60°

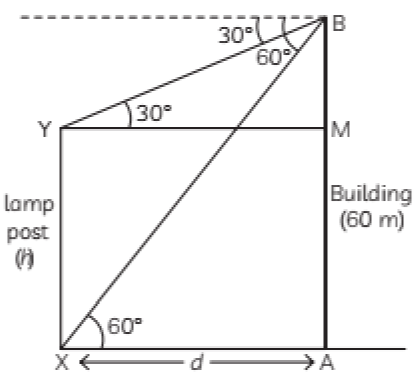
Let '$h$' metres be the height of the lamp-post and '$d$' metres be the horizontal distance between the lamp post and the building.

Then,

From right $\Delta$BMY, we have:

$$\frac{BM}{YM} = \tan 30°$$

$$\Rightarrow \quad \frac{60-h}{d} = \frac{1}{\sqrt{3}}$$




$$\Rightarrow \quad d = \sqrt{3}(60-h) \quad \text{.....(i)}$$

From right $\Delta$BAX, we have:

$$\frac{BA}{XA} = \tan 60°$$

$$\Rightarrow \quad \frac{60}{d} = \sqrt{3}$$

$$\Rightarrow \quad d = \frac{60}{\sqrt{3}} = 20\sqrt{3} \quad \text{.....(ii)}$$

From (i) and (ii),

$$\sqrt{3}(60-h) = 20\sqrt{3}$$

$\Rightarrow \quad 60 - h = 20$

$\Rightarrow \quad h = 40$

Hence the height of lamp-post is 40 m.

Now: $\quad BY = \sqrt{YM^2 + BM^2}$

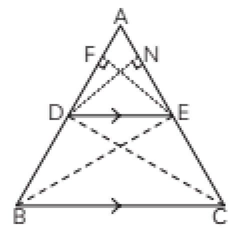
$$= \sqrt{d^2 + 20^2} = \sqrt{1200 + 400}$$

$$= \sqrt{1600} = 40 \text{ m}$$

Thus, the distance between the top of the building and the top of lamp-post is 40 m.

**33.** ABC is a triangle in which DE is a line parallel to side BC which cuts AB at D and AC at E.

We need to prove that:




$$\frac{AD}{DB} = \frac{AE}{EC}$$

Join BE and CD and draw EF $\perp$ AB and DN $\perp$ AC.

Now,

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD} \quad \text{...(i)}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC} \quad \text{...(ii)}$$

But $\Delta$BDE and $\Delta$CDE are on the same base DE and between the same parallels DE and BC.

So, $\quad ar(\Delta BDE) = ar(\Delta CDE) \quad$ ...(iii)

$$\Rightarrow \quad \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{ar(\varnothing ADE)}{ar(\varnothing CDE)}$$

Hence, by (i), (ii) and (iii), we have

$$\frac{AD}{BD} = \frac{AE}{EC}, \text{ or } \frac{AD}{DB} = \frac{AE}{EC}.$$

**OR**

Since BL and CM are medians, So L is the mid-point of CA; and M is the mid-point of AB.

*i.e.* $AL = \frac{1}{2}CA$ and $AM = \frac{1}{2}AB$

Using Pythagoras theorem in right triangle LAB, we have:

$$BL^2 = LA^2 + AB^2$$

$$\Rightarrow \quad BL^2 = \frac{CA^2}{4} + AB^2 \quad ...(i)$$

Similarly, in right triangle CAM, we have:

$$CM^2 = CA^2 + AM^2$$

$$\Rightarrow \quad CM^2 = CA^2 + \frac{AB^2}{4} \quad ...(ii)$$

Adding (i) and (ii), we get:

$$BL^2 + CM^2 = \left(\frac{CA^2}{4} + AB^2\right) + \left(CA^2 + \frac{AB^2}{4}\right)$$

$$= \frac{5}{4}CA^2 + \frac{5}{4}AB^2$$

$$= \frac{5}{4}(CA^2 + AB^2)$$

$$= \frac{5}{4}BC^2$$

(Using Pythagoras theorem in $\Delta ABC$)

$$\Rightarrow 4(BL^2 + CM^2) = 5\,BC^2$$

**34.**

| Marks | Frequency | Cumulative Frequency |
|---|---|---|
| 0–20 | 7 | 7 |
| 20–40 | 12 | 19 |
| 40–60 | 23 | 42 |
| 60–80 | 18 | 60 |
| 80–100 | 10 | 70 |

Here, N = 70, $\therefore \frac{N}{2} = 35$

Cumulative frequency just greater than 35 is 42 which belongs to class 40 – 60.

So, the median class is 40–60.

For this class,

$l = 40$, $h = 20$, $c.f. = 19$ and $f = 2$

$$\text{So,} \quad \text{Median} = l + \frac{\frac{N}{2} - c.f}{f} \times h$$

$$= 40 + \frac{35 - 19}{23} \times 20$$

= 40 + 13.9 (approx.)

= 53.9 (approx.)

**35.** Let average speed of passenger train

= $x$ km/h

Let average speed of express train

= $(x + 11)$ km/h

Time taken by passenger train to cover 132 km = $\frac{132}{x}$ hours

Time taken by express train to cover 132 km = $\left(\frac{132}{x+11}\right)$ hours

According to the given conditions,

$$\frac{132}{x} = \frac{132}{x+11} + 1$$

$$\Rightarrow \quad 132\left(\frac{1}{x} - \frac{1}{x+11}\right) = 1$$

$$\Rightarrow \quad 132\left(\frac{x+11-x}{x(x+11)}\right) = 1$$

$$\Rightarrow \quad 132\,(11) = x(x + 11)$$

$$\Rightarrow \quad 1452 = x^2 + 11x$$

$$\Rightarrow \quad x^2 + 11x - 1452 = 0$$

Comparing equation $x^2 + 11x - 1452 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = 11$ and $c = -1452$

Applying Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(1)(-1452)}}{2 \times 1}$$

$$\Rightarrow \quad x = \frac{-11 \pm \sqrt{121 + 5808}}{2}$$

$$\Rightarrow \quad x = \frac{-11 \pm \sqrt{5929}}{2}$$

$$\Rightarrow \quad x = \frac{-11 \pm 77}{2}$$

$$\Rightarrow \quad x = \frac{-11 \pm 77}{2}, \frac{-11 - 77}{2}$$

$$\Rightarrow \quad x = 33, -44$$

As speed cannot be in negative. Therefore, speed of passenger train = 33 km/h And, speed of express train = x + 11 = 33 + 11 = 44 km/h

**OR**

(A) Let fixed charge = ₹ x and let charge for every km = ₹ y

According to given conditions, we have

$$x + 0y = 105 \quad \text{...(i)}$$

$$x + 15 = 155 \quad \text{...(ii)}$$

Using equation (i), we can say that

$$x = 105 - 10y$$

Puttinmg this in equation (ii), we get

$$105 - 10y + 15y = 155$$

$$\Rightarrow \quad 5x = 50$$

$$\Rightarrow \quad y = 10$$

Putting value of $y$ in equation (i), we get

$$x + 10(10) = 105$$

$$\Rightarrow \quad x = 105 - 100$$

$$\Rightarrow \quad x = 5$$

Therefore, fixed charge = ₹ 5 and charge per km = ₹ 10

To travel distance of 25 km, person will have to pay = ₹ $(x + 25y)$

$$= ₹\ (5 + 25 \times 10)$$

$$= ₹\ (5 + 250)$$

$$= ₹\ 255$$

(B) Let cost of each bat = ₹ $x$

and let cost of each ball = ₹ $y$

According to given conditions, we have

$$7x + 6y = 3800 \quad \text{...(i)}$$

And, $$3x + 5y = 1750 \quad \text{...(ii)}$$

Using equation (i), we can say that

$$7x = 3800 - 6y$$

$$\Rightarrow \quad x = \frac{3800 - 6y}{7}$$

Putting this in eq. (ii), we get

$$3\left(\frac{3800 - 6y}{7}\right) + 5y = 1750$$

$$\Rightarrow \quad \left(\frac{11400 - 18y}{7}\right) = 1750$$

$$\Rightarrow \quad \frac{5y}{1} - \frac{18y}{7} = \frac{1750}{1} - \frac{11400}{7}$$

$$\Rightarrow \quad \frac{35y - 18y}{7} = \frac{12250 - 11400}{7}$$

$$\Rightarrow \quad 17y = 850$$

$$\Rightarrow \quad y = 50$$

Putting value of $y$ in (ii), we get

$$3x + 250 = 1750$$

$$\Rightarrow \quad 3x = 1500$$

$$\Rightarrow \quad x = 500$$

Therefore, cost of each bat = ₹ 500 and cost of each ball = ₹ 50.

## SECTION - E

**36.** (A) Different size of squares are = 18 × 18 × 1, 16 × 16 × 2, 14 × 14 × 3, 12 × 12 × 4, 10 × 10 ×5, 8 × 8 × 6

(B) As the side length of any value could be cut out from the square and it could be infinite in number.

(C) Let the width of square of each side be '$x$'

Then, sides of box are $20 - 2x$, $20 - 2x$ and $x$

$$\text{Volume} = lbh$$

$$= (20 - 2x)(20 - 2x)x$$

$$= (400 - 40x - 40x + 4x^2)x$$

$$= 4x^3 - 80x^2 + 400x$$

**OR**

On calculating the volume of the boxes given in the options. The box with dimension 14 × 14 × 3 has maximum volume as 588.

On calculating the volume of the boxes given in the option. The box with dimensions 18 × 18 × 1 has minimum volume.

**37.** (A)


There are 27 flags. So the middle most flag is 14th flag.

(B) Total distance travelled = 13 flags on left side + 13 flags on right side

$= 364 + 364$

$= 728$ m

(C) For placing first placing she go 2 m and come back 2 m. Then for second flag, she goes 4 m and come back 4 m and so one ....

Distance travalled = 4 , 8 , 12, ........

Then it forms an A.P. with $a$ = 4, $d$ = 4 and $n$ = 13

Then $S_{13} = \frac{13}{2}[8 + 12 \times 4]$

$= \frac{13}{2}(56) = 364$ m

### ⚠ Caution

➥ *Use the appropriate formula for finding the sum of first 'n' terms, as per the values mentioned in the question.*

**OR**

The maximum distance that she covered in placing will be the 13th flag on both side

Then, $a_{13} = a + (n - 1)d$

$= 4 + (13 - 1) \times 4$

$= 4 + 48 = 52$

∴ From carrying the flag to its position

she covers distance $= \frac{52}{2}$

$= 26$ m

**38.** (A) In the basis of given equation,

$h = -16t^2 + 64t + 80$

when, $t = 1$ second

$h = -16(1)^2 + 64(1) + 80$

$= -16 + 144 = 128$ m

(B) When ball hits the ground, $h = 0$

$-16t^2 + 64t + 80 = 0$

∴ $t^2 - 4t - 5 = 0$

$(t - 5)(t + 1) = 0$

$t = 5$ or $t = -1$

Since, time cannot be negative, so the time = 5 seconds.

(C) Since, $h = -16t^2 + 16t^2 + 80$

⇒ $128 = -16t^2 + 64t^2 + 80$

⇒ $16t^2 + 64t + 80 - 128 = 0$

⇒ $16t^2 + 64t - 48 = 0$

⇒ $t^2 - 4t + 3 = 0$

⇒ $t^2 + 3t - t + 3 = 0$

⇒ $(t - 3)(t - ) = 0$

⇒ $t = 3, 1$

**OR**

Rearrange the given equation, by completing the square, we get

$h = -16(t^2 - 4t - 5)$

$= -16[(t - 2)2 - 9]$

$= -16(t - 2)2 + 144$

Height is maximum, when $t = 2$

∴ Maximum height = 144 m