CBSE Test Paper 03

Chapter 3 Matrices

1. If a matrix A is symmetric as well as skew symmetric, then A is a
   a. none of these
   b. null matrix
   c. unit matrix
   d. diagonal matrix
2. The system of linear equations x + y + z = 2, 2x + y - z = 3, 3x + 2y - kz = 4 has a unique solution if,
   a. k = 0
   b. -1 < k < 1
   c. -2 < k < 2
   d. $k \neq 0$
3. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$, A + 2$A^t$ equals
   a. $2A^2$
   b. A^t
   c. A
   d. $-A^t$
4. The order of [x y z] $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is
   a. 3×1
   b. 1×1
   c. 1×3
   d. 3×3
5. The system of linear equations ax+ by= 0, cx + dy = 0 has a non-trival solution if
   a. ad - bc = 0
   b. ad - bc < 0
   c. ad - bc = 0.

d. ac + bd = 0

6. If A is a skew symmetric matrix, then A^2 is a ________.
7. If A and B are square matrices of the same order, then [k(A - B)]' = ________ where k is any scalar.
8. If A is skew-symmetric, then kA is a ________ where k is any scalar.
9. If matrix A = [1 2 3], then write AA'.
10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} \frac{1}{2} \end{bmatrix}$,$D = \begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$, then which of the sums A + B, B + C, C + D and B + D is defined?
11. Define square matrix.
12. Give an example of matrices A, B and C such that AB = AC, where A is non-zero matrix, but B ≠ C.
13. Verify that A^2= I, when $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$.
14. Let $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$; Show that f(x). f(y) = f(x+y).
15. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$.
16. Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.
17. Find X and Y, if $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$.
18. Using elementary transformation, find the inverse of the matrices $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

Solution

1. b. null matrix

   Explanation: Only a null matrix can be symmetric as well as skew symmetric.

   In Symmetric Matrix A^T = A,

   Skew Symmetric Matrix A^T = -A,

   Given that the matrix is satisfying both the properties.Therefore, Equating the RHS we get A = -A i.e 2A = 0.

   Therefore A=0,which is a null matrix.

2. d. $k \neq 0$

   Explanation: The given system of equation has a unique solution if :

   $$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & -k \end{vmatrix} \neq 0 \Rightarrow 1(-k+2)\ -1(-2k+3)+1(4-3) \neq 0 \Rightarrow k \neq 0$$

3. b. A^t

   Explanation: A + 2A' =

   $$\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -4 \\ -2 & 0 & -6 \\ 4 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix} = A'$$

4. b. 1×1

   Explanation: $[xyz]_{1\ x\ 3} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1}$ = $[A]_{1\ \times\ 1}$. (where ; matrix A denotes the product of three given matrices.)

5. a. ad – bc = 0

   Explanation: The given system of equations has a non – trivial solution if :

   $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0 \Rightarrow ad - bc = 0.$$

6. symmetric
7. k(A' - B')

8. skew-symmetric matrix

9. According to the question, A = [1 2 3]

$\therefore \quad A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ [interchange the elements of rows and columns]

Now, $AA' = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$= [(1 \times 1) + (2 \times 2) + (3 \times 3)]$

= [1 + 4 + 9] = [14]

10. Only B + D is defined since matrices of the same order can only be added.

11. A matrix in which the no. of rows is equal to no. of columns i.e. m = n.

12. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} [\because B \neq C]$

$\therefore AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$ ....(i)

And $\therefore AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$ .....(ii)

Thus, we see that AB = AC [using Eqs. (i) and (ii)]

Where, A is non-zero matrix but $B \neq C$

13. We have, $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} [\because A^2 = AA]$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ Hence proved.

14. L.H.S = f(x). f(y)

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x . \sin y + 0 & -\sin y \cos x - \sin x \cos y + 0 & 0+0+0 \\ \sin x \cos y + \cos x . \sin y + 0 & -\sin x . \sin y + \cos x \cos y + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x+y) = R.H.S.$$

15. We shall prove the result by the principle of mathematical induction.

We have P(n) : If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$, $n \in N$

P(1) : $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, so $A^1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Therefore, the result is true for n = 1.

Let the result be true for n = k. So

P(k) : $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$

Now, we prove that the rsult holds for n = k +1

Now, $A^{k+1} = A \cdot A^k = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta \cos k\theta - \sin\theta \sin k\theta & \cos\theta \sin k\theta + \sin\theta \cos k\theta \\ -\sin\theta \cos k\theta + \cos\theta \sin k\theta & -\sin\theta \sin k\theta + \cos\theta \cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta + k\theta) & \sin(\theta + k\theta) \\ -\sin(\theta + k\theta) & \cos(\theta + k\theta) \end{bmatrix} = \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Therefore, the result is true for n = k + 1. Thus by principle of mathematical induction.

We have $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$, holds for all natural numbers.

16. Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

On solving a + 4b = -7 and 2a + 5b = -8 & c + 4d = 2 and 2c + 5d = 4 we get a = 1, b = -2, c = 2, d = 0

$$X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

17. Given : $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ ....(i)

and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ ...(ii)

Multiplying eq. (i) by 2, $4X + 6Y = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$ .....(iii)

Multiplying eq. (ii) by 3, $9X + 6Y = 3\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$ ....(iv)

Eq. (iv) - Eq. (iii)

$$\Rightarrow 5X = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6-4 & -6-6 \\ -3-8 & 15-0 \end{bmatrix} = \begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{5}\begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

Now, From eq. (i), $3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2X$

$$= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

$$\Rightarrow 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\frac{4}{5} & 3+\frac{24}{5} \\ 4+\frac{22}{5} & 0-6 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

18. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$,

Since, A = IA

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$,

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_2$,

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$,

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \leftrightarrow R_3$,

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 0 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2 \; and \; R_3 \to R_3 + 2R_2$,

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_3 \; and \; R_2 \to R_2 - 3R_3$,

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$