Factorisation

- **Factorization** is the decomposition of an algebraic expression into product of factors. Factors of an algebraic term can be numbers or algebraic variables or algebraic expressions.

  For example, the factors of $2a^2b$ are 2, a, a, b, since $2a^2b = 2 \times a \times a \times b$

  The factors, 2, a, a, b, are said to be irreducible factors of $2a^2b$ since they cannot be expressed further as a product of factors.

  Also, $2a^2b = 1 \times 2 \times a \times a \times b$

  Therefore, 1 is also a factor of $2a^2b$. In fact, 1 is a factor of every term. However, we do not represent 1 as a separate factor of any term unless it is specially required.

  For example, the expression, $2x^2(x + 1)$, can be factorized as $2 \times x \times x \times (x + 1)$.

  Here, the algebraic expression $(x + 1)$ is a factor of $2x^2(x + 1)$.

- **Factorization of expressions by the method of common factors**

  This method involves the following steps.

  Step 1: Write each term of the expression as a product of irreducible factors.

  Step 2: Observe the factors, which are common to the terms and separate them.

  Step 3: Combine the remaining factors of each term by making use of distributive law.

  Example: Factorize $12p^2q + 8pq^2 + 18\,pq$.

  Solution: We have,

$12p^2q = 2 \times 2 \times 3 \times p \times p \times q$

$8pq^2 = 2 \times 2 \times 2 \times p \times q \times q$

$18pq = 2 \times 3 \times 3 \times p \times q$

The common factors are 2, p, and q.

$\therefore 12p^2q + 8pq^2 + 18pq$

$= 2 \times p \times q\ [(2 \times 3 \times p) + (2 \times 2 \times q) + (3 \times 3)]$

$= 2pq\ (6p + 4q + 9)$

- **Factorization by regrouping terms**

Sometimes, all terms in a given expression do not have a common factor. However, the terms can be grouped by trial and error method in such a way that all the terms in each group have a common factor. Then, there happens to occur a common factor amongst each group, which leads to the required factorization.

Example:

Factorize $2a^2 - b + 2a - ab$.

Solution:

$2a^2 - b + 2a - ab = 2a^2 + 2a - b - ab$

The terms,$2a^2$ and $2a$, have common factors, 2 and a.

The terms, $-b$ and $-ab$ have common factors, -1 and b.

Therefore,

$2a^2 - b + 2a - ab = 2a^2 + 2a - b - ab$

$= 2a\ (a + 1) - b\ (1 + a)$

$= (a + 1)\ (2a - b)$ (As the factor, $(1 + a)$, is common to both the terms)

Thus, the factors of the given expression are $(a + 1)$ and $(2a – b)$.

- Factorisation of quadratic polynomials of the form $ax^2 + bx + c$ can be done using Factor theorem and splitting the middle term.

Example 1:

Factorize x^2 – 7x + 10 using the factor theorem.

Solution:

Let $p(x) = x^2 - 7x + 10$

The constant term is 10 and its factors are ±1, ±2, ±5and ±10.

Let us check the value of the polynomial for each of these factors of 10.

$p(1) = 1^2 - 7 \cdot 1 + 10 = 1 - 7 + 10 = 4 \neq 0$

Hence, $x - 1$ is not a factor of $p(x)$.

$p(2) = 2^2 - 7 \cdot 2 + 10 = 4 - 14 + 10 = 0$

Hence, $x - 2$ is a factor of $p(x)$.

$p(5) = 5^2 - 7 \cdot 5 + 10 = 25 - 35 + 10 = 0$

Hence, $x - 5$ is a factor of $p(x)$.

We know that a quadratic polynomial can have a maximum of two factors. We have obtained the two factors of the given polynomial, which are $x – 2$ and $x – 5$.

Thus, we can write the given polynomial as:

$$p(x) = x^2 - 7x + 10 = (x - 2)(x - 5)$$

Example 2:

Factorize $2x^2$ – 11x + 15 by splitting the middle term.

Solution:

The given polynomial is $2x^2 - 11x + 15$.

Here, $a\,c$= 2 × 15 = 30. The middle term is − 11. Therefore, we have to split − 11 into two numbers such that their product is 30 and their sum is –11. These numbers are − 5 and − 6 [As (–5) + (–6) = –11 and (–5) × (–6) = 30].

Thus, we have:

$$\begin{aligned} 2x^2 - 11x + 15 &= 2x^2 - 5x - 6x + 15 \\ &= x(2x - 5) - 3(2x - 5) \\ &= (2x - 5)(x - 3) \end{aligned}$$

Note: A quadratic polynomial can have a maximum of two factors.

- $(a + b)(a - b) = a^2 - b^2$

Example:

Evaluate 95×105.

Solution:

We have, $95 \times 105 = (100 - 5) \times (100 + 5)$

$= (100)^2 - (5)^2$ [Using identity $(a + b)(a - b) = a^2 - b^2$]

$= 10000 - 25$

$= 9975$

- Identities for sum and difference of two cubes are:
  - $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
  - $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

For example, $x^6 - 729y^6$ can be factorized as:

$x^6 - 729y^6$

$= (x^3)^2 - (27y^3)^2$

$= (x^3 + 27y^3)(x^3 - 27y^3)$ [Using $a^2 - b^2 = (a + b)(a - b)$]

$= [(x)^3 + (3y)^3][(x)^3 - (3y)^3]$

$= (x + 3y)(x^2 + 9y^2 - 3xy)(x - 3y)(x^2 + 9y^2 + 3xy)$