Chapter 4 Matrices and Determinants

Ex 4.9

Answer 1e.

Consider a quadratic inequality $3x^2 - 2x + 5 > 0$. This inequality consists of only one variable x.

Thus, $3x^2 - 2x + 5 > 0$ is an example for quadratic inequality in one variable.

Another example is $y \geq x^2 + 4x - 8$. This quadratic inequality consists of two variable x and y.

Thus, $y \geq x^2 + 4x - 8$ is an example for quadratic inequality in two variables.

Answer 1gp.

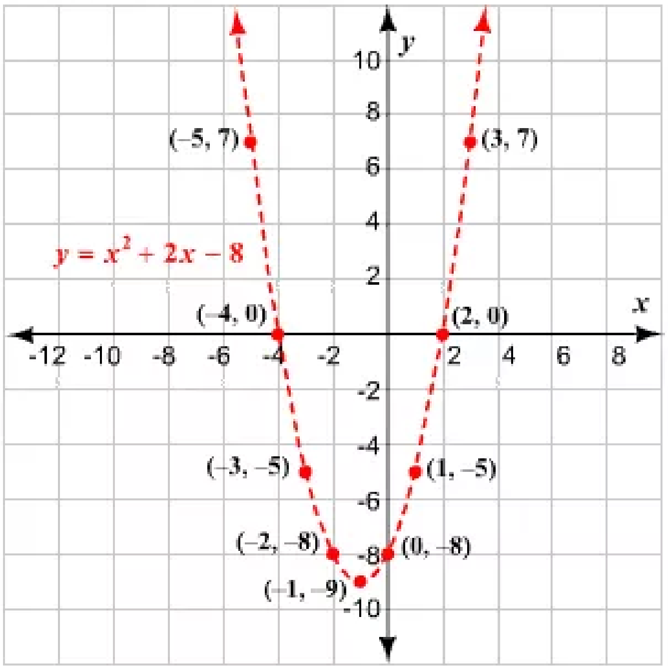
Step1 **Graph** $y = x^2 + 2x - 8$. For this, substitute some values for x, say, -1 and find the corresponding values for y.

$$y = (-1)^2 + 2(-1) - 8$$
$$= 1 - 2 - 8$$
$$= -9$$

Organize the results in a table.

x	-5	-4	-3	-2	-1	0	1	2	3
$y = x^2 + 2x - 8$	7	0	-5	-8	-9	-8	-5	0	7

Plot these points and join them using a smooth curve. Since the inequality is > use a dashed line to draw the curve.



Step 2 **Test** a point inside the parabola, say, (−1, 4).

$$y > x^2 + 2x - 8$$

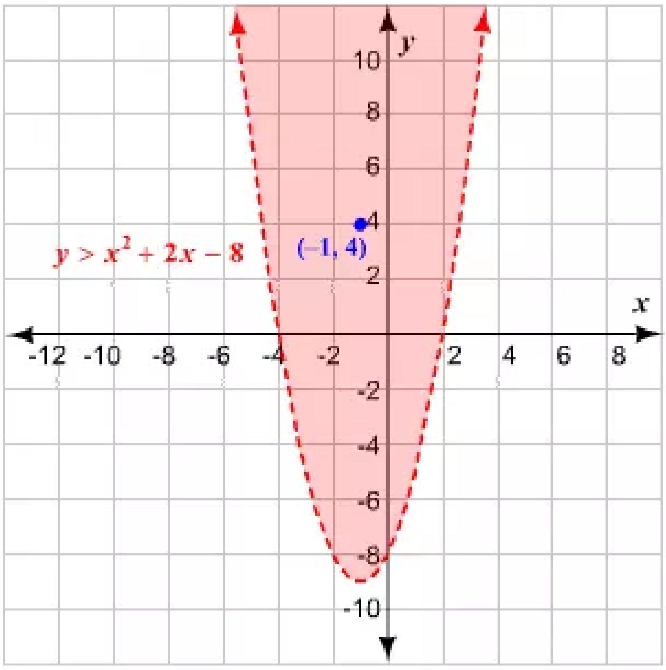
$$4 \overset{?}{>} (-1)^2 + 2(-1) - 8$$

$$4 \overset{?}{>} 1 - 2 - 8$$

$$4 > -9 \quad \checkmark$$

Thus, (−1, 4) is a solution of the inequality.

Step 3 **Shade** the region inside the parabola.



Answer 2gp.

Consider the inequality $y \le 2x^2 - 3x + 1$

Need to sketch the graph of inequality.

Step1:

Graph the parabola $y = 2x^2 - 3x + 1$.

Because the inequality symbol $\le$, make the parabola solid.

Step2:

Test a point inside a parabola, such as $(1,2)$.

$$y \le 2x^2 - 3x + 1$$

$$2 \le 2(1)^2 - 3(1) + 1$$

$$2 \le 2 - 3 + 1$$

$$2 \le 0$$

$$0 \le 1$$

This is false.

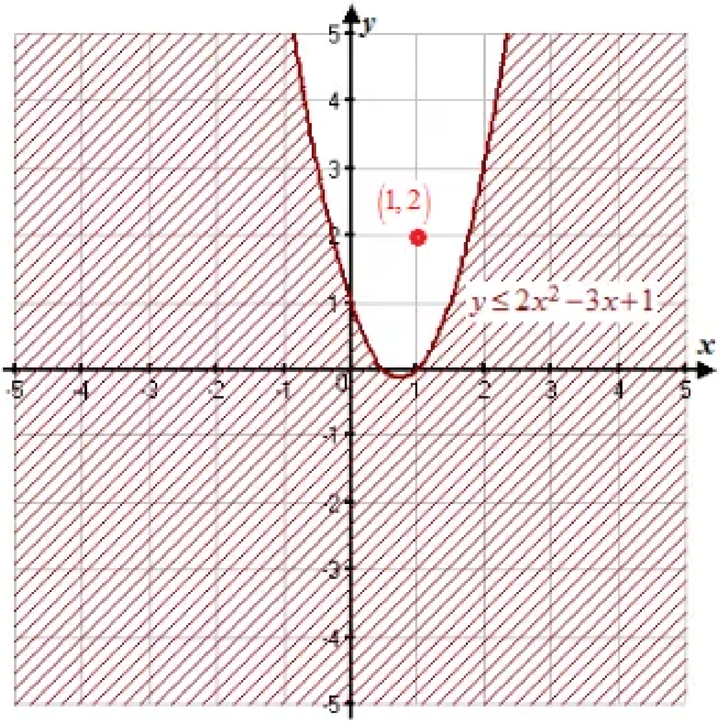
So $(1,2)$ is not a solution of the inequality.

Step3:

Shade the region outside the parabola, because $(1,2)$ is not a solution.

The graph of the inequality $y \le 2x^2 - 3x + 1$ is as follows:

The following diagram contains the graph of the inequality $y \le 2x^2 - 3x + 1$



Answer 3e.

Take a point inside the parabola, say, (2, 0) and check whether it satisfies the inequality.

$y \le x^2 + 4x + 3$

$0 \overset{?}{\le} (2)^2 + 4(2) + 3$

$0 \overset{?}{\le} 4 + 8 + 3$

$0 \le 15$ ×

Thus, (2, 0) is not a solution to the inequality. Thus, shading should be in the portion not containing this point. Also, since $\le$ is the inequality used, a solid curve should be used. This matches with graph in **choice C**.

Answer 3gp.

Step1 **Graph** $y = -x^2 + 4x + 2$.

For this, substitute some values for x, say, 0 and find the corresponding values for y.

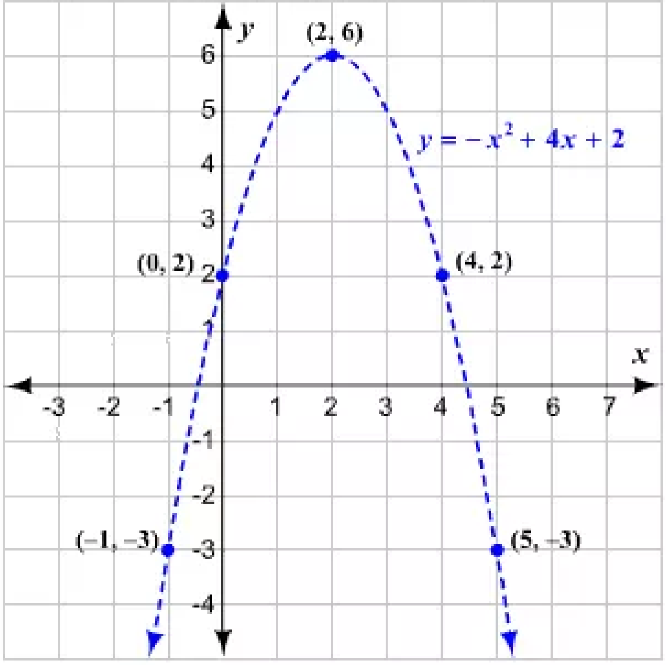
$y = -(0)^2 + 4(0) + 2$

$= 2$

Organize the results in a table.

x	-1	0	2	4	5
$y = -x^2 + 4x + 2$	-3	2	6	2	-3

Plot these points and join them using a smooth curve. Since < is the inequality, use a dashed line to draw the curve.



Step 2 **Test** a point inside the parabola, say, (1, 1).

$y < -x^2 + 4x + 2$

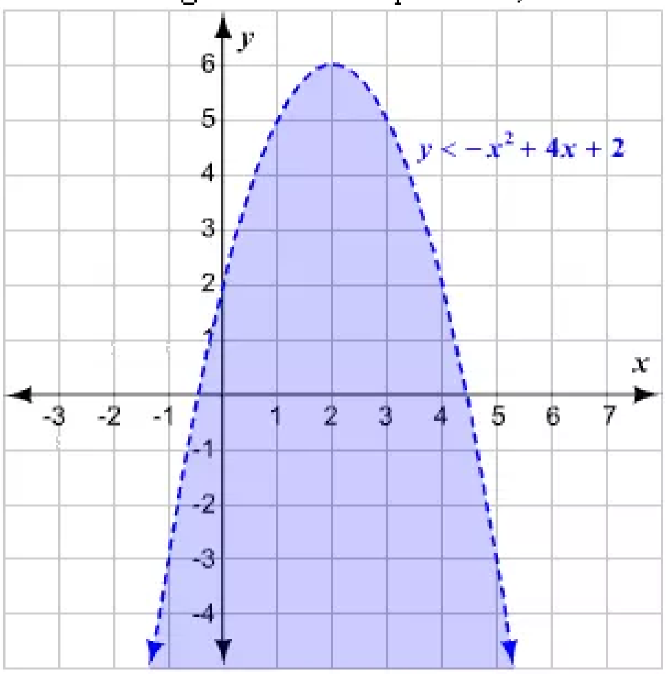
$1 \overset{?}{<} -(1)^2 + 4(1) + 2$

$1 \overset{?}{<} -1 + 4 + 2$

$1 < 5$ TRUE

Thus, (1, 1) is a solution of the inequality.

Step 3 **Shade** the region inside the parabola, which contains the test point.



Answer 4gp.

Consider the quadratic inequalities

$y \geq x^2$

$y < -x^2 + 5$

Need to sketch the graph of the inequalities.

Step1:

Graph $y \geq x^2$.

The graph is the green region inside and including the parabola $y = x^2$.

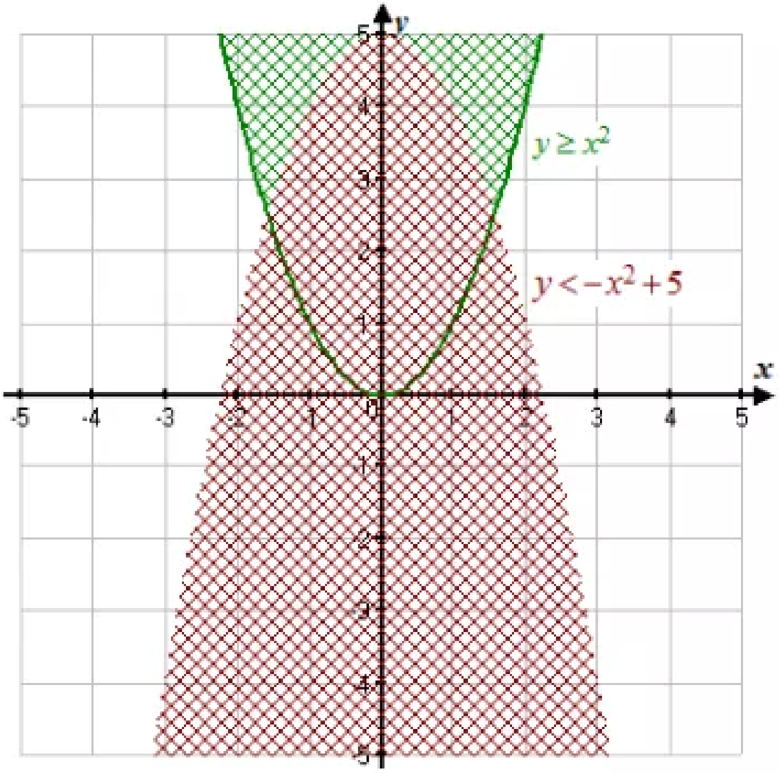
Step2:

Graph $y < -x^2 + 5$.

The graph is the red region inside but not including the parabola $y = -x^2 + 5$

Step3:

Identify the mixed red and green region where the two graphs overlap.
This region is the graph of the system.
The graph of the system is as follows:



Answer 5e.

Take a point inside the parabola, say, (2, 0) and check whether it satisfies the inequality.

$y < x^2 + 4x + 3$

$0 \overset{?}{<} (2)^2 + 4(2) + 3$

$0 \overset{?}{<} 4 + 8 + 3$

$0 < 15 \quad \times$

Thus, (2, 0) is not a solution to the inequality. Thus, shading should be in the portion not containing this point. Also, since the inequality is < a dashed curve should be used. This matches with graph in **choice A**.

Answer 5gp.

First, we can solve the inequality using a table.

Rewrite the inequality such that the right side of the inequality is 0. For this, subtract 3 from both the sides.

$2x^2 + 2x - 3 \le 3 - 3$

$2x^2 + 2x - 3 \le 0$

Substitute some values for x and find the corresponding values of y. Organize the results in a table.

x	-2	-1.825	-1	0	0.825	1
$2x^2 + 2x - 3$	1	0	-3	-3	0	1

From the table, we can see that $2x^2 + 2x - 3 \le 0$ when x takes the values greater than or equal to -1.825 and less than or equal to 0.825.

Therefore, the solution of the inequality is $-1.825 \le x \le 0.825$.

Now, we can solve the inequality using a graph.

The solution to the given inequality consists of the x-values for which the graph of $y = 2x^2 + 2x - 3$ lies on or below the x-axis.

Find the x-intercepts by substituting 0 for y.

$0 = 2x^2 + 2x - 3$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \ne 0$.

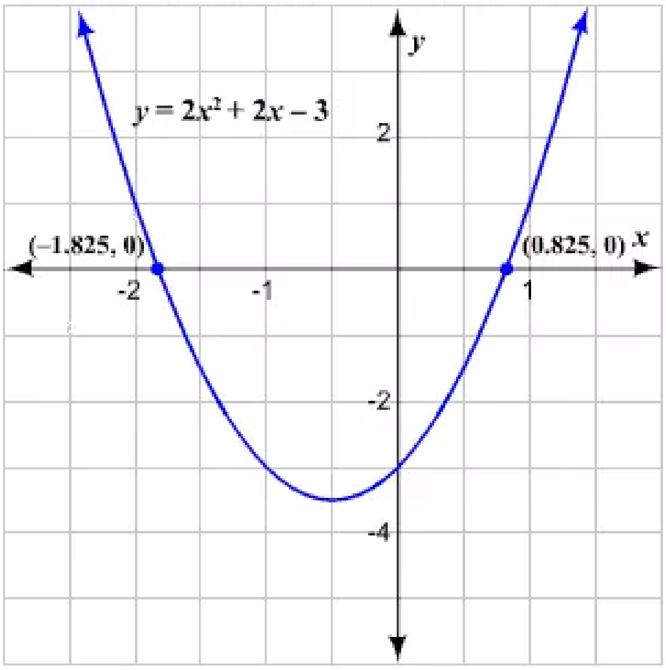
Substitute 2 for a, 2 for b, and -3 for c in the formula.

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-3)}}{2(2)}$$

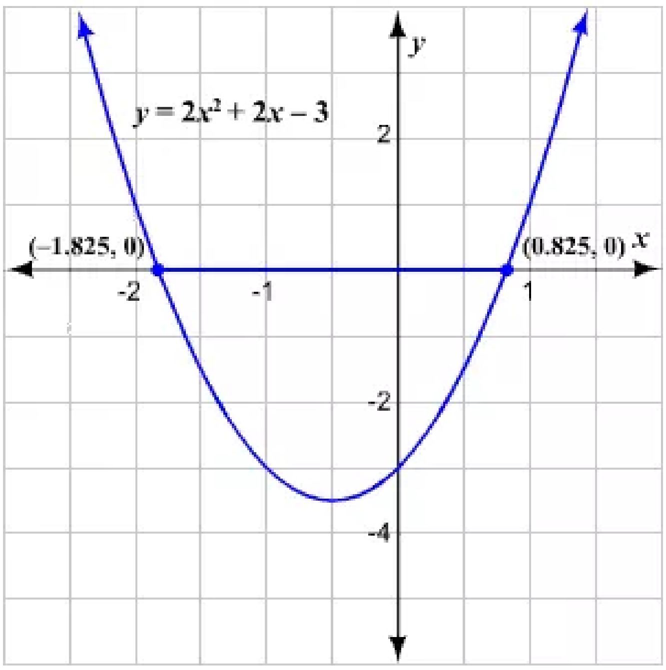
Evaluate.

$$x = \frac{-2 \pm \sqrt{4 + 24}}{4}$$
$$= \frac{-2 \pm \sqrt{28}}{4}$$
$$= \frac{-2 \pm 5.3}{4}$$
$$\approx 0.825 \text{ or } -1.825$$

Sketch a parabola that opens up and has 0.825 and -1.825 as x-intercepts.



The graph lies on or below the x-axis to the right of $x = -1.825$ and to the left of $x = 0.825$.



Thus, the solution is $-1.825 \leq x \leq 0.825$.

Answer 6gp.

Consider the number of teams T that have participated in a robot – building competition can be modeled by $T(x) = 7.51x^2 - 16.4x + 35.0$; $0 \le x \le 9$

Where x is the number of years since 1992

Need to determine in what years at least 200 teams participated in the robot building competition.

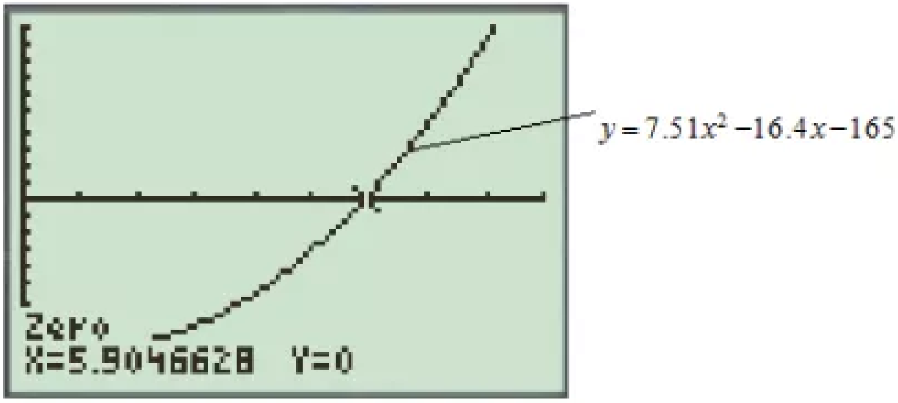
We need to solve $T(x) \ge 200$

$$7.51x^2 - 16.4x + 35.0 \ge 200$$

$$7.51x^2 - 16.4x - 165 \ge 0 \quad \text{Subtract 200 from each side}$$

Let us graph $y = 7.51x^2 - 16.4x - 165$ and check where the graph cuts x-axis.

The following diagram contains the graph of the equation $y = 7.51x^2 - 16.4x - 165$ on the domain $[0,9]$.



Clearly, the graph lies above x-axis when $5.9 < x \le 9$.

Therefore there were at least 200 teams participating in the year 1998 to 2001.

Answer 7e.

Step1 **Graph** $y = 4x^2$.

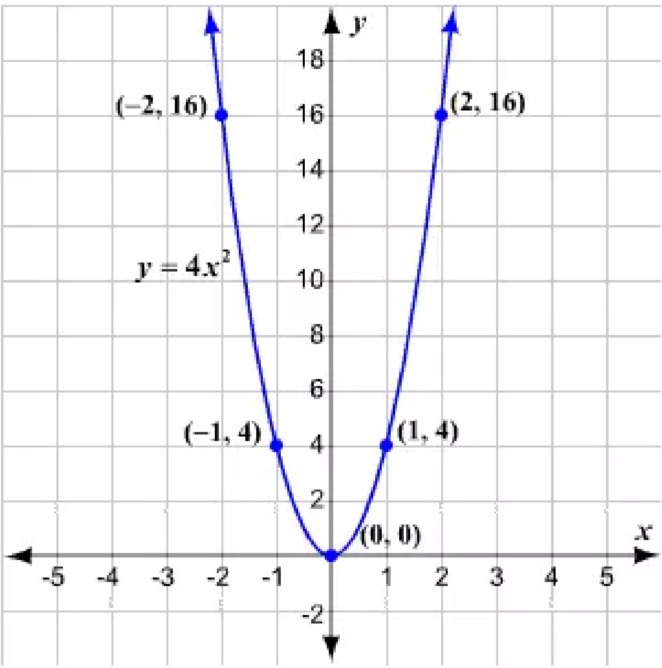
For this, substitute some values for x, say, -1 and find the corresponding values for y.

$$\begin{aligned} y &= (-1)^2 + 7(-1) + 12 \\ &= -1 - 7 + 12 \\ &= 4 \end{aligned}$$

Organize the results in a table.

x	-1	-2	-3	-4	-5	-6
$y = x^2 + 7x + 12$	6	2	0	0	2	6

Plot these points and join them using a smooth curve. Since ≥ is the inequality, draw the curve with a sold line.



Step 2 **Test** a point inside the parabola, say, (1, 8).

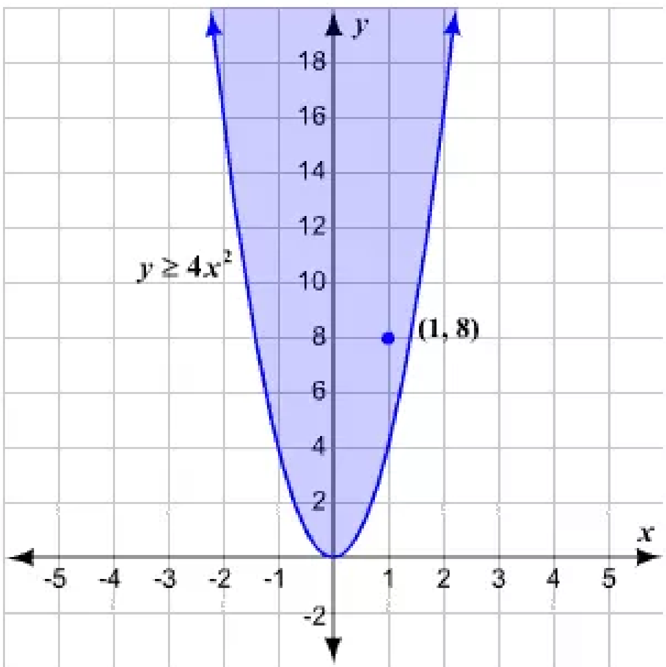
$y \geq 4x^2$

$8 \overset{?}{\geq} 4(1)^2$

$8 \geq 4$ ✓

Thus, (1, 8) is a solution of the inequality.

Step 3 **Shade** the region inside the parabola since it contains the test point.



Answer 7gp.

Write the equation that corresponds to the original inequality.

$2x^2 - 7x = 4$

Rewrite the equation such that the right side is 0. For this, subtract 4 from both the sides.

$$2x^2 - 7x - 4 = 4 - 4$$
$$2x^2 - 7x - 4 = 0$$

Factor the above equation. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 2 for a, -7 for b, and -4 for c in the formula.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-4)}}{2(2)}$$

Evaluate.

$$\begin{aligned} x &= \frac{7 \pm \sqrt{81}}{4} \\ &= \frac{7 \pm 9}{4} \\ &= \frac{16}{4} \text{ or } \frac{-2}{4} \\ &= 4 \text{ or } -0.5 \end{aligned}$$

The numbers 4 and -0.5 are the critical x-values of $2x^2 - 7x > 4$. Plot these points on a number line. Use open dots as these points do not satisfy the inequality.

The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$2x^2 - 7x > 4$
-2	$22 > 4$
0	$0 \not> 4$
5	$15 > 4$

We can see that the inequality is satisfied in the intervals $x < -0.5$ or $x > 4$.

Thus, the solution is $x < -0.5$ or $x > 4$.

Answer 9e.

Step1 **Graph** $y = x^2 + 5x$.

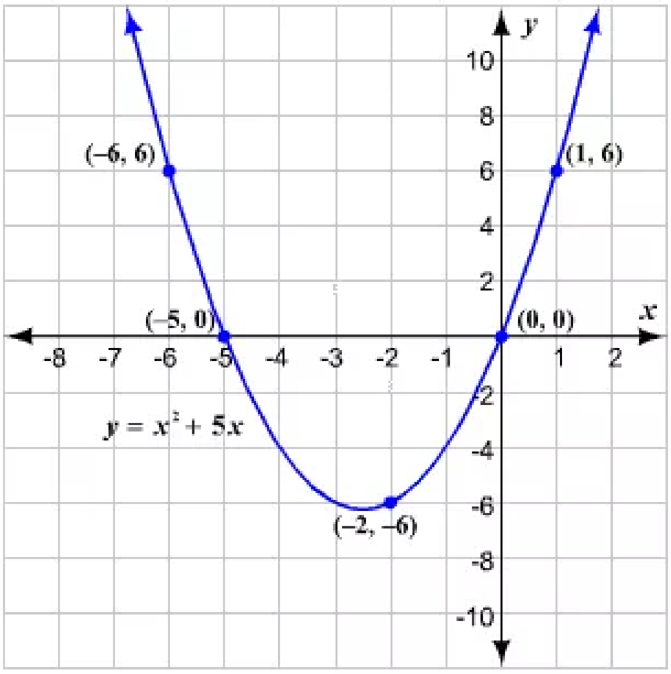
For this, substitute some values for x, say, 1 and find the corresponding values for y.

$$\begin{aligned} y &= (1)^2 + 5(1) \\ &= 1 + 5 \\ &= 6 \end{aligned}$$

Organize the results in a table.

x	1	0	−2	−5	−6
$y = x^2 + 5x$	6	0	−6	0	6

Plot these points and join them using a smooth curve. Since the inequality is ≤ use a sold line to draw the curve.

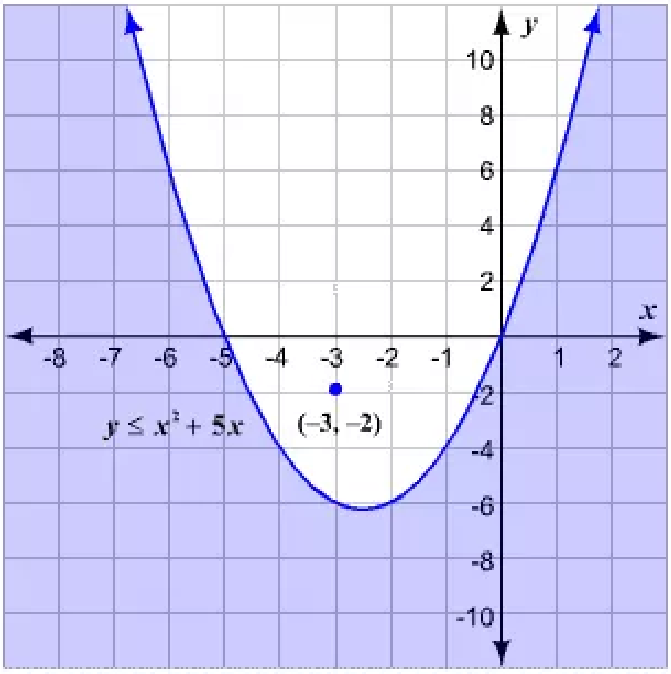


Step 2 **Test** a point inside the parabola, say, $(-3, -2)$.

$$y \le x^2 + 5x$$
$$-2 \overset{?}{\le} (-3)^2 + 5(-3)$$
$$-2 \le -6 \quad \times$$

Thus, $(-3, -2)$ is not a solution of the inequality.

Step 3 **Shade** the region outside the parabola since $(-3, -2)$ is not a solution of the inequality.



Answer 11e.

Step1 **Graph** $y = x^2 + 7x + 12$.

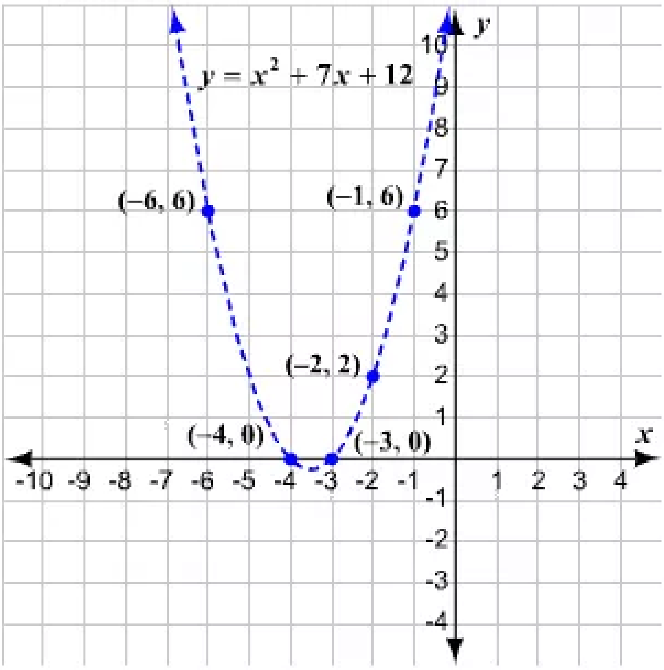
For this, substitute some values for x, say, -1 and find the corresponding values for y.

$$y = 4(-1)^2$$
$$= 4(1)$$
$$= 4$$

Organize the results in a table.

x	-2	-1	0	1	2
$y = 4x^2$	16	4	0	4	16

Plot these points and join them using a smooth curve. Since > is the inequality used, draw the curve with a dashed line.



Step 2 **Test** a point inside the parabola, say, (−3, 6).

$$y > x^2 + 7x + 12$$

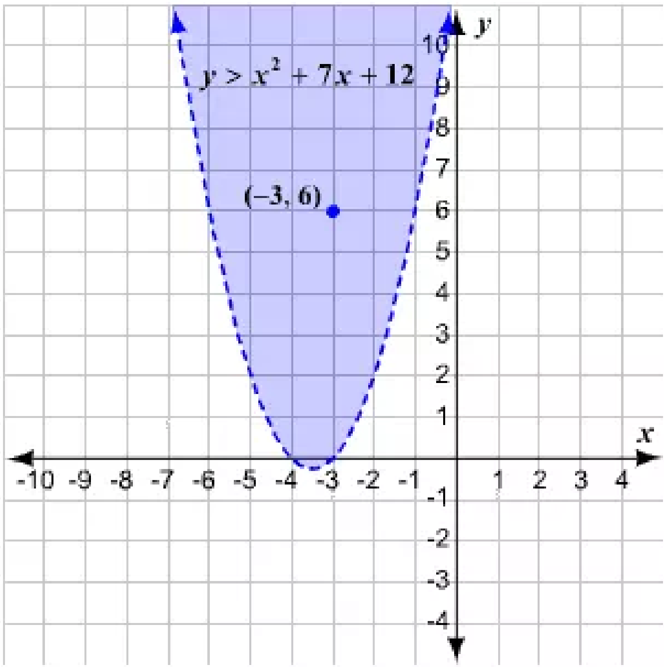
$$6 \overset{?}{>} (-3)^2 + 7(-3) + 12$$

$$6 \overset{?}{>} 9 - 21 + 12$$

$$6 > 0 \quad \checkmark$$

Thus, (−3, 6) is a solution of the inequality.

Step 3 **Shade** the region inside the parabola since it contains the test point.



Answer 12e.

Consider the inequality $y \le -x^2 + 3x + 10$

Need to sketch the graph of inequality.

Step1:

Graph the parabola $y \le -x^2 + 3x + 10$.

Because the inequality symbol $\le$, make the parabola solid.

Step2:

Test a point inside a parabola, such as $(1, 2)$.

$$y \le -x^2 + 3x + 10$$
$$2 \le -(1)^2 + 3(1) + 10$$
$$2 \le -1 + 3 + 10$$
$$2 \le 12$$

This is true.

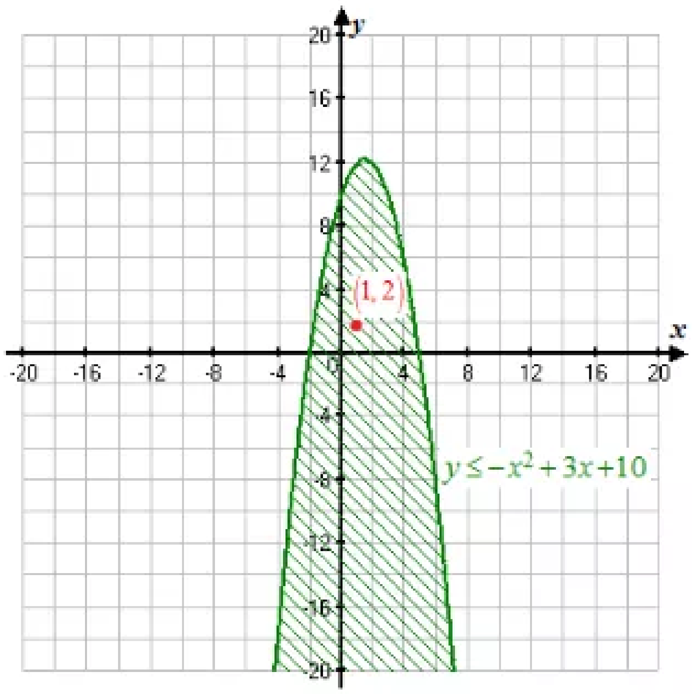
So $(1, 2)$ is a solution of the inequality.

Step3:

Shade the region inside the parabola, because $(1, 2)$ is a solution.

The graph of the inequality $y \le -x^2 + 3x + 10$ is as follows:

The following diagram contains the graph of the inequality $y \le -x^2 + 3x + 10$



Answer 13e.

Step1 **Graph** $y = 2x^2 + 5x - 7$.

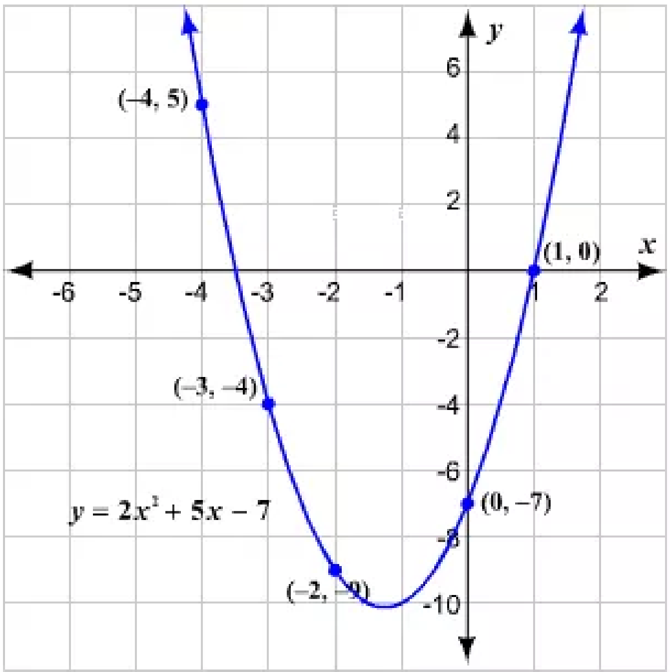
For this, substitute some values for x, say, 0 and find the corresponding values for y.

$$\begin{aligned} y &= 2(0)^2 + 5(0) - 7 \\ &= 0 + 0 - 7 \\ &= -7 \end{aligned}$$

Organize the results in a table.

x	1	0	-2	-3	-4
$y = 2x^2 + 5x - 7$	0	-7	-9	-4	5

Plot these points and join them using a smooth curve. Since $\geq$ is the inequality, use a sold line to draw the curve.



Step 2 **Test** a point inside the parabola, say, $(-2, -6)$.

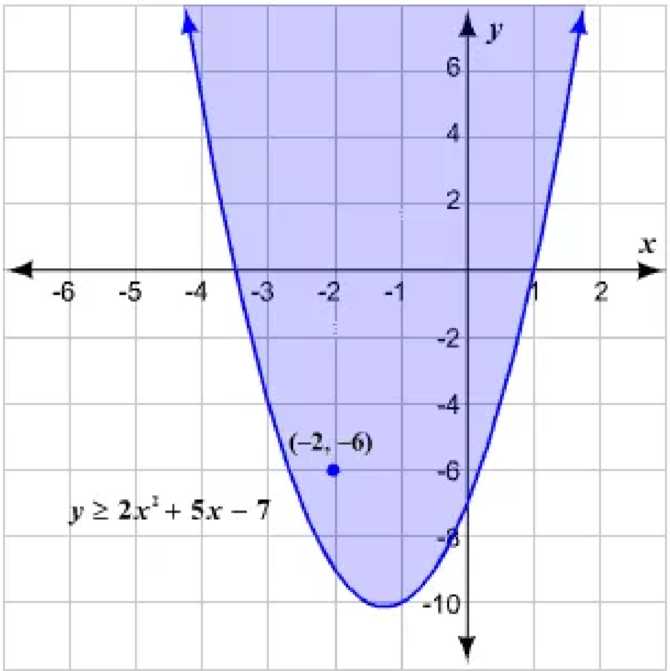
$$y \geq 2x^2 + 5x - 7$$

$$-6 \overset{?}{\geq} 2(-2)^2 + 5(-2) - 7$$

$$-6 \geq -9 \quad \checkmark$$

Thus, $(-2, -6)$ is a solution of the inequality.

Step 3 **Shade** the region inside the parabola since it contains the test point.



Answer 14e.

Consider the inequality $y \geq -2x^2 + 9x - 4$
Need to sketch the graph of inequality.
Step1:

Graph the parabola $y \geq -2x^2 + 9x - 4$.
Because the inequality symbol $\geq$, make the parabola solid.

Step2:

Test a point inside a parabola, such as $(1, 2)$.

$$y \geq -2x^2 + 9x - 4$$

$$2 \geq -2(1)^2 + 9(1) - 4$$

$$2 \geq -2 + 9 - 4$$

$$2 \geq 3$$

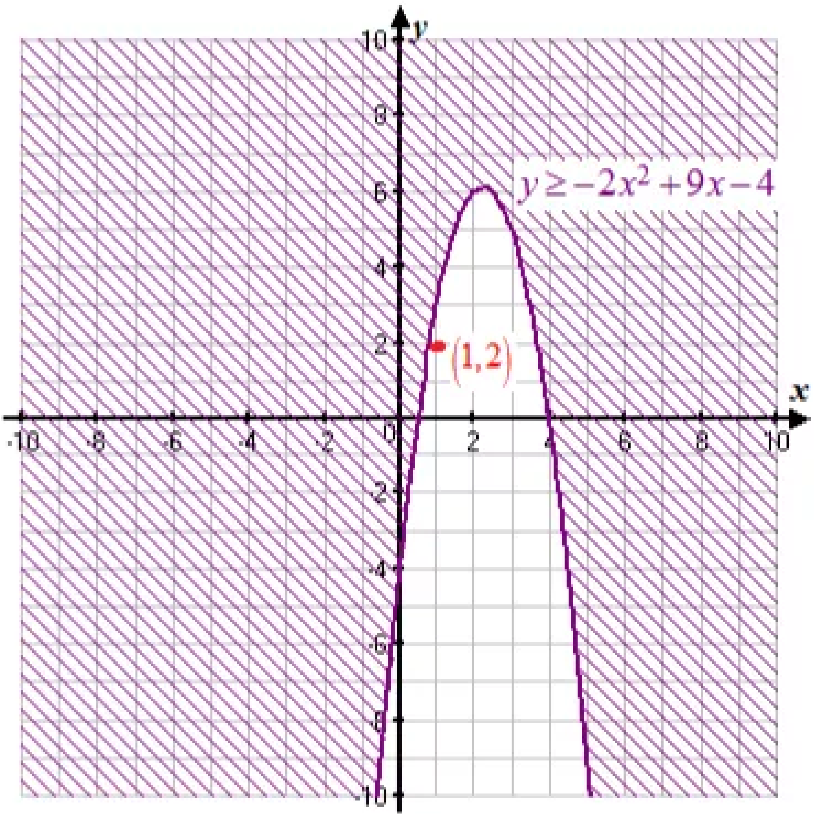
This is FALSE.
So $(1, 2)$ is not a solution of the inequality.

Step3:

Shade the region outside the parabola, because $(1,2)$ is not a solution.

The graph of the inequality $y \geq -2x^2 + 9x - 4$ is as follows:

The following diagram contains the graph of the inequality $y \geq -2x^2 + 9x - 4$



Answer 15e.

Step1 **Graph** $y = 4x^2 - 3x - 5$.

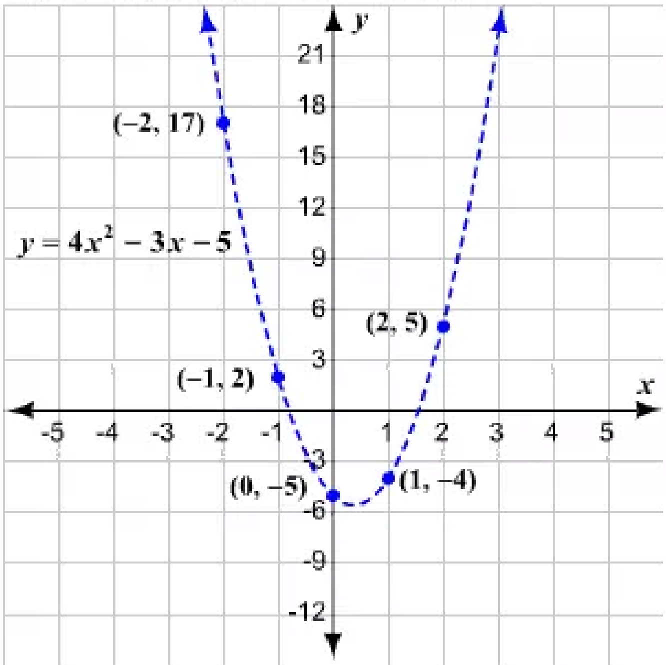
For this, substitute some values for x, say, -1 and find the corresponding values for y.

$$\begin{aligned} y &= 4(-1)^2 - 3(-1) - 5 \\ &= 4(1) + 3 - 5 \\ &= 2 \end{aligned}$$

Organize the results in a table.

x	-2	-1	0	1	2
$y = 4x^2 - 3x - 5$	17	2	-5	-4	5

Plot these points and join them using a smooth curve. Since > is the inequality, use a dashed line to draw the curve.



Step 2 **Test** a point inside the parabola, say, (3, 2).

$y < 4x^2 - 3x - 5$

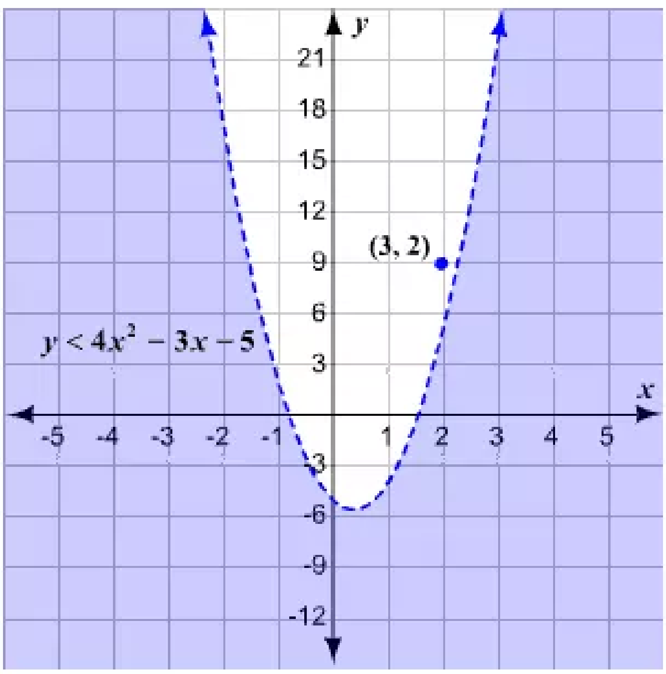
$2 \overset{?}{<} (3)^2 - 3(3) - 5$

$2 \overset{?}{<} 9 - 9 - 5$

$2 < -5$ False

Thus, (3, 2) is not a solution of the inequality.

Step 3 **Shade** the region outside the parabola since it does not contain the test point.



Answer 16e.

Consider the inequality $y > 0.1x^2 - x + 1.2$

Need to sketch the graph of inequality.

Step1:

Graph the parabola $y > 0.1x^2 - x + 1.2$.

Because the inequality symbol $>$, make the parabola line.

Step2:

Test a point inside a parabola, such as $(1,2)$.

$$y > 0.1x^2 - x + 1.2$$

$$2 > 0.1(1)^2 - (1) + 1.2$$

$$2 > 0.1 - 1 + 1.2$$

$$2 > 0.3$$

This is true.

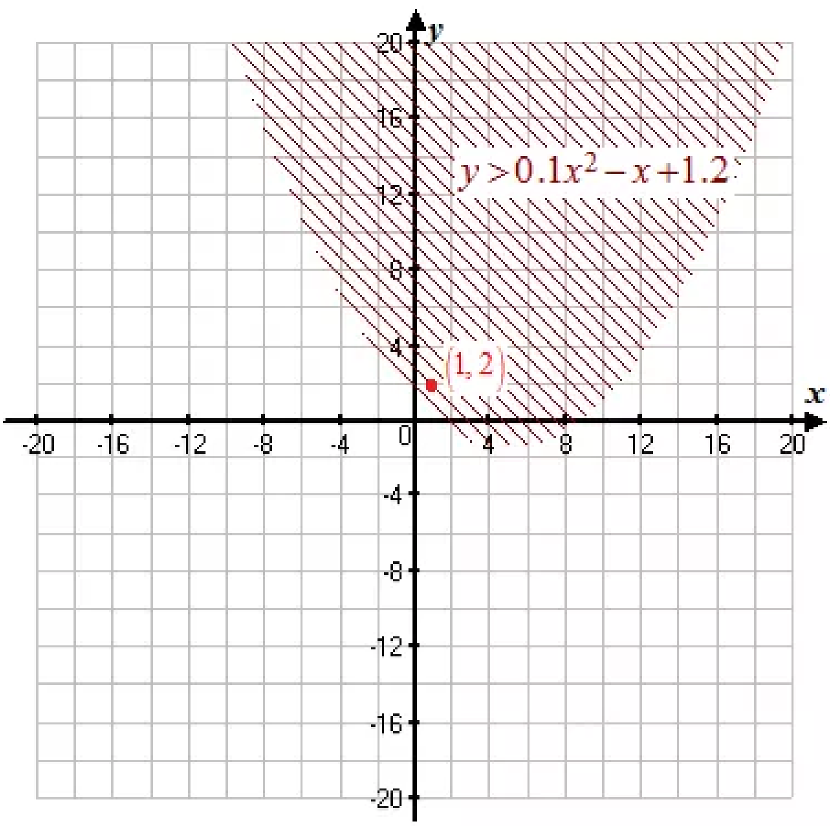
So $(1,2)$ is a solution of the inequality.

Step3:

Shade the region inside the parabola, because $(1,2)$ is a solution.

The graph of the inequality $y > 0.1x^2 - x + 1.2$ is as follows:

The following diagram contains the graph of the inequality $y > 0.1x^2 - x + 1.2$



Answer 17e.

Step1 **Graph** $y = -\frac{2}{3}x^2 + 3x + 1$.

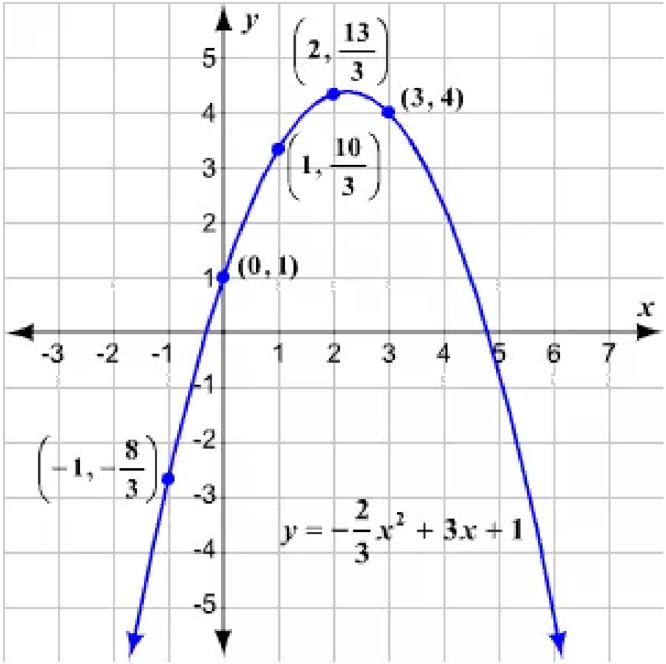
For this, substitute some values for x, say, -1 and find the corresponding values for y.

$$y = -\frac{2}{3}(-1)^2 + 3(-1) + 1$$
$$= -\frac{2}{3} - 3 + 1$$
$$= -\frac{8}{3}$$

Organize the results in a table.

x	-1	0	1	2	3
$y = -\frac{2}{3}x^2 + 3x + 1$	$-\frac{8}{3}$	1	$\frac{10}{3}$	$\frac{13}{3}$	4

Plot these points and join them using a smooth curve. Since $\leq$ is the inequality, use a solid line to draw the curve.



Step 2 **Test** a point inside the parabola, say, (2, 1).

$$y \le -\frac{2}{3}x^2 + 3x + 1$$

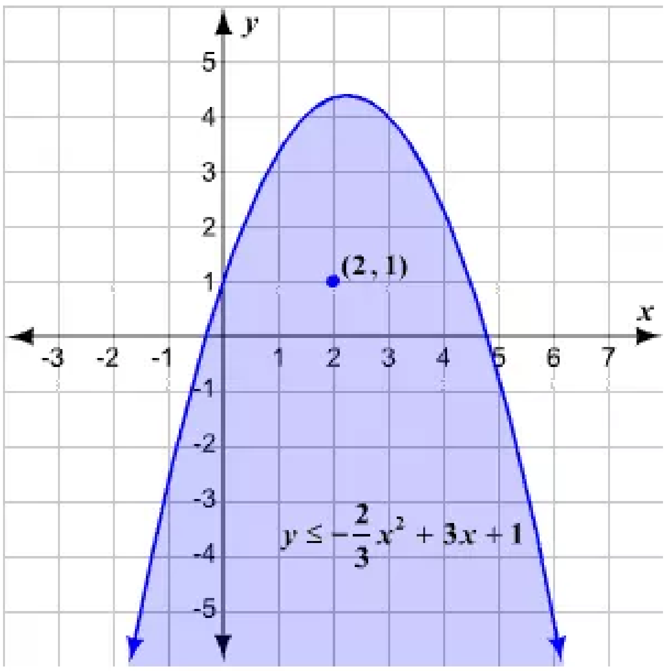
$$1 \overset{?}{\le} -\frac{2}{3}(2)^2 + 3(2) + 1$$

$$1 \overset{?}{\le} -\frac{8}{3} + 6 + 1$$

$$1 \le \frac{13}{3} \quad \checkmark$$

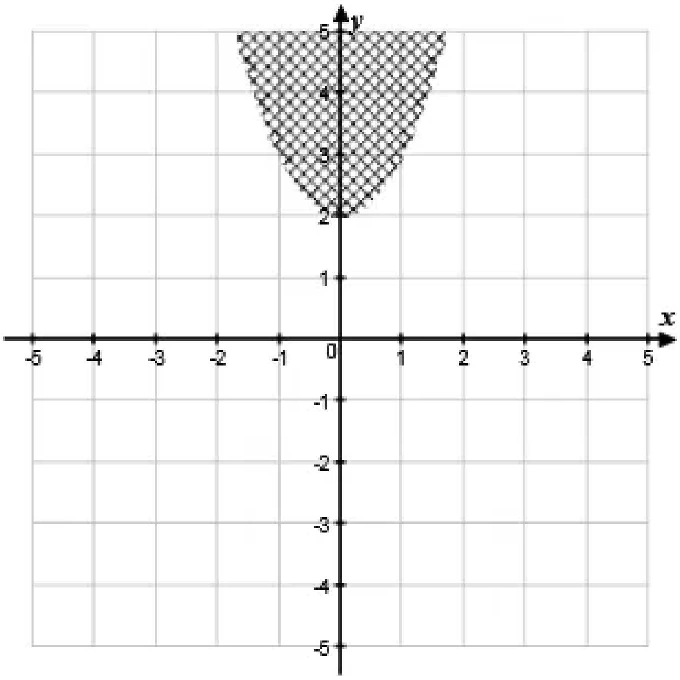
Thus, (2, 1) is a solution of the inequality.

Step 3 **Shade** the region inside the parabola since it contains the test point.



Answer 18e.

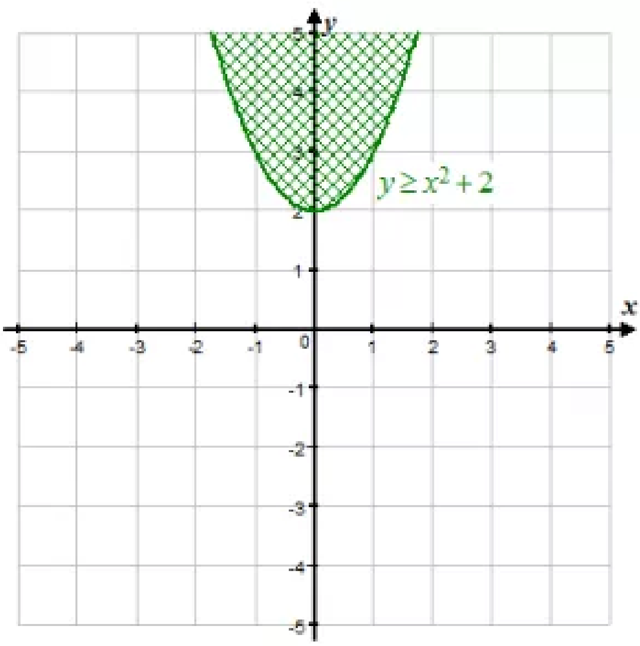
Consider the function $y \geq x^2 + 2$ and graph



In the given graph of the inequality $y \geq x^2 + 2$ the graph of the equation $y = x^2 + 2$ is not included.

Hence the error in the graph is that, it did not include the boundary.

The following diagram contains the graph of the inequality $y \geq x^2 + 2$.



Answer 19e.

The error is that the shading is to be done inside the parabola instead of outside.

Step1 **Graph** $y = x^2 + 2$.

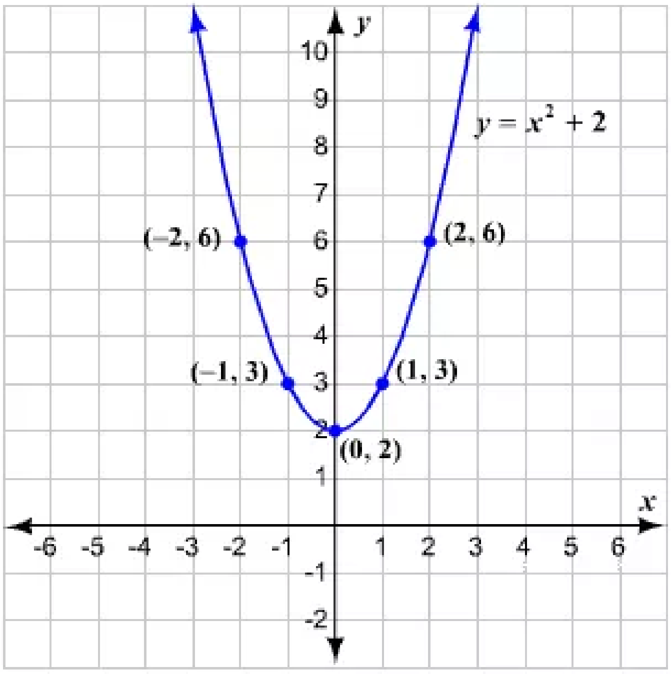
Substitute some value for x, say, 0 and find the corresponding value of y.

$$y = (0)^2 + 2$$
$$= 2$$

Organize the results in a table.

x	-2	-1	0	1	2
$y = x^2 + 2$	6	3	2	3	6

Plot these points and join them using a smooth curve. Since $\geq$ is the inequality, draw a solid curve.



Step 2 **Test** a point inside the parabola, say, (1, 6).

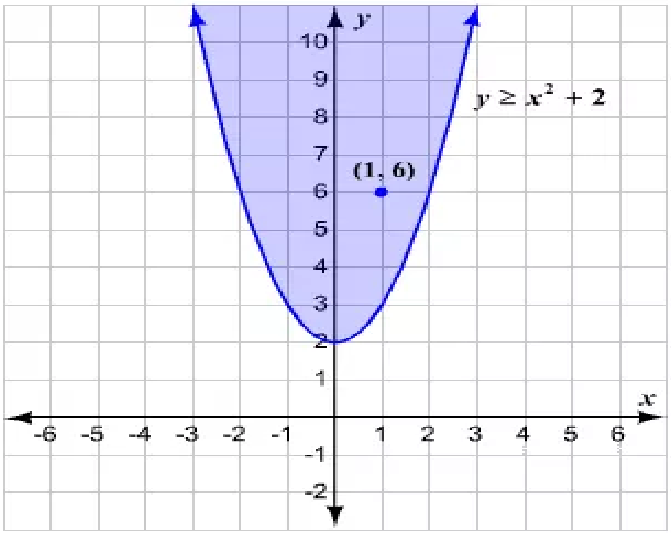
$$6 \geq x^2 + 2$$

$$6 \overset{?}{\geq} (1)^2 + 2$$

$$6 \geq 3 \quad \checkmark$$

Thus, (−2, −6) is a solution of the inequality.

Step 3 **Shade** the region inside the parabola since it contains the test point.



Answer 20e.

Consider the quadratic inequalities

$$y \geq 2x^2$$

$$y < -x^2 + 1$$

Need to sketch the graph of the inequalities.

Step1:

Graph $y \geq 2x^2$

The graph is the green region inside and including the parabola $y \geq 2x^2$.

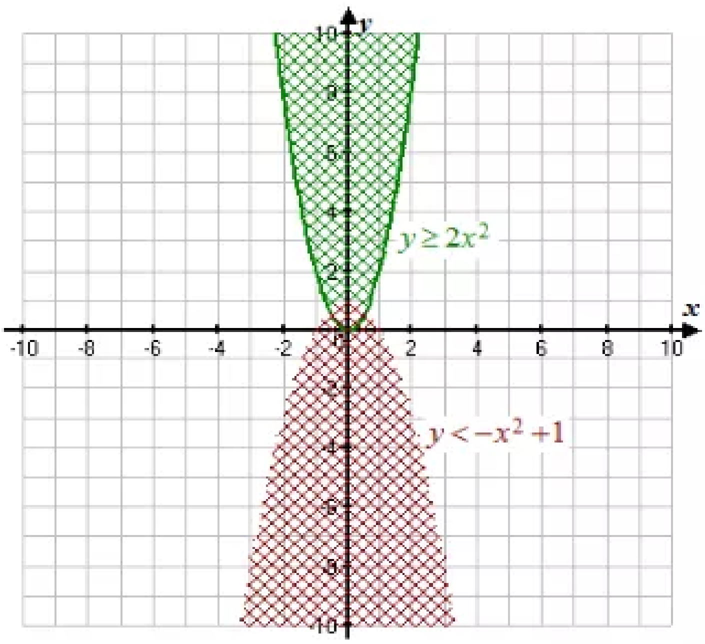
Step2:

Graph $y < -x^2 + 1$

The graph is the red region inside but not including the parabola $y = -x^2 + 1$

Step3:

Identify the mixed red and green region where the two graphs overlap.
This region is the graph of the system.
The graph of the system is as follows:



Answer 21e.

Step 1 **Graph** $y > -5x^2$. For this, first graph $y = -5x^2$.

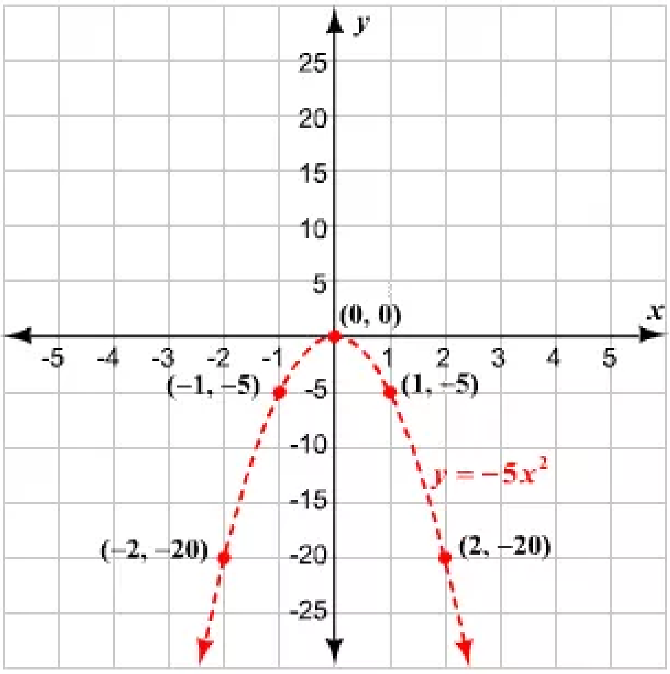
Substitute some value for x, say, 1 and find the corresponding value of y.

$$y = -5(1)^2$$
$$= -5$$

Organize the results in a table.

x	-2	-1	0	1	2
$y = -5x^2$	-20	-5	0	-5	-20

Plot these points and join them using a smooth curve. Since > is the inequality, draw a dashed curve.

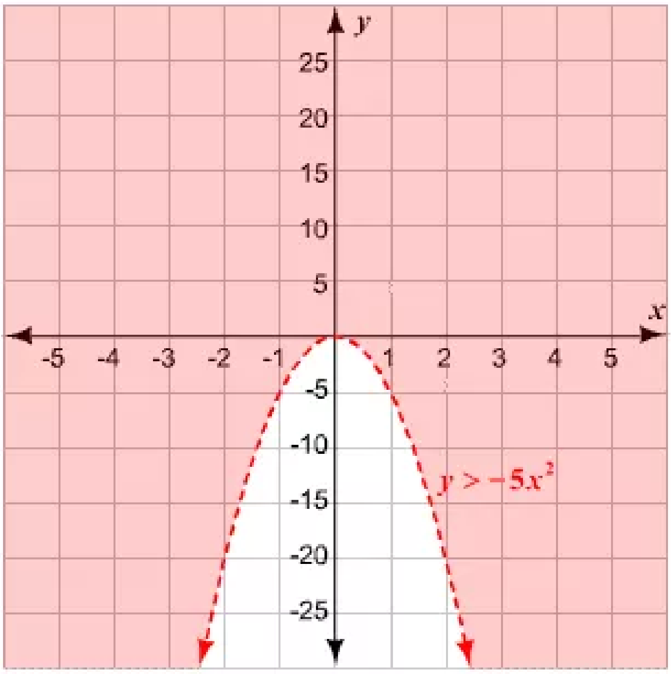


Test a point inside the parabola, say, $(1, -10)$.

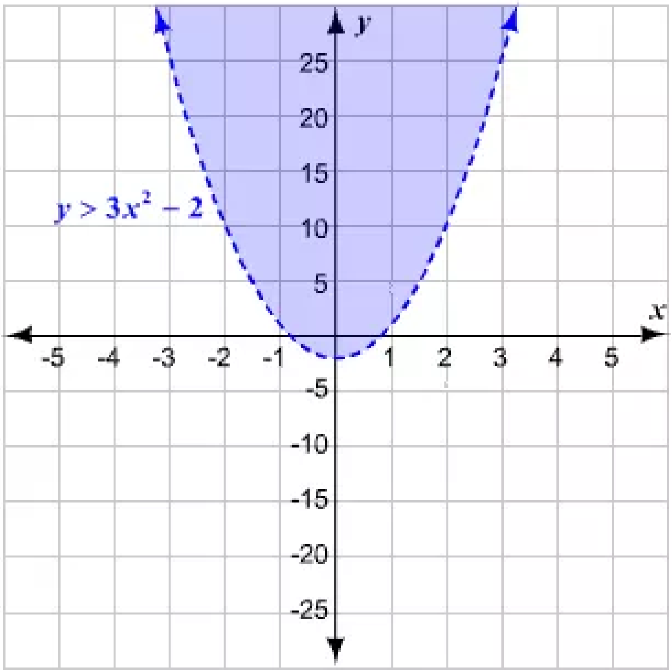
$$y > -5x^2$$
$$-10 \overset{?}{>} -5(1)^2$$
$$-10 > -5 \quad ✗$$

Thus, $(1, -10)$ is not a solution of the inequality.

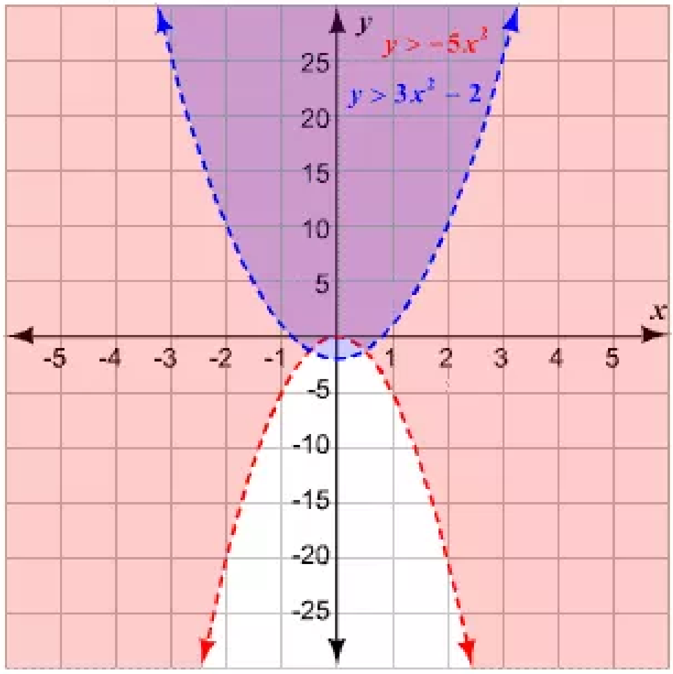
Shade the region outside the parabola since the test point is not a solution of the inequality.



Step 2 **Graph** $y > 3x^2 - 2$.



STEP 3 **Identify** the purple region where the two graphs intersect. This region is the graph of the system.



Answer 22e.

Consider the quadratic inequalities

$$y \geq x^2 - 4$$

$$y \leq -2x^2 + 7x + 4$$

Need to sketch the graph of the inequalities.

Step1:

Graph $y \geq x^2 - 4$

The graph is the green region inside and including the parabola $y = x^2 - 4$.

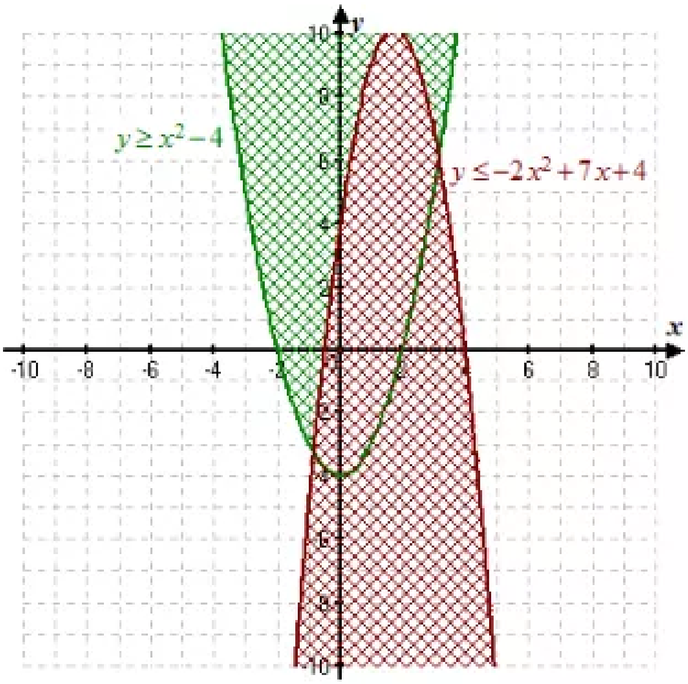
Step2:

Graph $y \le -2x^2 + 7x + 4$

The graph is the red region inside and including the parabola $y = -2x^2 + 7x + 4$

Step3:

Identify the mixed red and green region where the two graphs overlap.
This region is the graph of the system.
The graph of the system is as follows:



Answer 23e.

Step1 **Graph** $y \le -x^2 + 4x - 4$. For this, first graph $y = -x^2 + 4x - 4$.

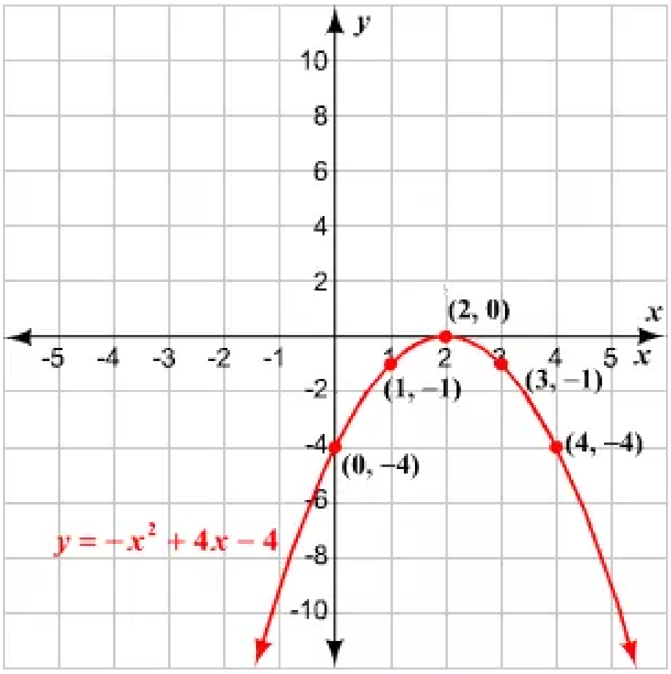
Substitute some value for x, say, 0 and find the corresponding value for y.

$$y = -(0)^2 + 4(0) - 4$$
$$= -4$$

Organize the results in a table.

x	0	1	2	3	4
$y = -x^2 + 4x - 4$	-4	-1	0	-1	-4

Plot these points and join them using a smooth curve. Since the inequality is $\leq$ use a solid line to draw the curve.



Test a point inside the parabola, say, (2, −4).

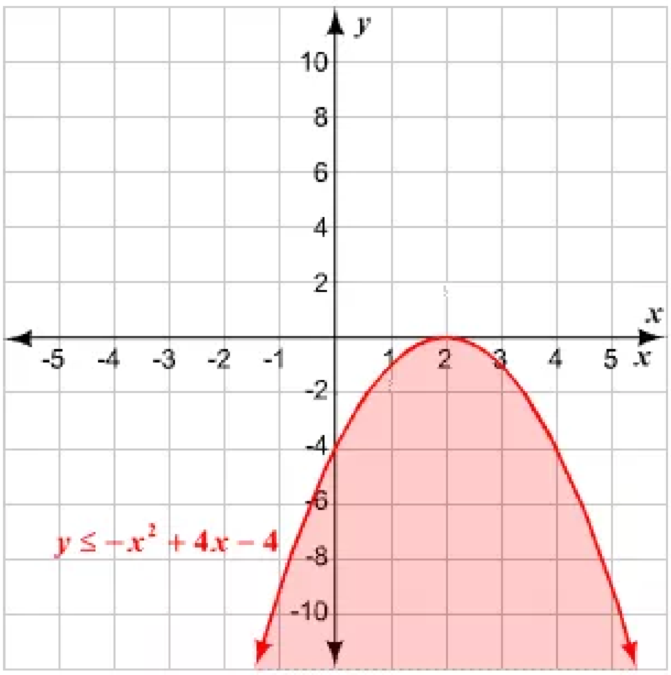
$y \leq -x^2 + 4x - 4$

$-4 \overset{?}{\leq} -(2)^2 + 4(2) - 4$

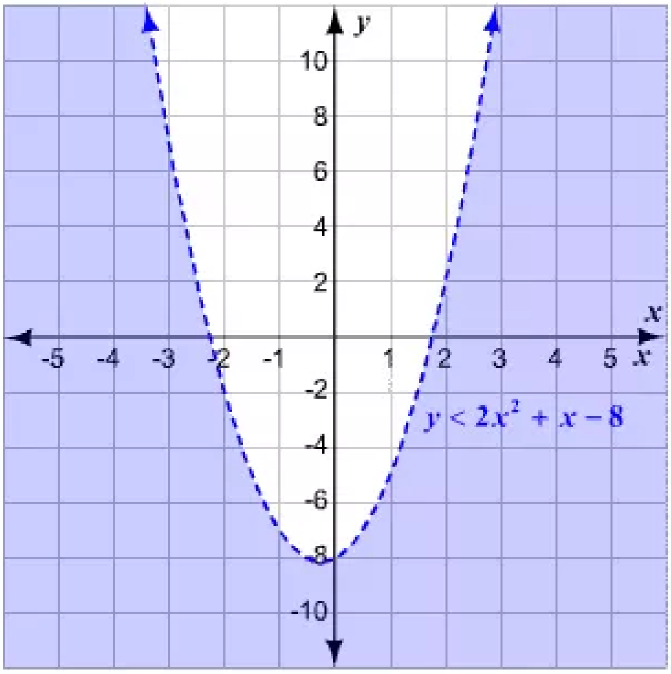
$-4 \leq 0$ ✓

Thus, (2, −4) is a solution of the inequality.

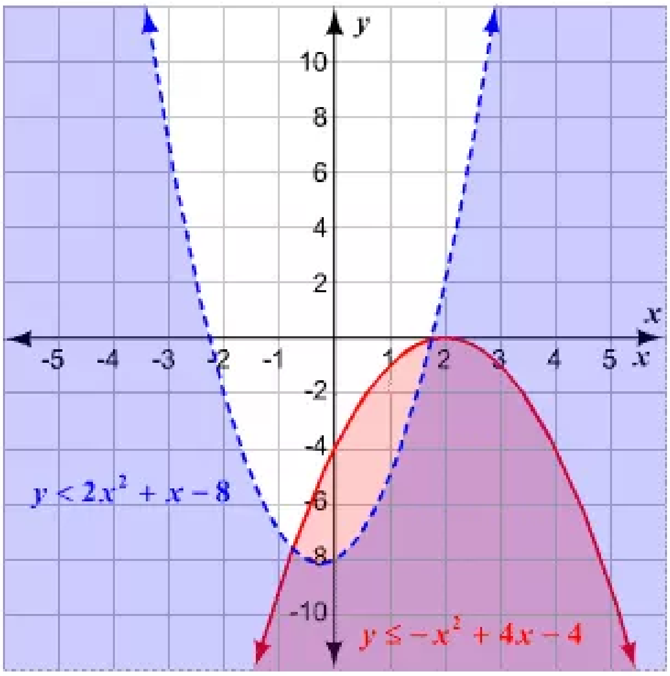
Shade the region inside the parabola since the test point is a solution of the inequality.



STEP 2 **Graph** $y < 2x^2 + x - 8$.



STEP 3 **Identify** the purple region where the two graphs intersect. This region is the graph of the system.



Answer 24e.

Consider the quadratic inequalities

$$y > 3x^2 + 3x - 5$$
$$y < -x^2 + 5x + 10$$

Need to sketch the graph of the inequalities.

Step1:

Graph $y > 3x^2 + 3x - 5$

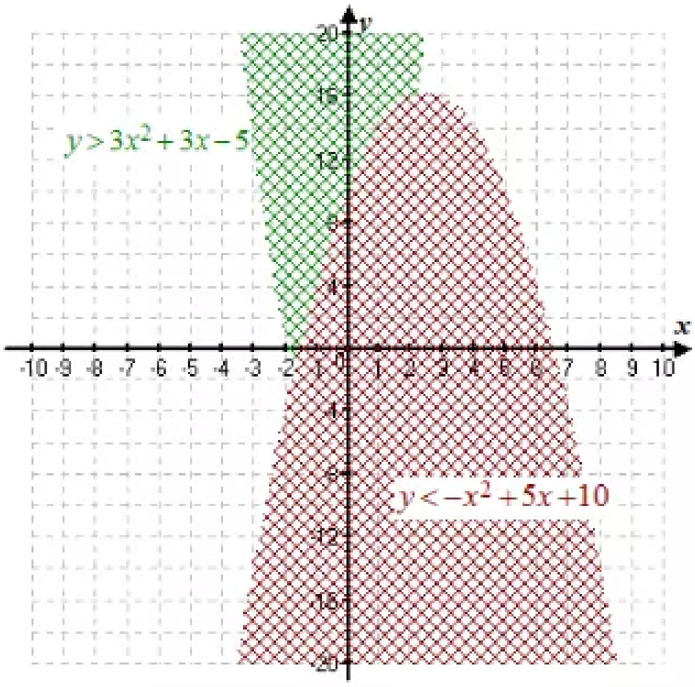
The graph is the green region inside but not including the parabola $y = 3x^2 + 3x - 5$.

Step2:

Graph $y < -x^2 + 5x + 10$
The graph is the red region inside but not including the parabola

Step3:

Identify the mixed red and green region where the two graphs overlap.
This region is the graph of the system.
The graph of the system is as follows:



Answer 25e.

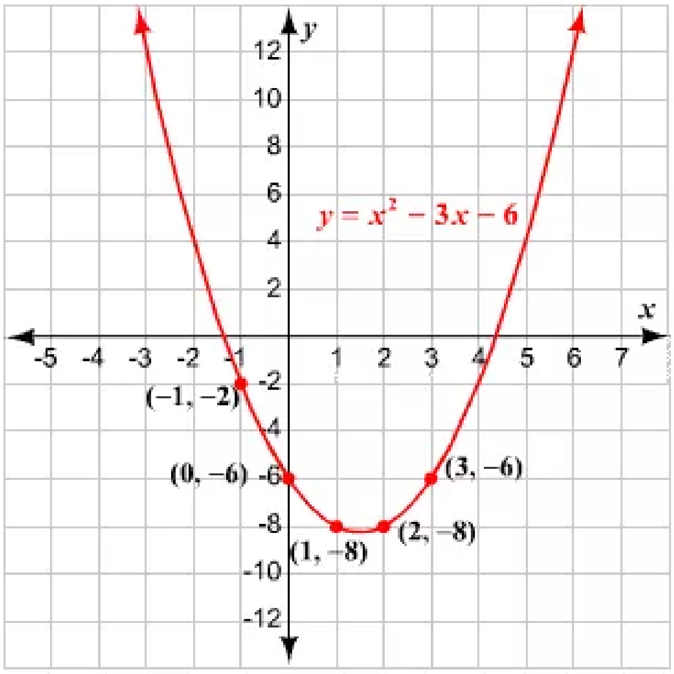
Step1 **Graph** $y \geq x^2 - 3x - 6$. For this, first graph $y = x^2 - 3x - 6$.
Substitute some value for x, say, 0 and find the corresponding value for y.

$$y = (0)^2 - 3(0) - 6$$
$$= -6$$

Organize the results in a table.

x	-1	0	1	2	3
$y = x^2 - 3x - 6$	-2	-6	-8	-8	-6

Plot these points and join them using a smooth curve. Since ≤ is the inequality, draw a solid curve.



Test a point inside the parabola, say, (2, −4).

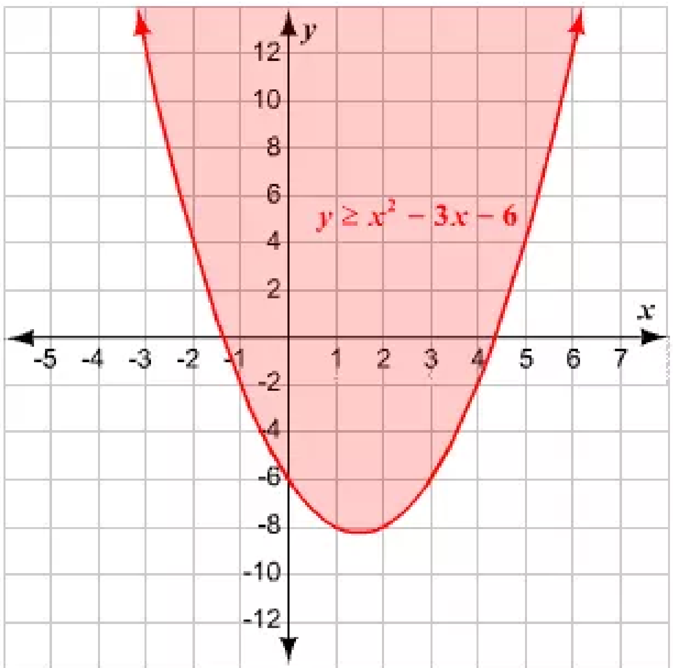
$y \geq x^2 - 3x - 6$

$-4 \overset{?}{\geq} (2)^2 - 4(2) - 6$

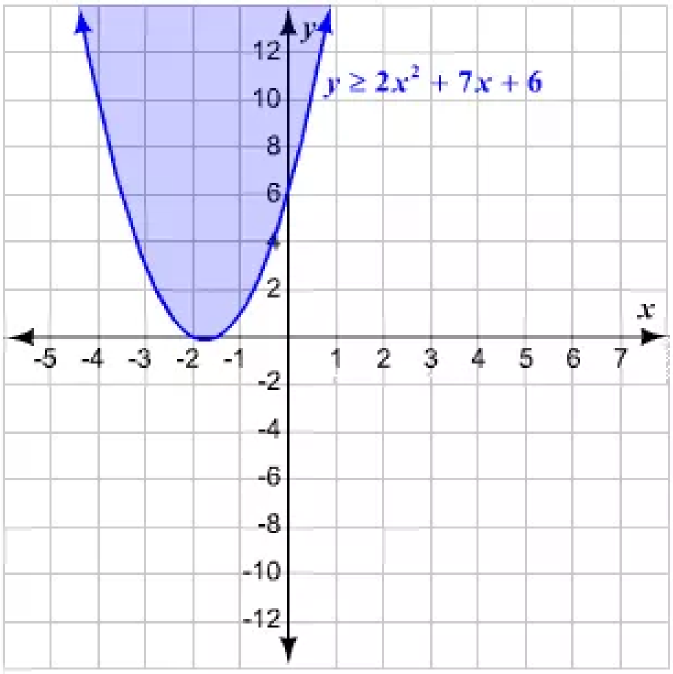
$-4 \geq -10$ ✓

Thus, (2, −4) is a solution of the inequality.

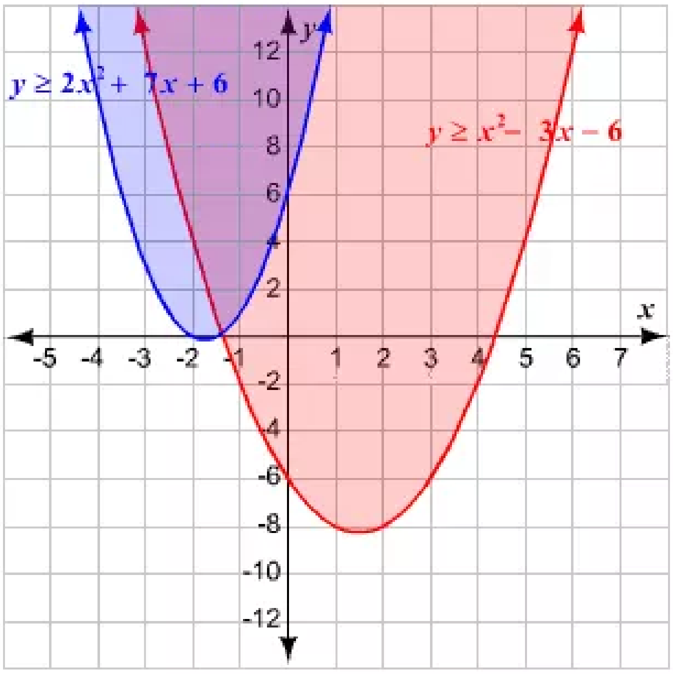
Shade the region inside the parabola since the test point is a solution of the inequality.



STEP 2 **Graph** $y \geq 2x^2 + 7x + 6$.



STEP 3 **Identify** the purple region where the two graphs intersect. This region is the graph of the system.



Answer 26e.

Consider the equation

$$x^2 - 5x < 0$$

Make the table of values.

x	-2	-1	0	1	2	3	4	5	6
$x^2 - 5x$	14	6	0	-4	-6	-6	-4	0	6

The solution of the inequality is exclusive since the inequality does not contain equality sign. Therefore, the points 0 and 5 are excluded from the solution set. The solution set to the inequality is an open set.

Therefore, the solution set of the given inequality is $\boxed{(0,5)}$.

Answer 27e.

In the given inequality, the right side is already 0.

Substitute some values for x and find the corresponding values of y. Organize the results in a table.

x	-5	-4	-3	-2	-1	0	1	2	3
$x^2 + 2x - 3$	12	5	0	-3	-4	-3	0	5	12

From the table we can see that $x^2 + 2x - 3 > 0$ when x takes the values less than -3 and values greater than 1.

Therefore, the solution of the inequality is $x < -3$ and $x > 1$.

Answer 28e.

Consider the equation

$$x^2 + 3x \le 10$$

Rewrite the inequality as $x^2 + 3x - 10 \le 0$

Make the table of values.

x	-6	-5	-4	-3	-2	-1	0	1	2	3
$x^2 + 3x - 10$	8	0	-6	-10	-12	-12	-10	-6	0	8

The solution of the inequality is inclusive since the inequality contains equality sign. Therefore, the points -5 and 2 are included into the solution set. The solution set to the inequality is a closed set.

Therefore, the solution set of the given inequality is $\boxed{[-5, 2]}$.

Answer 29e.

Rewrite the inequality such that the right side of the inequality is 0. For this, subtract 8 from both the sides.

$$x^2 - 2x - 8 \ge 8 - 8$$

$$x^2 - 2x - 8 \ge 0$$

Substitute some values for x and find the corresponding values of y. Organize the results in a table.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
$x^2 - 2x - 8$	26	7	0	-5	-8	-9	-8	-5	0	7	16

From the table, we can see that $x^2 - 2x - 8 \ge 0$ when x takes the values less than or equal to -2 and greater than or equal to 4.

Therefore, the solution of the inequality is $x \le -2$ and $x \ge 4$.

Answer 30e.

Consider the equation

$$-x^2+15x-50>0$$

Rewrite the inequality as $x^2-15x+50<0$

Make the table of values.

x	4	5	6	7	8	9	10	11
$x^2-15x+50$	6	0	–4	–6	–6	–4	0	6

The solution of the inequality is exclusive since the inequality does not contain equality sign. Therefore, the points 5 and 10 are excluded from the solution set. The solution set to the inequality is an open set.

Therefore, the solution set of the given inequality is $\boxed{(5,10)}$.

Answer 31e.

Rewrite the inequality such that the right side of the inequality is 0. For this, add 16 to both the sides.

$$x^2 - 10x + 16 < -16 + 16$$

$$x^2 - 10x + 16 < 0$$

Substitute some values for x and find the corresponding values of y. Organize the results in a table.

x	0	1	2	3	4	5	6	7	8	9	10
$x^2 - 10x + 16$	16	7	0	−5	−18	−9	−8	−15	0	7	16

From the table, it is clear that $x^2 - 10x + 16 < 0$ when x takes the values between 2 and 8.

Therefore, the solution of the inequality is $2 < x < 8$.

Answer 32e.

Consider the equation

$$x^2-4x>12$$

Rewrite the inequality as $x^2-4x-12>0$

Make the table of values.

x	–3	–2	–1	0	1	2	3	4	5	6	7
$x^2-4x-12$	9	0	–7	–12	–15	–16	–15	–12	–7	0	9

From the table it is clear that the inequality less than zero in between -2 and 6. So, the given inequality fails between -2 and 6 and true for all real numbers. Therefore the solution set of the given inequality is $\boxed{(-\infty,-2)\cup(6,\infty)}$.

Answer 33e.

Rewrite the inequality such that the right side is 0. For this, subtract 7 from both the sides.

$3x^2 - 6x - 2 - 7 \le 7 - 7$

$3x^2 - 6x - 9 \le 0$

Substitute some values for x and find the corresponding values of y.
Organize the results in a table.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
$3x^2 - 6x - 9$	63	36	15	0	-9	-12	-9	0	15	36	63

From the table, we can see that $3x^2 - 6x - 9 \le 0$ when x takes the values less than or equal to -1 and greater than or equal to 3.

Therefore, the solution of the inequality is $-1 \le x \le 3$.

Answer 34e.

Consider the equation

$2x^2 - 6x - 9 \ge 11$

Rewrite the inequality as $2x^2 - 6x - 20 \ge 0$

$x^2 - 3x - 10 \ge 0$

Make the table of values.

x	-3	-2	-1	0	1	2	3	4	5	6
$x^2 - 3x - 10$	8	0	-6	-10	-12	-12	-10	-6	0	8

From the table it is clear that the inequality less than zero in between -2 and 5. So, the given inequality fails between -2 and 6 and true for all real numbers. Therefore the solution set of the given inequality is $\boxed{(-\infty, -2] \cup [5, \infty)}$.

Answer 35e.

The graph of the given inequality is similar to the graph of $y = x^2 - 6x$, where solutions are the x-values for which the graph lies below the x-axis.

Find the x-intercepts by substituting $y = 0$.

$0 = x^2 - 6x$

Factor out the common term.

$x(x - 6) = 0$

Set each factor to zero.

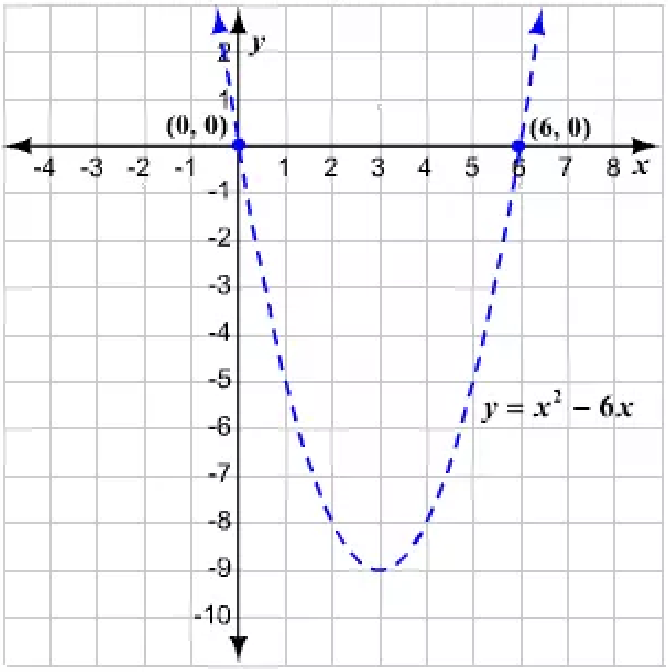
$x = 0 \quad \text{or} \quad x - 6 = 0$

Solve $x - 6 = 0$. For this, add 6 to both the sides.

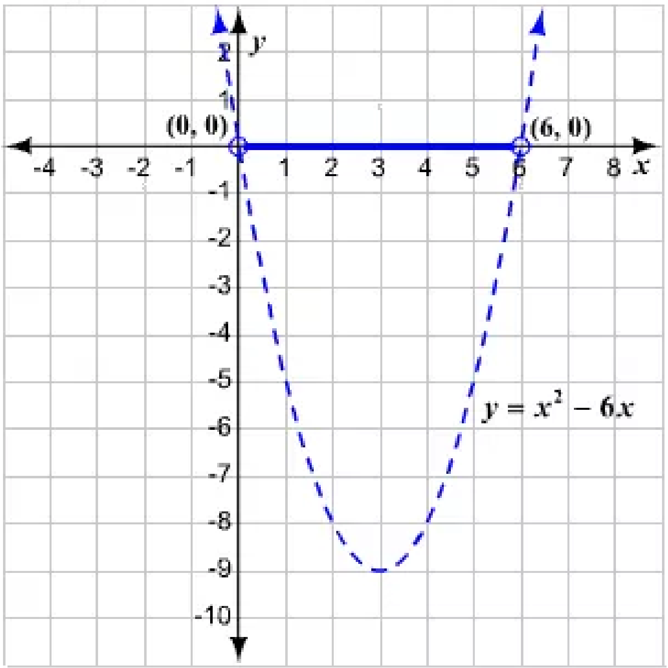
$x - 6 + 6 = 0 + 6$

$x = 6$

Sketch a parabola that opens up and has 0 and 6 as x-intercepts.



The graph lies below the x-axis between $x = 0$ and $x = 6$, excluding 0 and 6.



Thus, the solution is $0 < x < 6$.

Answer 36e.

Consider the equation

$$x^2 + 8x \le -7$$

Rewrite the equation as $x^2 + 8x + 7 \le 0$

Consider the following quadratic function

$$y = x^2 + 8x + 7$$

The solution set consists of x-values for which the graph of the function lies on the x-axis or below the x-axis. Find the x- intercepts by setting $y = 0$.

$$x^2 + 8x + 7 = 0$$

This is a quadratic equation. Solve this by using Quadratic formula.

The quadratic formula is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 1, b = 8, c = 7$

Substitute these values in quadratic formula.

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 1 \cdot 7}}{2(1)}$$

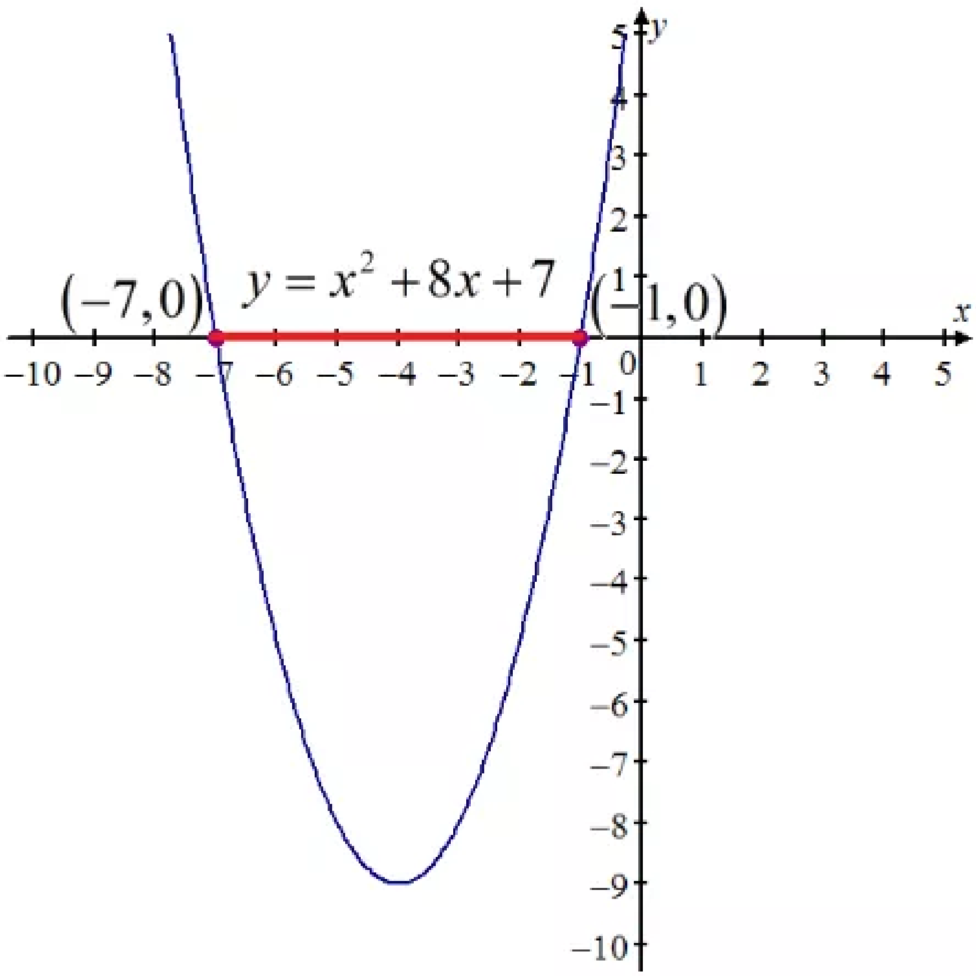
$$= \frac{-8 \pm \sqrt{64 - 28}}{2}$$

$$= \frac{-8 \pm \sqrt{36}}{2}$$

$$= \frac{-8 \pm 6}{2}$$

On simplifying, the x values are $-1, -7$

Plot these x-values in the graph and sketch a parabola opens up with these x-intercepts.



From the graph, the graph of the function lies on or below the x-axis to the left of $x = -1$ and to the right of $x = -7$.

Therefore, the solution set of the given inequality is $[-7, -1]$.

Answer 37e.

The solution to the given inequality consists of the x-values for which the graph of $y = x^2 - 4x + 2$ lies above the x-axis.

Find the x-intercepts by substituting $y = 0$.
$0 = x^2 - 4x + 2$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

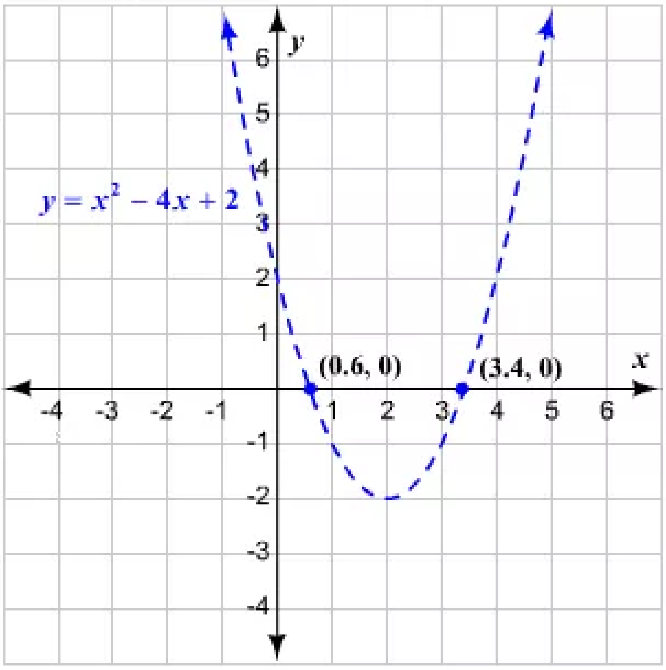
Substitute 1 for a, –4 for b, and 2 for c in the formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

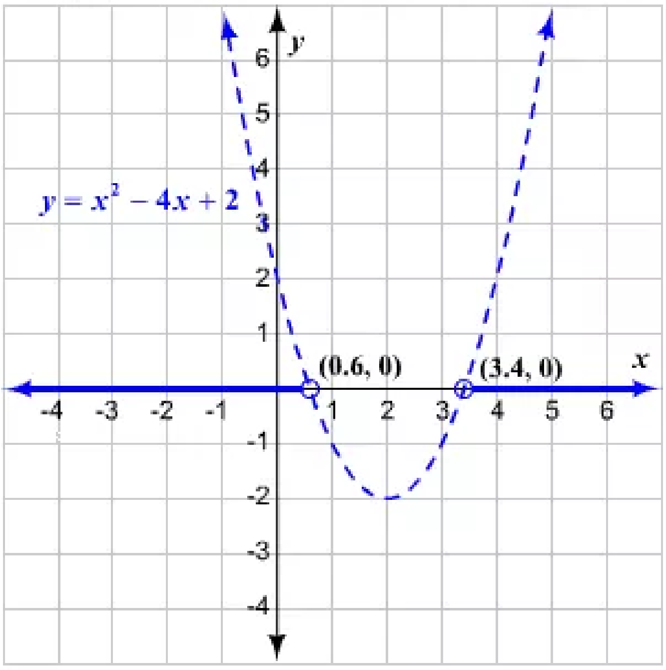
Evaluate.

$$\begin{aligned} x &= \frac{4 \pm \sqrt{16 - 8}}{2} \\ &= \frac{4 \pm \sqrt{8}}{2} \\ &= \frac{4 \pm 2.83}{2} \\ &= \frac{6.83}{2} \text{ or } \frac{1.17}{2} \\ &\approx 3.4 \text{ or } 0.6 \end{aligned}$$

Sketch a parabola that opens up and has 3.4 and 0.6 as x-intercepts.



The graph lies above the x-axis to the left of $x = 0.6$ and to the right of $x = 3.4$.



Thus, the solution is approximately $x < 0.6$ or $x > 3.4$.

Answer 38e.

Consider the equation

$$x^2 + 6x + 3 > 0$$

Consider the following quadratic function

$$y = x^2 + 6x + 3$$

The solution set consists of x-values for which the graph of the function lies on the x-axis or below the x-axis. Find the x- intercepts by setting $y = 0$.

$$x^2 + 6x + 3 = 0$$

This is a quadratic equation. Solve this by using Quadratic formula.

The quadratic formula is given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Here, $a=1, b=6, c=3$

Substitute these values in quadratic formula.

$$x=\frac{-6\pm\sqrt{6^2-4\cdot1\cdot3}}{2(1)}$$

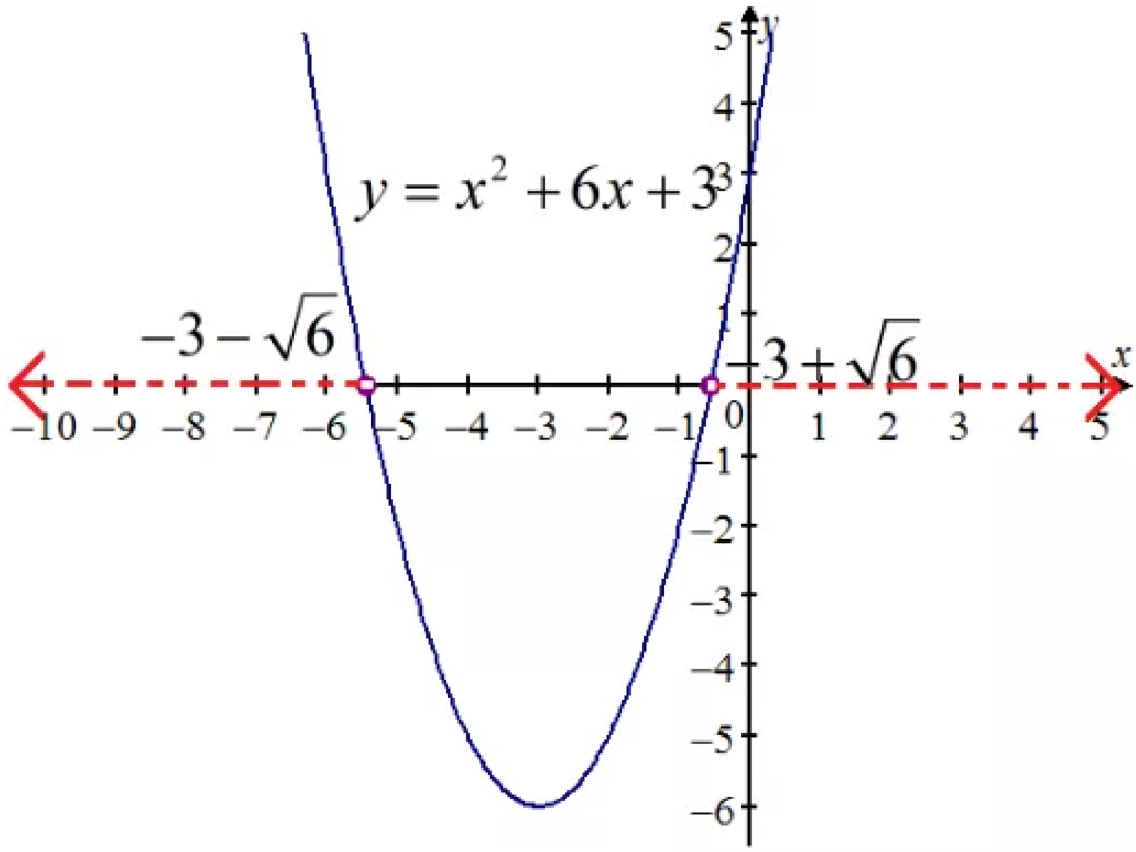
$$=\frac{-6\pm\sqrt{36-12}}{2}$$

$$=\frac{-6\pm\sqrt{24}}{2}$$

$$=\frac{-6\pm2\sqrt{6}}{2}$$

On simplifying, the x values are $-3+\sqrt{6}, -3-\sqrt{6}$

Plot these x-values in the graph and sketch a parabola opens up with these x-intercepts.



Answer 39e.

The graph of the given inequality is similar to the graph of $y = 3x^2 + 2x - 8$, where solutions are the x-values for which the graph lies on or above the x-axis.

Find the x-intercepts by substituting $y = 0$.

$0 = 3x^2 + 2x - 8$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

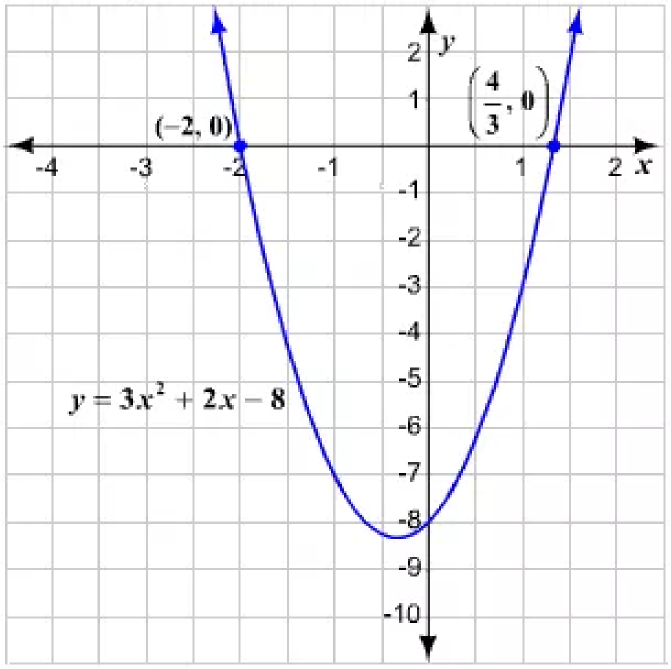
Substitute 3 for a, 2 for b, and -8 for c in the formula.

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-8)}}{2(3)}$$

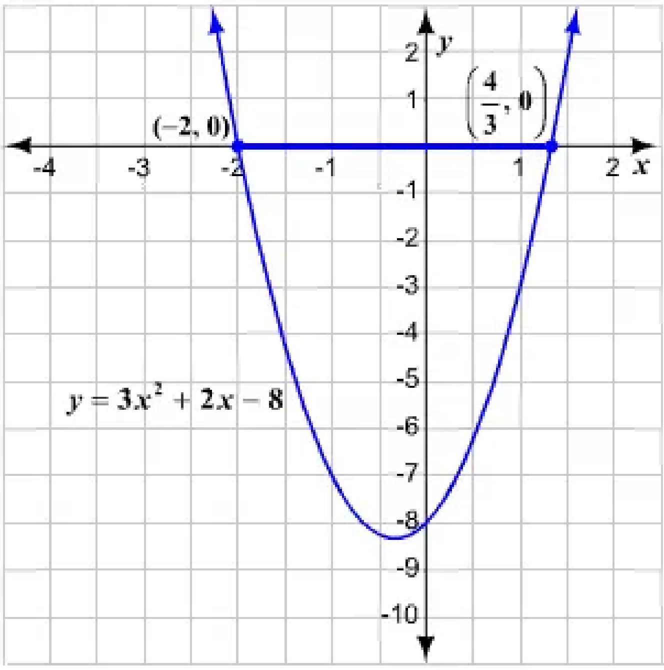
Evaluate.

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{4 + 96}}{6} \\ &= \frac{-2 \pm 10}{6} \\ &= \frac{8}{6} \text{ or } \frac{-12}{6} \\ &\approx 1.33 \text{ or } -2 \end{aligned}$$

Sketch a parabola that opens up and has 1.33 and -2 as x-intercepts.



The graph lies on or below the x-axis between $x = 1.33$ and $x = -2$, including the points 1.33 and -2.



Thus, the solution is $-2 \le x \le 1.33$.

Answer 40e.

Consider the equation

$$3x^2 + 5x - 3 < 1$$

Rewrite the inequality as $3x^2 + 5x - 4 < 0$

Consider the following quadratic function

$$y = 3x^2 + 5x - 4$$

The solution set consists of x-values for which the graph of the function lies on the x-axis or below the x-axis. Find the x- intercepts by setting $y = 0$.

$$3x^2 + 5x - 4 = 0$$

This is a quadratic equation. Solve this by using Quadratic formula.

The quadratic formula is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 3, b = 5, c = -4$

Substitute these values in quadratic formula.

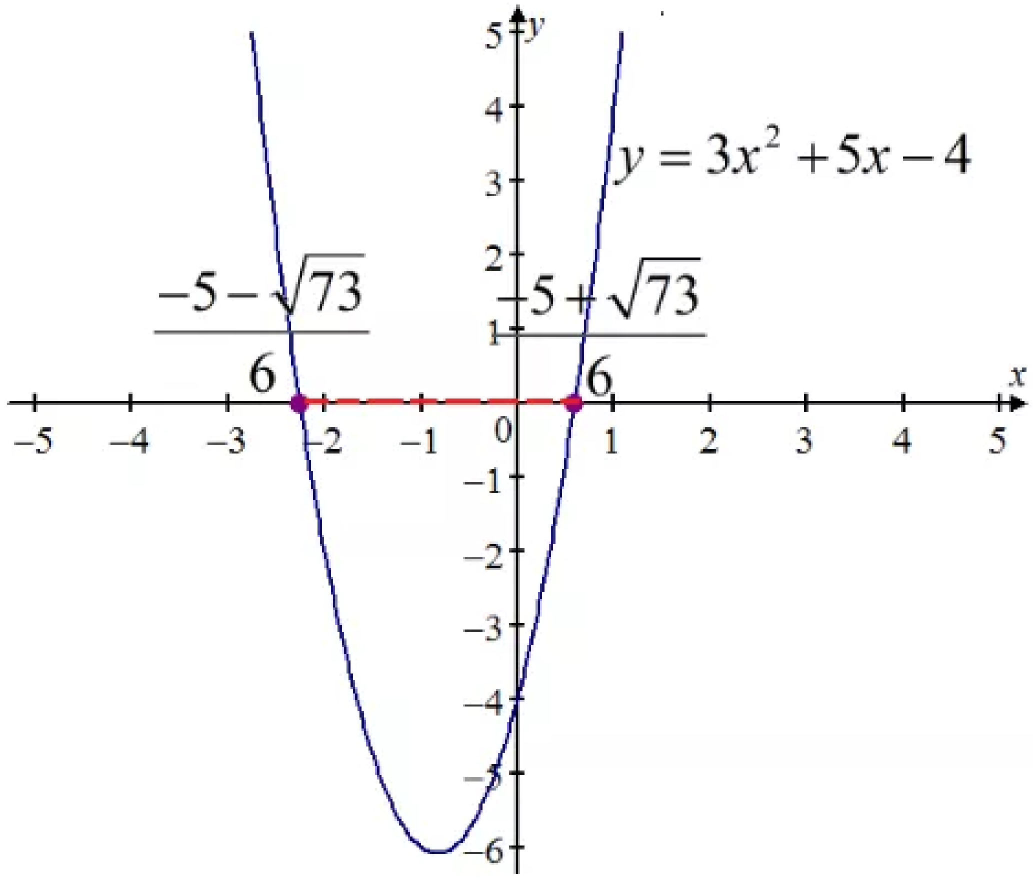
$$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 3 \cdot (-4)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{25 + 48}}{6}$$

$$= \frac{-5 \pm \sqrt{73}}{6}$$

On simplifying, the x values are $\frac{-5 + \sqrt{73}}{6}, \frac{-5 - \sqrt{73}}{6}$.

Plot these x-values in the graph and sketch a parabola opens up with these x-intercepts.



From the graph, the graph of the function lies on or below the x-axis to the left of $x = \frac{-5-\sqrt{73}}{6}$ and to the right of $x = \frac{-5+\sqrt{73}}{6}$.

Therefore the solution set of the given inequality is $\boxed{\left(\frac{-5-\sqrt{73}}{6}, \frac{-5+\sqrt{73}}{6}\right)}$.

Answer 41e.

Rewrite the inequality such that the right side of the inequality is 0. For this, subtract 10 from both the sides.

$-6x^2 + 19x - 10 \geq 10 - 10$

$-6x^2 + 19x - 10 \geq 0$

The solution to the given inequality consists of the x-values for which the graph of $y = -6x^2 + 19x - 10$ lies on or above the x-axis.

Find the x-intercepts by substituting $y = 0$.

$0 = -6x^2 + 19x - 10$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

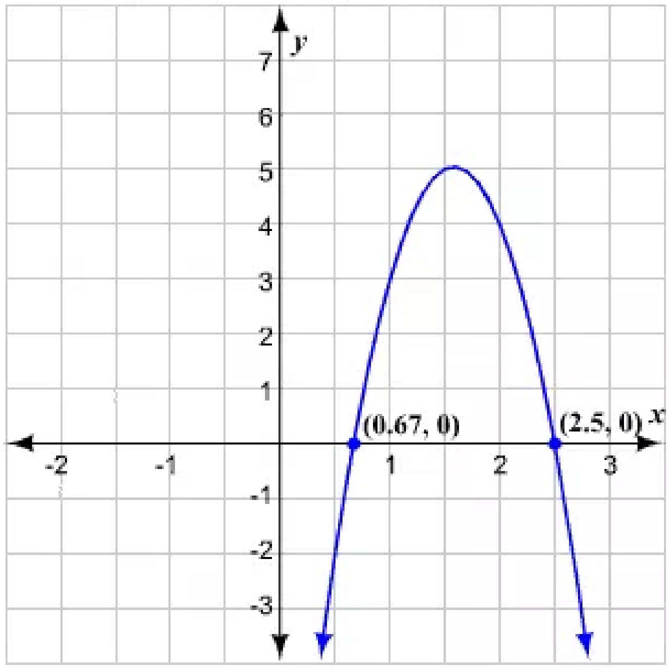
Substitute -6 for a, 19 for b, and -10 for c in the formula.

$$x = \frac{-19 \pm \sqrt{(19)^2 - 4(-6)(-10)}}{2(-6)}$$

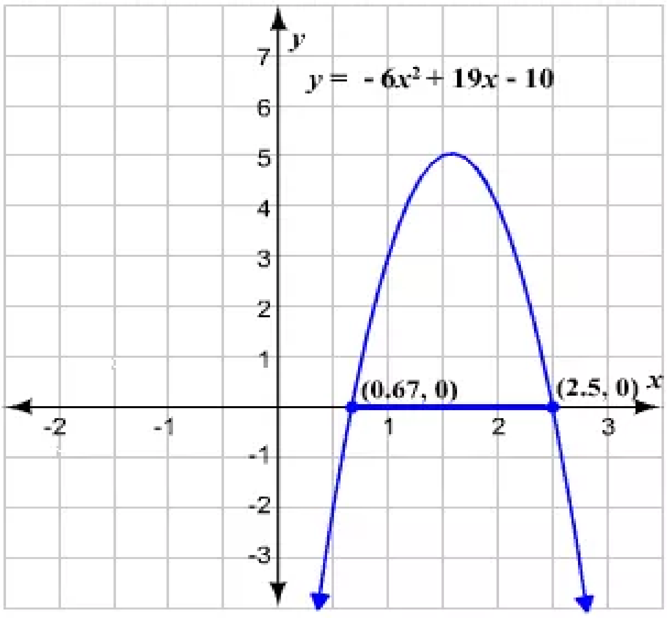
Evaluate.

$$\begin{aligned} x &= \frac{-19 \pm \sqrt{361-240}}{-12} \\ &= \frac{-19 \pm \sqrt{121}}{-12} \\ &= \frac{-19 \pm 11}{-12} \\ &= \frac{-8}{-12} \text{ or } \frac{-30}{-12} \\ &\approx 0.67 \text{ or } 2.5 \end{aligned}$$

Sketch a parabola that opens down and has 0.67 and 2.5 as x-intercepts.



The graph lies on or above the x-axis to the right of $x = 0.67$ and to the left of $x = 2.5$.



Thus, the solution is approximately $0.67 \le x \le 2.5$.

Answer 42e.

Consider the equation

$$-\frac{1}{2}x^2 + 4x \ge 1$$

Rewrite the inequality as $-x^2 + 8x \ge 2$

$$-x^2 + 8x - 2 \ge 0$$

$$x^2 - 8x + 2 \le 0$$

Consider the following quadratic function

$$y = x^2 - 8x + 2$$

The solution set consists of x-values for which the graph of the function lies on the x-axis or below the x-axis. Find the x- intercepts by setting $y = 0$.

$$x^2 - 8x + 2 = 0$$

This is a quadratic equation. Solve this by using Quadratic formula.

The quadratic formula is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 1, b = -8, c = 2$

Substitute these values in quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot (2)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64 - 8}}{2}$$

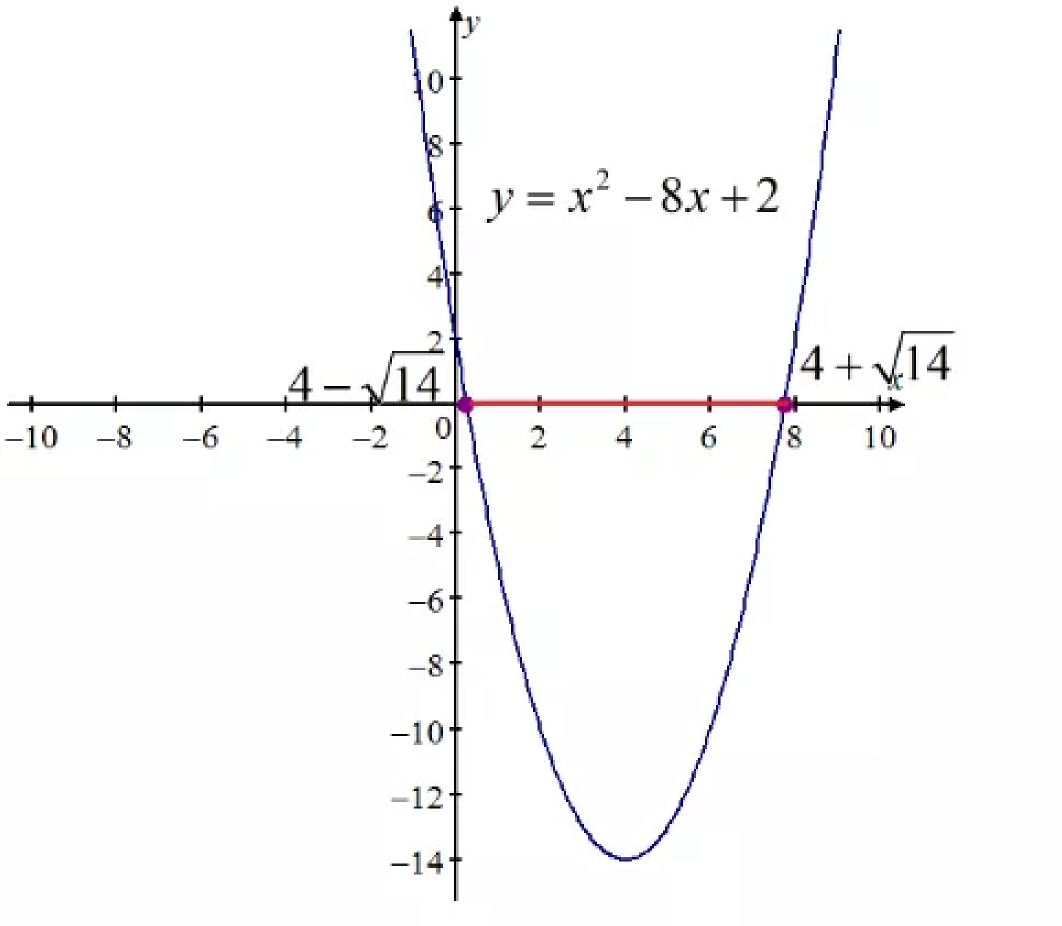
$$= \frac{8 \pm \sqrt{56}}{2}$$

$$= \frac{8 \pm 2\sqrt{14}}{2}$$

$$= 4 \pm \sqrt{14}$$

On simplifying, the x values are $4 + \sqrt{14}, 4 - \sqrt{14}$

Plot these x-values in the graph and sketch a parabola opens up with these x-intercepts.



From the graph, the graph of the function lies on or below the x-axis to the left of $x = 4 - \sqrt{14}$ and to the right of $x = 4 + \sqrt{14}$.

Therefore the solution set of the given inequality is $\boxed{\left[4-\sqrt{14}, 4+\sqrt{14}\right]}$.

Answer 43e.

Subtract 10 from both the sides.

$$4x^2 - 10x - 7 - 10 < 10 - 10$$
$$4x^2 - 10x - 17 < 0$$

The graph of the given inequality is similar to the graph of $y = 4x^2 - 10x - 17$, where solutions are the x-values for which the graph lies below the x-axis.

Find the x-intercepts by substituting $y = 0$.

$0 = 4x^2 - 10x - 17$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

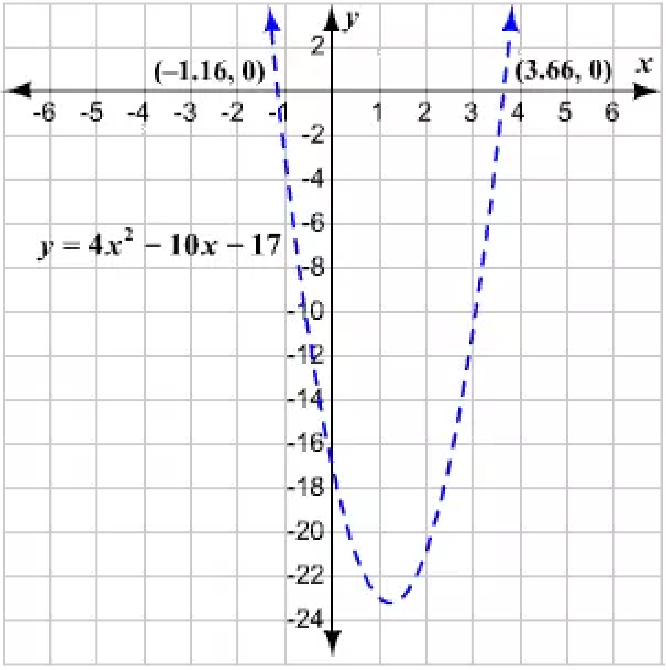
Substitute 4 for a, -10 for b, and -17 for c in the formula.

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(4)(-17)}}{2(4)}$$

Evaluate.

$$\begin{aligned} x &\approx \frac{10 \pm 19.28}{2(4)} \\ &\approx \frac{10 \pm 19.28}{8} \\ &\approx \frac{29.28}{8} \text{ or } \frac{-9.28}{8} \\ &\approx 3.66 \text{ or } -1.16 \end{aligned}$$

Sketch a parabola that opens up and has 3.66 and -1.16 as x-intercepts.



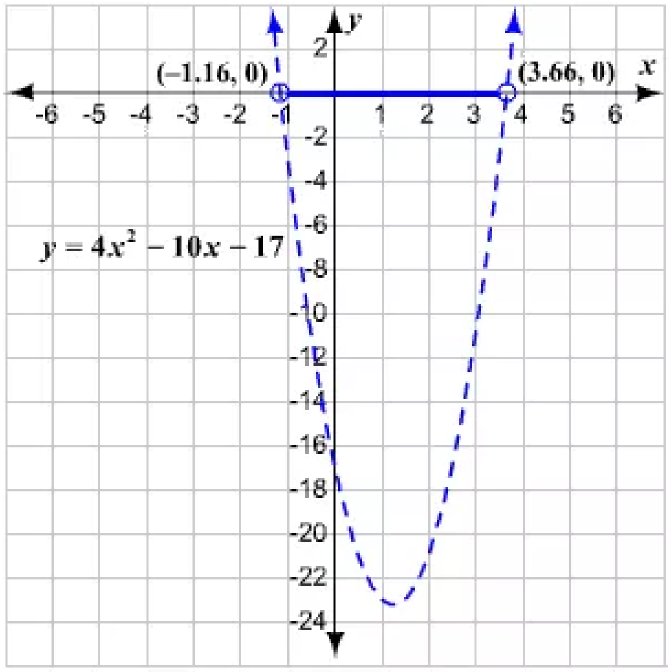
Test a point inside the parabola, say, (1, 1) to determine whether the point inside the parabola is a solution of the inequality.

Substitute 1 for x in the given inequality.
$4(1)^2 - 10(1) - 7 < 10$

Simplify.
$-13 < 10$

The inequality is true. Thus, (1, 1) is a solution of the inequality.

The graph lies below the x-axis between $x = 3.66$ and $x = -1.16$, excluding the points 3.66 and -1.16.



Thus, the solution is approximately $-1.16 < x < 3.66$.

Answer 44e.

Consider the equation

$$3x^2 - x - 4 > 0$$

Consider the following quadratic function

$$y = 3x^2 - x - 4$$

The solution set consists of x-values for which the graph of the function lies on the x-axis or below the x-axis. Find the x- intercepts by setting $y = 0$.

$$3x^2 - x - 4 = 0$$

This is a quadratic equation. Solve this by using Quadratic formula.

The quadratic formula is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 3, b = -1, c = -4$

Substitute these values in quadratic formula.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot (-4)}}{2(3)}$$

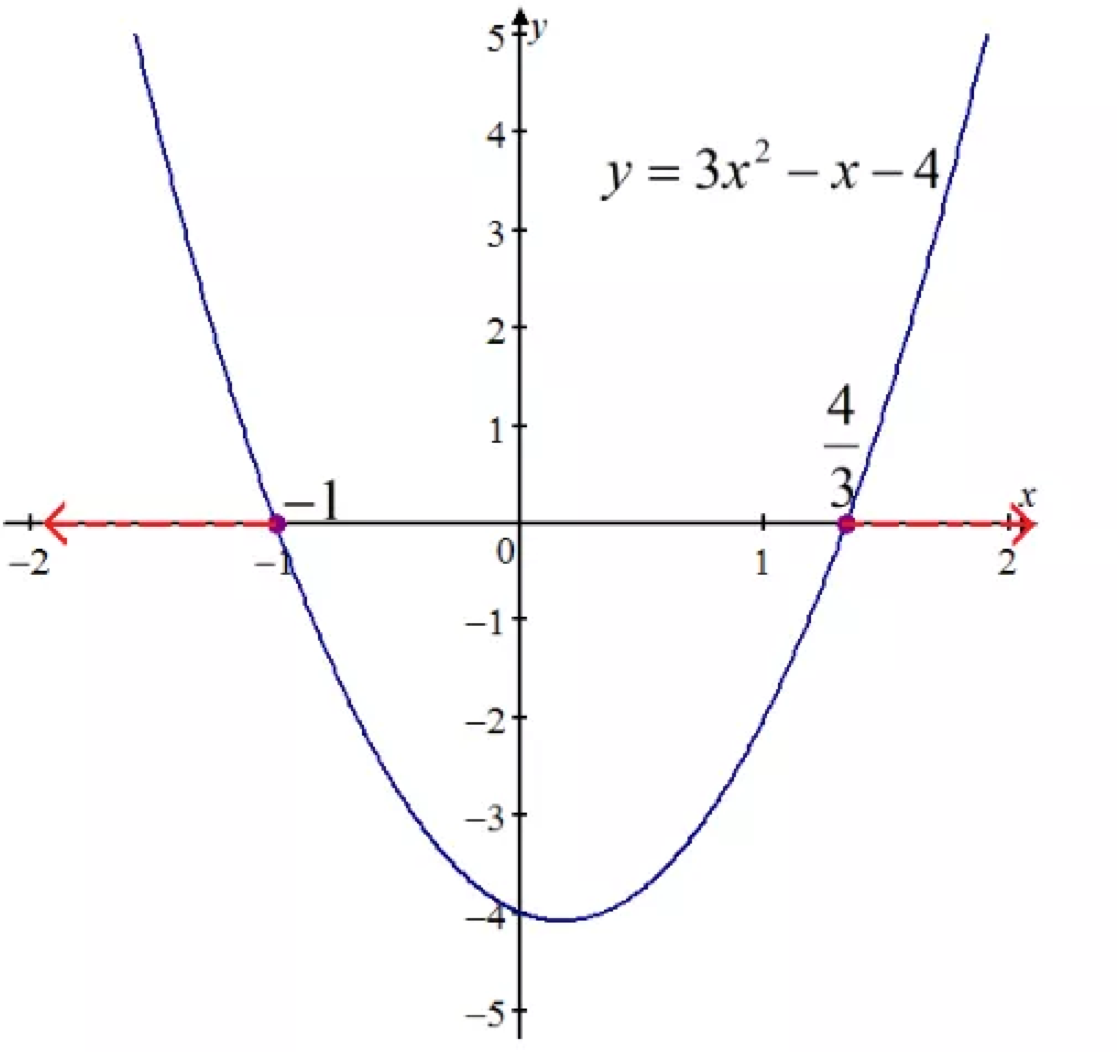
$$= \frac{1 \pm \sqrt{1+48}}{6}$$

$$= \frac{1 \pm \sqrt{49}}{6}$$

$$= \frac{1 \pm 7}{6}$$

On simplifying, the x values are $\frac{4}{3}, -1$

Plot these x-values in the graph and sketch a parabola opens up with these x-intercepts.



From the graph, the graph of the function lies on or above the x-axis to the left of $x = -1$ and to the right of $x = \frac{4}{3}$.

Therefore the solution set of the given inequality is $x < -1$ or $x > \frac{4}{3}$.

Thus the correct option is $\boxed{A}$.

Answer 45e.

Write equation that corresponds to the original inequality.
$2x^2 + 9x = 56$

Rewrite the inequality such that the right side of the inequality is 0. For this, subtract 56 from both the sides.

$$2x^2 + 9x - 56 = 56 - 56$$

$$2x^2 + 9x - 56 = 0$$

Factor the above equation. The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

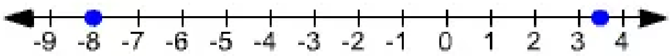
Substitute 2 for a, 9 for b, and -56 for c in the formula.

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4(2)(-56)}}{2(2)}$$

Evaluate.

$$\begin{aligned} x &= \frac{-9 \pm \sqrt{81+448}}{4} \\ &= \frac{-9 \pm \sqrt{529}}{4} \\ &= \frac{-9 \pm 23}{4} \\ &= \frac{14}{4} \text{ or } \frac{-32}{4} \\ &\approx 3.5 \text{ or } -8 \end{aligned}$$

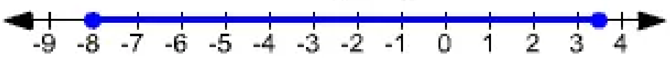
The numbers 3.5 and -8 are the critical x-values of $2x^2+9x \le 56$. Plot these points on a number line.



The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$2x^2+9x \le 56$
-9	$81 \not\le 56$
0	$0 \le 56$
4	$68 \not\le 56$

We can see that the inequality is satisfied in the interval $-8 \le x \le 3.5$.



The solution is $-8 \le x \le 3.5$. Therefore, the solution of the given inequality is the one in choice **B**.

Answer 46e.

To solve the following inequality algebraically,

$$4x^2 < 25$$

$$x^2 < \frac{25}{4}$$

$$x^2 - \left(\frac{5}{2}\right)^2 < 0$$

$$\left(x + \frac{5}{2}\right)\left(x - \frac{5}{2}\right) < 0$$

Then,

$x + \frac{5}{2} > 0$, and $x - \frac{5}{2} < 0$

$$-\frac{5}{2} < x < \frac{5}{2}$$

$$x \in \left(-\frac{5}{2}, \frac{5}{2}\right)$$

Therefore the solution set of the given inequality is $\boxed{\left(-\frac{5}{2}, \frac{5}{2}\right)}$

Answer 47e.

Write the equation that corresponds to the original inequality.
$x^2 + 10x + 9 = 0$

Factor the above equation. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

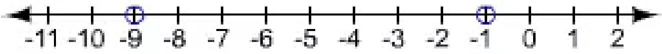
Substitute 1 for a, 10 for b, and 9 for c in the formula.

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(9)}}{2(1)}$$

Evaluate.

$$\begin{aligned} x &= \frac{-10 \pm \sqrt{100 - 36}}{2} \\ &= \frac{-10 \pm 8}{2} \\ &= \frac{-2}{2} \text{ or } \frac{-18}{2} \\ &= -1 \text{ or } -9 \end{aligned}$$

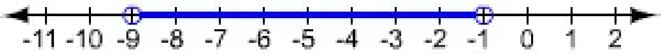
The numbers −1 and −9 are the critical x-values of the inequality $x^2 + 10x + 9 < 0$. Plot these points on a number line using open dots because the values do not satisfy the inequality.



The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$x^2 + 10x + 9 < 0$
-10	$9 \not< 0$
-2	$-7 < 0$
0	$9 \not< 0$

We can see that the inequality is satisfied in the interval $x = -9$ to $x = -1$.



Thus, the solution is $-9 < x < -1$.

Answer 48e.

To solve the following inequality algebraically,

$$x^2 - 11x \geq -28$$

$$x^2 - 11x + 28 \geq 0$$

$$x^2 - 4x - 7x + 28 \geq 0$$

$$x(x-4) - 7(x-4) \geq 0$$

$$(x-4)(x-7) \geq 0$$

$x - 4 \leq 0$ Or $x - 7 \geq 0$

$x \leq 4$ Or $x \geq 7$

$x \in (-\infty, 4]$ Or $x \in [7, \infty)$

$$x \in (-\infty, 4] \cup [7, \infty)$$

Therefore the solution set of the given inequality is $\boxed{(-\infty, 4] \cup [7, \infty)}$

Answer 49e.

Write the equation that corresponds to the original inequality.
$3x^2 - 13x = 10$

Rewrite the equation such that the right side is 0. For this, subtract 10 from both the sides.

$$3x^2 - 13x - 10 = 10 - 10$$

$$3x^2 - 13x - 10 = 0$$

Factor the above equation. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

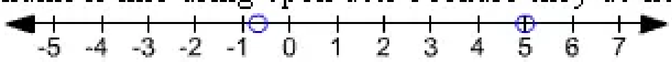
Substitute 3 for a, -13 for b, and 10 for c in the formula.

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(3)(-10)}}{2(3)}$$

Evaluate.

$$\begin{aligned} x &= \frac{13 \pm \sqrt{169 + 120}}{6} \\ &= \frac{13 \pm \sqrt{289}}{6} \\ &= \frac{13 \pm 17}{6} \\ &= \frac{30}{6} \text{ or } \frac{-4}{6} \\ &\approx 5 \text{ or } -0.67 \end{aligned}$$

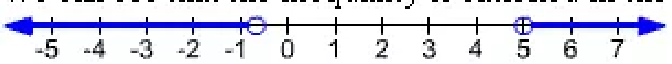
The numbers 5 and -0.67 are the critical x-values of $3x^2 - 13x > 10$. Plot these points on a number line using open dots because they do not satisfy the inequality.



The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$3x^2 - 13x > 10$
-1	$16 > 10$
0	$0 \not> 10$
6	$30 > 10$

We can see that the inequality is satisfied in the intervals $x < -0.6$ or $x > -5$.



Thus, the solution is $x < -0.6$ or $x > -5$.

Answer 50e.

To solve the following inequality algebraically,

$$2x^2-5x-3\le 0$$

$$2x^2+x-6x-3\le 0$$

$$x(2x+1)-3(2x+1)\le 0$$

$$(2x+1)(x-3)\le 0$$

$$\left(x+\frac{1}{2}\right)(x-3)\le 0$$

$x+\frac{1}{2}\ge 0$ And $x-3\le 0$

$$-\frac{1}{2}\le x\le 3$$

$$x\in\left[-\frac{1}{2},3\right]$$

Therefore the solution set of the given inequality is $\boxed{\left[-\frac{1}{2},3\right]}$

Answer 51e.

Write the equation that corresponds to the original inequality.
$4x^2+8x-21=0$

Factor the above equation. The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, where a, b, c are real numbers and $a\ne 0$.

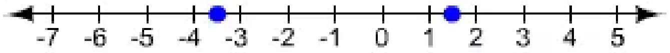
Substitute 4 for a, 8 for b, and -21 for c in the formula.

$$x = \frac{-8\pm\sqrt{8^2-4(4)(-21)}}{2(4)}$$

Evaluate.

$$\begin{aligned} x &= \frac{-8\pm 20}{8} \\ &= \frac{-8+20}{8} \text{ or } \frac{-8-20}{8} \\ &\approx 1.5 \text{ or } -3.5 \end{aligned}$$

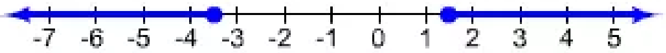
The numbers 1.5 and -3.5 are the critical x-values of $4x^2 - 8x - 21 \geq 0$. Plot these points on a number line using solid dots because the values satisfy the inequality.



The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$4x^2 + 8x - 21 \geq 0$
-4	$11 \geq 0$
1	$-9 \not\geq 0$
2	$11 \geq 0$

We can see that the inequality is satisfied in the interval $x \leq -3.5$ or $x \geq 1.5$.



Thus, the solution is $x \leq -3.5$ or $x \geq 1.5x$.

Answer 52e.

To solve the following inequality algebraically,

$$-4x^2 - x + 3 \leq 0$$

$$4x^2 + x - 3 \geq 0$$

$$4x^2 + 4x - 3x - 3 \geq 0$$

$$4x(x+1) - 3(x+1) \geq 0$$

$$(x+1)(4x-3) \geq 0$$

$$(x+1)\left(x - \frac{3}{4}\right) \geq 0$$

Then,

$x + 1 \leq 0$ Or $x - \frac{3}{4} \geq 0$

$x \leq -1$ Or $x \geq \frac{3}{4}$

$x \in (-\infty, -1]$ Or $x \in \left[\frac{3}{4}, \infty\right)$

$x \in (-\infty, -1] \cup \left[\frac{3}{4}, \infty\right)$

Therefore the solution set of the given inequality is $\boxed{(-\infty, -1] \cup \left[\frac{3}{4}, \infty\right)}$

Answer 53e.

Write the equation that corresponds to the original inequality.
$5x^2 - 6x - 2 = 0$

Factor the above equation. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

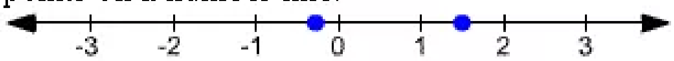
Substitute 5 for a, -6 for b, and -2 for c.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)}$$

Evaluate.

$$\begin{aligned} x &= \frac{6 \pm \sqrt{36 + 40}}{10} \\ &= \frac{6 \pm \sqrt{76}}{10} \\ &\approx \frac{6 \pm 8.72}{10} \\ &\approx \frac{14.72}{10} \text{ or } \frac{-2.72}{10} \\ &\approx 1.5 \text{ or } -0.27 \end{aligned}$$

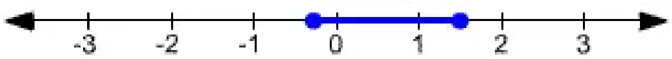
The numbers 1.5 and -0.27 are the critical x-values of $5x^2 - 6x - 2 \leq 0$. Plot these points on a number line.



The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$5x^2 - 6x - 2 \leq 0$
-1	$9 \not\leq 0$
0	$-2 \leq 0$
2	$6 \not\leq 10$

We can see that the inequality is satisfied in the interval $-0.27 \leq x \leq 1.5$.



Thus, the solution is $-0.27 \leq x \leq 1.5$.

Answer 54e.

To solve the following inequality algebraically,

$$-3x^2+10x>-2$$

$$9x^2-30x<6$$

$$(3x)^2-2(3x)(5)+5^2<6+5^2$$

$$(3x-5)^2-31<0$$

$$(3x-5)^2-\left(\sqrt{31}\right)^2<0$$

Then use the formula $a^2-b^2=(a+b)(a-b)$.

$$\left(3x-5+\sqrt{31}\right)\left(3x-5-\sqrt{31}\right)<0$$

$3x-5+\sqrt{31}>0$, and $3x-5-\sqrt{31}<0$

$$\frac{5-\sqrt{31}}{3}<x<\frac{5+\sqrt{31}}{3}$$

$$x\in\left(\frac{5-\sqrt{31}}{3},\frac{5+\sqrt{31}}{3}\right)$$

Therefore the solution set of the given inequality is $\boxed{\left(\frac{5-\sqrt{31}}{3},\frac{5+\sqrt{31}}{3}\right)}$

Answer 55e.

Write the equation that corresponds to the original inequality.
$-2x^2-7x=4$

Rewrite the inequality such that the right side of the inequality is 0. For this, subtract 4 from both the sides.

$$-2x^2-7x-4 = 4-4$$

$$-2x^2-7x-4 = 0$$

Factor the above equation. The solutions of a quadratic equation of the form

$ax^2+bx+c=0$ are $x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, where a, b, c are real numbers and $a\neq 0$.

Substitute -2 for a, -7 for b, and -4 for c in the formula.

$$x = \frac{-(-7)\pm\sqrt{(-7)^2-4(-2)(-4)}}{2(-2)}$$

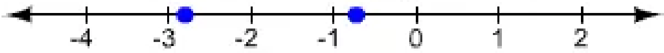
Evaluate.

$$x \approx \frac{7\pm 4.123}{2(-2)}$$

$$\approx \frac{7+4.123}{-4} \text{ or } \frac{7-4.123}{-4}$$

$$\approx -2.8 \text{ or } -0.72$$

The numbers -2.8 and -0.72 are the critical x-values of $-2x^2 - 7x \geq 4$. Plot these points on a number line using solid dots because the values satisfy the inequality.



The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$-2x^2 - 7x \geq 4$
-3	$3 \ngeq 4$
-1	$5 \geq 4$
1	$-9 \ngeq 4$

We can see that the inequality is satisfied in the interval $x = -2.8$ and $x = -0.72$.

Thus, the solution is $-2.8 \leq x \leq -0.72$.

Answer 56e.

To solve the following inequality algebraically,

$$3x^2 + 1 < 15x$$

$$3x^2 - 15x < -1$$

$$x^2 - 5x < -\frac{1}{3}$$

$$4x^2 - 20x < -\frac{4}{3}$$

$$(2x)^2 - 2(2x)(5) + 5^2 < 5^2 - \frac{4}{3}$$

$$(2x-5)^2 < \frac{71}{3}$$ Use the formula: $(a-b)^2 = a^2 - 2ab + b^2$

Then,

$$(2x-5)^2-\left(\sqrt{\frac{71}{3}}\right)^2<0$$

$$\left(2x-5+\sqrt{\frac{71}{3}}\right)\left(2x-5-\sqrt{\frac{71}{3}}\right)<0$$ Use the formula: $a^2-b^2=(a+b)(a-b)$

$$-\sqrt{\frac{71}{3}}<2x-5<\sqrt{\frac{71}{3}}$$

$$\frac{5-\sqrt{\frac{71}{3}}}{2}<x<\frac{5+\sqrt{\frac{71}{3}}}{2}$$

$$x\in\left(\frac{5-\sqrt{\frac{71}{3}}}{2},\frac{5+\sqrt{\frac{71}{3}}}{2}\right)$$

Therefore the solution set of the given inequality is $\boxed{\left(\frac{5-\sqrt{\frac{71}{3}}}{2},\frac{5+\sqrt{\frac{71}{3}}}{2}\right)}$

Answer 57e.

Write the equation that corresponds to the original inequality.
$6x^2-5=8x$

Rewrite the inequality such that the right side is 0. For this, subtract $8x$ from both the sides.

$$6x^2-5-8x = 8x-8x$$

$$6x^2-5-8x = 0$$

Rearrange the terms.
$6x^2-8x-5=0$

Factor the above equation. The solutions of a quadratic equation of the form

$ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 6 for a, −8 for b, and −5 for c in the formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(6)(-5)}}{2(6)}$$

Evaluate.

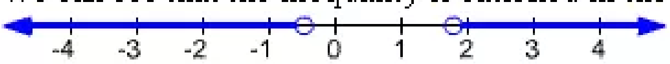
$$\begin{aligned} x &= \frac{8 \pm \sqrt{64 + 120}}{12} \\ &= \frac{8 \pm \sqrt{184}}{12} \\ &\approx \frac{8 \pm 13.56}{12} \\ &\approx \frac{21.56}{12} \text{ or } \frac{-5.56}{12} \\ &\approx 1.8 \text{ or } -0.46 \end{aligned}$$

The numbers 1.8 and −0.46 are the critical x-values of $6x^2 - 5 > 8x$. Plot these points on a number line using open dots because the values do not satisfy the inequality.

The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$6x^2 - 5 > -8$
−1	$1 > -8$
0	$-5 \not> 0$
2	$19 > 16$

We can see that the inequality is satisfied in the intervals $x < -0.46$ or $x > 1.8$.



Thus, the solution is $x < -0.46$ or $x > 1.8$.

Answer 58e.

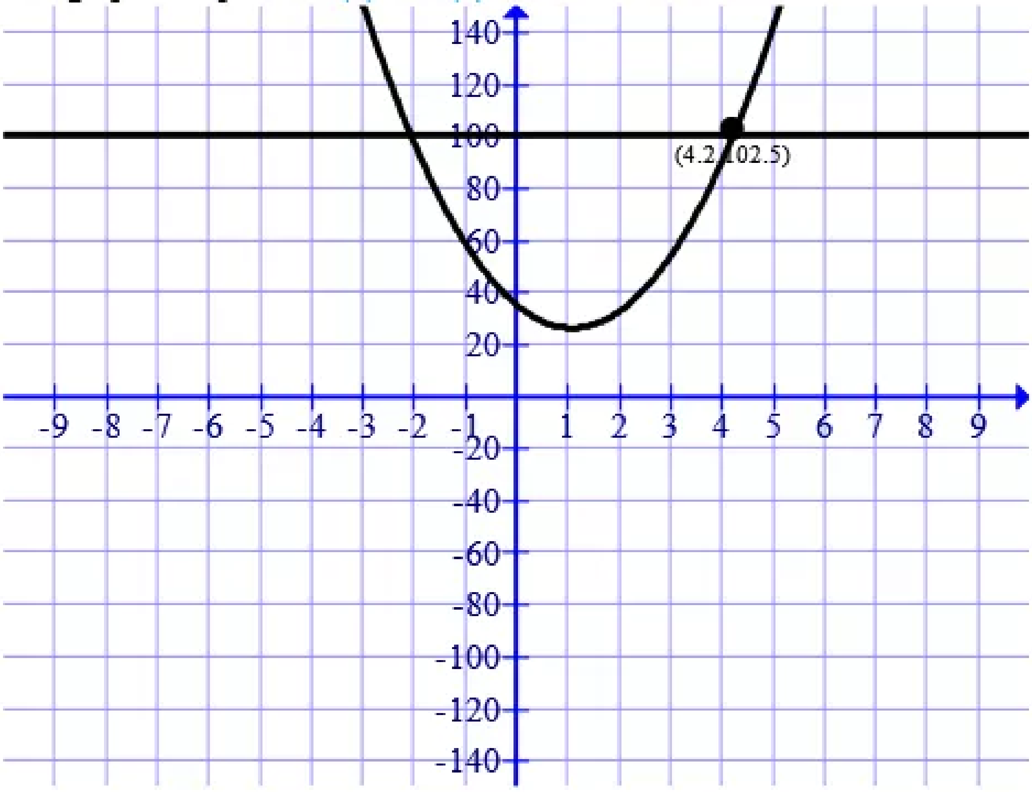
(a)

We need to draw the graph of the equation

$y = 7.5x^2 - 16.4x + 35$ ...... (1)

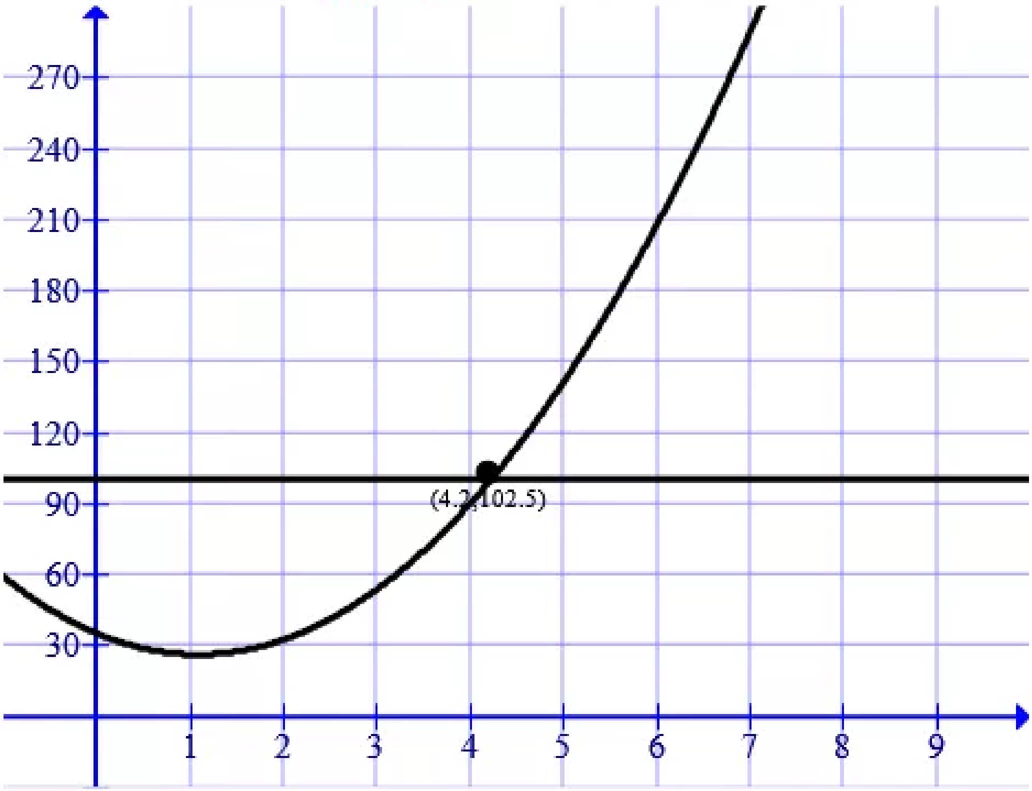
$y = 100$ ...... (2)

The graph of equations (1) and (2) is shown below:



(b)

The graph of the equations for (1) and (2) having $0 \le x \le 9, 0 \le y \le 300$ is shown below:



(c)

From the graph, it is seen that the intersection of points is $\boxed{4.2, 102.5}$

(d)

The number of participating teams greater than 100 will be after $\boxed{4 \text{ years}}$

Answer 59e.

Write the equation that corresponds to the original inequality.
$8x^2 - 3x + 1 = 10$

Rewrite the inequality such that the right side of the inequality is 0. For this, subtract 10 from both the sides.

$$\begin{aligned} 8x^2 - 3x + 1 - 10 &= 10 - 10 \\ 8x^2 - 3x - 9 &= 0 \end{aligned}$$

The graph of the given inequality is similar to the graph of $y = 8x^2 - 3x - 9$, where solutions are the x-values for which the graph lies on or above the x-axis.

Find the x-intercepts by substituting $y = 0$.
$0 = 8x^2 - 3x - 9$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

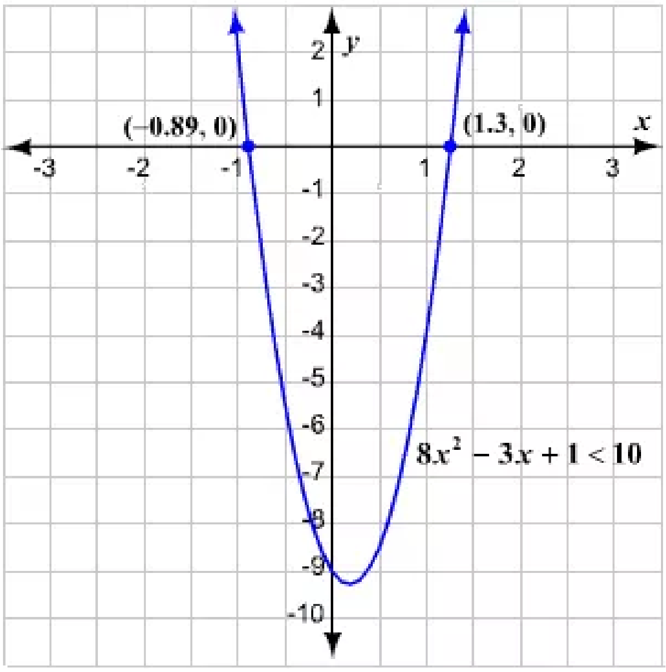
Substitute 8 for a, -3 for b, and -9 for c in the formula.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(8)(-9)}}{2(8)}$$

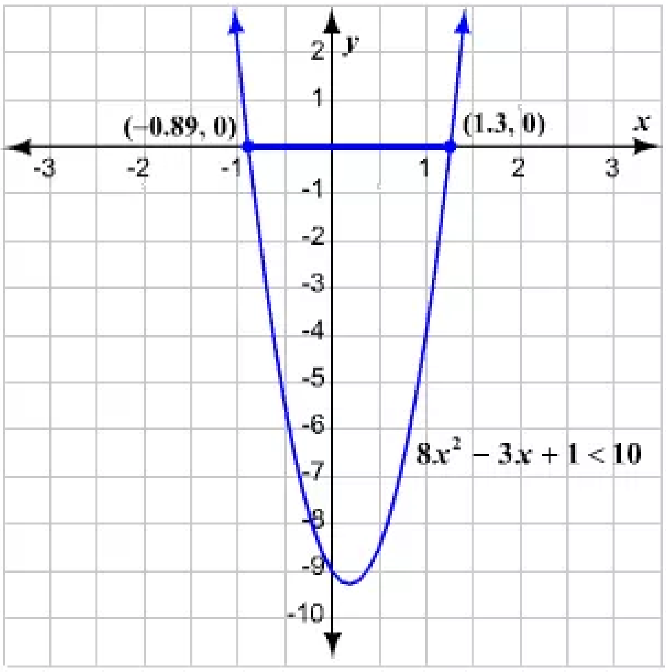
Evaluate.

$$\begin{aligned} x &\approx \frac{3 \pm 17.23}{16} \\ &\approx \frac{10 \pm 19.28}{8} \\ &\approx \frac{3 + 17.23}{16} \text{ or } \frac{3 - 17.23}{16} \\ &\approx 1.3 \text{ or } -0.89 \end{aligned}$$

Sketch a parabola that opens up and has 1.3 and -0.89 as x-intercepts.



The graph lies below the x-axis between $x=-0.89$ and $x=1.3$, excluding the points -0.89 and 1.13.



Thus, the solution is approximately $-0.89 < x < 1.3$.

Answer 60e.

To solve the following inequality algebraically,

$$4x^2+11x+3\geq -3$$

$$4x^2+11x+6\geq 0$$

$$4x^2+8x+3x+6\geq 0$$

$$4x(x+2)+3(x+2)\geq 0$$

$$(x+2)(4x+3)\geq 0$$

$$(x+2)\left(x+\frac{3}{4}\right)\geq 0$$ Divide both sides by 4

Then,

$x+2\le 0$ Or $x+\frac{3}{4}\ge 0$

$x\le -2$ Or $x\ge -\frac{3}{4}$

$x\in(-\infty,-2]$ Or $x\in\left[-\frac{3}{4},\infty\right)$

$x\in(-\infty,-2]\cup\left[-\frac{3}{4},\infty\right)$

Therefore the solution set of the given inequality is $\boxed{(-\infty,-2]\cup\left[-\frac{3}{4},\infty\right)}$

Answer 61e.

Rewrite the inequality such that the right side is 0. For this, subtract 2 from both the sides.

$$-x^2-2x-1-2>2-2$$
$$-x^2-2x-3>0$$

The solution to the given inequality consists of the x-values for which the graph of $y=-x^2-2x-3$ lies above the x-axis.

Find the x-intercepts by substituting 0 for y.
$0=-x^2-2x-3$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, where a, b, c are real numbers and $a\ne 0$.

Substitute -1 for a, -2 for b, and -3 for c in the formula.

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(-1)(-3)}}{2(-1)}$$

Evaluate.

$$x=\frac{2\pm\sqrt{4-12}}{-2}$$
$$=\frac{2\pm\sqrt{-8}}{-2}$$

Since $\sqrt{-8}$ is an imaginary number, there is no real solution for the given inequality.

Answer 62e.

To solve the following inequality algebraically,

$$-3x^2+4x-5\le 2$$

$$-3x^2+4x-5-2\le 0$$

$$3x^2-4x+7\ge 0$$

$$x^2-\frac{4}{3}x+\frac{7}{3}\ge 0$$

$$\left\{x^2-2\left(\frac{2}{3}\right)x+\left(\frac{2}{3}\right)^2\right\}+\left\{\frac{7}{3}-\left(\frac{2}{3}\right)^2\right\}\ge 0$$

$$\left(x-\frac{2}{3}\right)^2+\frac{17}{9}\ge 0$$

The above inequality is obviously true for any real x.

Therefore the solution set of the given inequality is $\boxed{\mathbb{R}}$

Answer 63e.

Write equation that corresponds to the original inequality.
$x^2-7x+4=5x-2$

Rewrite the inequality such that the right side of the inequality is 0. For this, subtract $5x-2$ from both the sides.

$$x^2-7x+4-(5x-2) = 5x-2-(5x-2)$$

$$x^2-7x+4-5x+2 = 5x-2-5x+2$$

$$x^2-12x+6 = 0$$

Factor the above equation. The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are $x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, where a, b, c are real numbers and $a\ne 0$.

Substitute 1 for a, -12 for b, and 6 for c in the formula.

$$x = \frac{-(-12)\pm\sqrt{(-12)^2-4(6)(1)}}{2(1)}$$

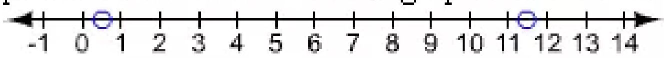
Evaluate.

$$x \approx \frac{12\pm 10.95}{2}$$

$$\approx \frac{12+10.95}{2} \text{ or } \frac{12-10.95}{2}$$

$$\approx 11.5 \text{ or } 0.52$$

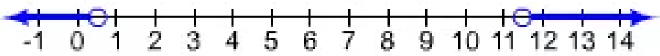
The numbers 11.5 and 0.52 are the critical x-values of $x^2 - 7x + 4 > 5x - 2$. Plot these points on a number line using open dots because the values do not satisfy the inequality.



The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$x^2 - 7x + 4 > 5x - 2$
-1	$12 > -7$
1	$-2 \not> 3$
20	$264 > 98$

We can see that the inequality is satisfied in the interval $x = 11.5$ to $x = 0.52$.



Thus, the solution is $x < 0.52$ or $x > 11.5$.

Answer 64e.

To solve the following inequality algebraically,

$$2x^2 + 9x - 1 \ge -3x + 1$$

$$2x^2 + 12x - 2 \ge 0$$

$$x^2 + 6x - 1 \ge 0$$

$$x^2 + 6x + 9 \ge 10$$

$$(x+3)^2 \ge 10$$

$$(x+3)^2 - \left(\sqrt{10}\right)^2 \ge 0$$

$\left(x+3+\sqrt{10}\right)\left(x+3-\sqrt{10}\right) \ge 0$ Use the formula: $a^2 - b^2 = (a+b)(a-b)$

So, we have

$x+3 \le -\sqrt{10}$ Or $x+3 \ge \sqrt{10}$

$x \le -3-\sqrt{10}$ Or $x \ge -3+\sqrt{10}$

$x \in \left(-\infty, -3-\sqrt{10}\right]$ Or $x \in \left[-3+\sqrt{10}, \infty\right)$

$$x \in \left(-\infty, -3-\sqrt{10}\right] \cup \left[-3+\sqrt{10}, \infty\right)$$

Therefore the solution set of the given inequality is $\boxed{\left(-\infty, -3-\sqrt{10}\right] \cup \left[-3+\sqrt{10}, \infty\right)}$

Answer 65e.

Rewrite the inequality such that the right side is 0. For this, subtract $-x^2 + 1$ from both the sides.

$$3x^2 - 2x + 1 - (-x^2 + 1) \le -x^2 + 1 - (-x^2 + 1)$$

$$3x^2 - 2x + 1 + x^2 - 1 \le 0$$

$$4x^2 - 2x \le 0$$

Substitute some values for x and find the corresponding values of y.
Organize the results in a table.

x	-1	-0.5	0	0.3	0.4	0.5	1
$4x^2 - 2x$	6	2	0	-0.24	-0.16	0	2

From the table, we can see that $4x^2 - 2x \le 0$ when x takes the values less than or equal to 0.5 and greater than or equal to 0.

Therefore, the solution of the inequality is $x < 0.5$ or $x > 0$.

Answer 66e.

To solve the following inequality algebraically,

$$5x^2 + x - 7 < 3x^2 - 4x$$

$$2x^2 + 5x - 7 < 0$$

$$2x^2 - 2x + 7x - 7 < 0$$

$$2x(x-1) + 7(x-1) < 0$$

$$(2x+7)(x-1) < 0$$

$$\left(x + \frac{7}{2}\right)(x-1) < 0$$ Divide both sides by 2

So, we have

$x + \frac{7}{2} > 0$ And $x - 1 < 0$

$x > -\frac{7}{2}$ And $x < 1$

$$-\frac{7}{2} < x < 1$$

$$x \in \left(-\frac{7}{2}, 1\right)$$

Therefore the solution set of the given inequality is $\boxed{\left(-\frac{7}{2}, 1\right)}$

Answer 67e.

Write the equation that corresponds to the original inequality.

$6x^2 - 5x + 2 = -3x^2 + x$

Rewrite the inequality such that the right side of the inequality is 0. For this, add $3x^2 - x$ from both the sides.

$$6x^2 - 5x + 2 + 3x^2 - x = -3x^2 + x + 3x^2 - x$$
$$9x^2 - 6x + 2 = 0$$

Factor the above equation. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 9 for a, -6 for b, and 2 for c in the formula.

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(9)(2)}}{2(9)}$$

Evaluate.

$$x = \frac{6 \pm \sqrt{-36}}{2(8)}$$

The square root of a negative number is not a real number. Thus, the given inequality has no real solution.

Answer 68e.

Consider

$$x < -2 \text{ Or } x > 5$$

Now write a quadratic inequality in one variable that has a solution of $x < -2$ or $x > 5$.

$$x < -2 \text{ Or } \quad x > 5$$
$$\Rightarrow \quad x + 2 < 0 \quad \text{Or} \quad x - 5 > 0$$
$$\Rightarrow \quad (x+2)(x-5) > 0$$
$$\Rightarrow \quad x^2 + 2x - 5x - 10 > 0$$
$$\Rightarrow \quad x^2 - 3x - 10 > 0$$

Therefore, the quadratic inequality that has a solution of

$x < -2$ or $x > 5$ is $\boxed{x^2 - 3x - 10 > 0}$

Answer 69e.

(a) In order to find the area, first graph $y \le -x^2 + 4x$. For this, graph $y = -x^2 + 4x$.

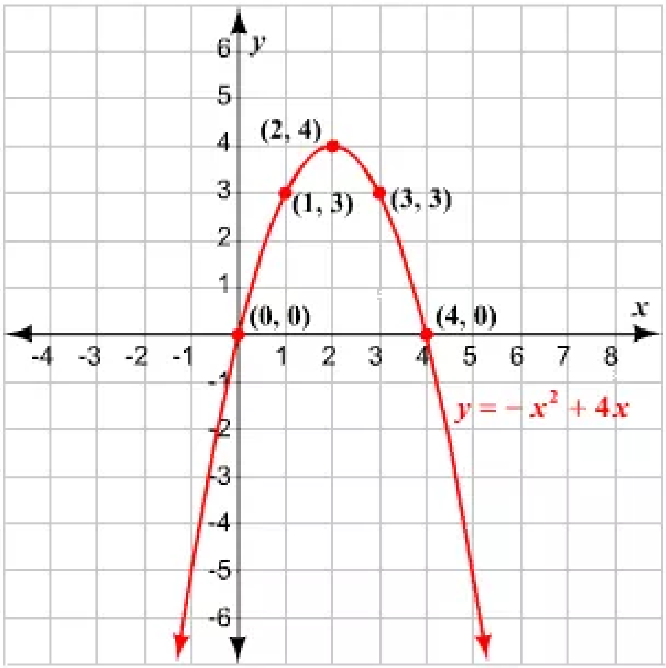
Substitute some value for x, say, 0 and find the corresponding value for y.

$$y = -(0)^2 + 4(0)$$
$$= 0$$

Organize the results in a table.

x	0	1	2	3	4
$y = -x^2 + 4x$	0	3	4	3	0

Plot these points and join them using a smooth curve. Since the inequality is $\le$ use a solid line to draw the curve.

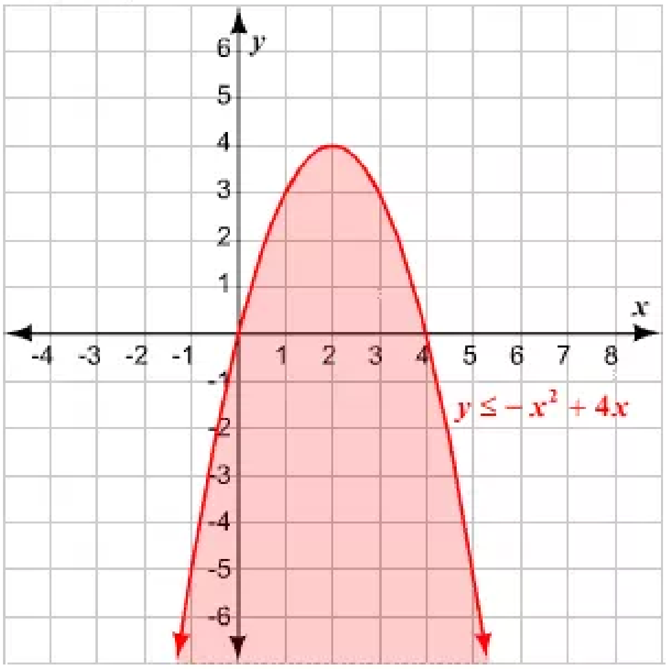


Test a point inside the parabola, say, (2, –4).

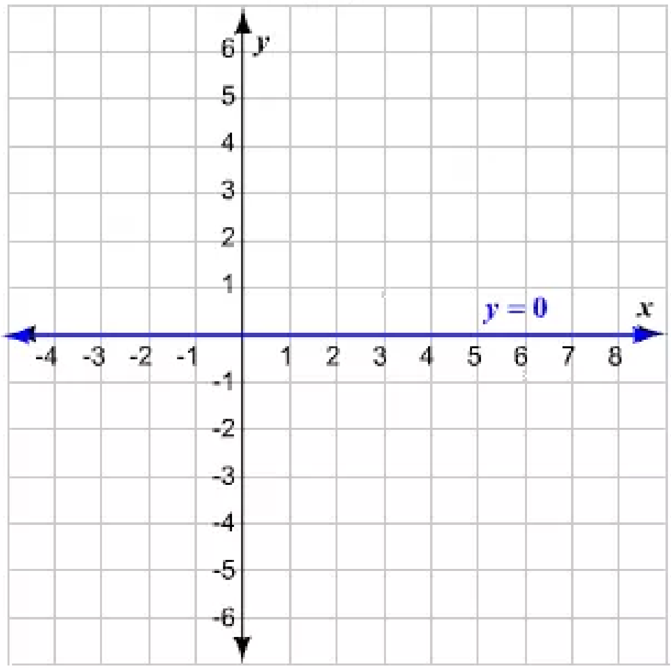
$$y \le -x^2 + 4x$$
$$-4 \overset{?}{\le} -(2)^2 + 4(2)$$
$$-4 \le 4 \quad \checkmark$$

Thus, (2, –4) is a solution of the inequality.

Shade the region inside the parabola since the test point is a solution of the inequality.



Now, graph $y \ge 0$. For this, first, graph $y = 0$. Since the inequality is $\ge$, use a solid line to draw the graph.



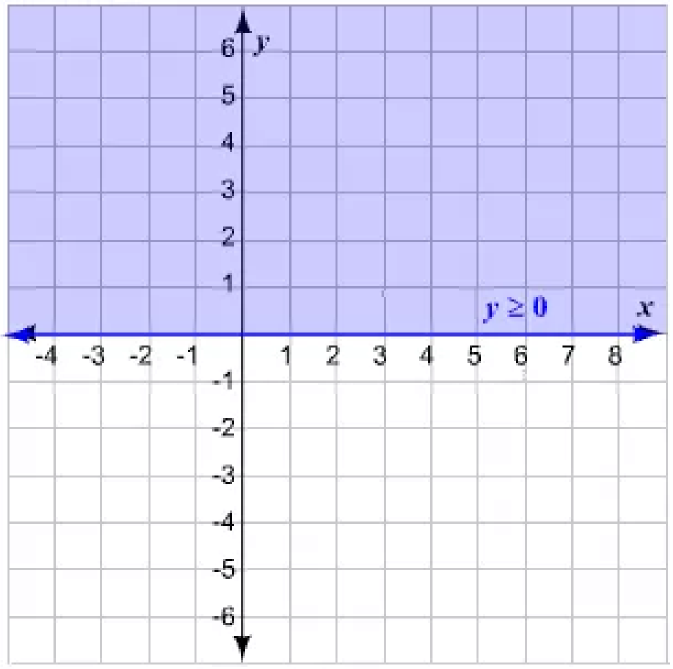
Test a point inside the graph, say, (2, 2).

$y \ge 0$

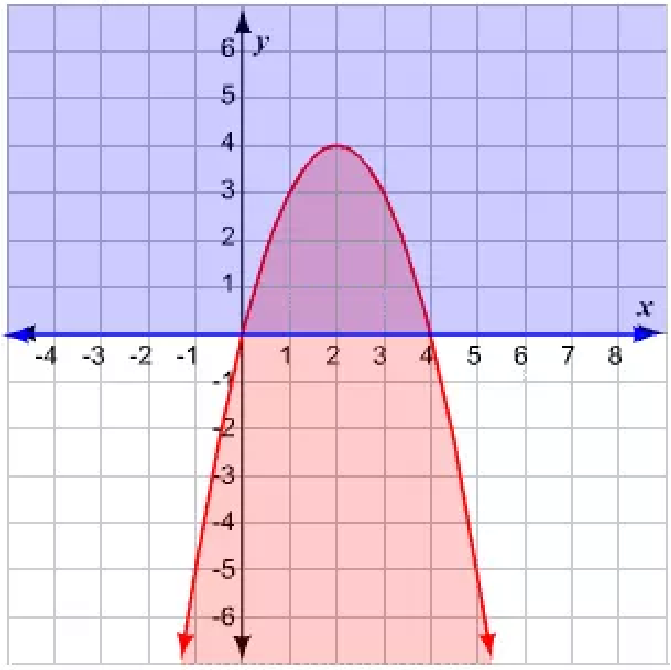
$2 \ge 0$ ✓

Thus, (2, 2) is a solution of the inequality.

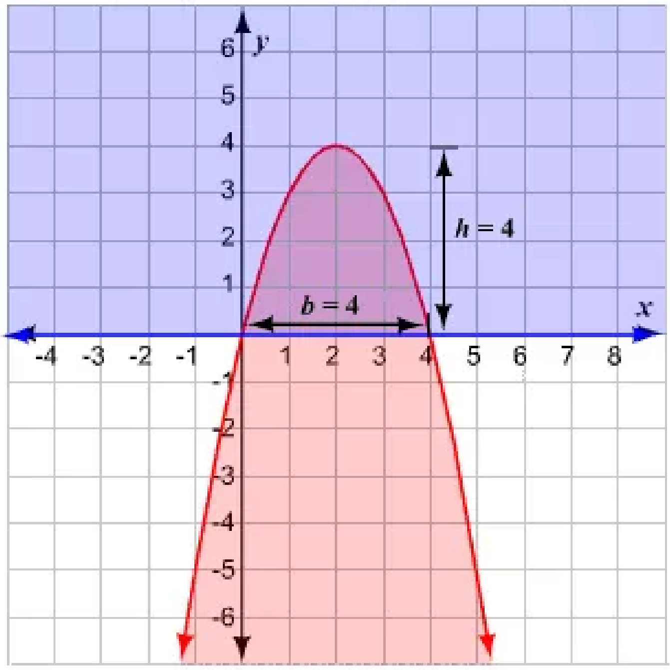
Shade the region containing the test point as the test point is a solution of the inequality.



Identify the purple region where the two graphs intersect. This region is the graph of the system.



On comparing the above graph with the given diagram, we get b and h as 4.

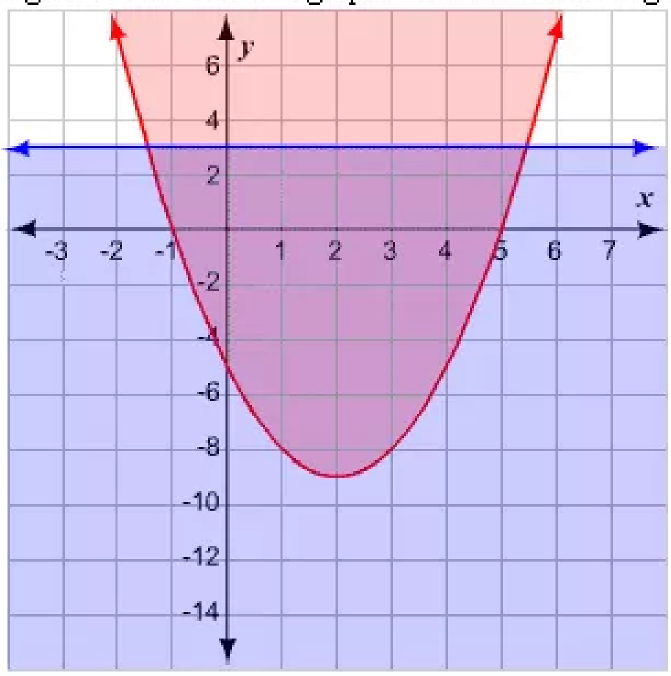


Find the area by substituting the values of b and h in $A = \frac{2}{3}bh$.

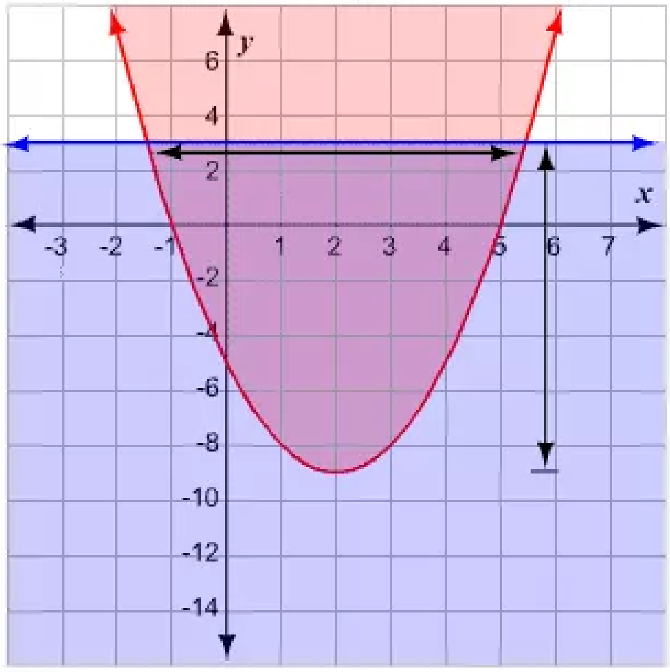
$$A = \frac{2}{3}(4)(4)$$
$$\approx 10.7$$

Thus, the area of the region determined by the given pair of inequality is about 10.7.

(b) In order to find the area, repeat the process in part **(a)** and identify the purple region where the two graphs intersect. This region is the graph of the system.



On comparing the above graph with the given diagram, we get $b = 7$ and $h = 4$.



Find the area by substituting the values of b and h in $A = \frac{2}{3}bh$.

$$A = \frac{2}{3}(7)(9)$$
$$\approx 42$$

Therefore, the area of the region determined by the given pair of inequality is about 42.

Answer 70e.

Consider the inequality

$$W \le 8000d^2$$

Where W is the weight (in pounds) and d is the rope's diameter (in inches)

Now sketch the graph for the above inequality

Graph $W \le 8000d^2$ for nonnegative values of d.

Because the inequality symbol is $\le$, make the parabola solid.

Test a point insides the parabola, such as $(1,16000)$

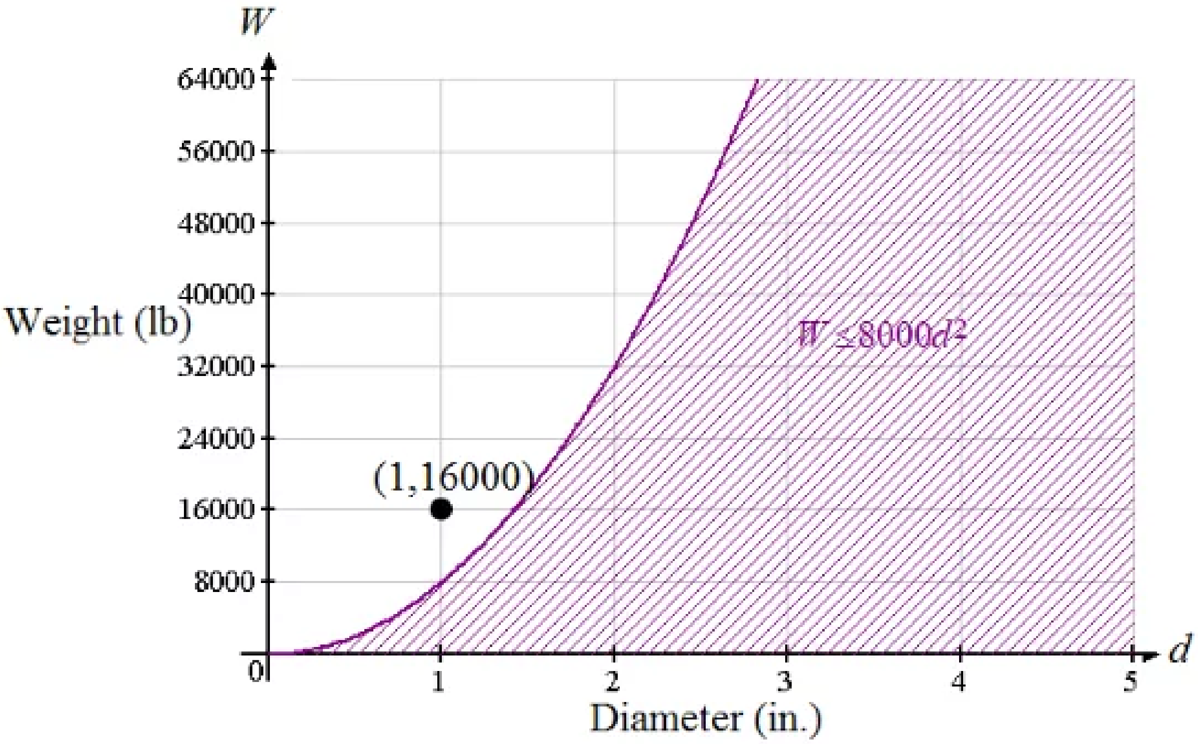
$$W \le 8000d^2$$

$$16000 \overset{?}{\le} 8000(1)^2$$

$$16000 \le 8000, \text{ false}$$

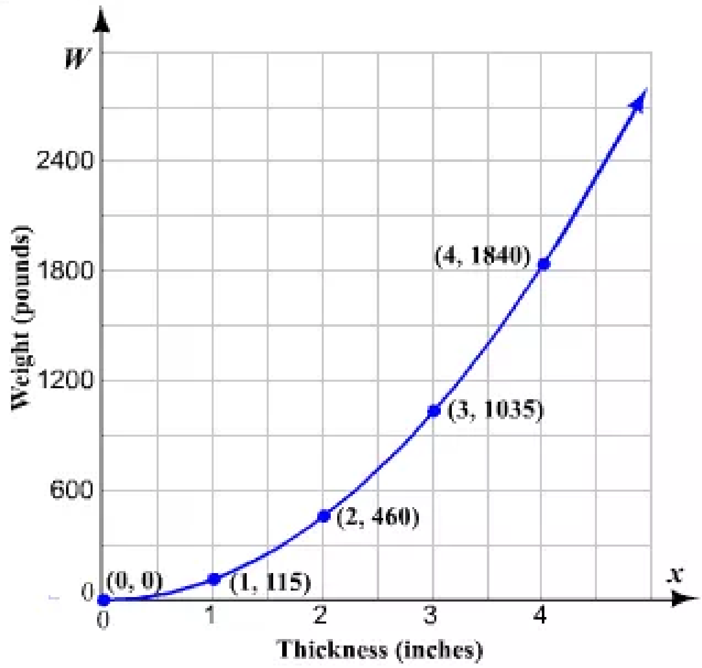
Because $(1,16000)$ is not a solution, shade the region below the parabola

The graph of $W \le 8000d^2$ as shown below:



Answer 71e.

Graph $W \le 115x^2$ for nonnegative values of x. Since $\le$ is the inequality symbol, the parabola will have a solid curve.

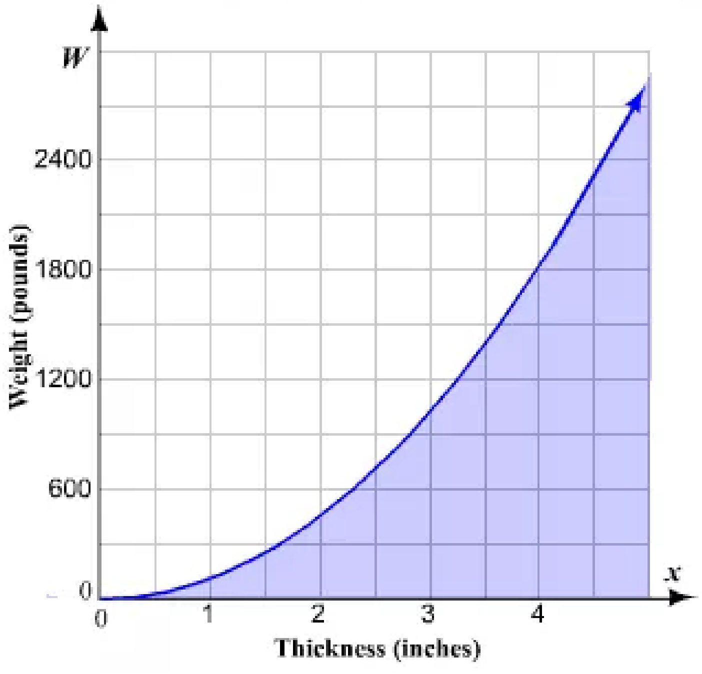


Test a point inside the parabola. Let the point be (0.5, 100).

$100 \le 115(0.5)^2$

$100 \le 28.75$ ×

We can see that (0.5, 100) is not a solution. Thus, shade the region below the parabola.



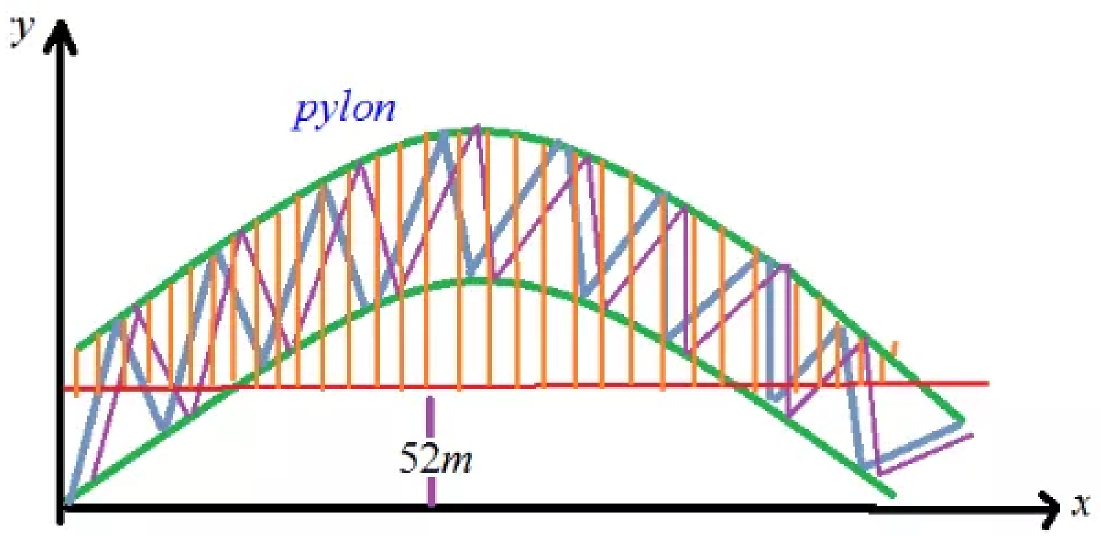
Answer 72e.

Observe the following diagram

The arch of the Sydney harbor bridge in Sydney can be modeled by

$y = -0.00211x^2 + 1.06x$ Where x is the distance (in meters)

y is the height (in meters)



Now Solve the inequality $y > 52$ where

$y = -0.00211x^2 + 1.06x$

$-0.00211x^2 + 1.06x > 52$

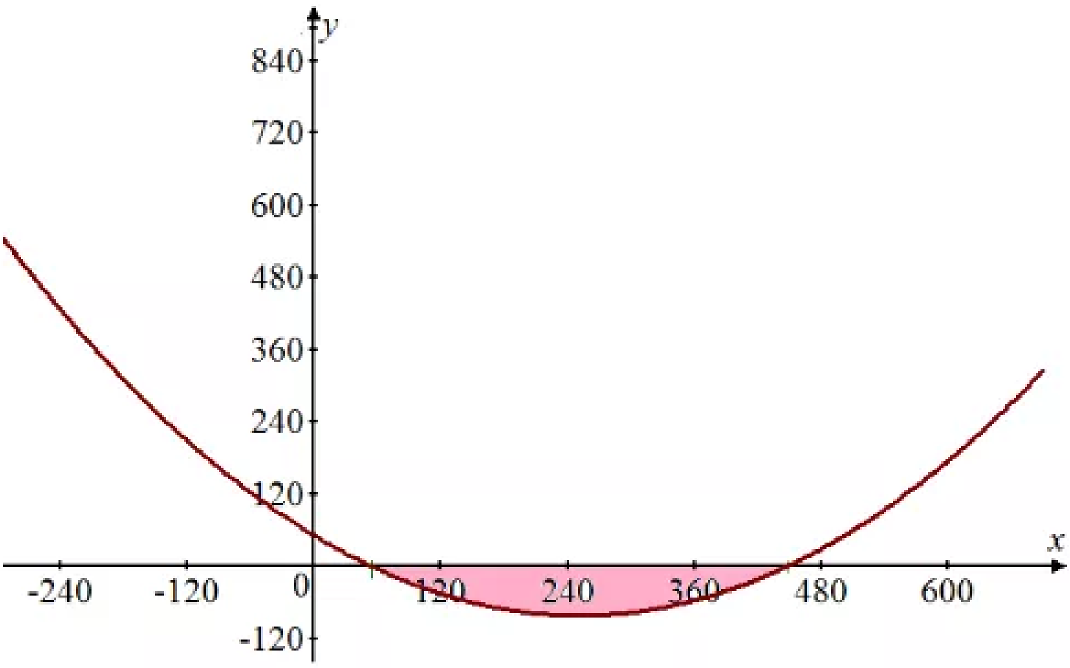
$0.00211x^2 - 1.06x + 52 < 0$

Solve the following inequality by graphing.

$0.00211x^2 - 1.06x + 52 < 0$

The following diagram contains the graph of the inequality $0.00211x^2 - 1.06x + 52 < 0$

As shown below



Therefore

The solution set of the given inequality is $(60, 450)$

Answer 73e.

It is given that the larvae's length is greater than 10 millimeters. Thus, $L(x) > 10$.

Substitute $0.00170x^2 + 0.145x + 2.35$ for $L(x)$.

$0.00170x^2 + 0.145x + 2.35 > 10$

Subtract 10 from each side.

$$0.00170x^2 + 0.145x + 2.35 - 10 > 10 - 10$$

$$0.00170x^2 + 0.145x - 7.65 > 0$$

In order to solve the inequality, replace the inequality sign with = sign.

$0.00170x^2 + 0.145x - 7.65 = 0$

Factor the equation. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 0.00170 for a, 0.145 for b, and -7.65 for c in the formula.

$$x = \frac{-0.145 \pm \sqrt{(0.145)^2 - 4(0.00170)(-7.65)}}{2(0.00170)}$$

Evaluate the square root and simplify.

$$x \approx \frac{-0.145 \pm 0.270}{2(0.00170)}$$

Simplify.

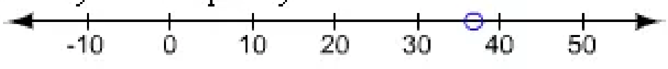
$$x = \frac{-0.145 + 0.270}{2(0.00170)} \approx 36.84$$

or

$$x = \frac{-0.145 - 0.270}{2(0.00170)} \approx -122.13$$

Since x represents the ages of the larvae in days, discard the negative value of x. Thus, the critical x value is 36.84.

Draw a number line. Plot the critical x-value 36.84 using open dots as the value does not satisfy the inequality.



The domain restricts the number of days to less than or equal to 40. Thus, $37 \leq x \leq 40$.

Therefore, the larvae's length tends to be greater than 10 millimeters between 37 to 40 days.

Answer 74e.

Suppose a study found that a driver's reaction time $A(x)$ to audio stimuli and his or her reaction time $V(x)$ to visual stimuli (both in milliseconds) can be modulated by:

$$A(x) = 0.0051x^2 - 0.319x + 15, 16 \le x \le 70$$

$$V(x) = 0.005x^2 - 0.23x + 22, 16 \le x \le 70$$

where x is the driver's age (in years).

(a)

We need to write an inequality that we can find the x-values for which $A(x)$ is less than $V(x)$.

Since $A(x)$ is less than $V(x)$. Therefore the inequality:

$$A(x) < V(x)$$

$$0.0051x^2 - 0.319x + 15 < 0.005x^2 - 0.23x + 22$$

$$0.0051 - 0.319x + 15 - 0.005x^2 + 0.23x - 22 < 0$$

$$0.0001x^2 - 0.089x - 7 < 0$$

Therefore the inequality is: $\boxed{0.0001x^2 - 0.089x - 7 < 0}$.

(b)

We need to make a table to find the solution of the inequality from part (a). The table will be containing x-values from 16 to 70 in increment of 6.

From part (a), we have

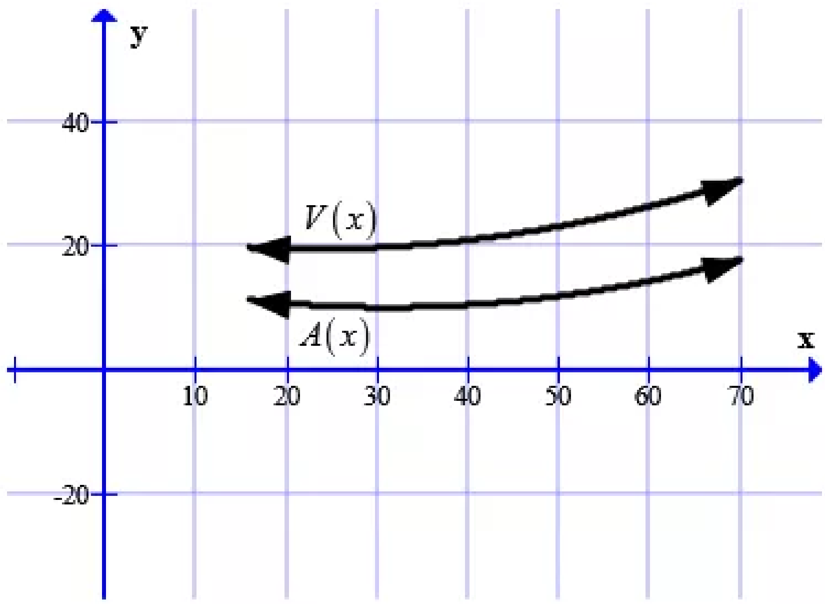
$$y = 0.0001x^2 - 0.089x - 7$$

The table is shown below:

y_{16}	-8.3984
y_{22}	-8.9096
y_{28}	-9.4136
y_{34}	-9.9104
y_{40}	-10.4000
y_{46}	-10.8824
y_{52}	-11.3576
y_{58}	-11.8256
y_{64}	-12.2864
y_{70}	-12.7400

(c)
We need to check the solution we found in part (b) by using a graphing calculator to solve the inequality $A(x) < V(x)$ graphically and the reasonable solution.

The graph of $A(x) = 0.0051x^2 - 0.319x + 15, 16 \le x \le 70$ and
$V(x) = 0.005x^2 - 0.23x + 22, 16 \le x \le 70$ is shown below:



Therefore the reasonable solution is: $\boxed{16 \le x \le 70}$.

(d)
We need to explain that based on our results from part (b) and (c) we need to say what we think a driver would react more quickly to a traffic light changing from green to yellow or to the siren of an approaching ambulance.
Based on our result from part (b) and (c) we think a driver would react more quickly the siren of an approaching ambulance. Because the audio stimuli reaction time $A(x)$ is less than the visual stimuli reaction time $V(x)$.

Answer 75e.

a. The height of the ball must be less than 8 feet to go into goal since the soccer goal is 8 feet high.
$y < 8$

Substitute $0.0540x^2 + 1.43x$ for y.
$0.0540x^2 + 1.43x < 8$.

Replace the inequality sign with = sign.
$0.0540x^2 + 1.43x = 8$

Subtract 8 from both the sides.
$$0.0540x^2 + 1.43x - 8 = 8 - 8$$
$$0.0540x^2 + 1.43x - 8 = 0$$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

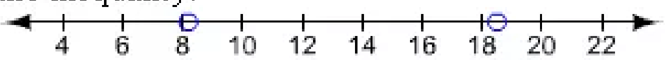
Substitute 0.0540 for a, 1.43 for b, and −8 for c in the formula and evaluate.
$$x = \frac{-1.43 \pm \sqrt{(1.43)^2 - 4(-0.0540)(-8)}}{2(-0.0540)}$$
$$x \approx \frac{-1.43 \pm 0.5629}{2(-0.0540)}$$

Simplify.
$$x \approx \frac{-1.43 + 0.5629}{-0.108}$$
$$\approx 8.02$$

or

$$x \approx \frac{-1.43 - 0.5629}{-0.108}$$
$$\approx 18.45$$

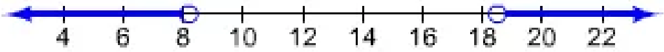
The numbers 8.02 and 18.45 are the critical x-values of $0.0540x^2 + 1.43x < 8$. Plot these points on a number line using open dots because the values do not satisfy the inequality.



The critical x-values partition the number line into three intervals. Test an x-value in each interval to see if it satisfies the inequality.

x	$0.0540x^2 + 1.43x < 8$
6	$6.636 < 8$
12	$9.384 \not< 8$
20	$7 < 8$

We can see that the inequality is satisfied in the interval $x = 8.02$ or $x = 18.45$.



Thus, the solution is $x < 8.02$ or $x > 18.45$. The values of x for which the ball is low enough to go into goal are $x \le 8$ or $x \ge 18.5$.

b. We know that the values of x for which the ball is low enough to go into goal are $x \le 8$ or $x \ge 18.5$.

It is given that a soccer player kicks the ball toward the goal from a distance of 15 feet. The kick from the horizontal distances less than 8 or greater than 18.5 will only score a goal.

Thus, if the player kicks ball from a distance of 15 feet, he will not score a goal.

Substitute 15 for x in $y = -0.0540x^2 + 1.43x$ to find the height corresponding to a horizontal distance of 15 feet.
$y = -0.0540(15)^2 + 1.43(15)$

Simplify.
$y = 9.3$

In order to find the distance by which the ball will go over the goal, subtract height of the soccer goal from 9.3 feet.
$9.3 - 8 = 1.3$

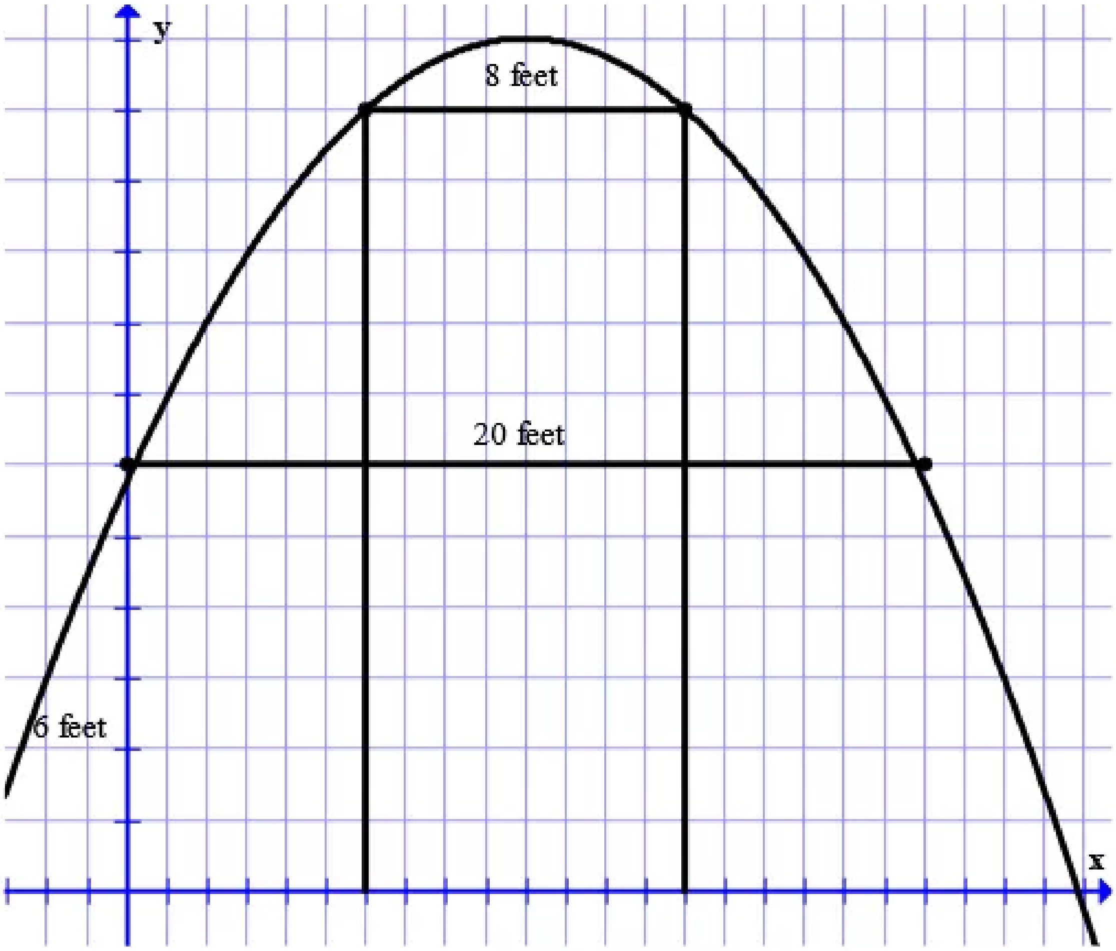
Therefore, the ball will go over the goal by 1.3 feet.

Answer 76e.

Given that a truck that is 11 feet tall and 7 feet wide is travelling under an arch.
The arch can be modeled by $y = -0.0625x^2 + 1.25x + 5.75$, where x and y are measured in feet.

(a)

Considering the graph below:



From, the graph it is seen that ,the truck 11 feet tall and 7 feet wide

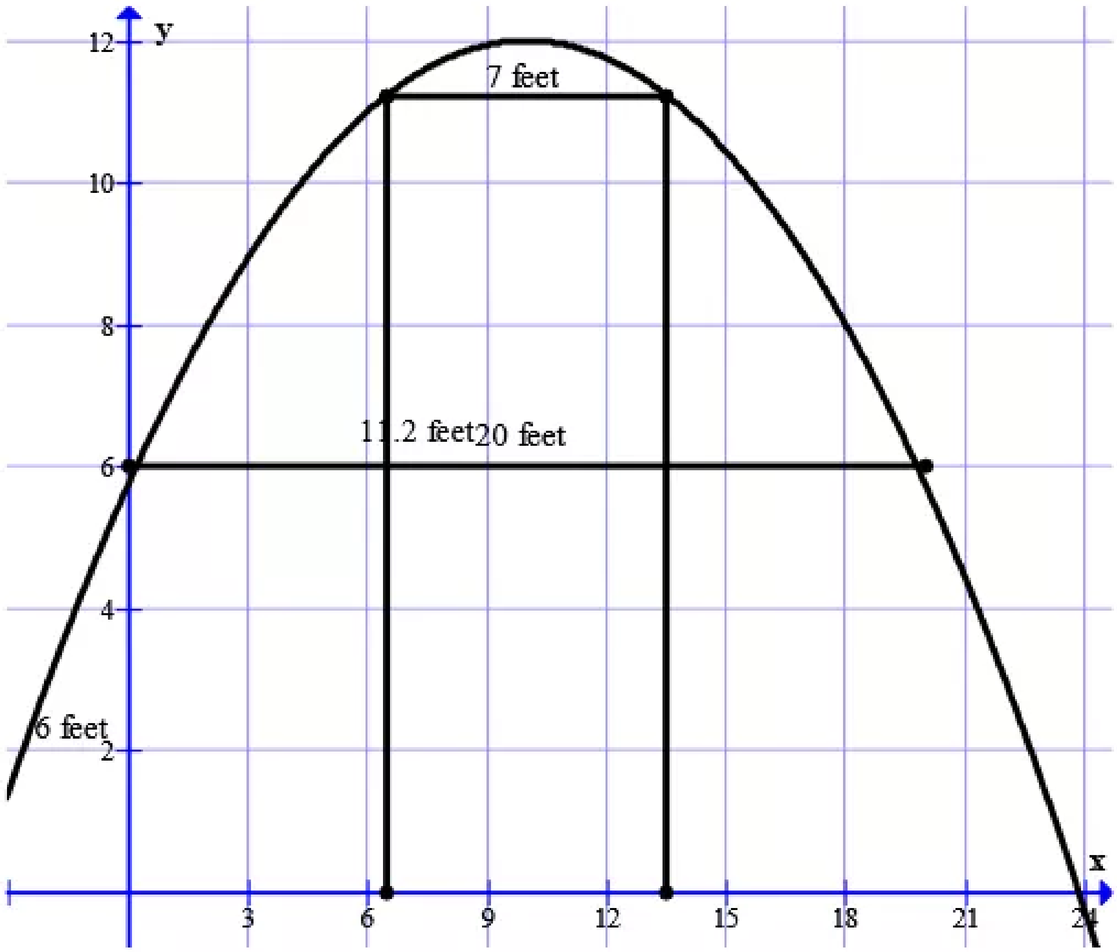
will fit under thearch since 8 feet width is available for any 11 feet tall object.

(b)

The maximum width is 8 feet for a 11 feet tall truck to fit under the arch.

©

The maximum height that a truck 7 feet wide can have is $\boxed{11.2\text{ feet}}$ as seen from the graph below:



Answer 77e.

(a) In order to find the thickness of ice that can support a weight of 20 tons, substitute 20 for $w(x)$ in the given equation.

$20 = 0.1x^2 - 0.5x - 5$

Rewrite the equation such that the right side of the inequality is 0. For this, subtract 20 from both the sides.

$$20 - 20 = 0.1x^2 - 0.5x - 5 - 20$$

$$0 = 0.1x^2 - 0.5x - 25$$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

Substitute 0.1 for a, -0.5 for b, and -25 for c in the formula.

$$x = \frac{-(-0.5) \pm \sqrt{(-0.5)^2 - 4(0.1)(-25)}}{2(0.1)}$$

Evaluate.

$$x = \frac{0.5 \pm \sqrt{0.25 + 10}}{0.2}$$
$$= \frac{0.5 \pm \sqrt{10.25}}{0.2}$$
$$= \frac{0.5 \pm 3.20}{0.2}$$
$$= \frac{3.7}{0.2} \text{ or } \frac{-2.7}{0.2}$$
$$\approx 18.5 \text{ or } -13.5$$

Since the thickness can never be negative, ignore $x = -13.5$.

Thus, the thickness is 18.5 inches.

(b) We know that w is the maximum weight the ice can support. The function $w(x)$ will give meaningful results only when it is greater than or equal to zero.

$0.1x^2 - 0.5x - 5 \geq 0$

The solution to the given inequality consists of the x-values for which the graph of $y = 0.1x^2 - 0.5x - 5$ lies on or above the x-axis.

Find the x-intercepts by substituting $y = 0$.

$0 = 0.1x^2 - 0.5x - 5$

The above equation is in standard form. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where a, b, c are real numbers and $a \neq 0$.

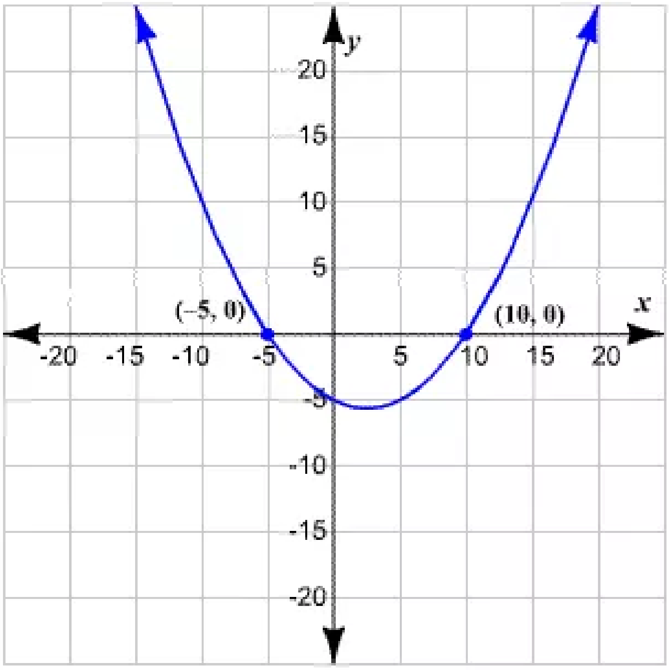
Substitute 0.1 for a, –0.5 for b, and –5 for c in the formula.

$$x = \frac{-(-0.5) \pm \sqrt{(-0.5)^2 - 4(0.1)(-5)}}{2(0.1)}$$

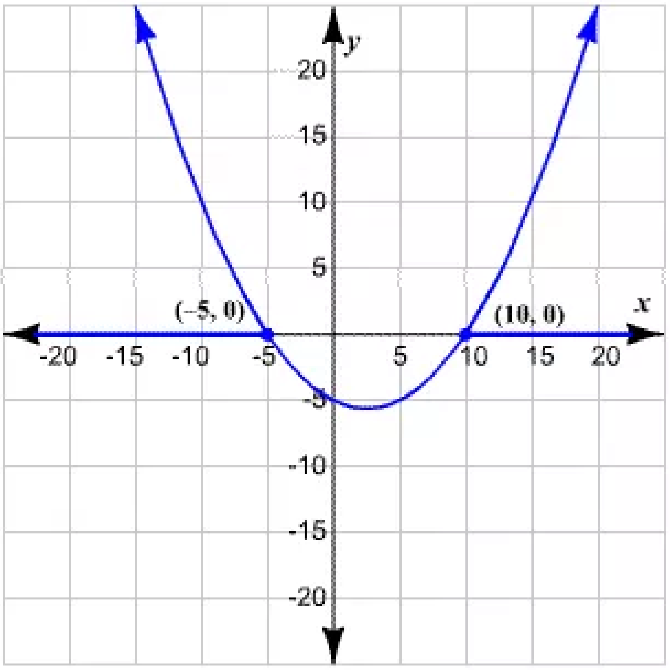
Evaluate.

$$x = \frac{0.5 \pm \sqrt{0.25 + 2}}{0.2}$$
$$= \frac{0.5 \pm \sqrt{2.25}}{0.2}$$
$$= \frac{0.5 \pm 1.5}{0.2}$$
$$\approx 10 \text{ or } -5$$

Sketch a parabola that opens up and has 10 and -5 as the x-intercepts.



The graph lies on or above the x-axis to the left of $x = -5$ and to the right of $x = 10$.



The solution is $x \le -5$ or $x \ge 10$. We know that x is the thickness and can never be negative.

Thus, the minimum value for which the function gives meaningful results is $x = 10$.

Answer 78e.

Consider the function

$$y = 3x + 7$$

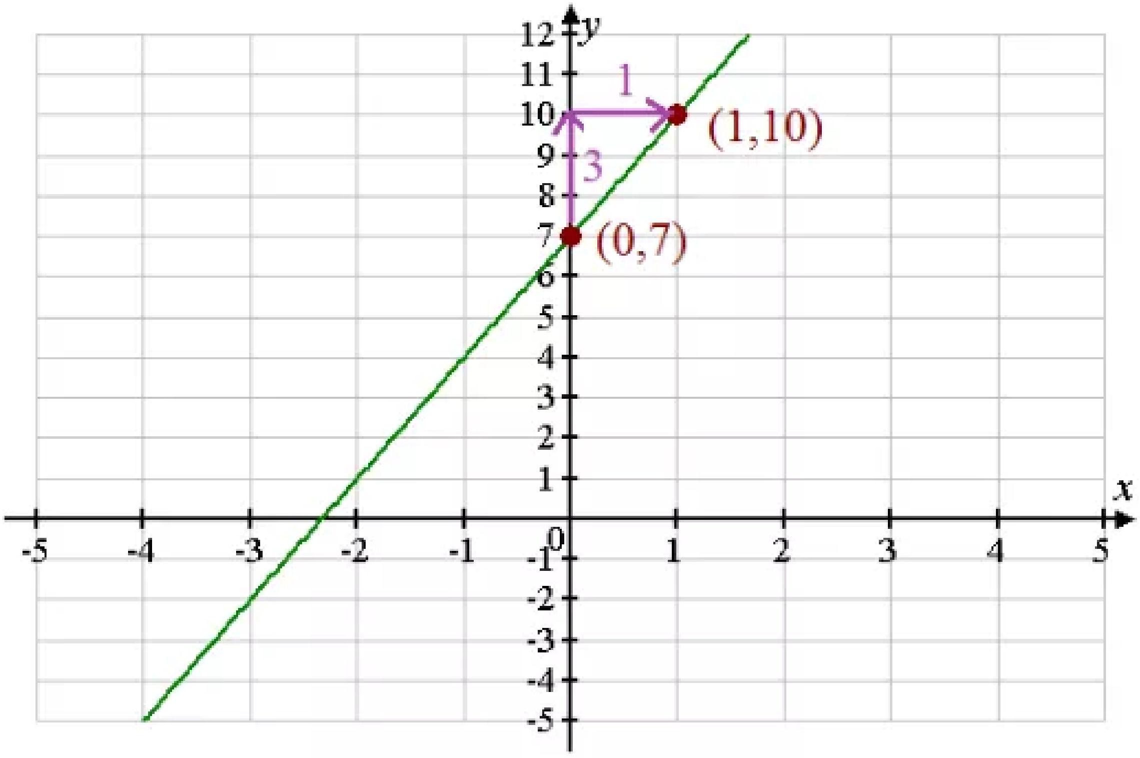
Now sketch the graph of the above function as follows:

Step1: The equation is already in slope-intercept form.

Step2: Identify the y-intercept. The y-intercept is 7, so plot the point $(0, 7)$ where the crosses the y-axis

Step3: Identify the slope. The slope is 3, so plot a second point on the line by starting at $(0, 7)$ and then moving up 3 units and right 1 unit. The second point is $(1, 10)$

Step4: Draw a line through the two points.



Answer 79e.

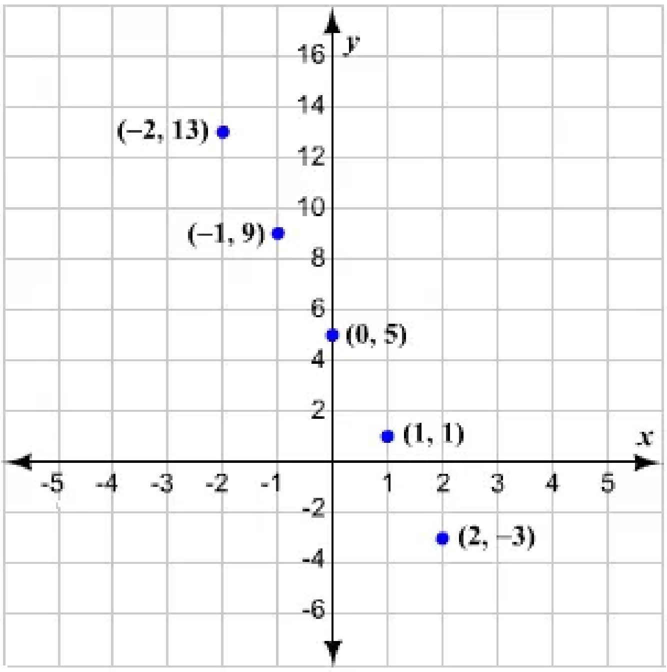
In order to graph the given equation, first find some points with coordinates that are solutions of the equation.

Choose some values for x and find the corresponding values of y.
Organize the results in a table.

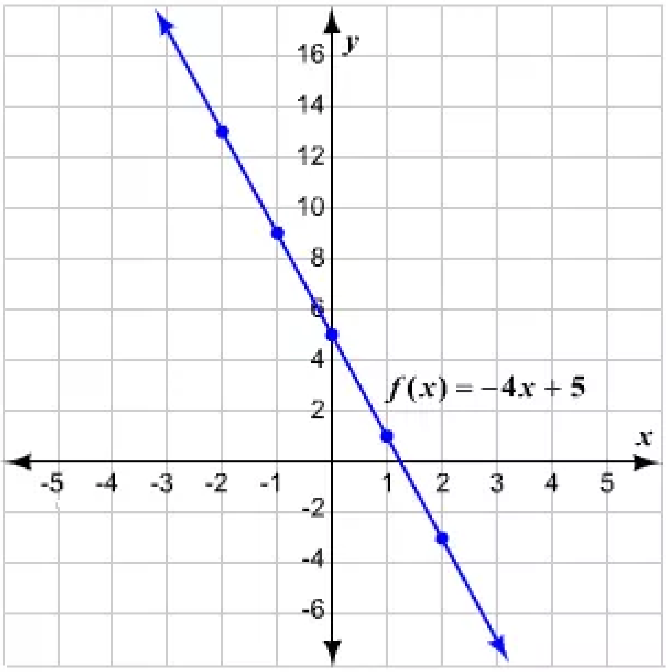
x	-2	-1	0	1	2
y	13	9	5	1	-3

The points are $(-2, 13)$, $(-1, 9)$, $(0, 5)$, $(1, 1)$, and $(2, -3)$.

Plot the points on a coordinate plane.



Connect the points with a straight line.



Answer 80e.

Consider the function

$$y=\frac{1}{2}|x| \quad \ldots\ldots (1)$$

Now sketch the graph of the above function as follows:

The graph of $y=\frac{1}{2}|x|$ is the graph of $y=|x|$ vertically shrunk by a factor of vertical stretch graph of $\frac{1}{2}$.

The standard function is of the form $y=a|x-h|+k$, where the vertex is (h,k) and the graph is symmetry about $x=h$.

Comparing equation (1) with the standard form, then the vertex is

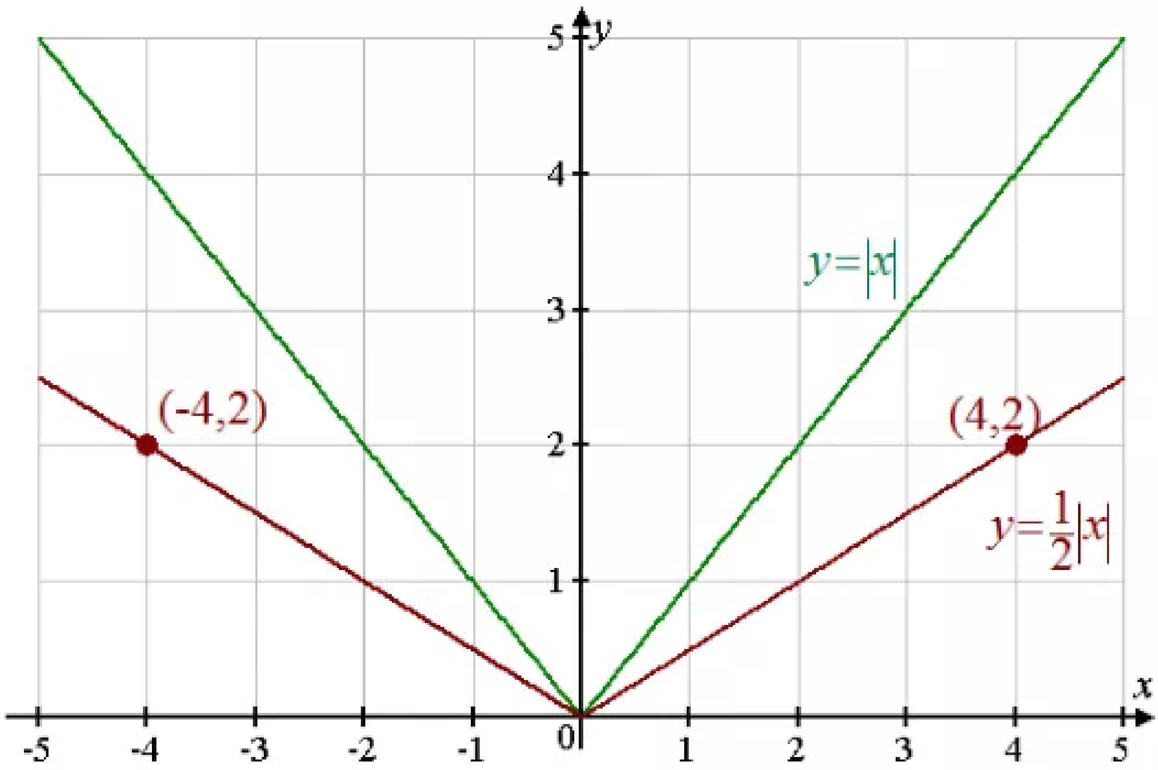
$$(h,k)=(0,0)$$

The graph of (1) is symmetric about $x=0$ and the graph is V-shaped.

Substitute 4 for x in the equation $y=\frac{1}{2}|x|$, then $y=2$

Now plot the point $(4,2)$ on the graph and by using symmetry plot another point as $(-4,2)$. Then connect the points with V-shaped graph.

Thus, the graph has vertex $(0,0)$ and passes through $(-4,2)$ and $(4,2)$

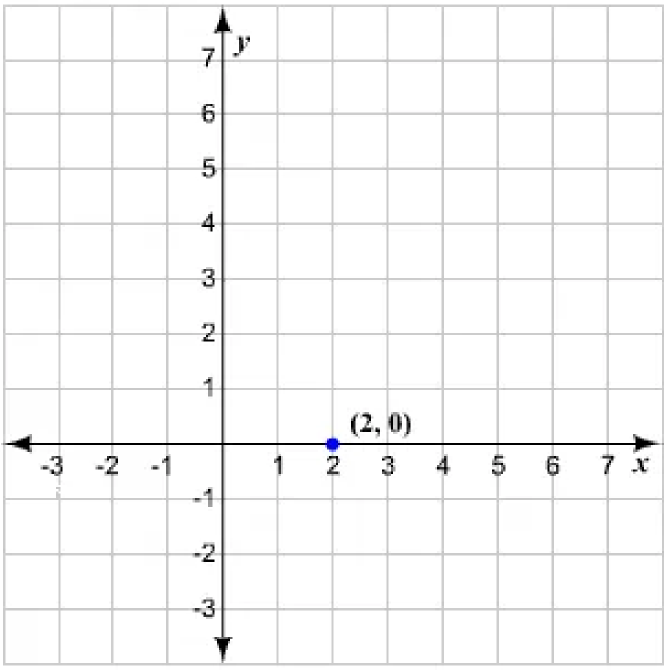


Answer 81e.

Step 1 The given function is of the form $y = |x - h| + k$, where (h, k) is the vertex of the function's graph.

We get the value of h as 2 and that of k as 0. Thus, the vertex is (2, 0).

Plot the vertex.



Step 2 Use symmetry to find two more points.
Substitute any value, say, 2 for y in the given function.
$2 = |x - 2|$

We get $x - 2 = 2$ and $x - 2 = -2$.

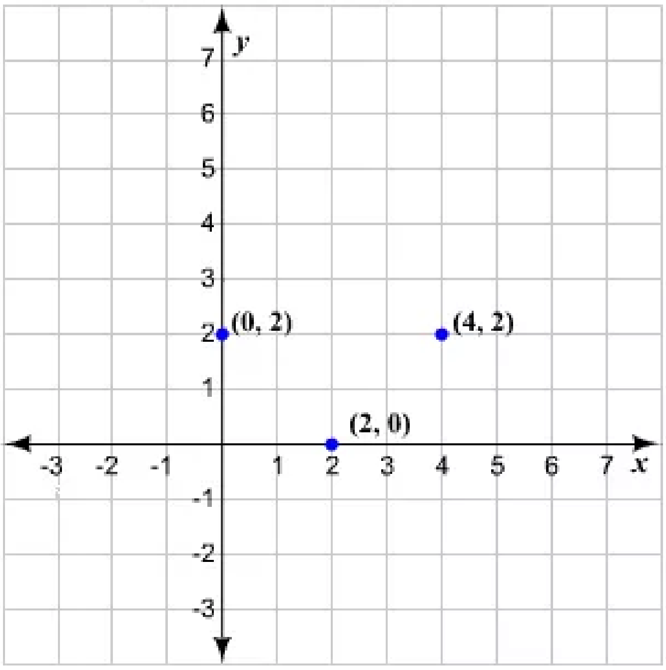
Add 2 to both the sides of the two equations.

$x - 2 + 2 = 2 + 2$ and $x - 2 + 2 = -2 + 2$

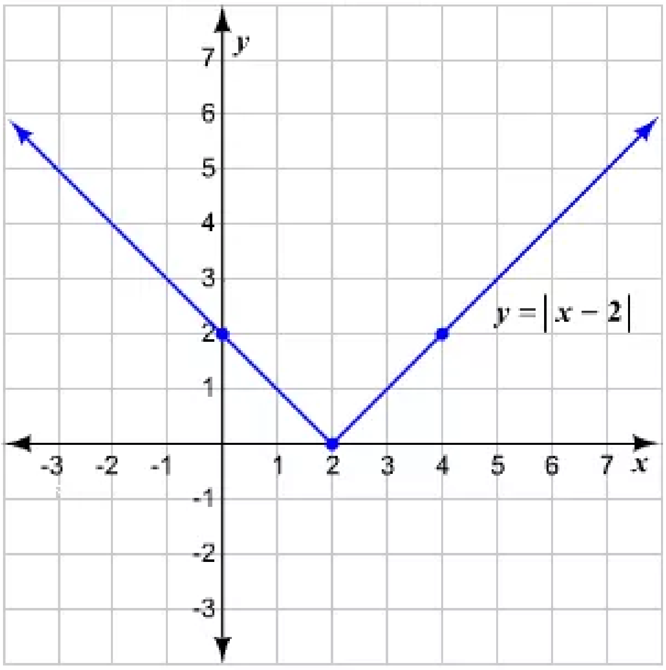
$x = 4$ and $x = 0$

The two points are (4, 2) and (0, 2).

Plot these points on the graph.



Step 3 Connect the points using straight lines to obtain a V-shaped graph.



Answer 82e.

Consider the function

$$y = |x+6| - 1 \quad \text{...... (1)}$$

Now sketch the graph of the above function as follows:

The standard function is of the form $y = a|x-h|+k$, where the vertex is (h,k)

Comparing equation (1) with the standard form, then the vertex is

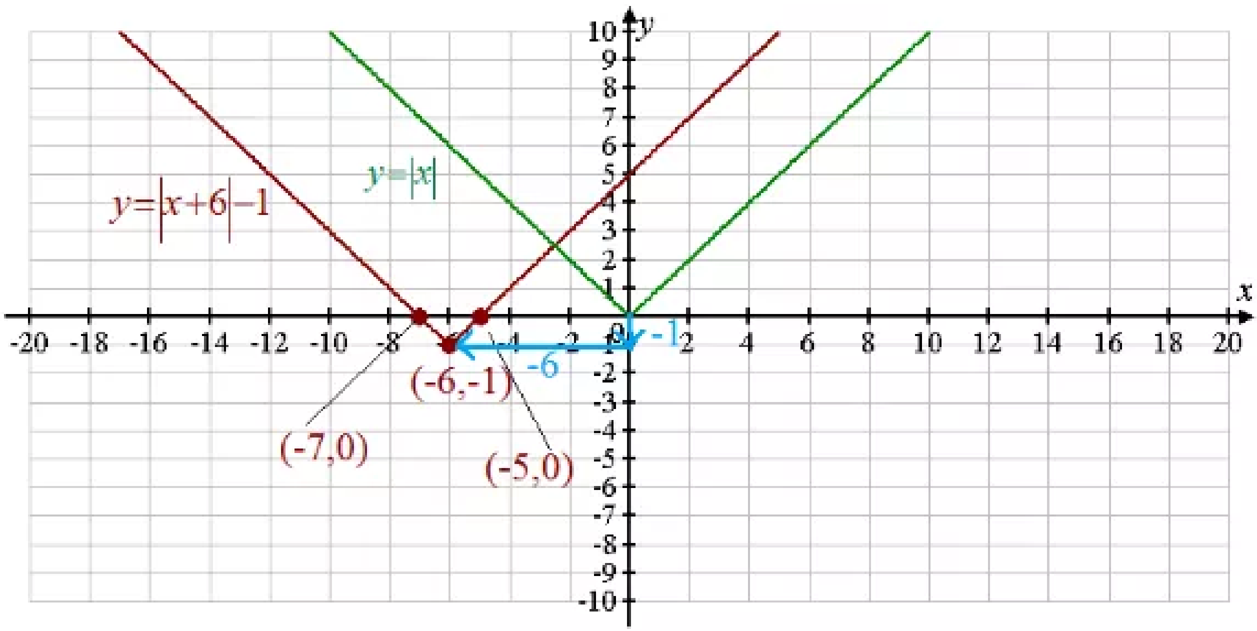
$$(h,k) = (-6,-1)$$

Step1: plot the vertex $(-6,-1)$

Step2: plot another point on the graph, such as $(-5,0)$. Use symmetry to plot another point, $(-7,0)$

Step3: connect the points with a V-shaped graph.

Step4: The graph of $y = |x+6| - 1$ is the graph of $y = |x|$ translated down 1 unit and left 6 units.



Answer 83e.

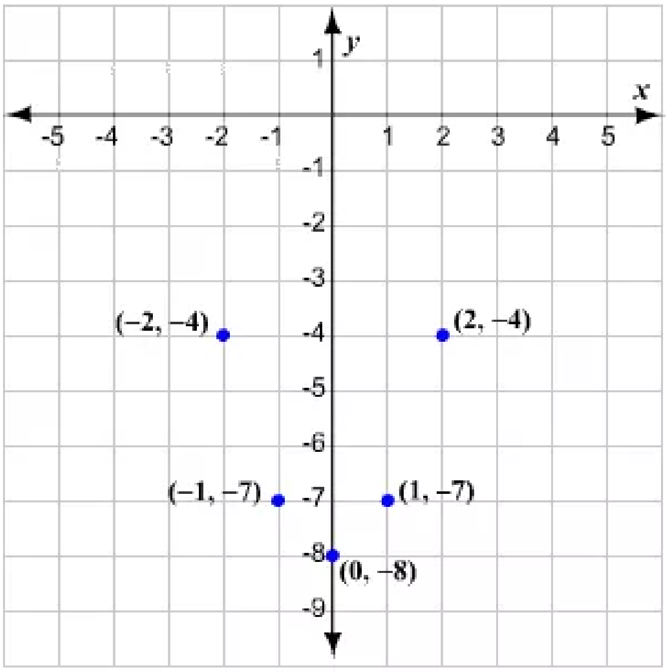
In order to graph the given equation, first find some points with coordinates that are solutions of the equation.

Choose some values for x and find the corresponding values of y.
Organize the results in a table.

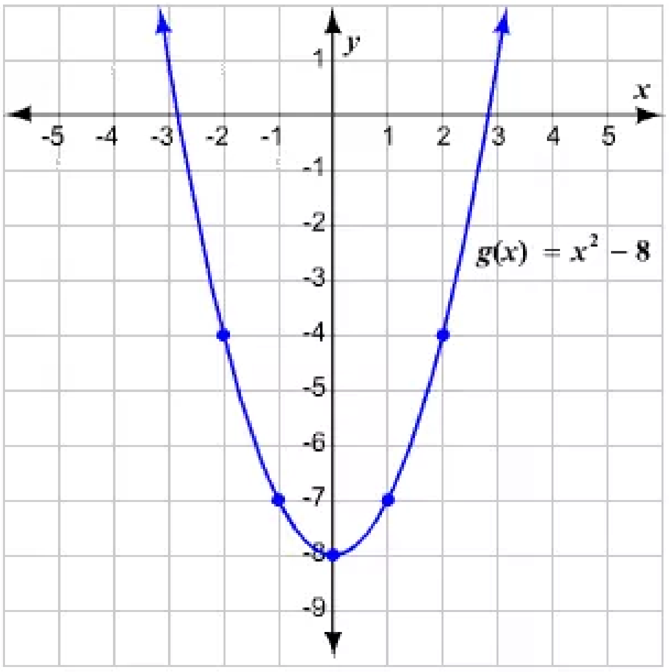
x	-2	-1	0	1	2
y	-4	-7	-8	-7	-4

The points are $(-2, -4)$, $(-1, -7)$, $(0, -8)$, $(1, -7)$, and $(2, -4)$.

Plot the points on a coordinate plane.



Connect the points with a smooth curve.



Answer 84e.

Consider the function

$$y = x^2 + 4x + 3$$

Now sketch the graph of the above function as follows:

Step1: Identify the coefficients of the function. The coefficients are

$$a = 1, b = 4, \text{ and } c = 3$$

Because $a > 0$, the parabola opens up

Step2: Find the vertex. Calculate the x-coordinate.

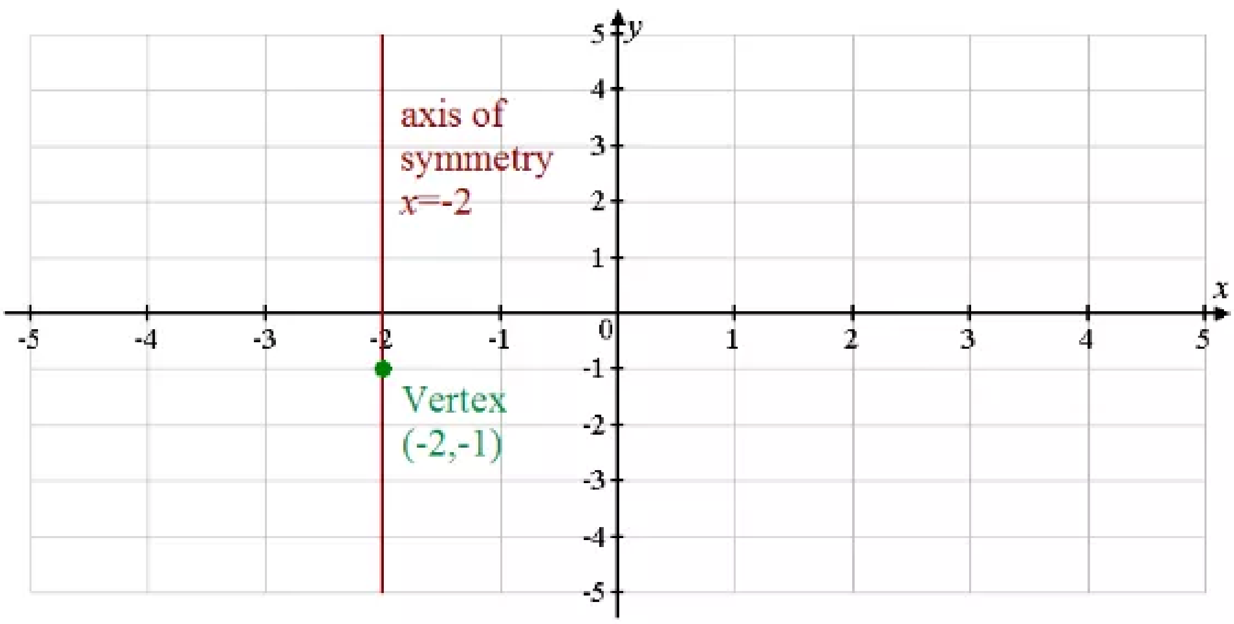
$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{4}{2(1)} \\ &= -2 \end{aligned}$$

Then find the y-coordinate of the vertex.

$$\begin{aligned} y &= x^2 + 4x + 3 \\ &= (-2)^2 + 4(-2) + 3 \\ &= 4 - 6 + 3 \\ &= 1 \end{aligned}$$

So, the vertex is $(-2, 1)$. Plot the point

Step3: Draw the axis of symmetry $x = -2$



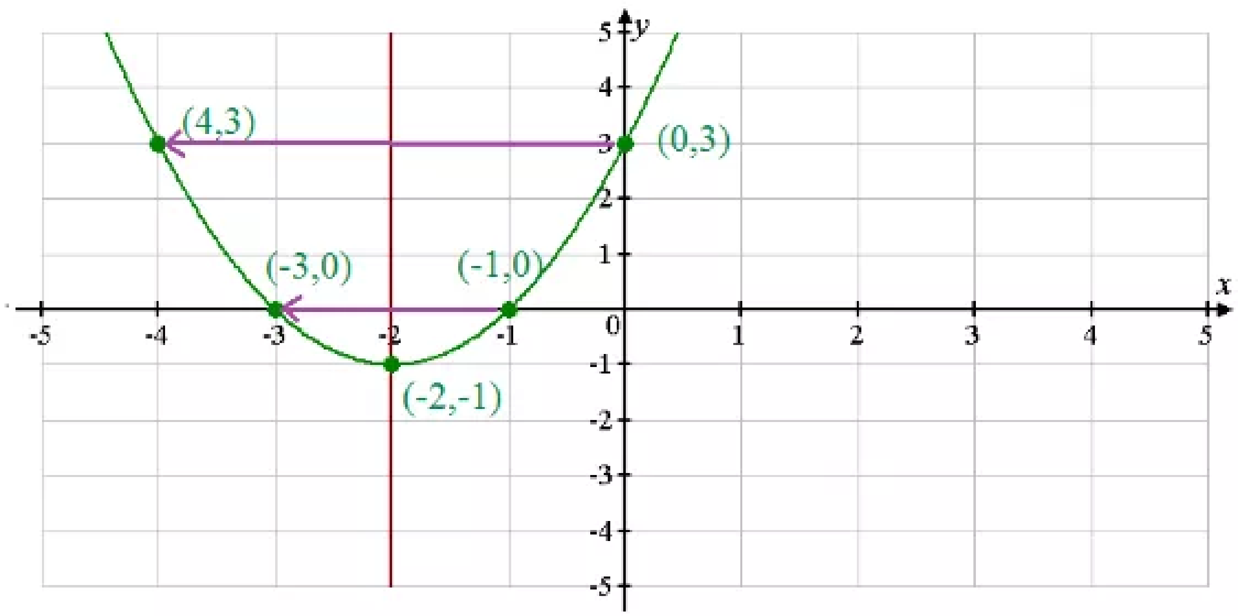
Step4: Identify the y-coordinate c, which is3. Plot the point $(0,3)$. Then reflect this point in the axis of symmetry to plot another point, $(4,3)$

Step5: Evaluate the function for another value of x, such as $x=-1$

$$\begin{aligned} y &= x^2+4x+3 \\ &= (-1)^2+4(-1)+3 \\ &= 1-4+3 \\ &= 0 \end{aligned}$$

Plot the point $(-1,0)$ and its reflection $(-3,0)$ in the axis of symmetry

Step6: Draw a parabola through the plotted points.



Answer 85e.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y = ax^2 + bx + c$. On comparing, we have a is 2, b is -9, and c is 4. Since $a = 2 > 0$, the graph opens up.

STEP 2 Find the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute 2 for a, and -9 for b and evaluate.

$$\begin{aligned} -\frac{b}{2a} &= -\frac{(-9)}{2(2)} \\ &= 2.25 \end{aligned}$$

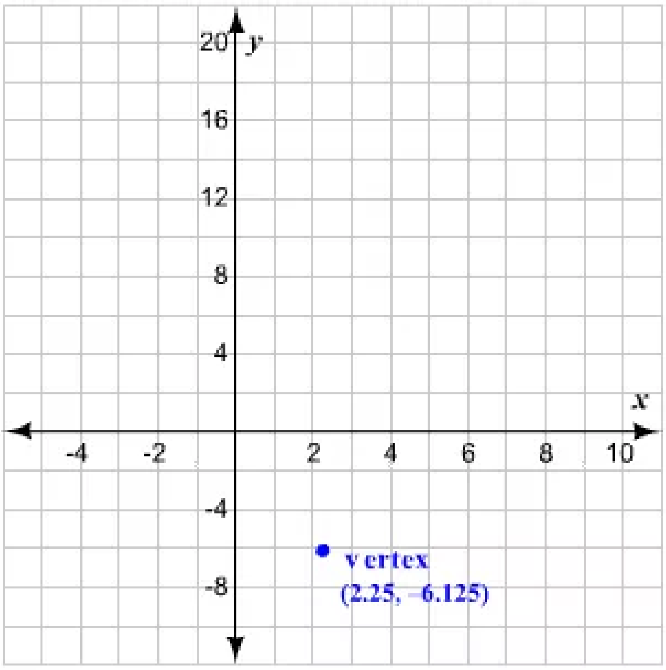
The x-coordinate of the vertex is 2.25.

Substitute 2.25 for x in the given function to find the y-coordinate.

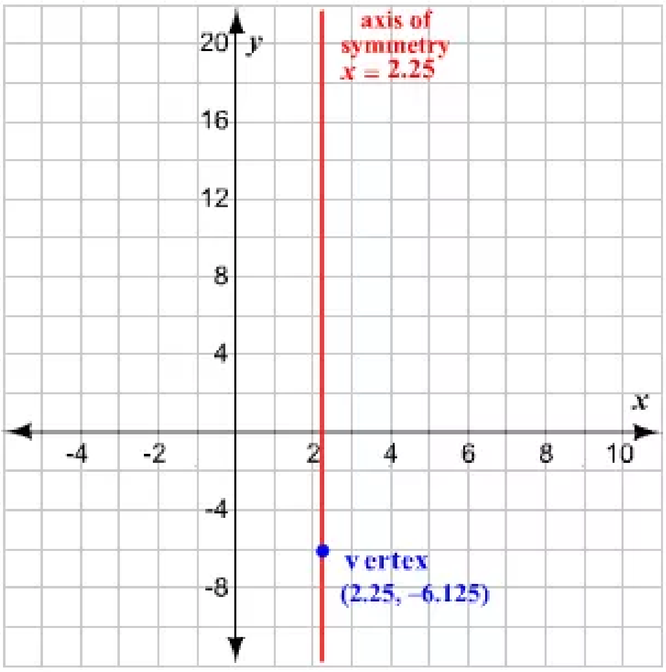
$$\begin{aligned} y &= 2(2.25)^2 - 9(2.25) + 4 \\ &\approx -6.125 \end{aligned}$$

Thus, the vertex of the graph of the given function is (2.25, −6.125).

Plot the vertex on a coordinate plane.

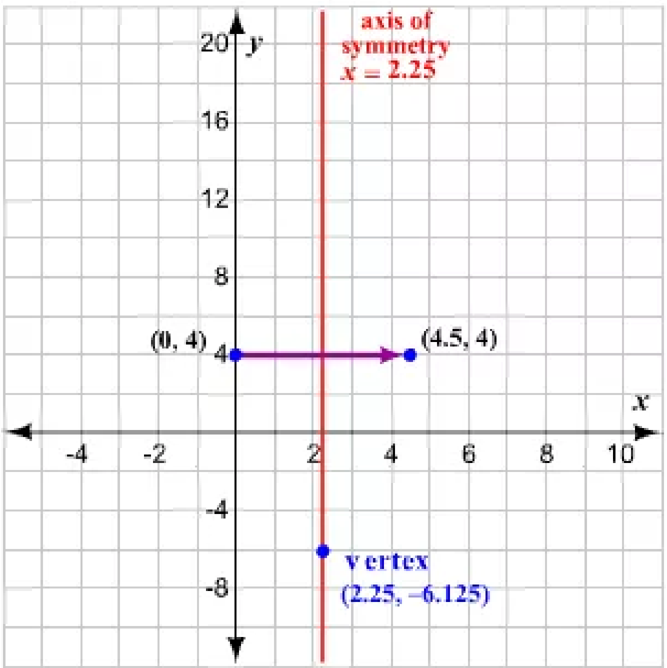


STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$. The axis of symmetry of the given function is the line $x = 2.25$. Now, draw the axis of symmetry $x = 2.25$.



STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola. Thus, the y-intercept of the given function is 2 and (0, 4) is on the parabola.

Now, reflect the point (0, 4) in the axis of symmetry to get another point.



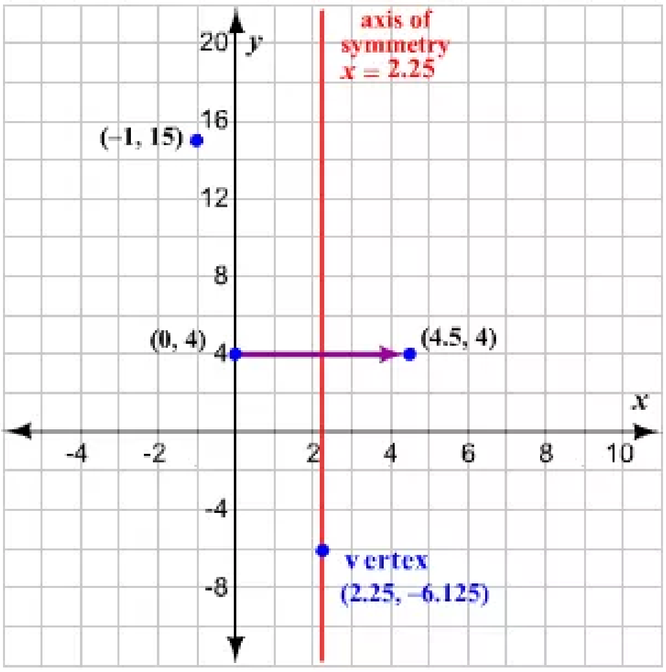
STEP 5 Evaluate the given function for another value of x, say, -1.

Substitute -1 for x in the function and simplify.

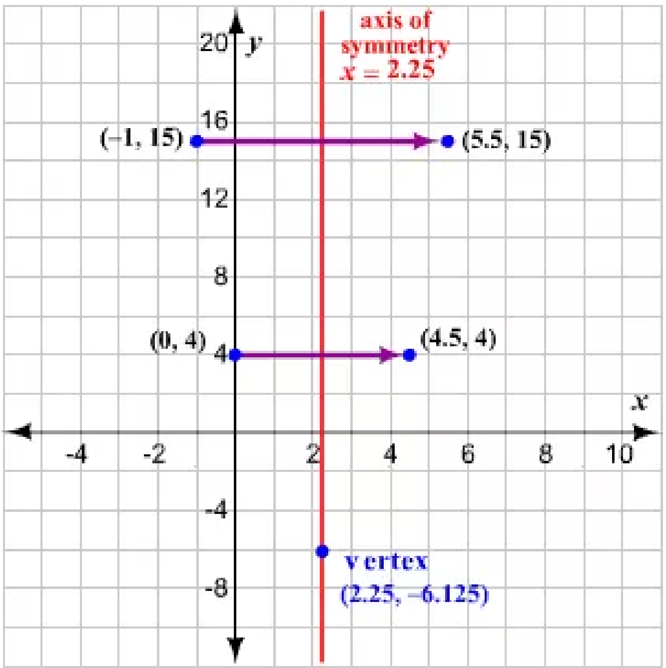
$$\begin{aligned} y &= 2(-1)^2 - 9(-1) + 4 \\ &= 2 + 9 + 4 \\ &= 15 \end{aligned}$$

Thus, the point $(-1, 15)$ lies on the graph.

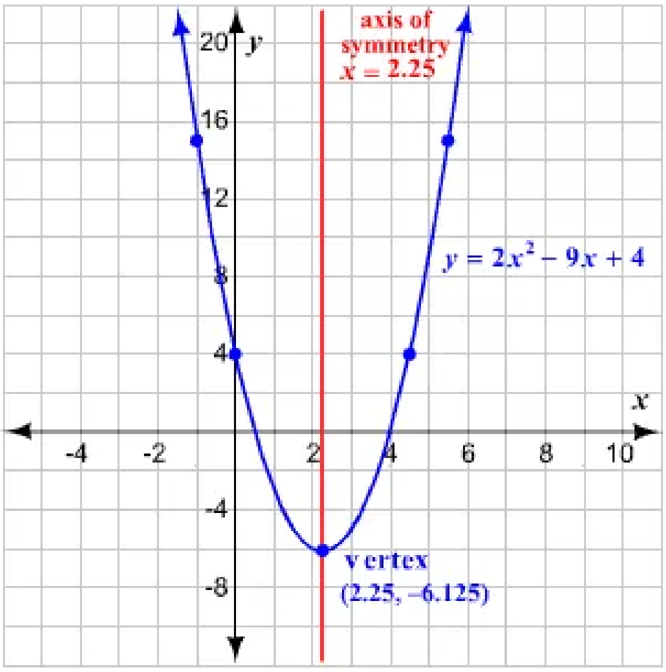
Plot the point on the coordinate plane.



Reflect $(-1, 15)$ in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 86e.

Consider the function

$$y=\frac{1}{4}x^2-2x+1$$

Now sketch the graph of the above function as follows:

Step1: Identify the coefficients of the function. The coefficients are

$$a=\frac{1}{4}, b=-2, \text{ and } c=1$$

Because $a>0$, the parabola opens up

Step2: Find the vertex. Calculate the x-coordinate.

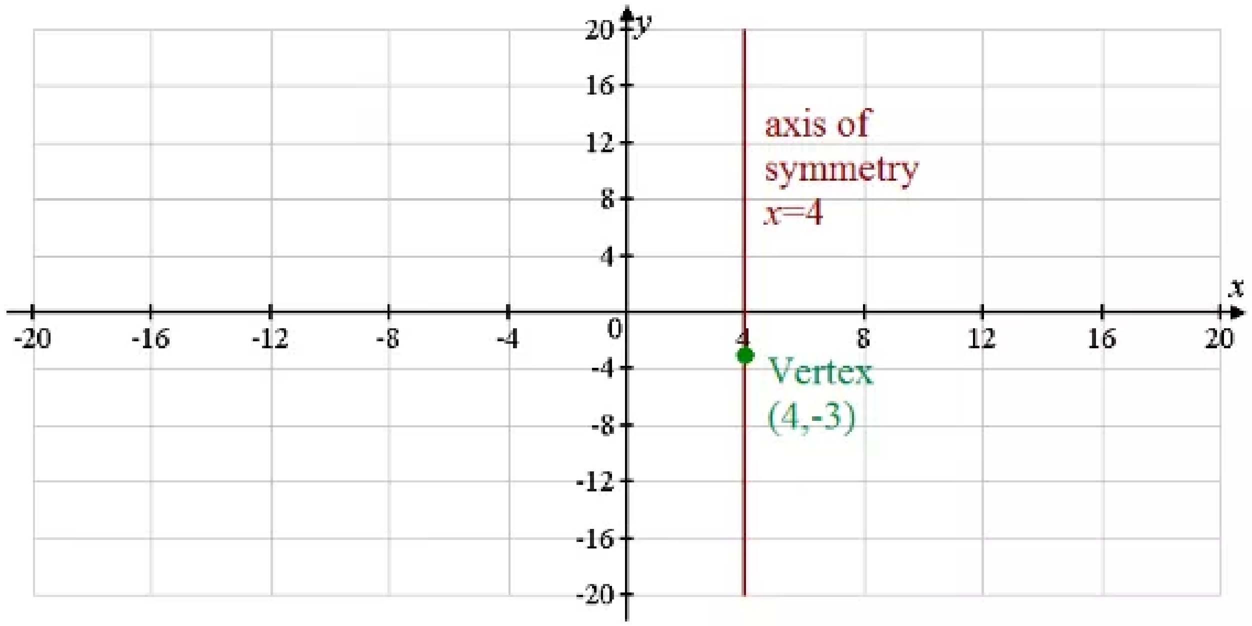
$$x=-\frac{b}{2a}$$
$$=-\frac{(-2)}{2\left(\frac{1}{4}\right)}$$
$$=4$$

Then find the y-coordinate of the vertex.

$$y=\frac{1}{4}x^2-2x+1$$
$$=\frac{1}{4}(4)^2-2(4)+1$$
$$=4-8+1$$
$$=-3$$

So, the vertex is $(4,-3)$. Plot the point

Step3: Draw the axis of symmetry $x = 4$



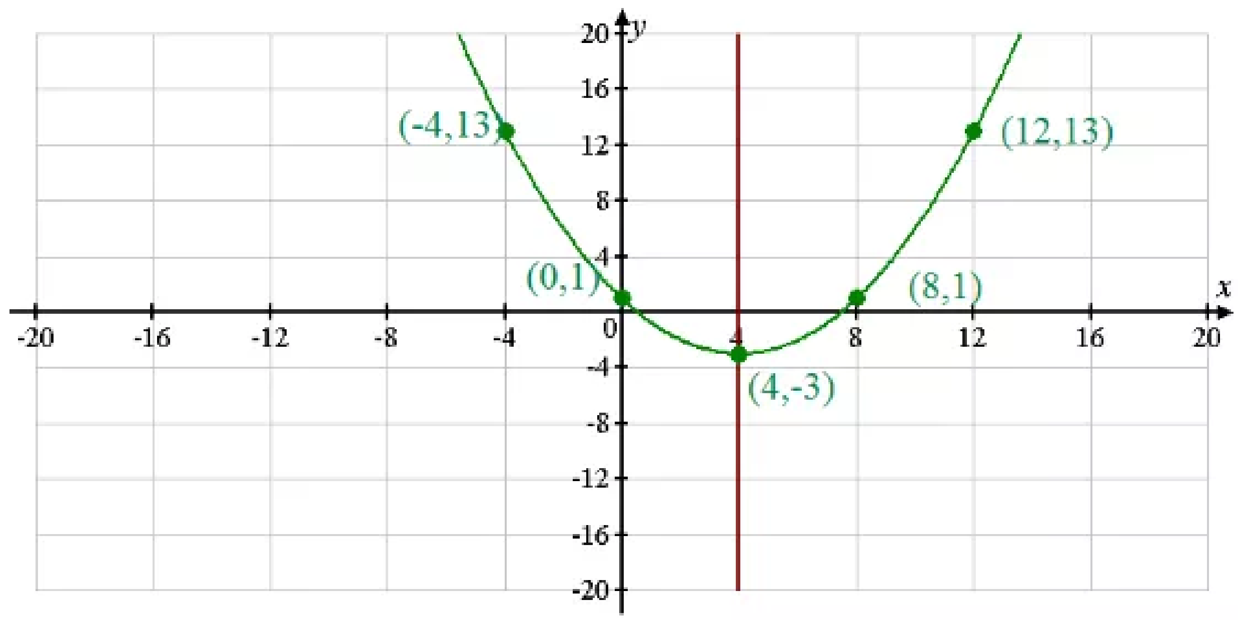
Step4: Identify the *y*-coordinate *c*, which is 1. Plot the point $(0,1)$. Then reflect this point in the axis of symmetry to plot another point, $(8,1)$

Step5: Evaluate the function for another value of *x*, such as $x = -4$

$$\begin{aligned} y &= \frac{1}{4}x^2 - 2x + 1 \\ &= \frac{1}{4}(-4)^2 - 2(-4) + 1 \\ &= 4 + 8 + 1 \\ &= 13 \end{aligned}$$

Plot the point $(-4,13)$ and its reflection $(12,13)$ in the axis of symmetry

Step6: Draw a parabola through the plotted points.



Answer 87e.

Number the equations.

$x + y + z = -2$ Equation 1

$4x + 2y + z = 3$ Equation 2

$z = -3$ Equation 3

Solve the system of equations.

Add –4 times Equation 1 to Equation 2.

$$\begin{array}{rcrcrcl} -4x & - & 4y & - & 4z & = & 8 \\ 4x & + & 2y & + & z & = & 3 \\ \hline & - & 2y & - & 3z & = & 11 \end{array}$$ Equation 4

Substitute –3 for z in Equation 4.

$-2y - 3(-3) = 11$

Solve for y.

$$\begin{aligned} -2y + 9 &= 11 \\ -2y &= 2 \\ y &= -1 \end{aligned}$$

Substitute –1 for y, and –3 for z in Equation 1.

$x - 1 - 3 = -2$

Solve for x.

$$\begin{aligned} x - 4 &= -2 \\ x &= 2 \end{aligned}$$

The solution of the given system of equations is (2, –1, –3).

Answer 88e.

Consider the system of equations

$x + y + z = 3$ ...... (1)

$2x + 3y - z = -8$ ...... (2)

$z = 4$ ...... (3)

Now solve the above system as follows:

Substitute equation (3) in equations (1),

$x + y + z = 3$ First equation

$x + y + 4 = 3$ Substitute 4 for z

$x + y = -1$ ...... (4)

Substitute equation (3) in equations (2),

$2x + 3y - 2 = -8$ Second equation

$2x + 3y - 4 = -8$ Substitute 4 for z

$2x + 3y = -4$ ...... (5)

Now solve equations (4) and (5), by multiply equation (4) with -2 and then add

$$-2x-2y=2 \quad \text{Multiply equation (4) by -2.}$$
$$2x+3y=-4 \quad \text{Equation (5)}$$
$$y=-2 \quad \text{Add.}$$

Substitute -2 for y in equation (4),

$$x+y=-1$$
$$x-2=-1$$
$$x=1$$

Therefore, the solution of the given system is $\boxed{(1,-2,4)}$

Check:

Substitute 1 for x, -2 for y, and 4 for z in equation (1),

$$x+y+z=3$$
$$1-2+4\stackrel{?}{=}3$$
$$3=3, \text{ true}$$

Substitute 1 for x, -2 for y, and 4 for z in equation (2),

$$2x+3y-z=-8$$
$$2(1)+3(-2)-4\stackrel{?}{=}-8$$
$$2-6-4\stackrel{?}{=}-8$$
$$-8=-8, \text{ true}$$

Answer 89e.

Number the equations in the given system.

$$4x+2y+z=-6 \qquad \text{Equation 1}$$
$$x+y+z=-3 \qquad \text{Equation 2}$$
$$16x+4y+z=0 \qquad \text{Equation 3}$$

STEP 1 **Rewrite** the system as a linear system in two variables.

Let the two variables be x and y. Add -1 times of Equation 2 to Equation 1 to eliminate z.

$$4x+2y+z=-6$$
$$-x-y-z=3$$
$$3x+y=-3 \qquad \text{new Equation 1}$$

Add -1 times of Equation 3 to Equation 1 to eliminate z again.

$$4x+2y+z=-6$$
$$-16x-4y-z=0$$
$$-12x-2y=-6 \qquad \text{new Equation 2}$$

STEP 2 **Solve** the new linear system for both of its variables.

Add 2 times new Equation 1 to new Equation 2 to eliminate y.

$$\begin{array}{rcrcr} 6x & + & 2y & = & -6 \\ -12x & - & 2y & = & -6 \\ \hline -6x & & & = & -12 \end{array}$$

Divide both the sides by −6 to isolate x.

$$\frac{-6x}{-6} = \frac{-12}{-6}$$
$$x = 2$$

Replace x with 2 in new Equation 1 and solve for y.

$$3(2) + y = -3$$
$$6 + y = -3$$
$$y = -9$$

STEP 3 **Substitute** −9 for y, and 2 for x into an original equation and solve for x.

Use Equation 2.

$$2 + (-9) + z = -3$$

Solve for x.

$$2 - 9 + z = -3$$
$$-7 + z = -3$$
$$z = 4$$

The solution to the given system of equations is (2, −9, 4).

Answer 90e.

Consider the system of equations

$$x + y + z = 8 \quad \ldots\ldots (1)$$
$$9x - 3y + z = 0 \quad \ldots\ldots (2)$$
$$4x - 2y + z = -1 \quad \ldots\ldots (3)$$

Now solve the above system as follows:

First solve the equations (1) and (2):

$$-x - y - z = -8 \quad \text{Multiply equation (1) by -1.}$$
$$9x - 3y + z = 0 \quad \text{Equation (2)}$$
$$8x - 4y = -8 \quad \text{Add}$$

Divide both sides by 4,

$$2x - y = -2 \quad \ldots\ldots (4)$$

Next, solve the equations (1) and (3):

$$\begin{array}{ll} -x-y-z=-8 & \text{Multiply equation (1) by -1.} \\ \underline{4x-2y+z\ =-1} & \text{Equation (3)} \\ 3x-3y\quad =-9 & \text{Add} \end{array}$$

Divide both sides by 3,

$$x-y=-3 \qquad \text{...... (5)}$$

Now solve the equations (4) and (5):

$$\begin{array}{ll} -2x+y=2 & \text{Multiply equation (4) by -1.} \\ \underline{\ x-y=-3} & \text{Equation (5)} \\ -x\quad =-1 & \text{Add} \end{array}$$

Divide both sides by -1,

$$x=\boxed{1}$$

Substitute $x=1$ into equation (5),

$$\begin{aligned} x-y&=-3 \\ 1-y&=-3 \\ -y&=-4 \\ y&=\boxed{4} \end{aligned}$$

Substitute $x=1$ and $y=4$ into equation (1),

$$\begin{aligned} x+y+z&=8 \\ 1+4+z&=8 \\ z&=\boxed{3} \end{aligned}$$

Therefore, the solution of the given system is $\boxed{(1,4,3)}$

Check:

Substitute 1 for x, 4 for y, and 3 for z in equation (1),

$$\begin{aligned} x+y+z&=8 \\ 1+4+3&\overset{?}{=}8 \\ 8&=8, \text{ true} \end{aligned}$$

Substitute 1 for x, 4 for y, and 3 for z in equation (2),

$$\begin{aligned} 9x-3y+z&=0 \\ 9(1)-3(4)+3&\overset{?}{=}0 \\ 9-12+3&\overset{?}{=}0 \\ 0&=0, \text{ true} \end{aligned}$$

Substitute 1 for x, 4 for y, and 3 for z in equation (3),

$$\begin{aligned} 4x-2y+z&=-1 \\ 4(1)-2(4)+3&\overset{?}{=}-1 \\ 4-8+3&\overset{?}{=}-1 \\ -1&=-1, \text{ true} \end{aligned}$$

Answer 91e.

Number the equations in the given system.

$$\begin{aligned} x + y + z &= 5 \quad \text{Equation 1} \\ 2x - 3y + 3z &= 9 \quad \text{Equation 2} \\ -x + 7y - z &= 11 \quad \text{Equation 3} \end{aligned}$$

Eliminate x and z from Equations 1 and 3 by adding Equation 3 to Equation 1.

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 5 \\ -x & + & 7y & - & z & = & 11 \\ \hline & & 8y & & & = & 16 \end{array}$$

Divide both the sides by 8.

$$\frac{8y}{8} = \frac{16}{8}$$

$$y = 2$$

Add Equation 2 to −2 times Equation 1.

$$\begin{array}{rcrcrcr} -2x & - & 2y & - & 2z & = & -10 \\ 2x & - & 3y & + & 3z & = & 9 \\ \hline & - & 5y & + & z & = & -1 \end{array}$$

Substitute 2 for y in the equation.

$$-5(2) + z = -1$$

Solve for z.

$$\begin{aligned} -10 + z &= -1 \\ z &= 9 \end{aligned}$$

Now, substitute 2 for y, and 9 for z in equation 1 to find the value of x.

$$x + 2 + 9 = 5$$

Solve for x.

$$\begin{aligned} x + 11 &= 5 \\ x &= -6 \end{aligned}$$

The solution to the given system of equations is $x = -6$, $y = 2$, and $z = 9$, or the ordered triple $(-6, 2, 9)$.

Answer 92e.

Consider the system of equations

$$x + y + z = 1 \quad \text{......}(1)$$

$$x - y + z = 1 \quad \text{......}(2)$$

$$3x + y + 3z = 3 \quad \text{......}(3)$$

Now, solve the system for x.

Step 1:

Subtract (3) from $(1)\times 3$

$$3x + 3y + 3z = 3$$

$$\underline{3x + 3y + 3z = 3}$$

$$0 = 0$$

Because, here obtained $0 = 0$, the system has infinitely many solutions.

Step 2:

Describe the solution of the system. One way to do this, substitute $x + z = 1 - y$ in equation (2) it produces $y = 0$. Any ordered triple of the form $(x, 0, 1 - x)$ is a solution of the system.